

Supplementary material

20 de octubre de 2017

In this appendix, the parameters $\Pi_i (i = 1, \dots, 25)$ that appear in (5.17) as a function of $\tilde{\kappa}$ are defined as follows:

$$\Pi_1 = -\frac{2}{15\tilde{\kappa}^5} (15\tilde{\kappa} - 5\tilde{\kappa}^3 + 2\tilde{\kappa}^5 - 15 \tanh \tilde{\kappa}) \quad (1)$$

$$\Pi_2 = \frac{1}{6\tilde{\kappa}^3} \left[2\tilde{\kappa} (\tilde{\kappa}^2 - 6) - \frac{3\tilde{\kappa}}{\cosh^2 \tilde{\kappa}} + 15 \tanh \tilde{\kappa} \right] \quad (2)$$

$$\Pi_3 = \frac{58}{945\tilde{\kappa}^2} \quad (3)$$

$$\begin{aligned} \Pi_4 = & \left\{ \frac{1}{5\tilde{\kappa}^3} \left(\frac{\tilde{\kappa}^2 + 2}{\tilde{\kappa}} - 2 \tanh \tilde{\kappa} \right) + \frac{1}{\tilde{\kappa}^9} [9 (28 + 14\tilde{\kappa}^2 + \tilde{\kappa}^4) \tanh \tilde{\kappa} \right. \\ & \left. - \tilde{\kappa} (252 + 42\tilde{\kappa}^2 + \tilde{\kappa}^4)] + \frac{2 \tanh \tilde{\kappa}}{\tilde{\kappa}^3} - \frac{\tilde{\kappa}^2 + 2}{\tilde{\kappa}^4} + \frac{3}{\tilde{\kappa}^7} [20\tilde{\kappa} + \tilde{\kappa}^3 - (20 + 7\tilde{\kappa}^2) \tanh \tilde{\kappa}] \right\} \end{aligned} \quad (4)$$

$$\begin{aligned} \Pi_5 = & \left\{ \frac{1}{1344\tilde{\kappa}^7 \cosh^2 \tilde{\kappa}} [21 (9 + 18\tilde{\kappa}^2 + 4\tilde{\kappa}^4) \sinh 2\tilde{\kappa} - 126\tilde{\kappa} (3 + 2\tilde{\kappa}^2) \cosh 2\tilde{\kappa} \right. \\ & \left. - 16\tilde{\kappa}^7] + \frac{\cosh 2\tilde{\kappa} - 2\tilde{\kappa} (\sinh 2\tilde{\kappa} - \tilde{\kappa})}{40\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} + \frac{1}{20 \cosh^2 \tilde{\kappa}} + \frac{3 (\tanh \tilde{\kappa} - \tilde{\kappa}) (\tilde{\kappa} \tanh \tilde{\kappa} - 1)}{8\tilde{\kappa}^5} \right. \\ & \left. + \frac{2\tilde{\kappa} (\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{8\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} \right\} \end{aligned} \quad (5)$$

$$\begin{aligned} \Pi_6 = & \frac{1}{3} \left\{ -\frac{4 \cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{5 \cdot 12 \cosh^3 \tilde{\kappa}} - \frac{1}{243\tilde{\kappa}^5} [6\tilde{\kappa} + 9\tilde{\kappa}^3 - (2 + 9\tilde{\kappa}^2) \tanh \tilde{\kappa} \right. \\ & \left. + \frac{1}{\cosh^2 \tilde{\kappa}} (-9\tilde{\kappa} (122 + 21\tilde{\kappa}^2) + (1094 + 549\tilde{\kappa}^2) \tanh \tilde{\kappa}) \right] + \frac{3}{27\tilde{\kappa}^2} \\ & \left. + \frac{61 \tanh \tilde{\kappa} - 63\tilde{\kappa}}{27\tilde{\kappa}^3 \cosh^2 \tilde{\kappa}} - \frac{\tanh \tilde{\kappa}}{27\tilde{\kappa}^3} \right\} \end{aligned} \quad (6)$$

