Other supplementary material

Madhu V. Majji[1,2,3], Sanjoy Banerjee[2,3] and Jeffrey F. Morris[1,3]†

[1]Benjamin Levich Institute, CUNY City College of New York, New York, NY10031, USA
[2]Energy Institute, CUNY City College of New York, New York, NY10031, USA
[3]Department of Chemical Engineering, CUNY City College of New York, New York, NY10031, USA

1. Linear stability analysis of a fluid with radially varying viscosity

A linear stability analysis of the circular Couette flow is performed for a fluid with radially varying viscosity profile in the annular region to take into account concentration variation due to either inertial or shear-induced migration. Three profiles of radially varying relative viscosity, $f(r)$, corresponding to the three concentration profiles are considered in figure 1. Two center peaking concentration profiles for $\phi_{avg} = 0.025$ and 0.1, to simulate the steady concentration profiles taking into account migration effects, and one uniform concentration profile with zero concentration near the walls for $\phi_{avg} =$ 0.1, to simulate a profile corresponding to instant rise of Re from rest to a given value (with only wall exclusion effects causing concentration variation), were considered. For $\phi_{avg} = 0.1$, the maximum concentration of the center peak profile is 0.4. While in reality, it is unlikely to reach such elevated concentration, we consider the extreme case to capture the deviation from pure fluid. The azimuthal velocity profiles across the annular gap of the base flow (circular Couette flow) for the three cases are shown in figure 2 along with the circular Couette flow for a Newtonian fluid. For the cases with a central concentration peak, the velocity has an inflection near the middle of the annulus, and the deviation from the Newtonian fluid increased near the middle of the annulus with increase in ϕ_{avg}. However, with increasing uniformity of the concentration profile, for $\phi_{avg} = 0.1$, the deviation was reduced and the location of maximum deviation moved closer to the boundaries.

Linear stability analyses of the three velocity profiles were performed and the neutral stability curves for $k_\theta = 0$, 1, 2, and 3 are shown in plots of Re versus k_z in figure 3. Note that the effective viscosity of the suspension corresponding to ϕ_{avg} is used in computing Re. As the Re is increased from that corresponding to the circular Couette flow, the first mode of disturbance that amplifies is that corresponding to the lowest Re point on the neutral stability curves in each plot and the corresponding values of (k_θ, Re) are (0,125.94), (0, 169.94) and (0, 122.82) for the two center peak concentration profiles of $\phi_{avg} = 0.025$ and 0.1 and one uniform concentration profile with particle free boundaries of $\phi_{avg} = 0.1$, respectively. The transition Re for the suspension is predicted to be higher than that of the pure fluid whereas the experiments showed a lower Re for the suspension. The predicted transition Re increased with increase in ϕ_{avg} for the center peaked profiles; the increase was smaller for the more uniform concentration profile for $\phi_{avg} = 0.1$.

In all the three cases the mode of instability is that of $k_\theta = 0$ which is Taylor vortex flow

† Email address for correspondence: morris@ccny.cuny.edu

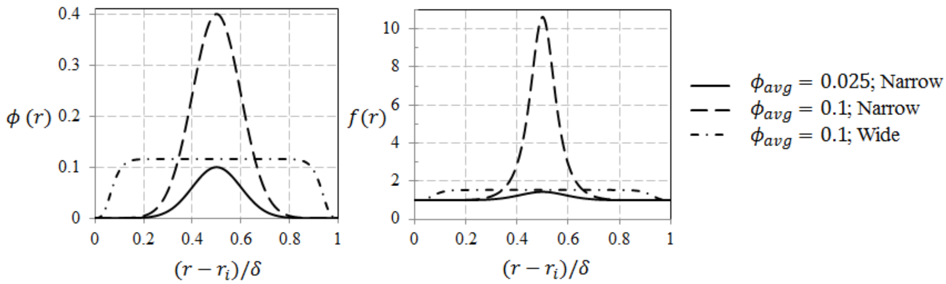

Figure 1: The concentration profiles studied (left plot) for $\phi_{avg} = 0.025$ and 0.1 assuming migration of particles to the annular mid gap (Narrow) and for $\phi_{avg} = 0.1$ with uniform particle distribution in the annular region with particle free jones near the walls (Wide). Profiles of relative viscosity (right plot) $f(r) = \mu_s/\mu_l$ corresponding to the considered concentration profiles.

