> *# The purpose of this supplementary document is to illustrate the key steps required to obtain droplet velocity and shape in the presence of arbitrary Stokes flow, for compound droplets. In section S1 we delineate the MAPLE scripts to evaluate the flow variables for concentric drop theory. Subsequently the script files for specific cases of imposed extensional flow, Poiseuille flow and evaluation of effective viscosity have been appended. Additionally, we have also given the MATLAB script files to evaluate the flow field and the drop migration velocities for eccentric drop configurations.*

> **# *S1. Calculation of flow field, surfactant concentration, migration velocity* and *shape deformation* for *arbitrary Stokes flow***

> *restart* :

> $p_{-n-1,\,1} := A_{-n-1,\,m}$:

> $\Phi_{-n-1,\,1} := B_{-n-1,\,m}$:

> $\chi_{-n-1,\,1} := C_{-n-1,\,m}$:

> $p_{n,\,inf} := \frac{2\cdot(2\cdot n+3)}{n}\cdot \zeta b_{n,\,m}$:

> $\Phi_{n,\,inf} := \frac{1}{n}\cdot \eta b_{n,\,m}$:

> $\chi_{n,\,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{n,\,m}$:

> $p_{-n-1,\,inf} := \frac{2\cdot(2\cdot n-1)}{n+1}\cdot \zeta b_{-n-1,\,m}$:

> $\Phi_{-n-1,\,inf} := -\frac{1}{n+1}\cdot \eta b_{-n-1,\,m}$:

> $\chi_{-n-1,\,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{-n-1,\,m}$:

> $p_{n,\,2} := \lambda_2\cdot E_{n,\,m}$:

> $\Phi_{n,\,2} := F_{n,\,m}$:

> $\chi_{n,\,2} := G_{n,\,m}$:

> $p_{-n-1,\,2} := \lambda_2\cdot E_{-n-1,\,m}$:

> $\Phi_{-n-1,\,2} := F_{-n-1,\,m}$:

> $\chi_{-n-1,\,2} := G_{-n-1,\,m}$:

> *# equations 1 - 7 at the inner interface r=1*

> $$eq1 := \frac{n}{2\cdot(2\cdot n+3)}\cdot p_{n,\,inf}+n\cdot\Phi_{n,\,inf}+\frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,\,inf}-(n+1)\cdot\Phi_{-n-1,\,inf} + \frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,\,1}-(n+1)\cdot\Phi_{-n-1,\,1}=0 :$$

> $$eq2 := \frac{n}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2}+n\cdot\Phi_{n,\,2}+\frac{(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2}-(n+1)\cdot\Phi_{-n-1,\,2} =0 :$$

> $$eq3 := -\left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,1}+(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1}+\frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)}\cdot p_{n,\,inf}+n$$

$\cdot (n-1)\cdot \Phi_{n,\,inf} - \left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,inf} + (n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf}$
$= \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2} + n\cdot(n-1)\cdot\Phi_{n,\,2} - \left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\right)\cdot p_{-n-1,\,2} + (n+1)$
$\cdot(n+2)\cdot\Phi_{-n-1,\,2}:$

> $eq4 := n\cdot(n+1)\cdot\chi_{-n-1,\,1} + n\cdot(n+1)\cdot\chi_{n,\,inf} + n\cdot(n+1)\cdot\chi_{-n-1,\,inf} = n\cdot(n+1)\cdot\chi_{n,\,2} + n$
$\cdot(n+1)\cdot\chi_{-n-1,\,2}:$

> $eq5 := -\frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,1} + 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1}$
$-\frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,inf} + 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf} - \frac{n^2\cdot(n+2)}{2\cdot n+3}\cdot p_{n,\,inf}$
$-2\cdot(n-1)\cdot n\cdot(n+1)\cdot\Phi_{n,\,inf} + \frac{(n+1)^2\cdot(n-1)}{(2\cdot n-1)}\cdot p_{-n-1,\,2} - 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\lambda_2$
$\cdot\Phi_{-n-1,\,2} + \frac{n^2\cdot(n+2)}{(2\cdot n+3)}\cdot p_{n,\,2} + 2\cdot(n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\Phi_{n,\,2} = -\omega_2\cdot n\cdot(n+1)$
$\cdot\Gamma_{12,\,n,\,m}:$