$$\begin{aligned} \Pi_7 = & \frac{1}{10\tilde{\kappa}^2} \left\{ -\frac{32}{21} + \frac{5 (24 + 12\tilde{\kappa}^2 + \tilde{\kappa}^4) \tanh \tilde{\kappa} - \tilde{\kappa} (120 + 20\tilde{\kappa}^2 + \tilde{\kappa}^4)}{\tilde{\kappa}^7} \right. \\ & \left. + \frac{5 (\tilde{\kappa} - \tanh \tilde{\kappa})}{\tilde{\kappa}^3} \right\} \end{aligned} \quad (7)$$

$$\begin{aligned} \Pi_8 = 6 \left\{ \left(1 + \frac{2}{\tilde{\kappa}^2} - 2 \frac{\tanh \tilde{\kappa}}{\tilde{\kappa}} \right) \left(\frac{1}{3\tilde{\kappa}^2} - \frac{1}{\tilde{\kappa}^4} + \frac{\tanh \tilde{\kappa}}{\tilde{\kappa}^5} \right) \right. \\ \left. + \frac{7\tilde{\kappa}^2 \tanh \tilde{\kappa} - 20\tilde{\kappa} - \tilde{\kappa}^3 + 20 \tanh \tilde{\kappa}}{\tilde{\kappa}^7} - \frac{1}{12\tilde{\kappa}^7} \left[30\tilde{\kappa} + \frac{\tilde{\kappa} (21 + 2\tilde{\kappa}^2)}{\cosh^2 \tilde{\kappa}} \right. \right. \\ \left. \left. - (51 + 6\tilde{\kappa}^2) \tanh \tilde{\kappa} \right] \right\} \quad (8) \end{aligned}$$

$$\begin{aligned} \Pi_9 = 6 \left\{ \frac{1}{8} \frac{(\tilde{\kappa} - \tanh \tilde{\kappa})(\tilde{\kappa} \tanh \tilde{\kappa} - 1)}{\tilde{\kappa}^5} - \frac{1}{60 \cosh^2 \tilde{\kappa}} \right. \\ \left. - \frac{1}{72\tilde{\kappa}^5} \left[6\tilde{\kappa} - 8 \tanh \tilde{\kappa} + \frac{1}{\cosh^2 \tilde{\kappa}} (-3\tilde{\kappa}(15 + 2\tilde{\kappa}^2) + (47 + 18\tilde{\kappa}^2) \tanh \tilde{\kappa}) \right] \right. \\ \left. + \left(\frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{8\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} \right) \left(\frac{\tilde{\kappa} - \tanh \tilde{\kappa}}{\tilde{\kappa}^3} - \frac{1}{3} \right) \right\} \quad (9) \end{aligned}$$

$$\begin{aligned} \Pi_{10} = 2 \left\{ \left(\frac{\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{12 \cosh^3 \tilde{\kappa}} \right) \left(\frac{1}{3} - \frac{\tilde{\kappa} - \tanh \tilde{\kappa}}{\tilde{\kappa}^3} \right) - \frac{1}{27\tilde{\kappa}^2} + \frac{\tanh \tilde{\kappa}}{81\tilde{\kappa}^3} \right. \\ \left. + \frac{108\tilde{\kappa} - 56 \sinh 2\tilde{\kappa} + \sinh 4\tilde{\kappa}}{288\tilde{\kappa}^3 \cosh^4 \tilde{\kappa}} \right\} \quad (10) \end{aligned}$$

$$\Pi_{11} = \frac{404}{5775\tilde{\kappa}^4} \quad (11)$$

$$\begin{aligned} \Pi_{12} = \frac{12}{10\tilde{\kappa}^2} \left\{ -\frac{32}{21} \left(\frac{1}{\tilde{\kappa}^2} + \frac{2}{\tilde{\kappa}^4} - 2 \frac{\tanh \tilde{\kappa}}{\tilde{\kappa}^3} \right) - \frac{1}{\tilde{\kappa}^{11}} [\tilde{\kappa} (8640 + 1440\tilde{\kappa}^2 + 72\tilde{\kappa}^4 + \tilde{\kappa}^6) \right. \\ \left. - (8640 + 4320\tilde{\kappa}^2 + 360\tilde{\kappa}^4 + 11\tilde{\kappa}^6) \tanh \tilde{\kappa}] + \frac{5}{\tilde{\kappa}^7} [\tilde{\kappa}(20 + \tilde{\kappa}^2) - (20 + 7\tilde{\kappa}^2) \tanh \tilde{\kappa}] \right\} \quad (12) \end{aligned}$$