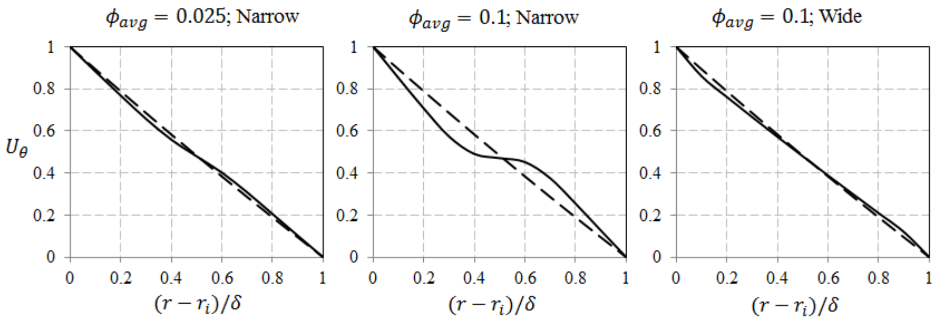

Figure 2: Computed base state velocity field for suspensions with concentration profiles as shown in 1. Dotted line shows the circular Couette flow for pure fluid and the solid line shows the modified velocity profile due to the concentration profile.

but it is interesting and potentially significant that the wave number for time modulation k_{ti} is non-zero. Figure 4 shows the wave speed of the time modulating flow structure $k_{ti}r_i/Re\delta$ for the three cases. Note that, for a pure fluid with uniform viscosity across the annular gap, the wave speeds for $k_\theta = 0$ (TVF) and +/-1 (L-SVF/R-SVF) are 0 and +/-0.5, respectively. For the central peak ϕ profiles, the absolute value of the wave speed was observed to increase with increase in ϕ_{avg}. For $\phi_{avg} = 0.1$, the absolute value of the wave speed reduced towards zero with increasing uniformity of the concentration profile. The height of the vortex relative to the annular gap (π/k_z) for the three cases is plotted in figure 4. It increases with increase in ϕ_{avg} for the central peak concentration profiles and decreases with increase in uniformity of the concentration profile at $\phi_{avg} = 0.1$.

The results of the linear stability analysis show that a simple effective fluid model with radially varying viscosity profiles fails to explain the experimentally observed deviations of the suspension from pure fluid in terms of either the flow transition Re or the flow structures. The analysis predicted that the modified viscosity profiles due to potential migration stabilizes the flow whereas the experiments showed that the addition of particles destabilized the flow. We feel that a most likely candidate for the role of suspended particles is the finite size perturbations they create.

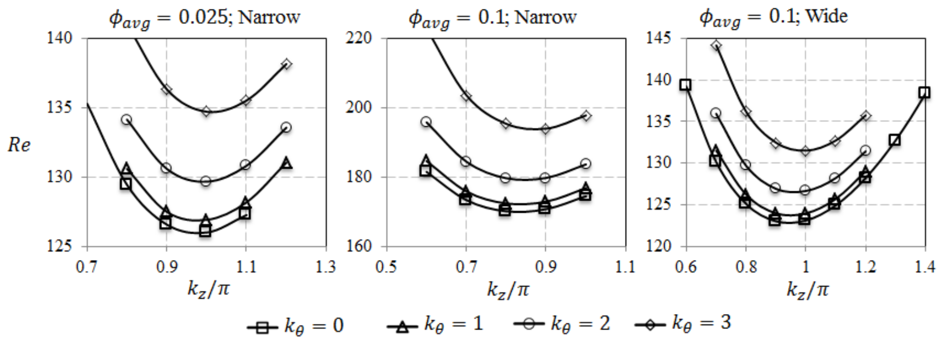

Figure 3: Neutral stability curves obtained from linear stability analysis of the flow profiles shown in (2). The flow structure corresponding to the minimum Re point for all the concentrations is of $k_\theta = 0$ and the structures have non zero time modulation ($k_{ti} \neq 0$) as shown in (4).

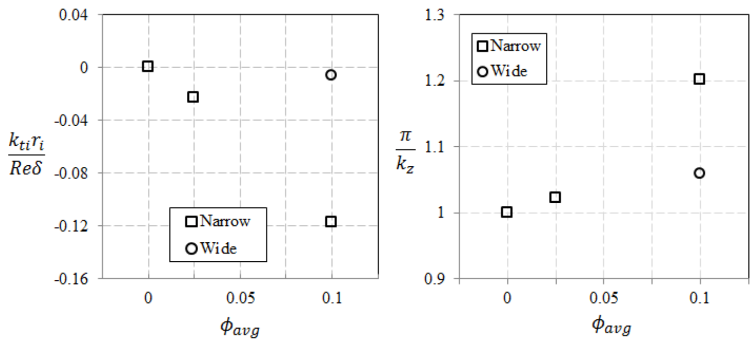

Figure 4: Wave speeds of the time modulating flow structure and the height of the vortex relative to the annular gap at the minimum *Re* point on the neutral stability plot, figure 3, of each of the three concentration profiles considered. Values for the pure fluid are plotted at $\phi_{avg} = 0$ for comparison. Time modulation increased with increase in ϕ_{avg} if inertial migration is considered. For $\phi_a vg = 0.1$, as the concentration profile gets more uniform the flow structure after instability resembles closer to that of the pure fluid.