> $eq6 := (n-1)\cdot n\cdot(n+1)\cdot\chi_{n,\,inf} - n\cdot(n+1)\cdot(n+2)\cdot\chi_{-n-1,\,inf} - n\cdot(n+1)\cdot(n+2)$
$\cdot\chi_{-n-1,\,1} - (n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\chi_{n,\,2} + n\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\chi_{-n-1,\,2} = 0:$

> $eq7 := \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2} + n\cdot(n-1)\cdot\Phi_{n,\,2} - \frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2} + (n+1)\cdot(n$
$+2)\cdot\Phi_{-n-1,\,2} = n\cdot(n+1)\cdot\Gamma_{12,\,n,\,m}:$

> *# equations 8 - 14 at the outer interface r=R*

> *# redefining solid harmonics at r=R*

> $p_{n,\,3} := \lambda_3\cdot A_{n,\,m}\cdot R^n:$

> $\Phi_{n,\,3} := B_{n,\,m}\cdot R^n:$

> $\chi_{n,\,3} := C_{n,\,m}\cdot R^n:$

> $p_{n,\,2} := \lambda_2\cdot E_{n,\,m}\cdot R^n:$

> $\Phi_{n,\,2} := F_{n,\,m}\cdot R^n:$

> $\chi_{n,\,2} := G_{n,\,m}\cdot R^n:$

> $p_{-n-1,\,2} := \lambda_2\cdot E_{-n-1,\,m}\cdot R^{-n-1}:$

> $\Phi_{-n-1,\,2} := F_{-n-1,\,m}\cdot R^{-n-1}:$

> $\chi_{-n-1,\,2} := G_{-n-1,\,m}\cdot R^{-n-1}:$

> $eq8 := \frac{n\cdot R}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2} + \frac{n}{R}\cdot\Phi_{n,\,2} + \frac{(n+1)\cdot R}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2} - \frac{(n+1)}{R}\cdot\Phi_{-n-1,\,2}$
$= \left(U3_{dz} - U2_{dz}\right)\cdot f_{1,\,0} + \left(U3_{dx} - U2_{dx}\right)\cdot f_{1,\,1}:$

$> eq9 := \frac{n \cdot R}{2 \cdot (2 \cdot n + 3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n}{R} \cdot \Phi_{n,3} = (U3_{dz} - U2_{dz}) \cdot f_{1,0} + (U3_{dx} - U2_{dx}) \cdot f_{1,1}:$

$> eq10 := \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n + 3) \cdot \lambda_2} \cdot p_{n,2} + \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,2} - \left(\frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n - 1) \cdot \lambda_2} \right) \cdot p_{-n-1,2} + \frac{(n+1) \cdot (n+2)}{R} \cdot \Phi_{-n-1,2} - \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n + 3) \cdot \lambda_3} \cdot p_{n,3} - \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,3} = 0:$

$> eq11 := n \cdot (n+1) \cdot (\chi_{n,2} + \chi_{-n-1,2}) - n \cdot (n+1) \cdot \chi_{n,3} = 0:$

$> eq12 := -\frac{(n+1)^2 \cdot (n-1) \cdot R}{(2 \cdot n - 1)} \cdot p_{-n-1,2} + 2 \cdot n \cdot (n+1) \cdot \frac{(n+2)}{R} \cdot \lambda_2 \cdot \Phi_{-n-1,2} - \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n + 3)} \cdot p_{n,2} - \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_2 \cdot \Phi_{n,2} + \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n + 3)} \cdot p_{n,3} + \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_3 \cdot \Phi_{n,3} = -\alpha \cdot \delta_2 \cdot n \cdot (n+1) \cdot \Gamma_{23,n,m}:$

$> eq13 := \lambda_2 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,2} - \lambda_2 \cdot n \cdot (n+1) \cdot (n+2) \cdot \chi_{-n-1,2} - \lambda_3 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,3} = 0:$