$$\begin{aligned} \Pi_{13} = 36 \left\{ \frac{1}{3\tilde{\kappa}^8} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa})^2 - \frac{1}{\tilde{\kappa}^{11}} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa}) (40\tilde{\kappa} + 2\tilde{\kappa}^3 \right. \\ \left. - (40 + 14\tilde{\kappa}^2) \tanh \tilde{\kappa}) + \frac{1}{\tilde{\kappa}^{10} \cosh^2 \tilde{\kappa}} \left[-18 + \frac{2\tilde{\kappa}^2}{3} - \frac{\tilde{\kappa}^4}{10} - \frac{1}{4}(91 + 10\tilde{\kappa}^2) \cosh 2\tilde{\kappa} \right. \right. \\ \left. \left. + \frac{(163 + 86\tilde{\kappa}^2 + 2\tilde{\kappa}^4) \sinh 2\tilde{\kappa}}{8\tilde{\kappa}} \right] \right\} \quad (13) \end{aligned}$$

$$\Pi_{14} = \frac{12}{10\tilde{\kappa}^2} \left\{ \frac{1}{432\tilde{\kappa}^9 \cosh^2 \tilde{\kappa}} [-4\tilde{\kappa}^9 - 54\tilde{\kappa}(15 + 10\tilde{\kappa}^2 + 2\tilde{\kappa}^4) \cosh 2\tilde{\kappa} + 27(15 + 30\tilde{\kappa}^2 + 10\tilde{\kappa}^4 + \tilde{\kappa}^6) \sinh 2\tilde{\kappa}] + \frac{64\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - 32 \cosh 2\tilde{\kappa}}{168\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} + \frac{1}{12 \cosh^2 \tilde{\kappa}} - \frac{15(\tilde{\kappa} - \tanh \tilde{\kappa})(\tilde{\kappa} \tanh \tilde{\kappa} - 1)}{25\tilde{\kappa}^5} \right\} \quad (14)$$

$$\Pi_{15} = \frac{2}{5\tilde{\kappa}^2} \left\{ -\frac{32 \cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{21 \cdot 12 \cosh^3 \tilde{\kappa}} - \frac{1}{8748\tilde{\kappa}^7 \cosh^3 \tilde{\kappa}} [-6561\tilde{\kappa}(120 + 20\tilde{\kappa}^2 + \tilde{\kappa}^4) \cosh \tilde{\kappa} + 3\tilde{\kappa}(40 + 60\tilde{\kappa}^2 + 27\tilde{\kappa}^4) \cosh 3\tilde{\kappa} + 32805(24 + 12\tilde{\kappa}^2 + \tilde{\kappa}^4) \sinh \tilde{\kappa} - 5(8 + 36\tilde{\kappa}^2 + 27\tilde{\kappa}^4) \sinh 3\tilde{\kappa}] - \frac{5}{81\tilde{\kappa}^3} \left[\tanh \tilde{\kappa} - 3\tilde{\kappa} + \frac{63\tilde{\kappa} - 61 \tanh \tilde{\kappa}}{\cosh^2 \tilde{\kappa}} \right] \right\} \quad (15)$$

$$\Pi_{16} = 72 \left\{ \frac{\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa}}{120\tilde{\kappa}^9} \left[15(\tilde{\kappa} - \tanh \tilde{\kappa})(\tilde{\kappa} \tanh \tilde{\kappa} - 1) - \frac{2\tilde{\kappa}^5}{\cosh^2 \tilde{\kappa}} \right] - \frac{2\tilde{\kappa}(\sinh \tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{24\tilde{\kappa}^6 \cosh^2 \tilde{\kappa}} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa}) - \frac{1}{72\tilde{\kappa}^9} [6\tilde{\kappa}(14 + \tilde{\kappa}^2) - 4(16 + 9\tilde{\kappa}^2) \tanh \tilde{\kappa} - \frac{3\tilde{\kappa}(528 + 87\tilde{\kappa}^2 + 2\tilde{\kappa}^4)}{\cosh^2 \tilde{\kappa}} + (1564 + 765\tilde{\kappa}^2 + 54\tilde{\kappa}^4) \tanh \tilde{\kappa}] + \frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{8\tilde{\kappa}^9 \cosh^2 \tilde{\kappa}} [\tilde{\kappa}(20 + \tilde{\kappa}^2) - (20 + 7\tilde{\kappa}^2) \tanh \tilde{\kappa}] \right\} \quad (16)$$

$$\Pi_{17} = 24 \left\{ \frac{1}{36\tilde{\kappa}^4 \cosh^3 \tilde{\kappa}} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa})(\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}) + \frac{1}{81\tilde{\kappa}^7} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa}) \left(\tanh \tilde{\kappa} - 3\tilde{\kappa} + \frac{63\tilde{\kappa} - 61 \tanh \tilde{\kappa}}{\cosh^2 \tilde{\kappa}} \right) - \frac{\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{12\tilde{\kappa}^7 \cosh^3 \tilde{\kappa}} [\tilde{\kappa}(20 + \tilde{\kappa}^2) - (20 + 7\tilde{\kappa}^2) \tanh \tilde{\kappa}] + \frac{1}{2304\tilde{\kappa}^7 \cosh^4 \tilde{\kappa}} [288\tilde{\kappa}(18 + \tilde{\kappa}^2) + 2112\tilde{\kappa} \cosh 2\tilde{\kappa} - 36\tilde{\kappa} \cosh 4\tilde{\kappa} - (3744 + 16\tilde{\kappa}^2 \cosh 2\tilde{\kappa} - 448\tilde{\kappa}^2) \sinh 2\tilde{\kappa} + 57 \sinh 4\tilde{\kappa}] \right\} \quad (17)$$

$$\begin{aligned} \Pi_{18} = 36 \left\{ \frac{1}{576\tilde{\kappa}^6 \cosh^4 \tilde{\kappa}} \left[-\frac{9}{2} + \frac{3\tilde{\kappa}^2}{2} + \frac{4\tilde{\kappa}^6}{7} + 9(3 + 2\tilde{\kappa}^2) \cosh 2\tilde{\kappa} \right. \right. \\ \left. - \frac{45}{16} \cosh 4\tilde{\kappa} - \frac{3(9 + 18\tilde{\kappa}^2 + 4\tilde{\kappa}^4) \sinh \tilde{\kappa} \cosh \tilde{\kappa}}{\tilde{\kappa}} + \frac{9(13 + 8\tilde{\kappa}^2) \sinh 4\tilde{\kappa}}{64\tilde{\kappa}} \right] \\ \left. - \frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{480\tilde{\kappa}^7 \cosh^2 \tilde{\kappa}} \left[15(\tilde{\kappa} - \tanh \tilde{\kappa})(\tilde{\kappa} \tanh \tilde{\kappa} - 1) - \frac{2\tilde{\kappa}^5}{\cosh^2 \tilde{\kappa}} \right] \right. \\ \left. + \left[\frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{72\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} \right]^2 \right\} \end{aligned} \quad (18)$$

$$\begin{aligned} \Pi_{19} = 24 \left\{ \frac{\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{1440\tilde{\kappa}^5 \cosh^3 \tilde{\kappa}} \left[15(\tilde{\kappa} - \tanh \tilde{\kappa})(\tilde{\kappa} \tanh \tilde{\kappa} - 1) - \frac{2\tilde{\kappa}^5}{\cosh^2 \tilde{\kappa}} \right] \right. \\ \left. - \frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{288\tilde{\kappa}^2 \cosh^5 \tilde{\kappa}} (\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}) - \frac{1}{145800\tilde{\kappa}^5} [162(5\tilde{\kappa} - 6 \tanh \tilde{\kappa}) \right. \\ \left. + \frac{1}{\cosh^2 \tilde{\kappa}} \left(-10425\tilde{\kappa} - 450\tilde{\kappa}^3 + \frac{225(307 + 42\tilde{\kappa}^2)}{\cosh^2 \tilde{\kappa}} \right) \right. \\ \left. + \left(12979 + 450\tilde{\kappa}^2 - \frac{71467 + 27450\tilde{\kappa}^2}{\cosh^2 \tilde{\kappa}} \right) \tanh \tilde{\kappa} \right] \\ \left. - \frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{648\tilde{\kappa}^5 \cosh^2 \tilde{\kappa}} \left(-3\tilde{\kappa} + \frac{63\tilde{\kappa} - 61 \tanh \tilde{\kappa}}{\cosh^2 \tilde{\kappa}} + \tanh \tilde{\kappa} \right) \right\} \end{aligned} \quad (19)$$

$$\begin{aligned} \Pi_{20} = 4 \left\{ \left(\frac{\sinh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{36 \cosh^3 \tilde{\kappa}} \right)^2 + \frac{2 \cosh 3\tilde{\kappa} - 18 \cosh \tilde{\kappa}}{972\tilde{\kappa}^3 \cosh^3 \tilde{\kappa}} \left[-3\tilde{\kappa} + \frac{63\tilde{\kappa} - 61 \tanh \tilde{\kappa}}{\cosh^2 \tilde{\kappa}} + \tanh \tilde{\kappa} \right] \right. \\ \left. + \frac{1}{15552\tilde{\kappa}^3 \cosh^6 \tilde{\kappa}} [-4380\tilde{\kappa} + 2349 \sinh 2\tilde{\kappa} - 81 \sinh 4\tilde{\kappa} + \sinh 6\tilde{\kappa}] \right\} \end{aligned} \quad (20)$$

$$\Pi_{21} = \frac{1}{3} + \frac{\tanh \tilde{\kappa} - \tilde{\kappa}}{\tilde{\kappa}^3}, \quad \Pi_{22} = -\frac{16}{105\tilde{\kappa}^2} \quad (21)$$

$$\Pi_{23} = 6 \left\{ \frac{1}{3\tilde{\kappa}^4} (\tilde{\kappa}^2 + 2 - 2\tilde{\kappa} \tanh \tilde{\kappa}) - \frac{20\tilde{\kappa} + \tilde{\kappa}^3 - 20 \tanh \tilde{\kappa} - 7\tilde{\kappa}^2 \tanh \tilde{\kappa}}{\tilde{\kappa}^7} \right\} \quad (22)$$

$$\Pi_{24} = 6 \left\{ \frac{1}{120\tilde{\kappa}^5} \left[15 (\tilde{\kappa} - \tanh \tilde{\kappa}) (\tilde{\kappa} \tanh \tilde{\kappa} - 1) - \frac{2\tilde{\kappa}^5}{\cosh^2 \tilde{\kappa}} \right] - \frac{2\tilde{\kappa}(\sinh 2\tilde{\kappa} - \tilde{\kappa}) - \cosh 2\tilde{\kappa}}{24\tilde{\kappa}^2 \cosh^2 \tilde{\kappa}} \right\} \quad (23)$$

$$\Pi_{25} = 2 \left\{ \frac{\cosh 3\tilde{\kappa} - 9 \cosh \tilde{\kappa}}{36 \cosh^3 \tilde{\kappa}} + \frac{1}{81\tilde{\kappa}^3} \left[\frac{63\tilde{\kappa} - 61 \tanh \tilde{\kappa}}{\cosh^2 \tilde{\kappa}} - 3\tilde{\kappa} + \tanh \tilde{\kappa} \right] \right\}. \quad (24)$$