$> eq14 := \frac{1}{d} \cdot \left(\frac{n \cdot (n+1)}{2 \cdot (2 \cdot n + 3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n \cdot (n-1)}{R^2} \cdot \Phi_{n,3} \right) = n \cdot (n+1) \cdot \Gamma_{23,n,m}:$

$> sol1 := solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13, eq14\}, \{A_{n,m}, B_{n,m}, C_{n,m}, A_{-n-1,m}, B_{-n-1,m}, C_{-n-1,m}, E_{n,m}, F_{n,m}, G_{n,m}, E_{-n-1,m}, F_{-n-1,m}, G_{-n-1,m}, \Gamma_{12,n,m}, \Gamma_{23,n,m}\}):$

$> A_{n,m} := simplify(rhs(sol1[1])):$

$> A_{-n-1,m} := simplify(rhs(sol1[2])):$

$> B_{n,m} := simplify(rhs(sol1[3])):$

$> B_{-n-1,m} := simplify(rhs(sol1[4])):$

$> C_{n,m} := simplify(rhs(sol1[5])):$

$> C_{-n-1,m} := simplify(rhs(sol1[6])):$

$> E_{n,m} := simplify(rhs(sol1[7])):$

$> E_{-n-1,m} := simplify(rhs(sol1[8])):$

$> F_{n,m} := simplify(rhs(sol1[9])):$

$> F_{-n-1,m} := simplify(rhs(sol1[10])):$

$> G_{n,m} := simplify(rhs(sol1[11])):$

$> G_{-n-1,m} := simplify(rhs(sol1[12])):$

$> \Gamma 12_{n,m} := simplify(rhs(sol1[13])):$

$> \Gamma 23_{n,m} := simplify(rhs(sol1[14])):$

$> A_{-2,0} := simplify(subs(f_{1,0} = 1, f_{1,1} = 0, n = 1, m = 0, A_{-n-1,m})):$

$> A_{-2,1} := simplify(subs(f_{1,0} = 0, f_{1,1} = 1, n = 1, m = 1, A_{-n-1,m})):$

> $Ac_{-2,\,1} := simplify\Big(subs\Big(f_{1,\,0}=0, f_{1,\,1}=1,\, U2_{dx}=U2_{dy},\, U3_{dx}=U3_{dy},\, \zeta b_{n,\,m}=\zeta bc_{n,\,m},\, \eta b_{n,\,m} = \eta bc_{n,\,n},\, \psi b_{n,\,m}=\psi bc_{n,\,m},\, \zeta b_{-n-1,\,m}=\zeta bc_{-n-1,\,m},\, \eta b_{-n-1,\,m}=\eta bc_{-n-1,\,n},\, \psi b_{-n-1,\,m} = \psi bc_{-n-1,\,m}, A_{-2,\,1}\Big)\Big):$

> $\eta b_{1,\,0} := \eta t_{1,\,0} - U2_{dz}:$

> $\eta b_{1,\,1} := \eta t_{1,\,1} - U2_{dx}:$

> $\eta bc_{1,\,1} := \eta ct_{1,\,1} - U2_{dy}:$

> $rel1 := A_{-2,\,0}=0:$

> $rel2 := A_{-2,\,1}=0:$

> $rel3 := Ac_{-2,\,1}=0:$

> $E_{-2,\,0} := simplify\big(subs\big(f_{1,\,0}=1, f_{1,\,1}=0,\, n=1,\, m=0,\, E_{-n-1,\,m}\big)\big):$

> $E_{-2,\,1} := simplify\big(subs\big(f_{1,\,0}=0, f_{1,\,1}=1,\, n=1,\, m=1,\, E_{-n-1,\,m}\big)\big):$

> $Ec_{-2,\,1} := simplify\Big(subs\Big(f_{1,\,0}=0, f_{1,\,1}=1,\, U2_{dx}=U2_{dy},\, U3_{dx}=U3_{dy},\, \zeta b_{n,\,m}=\zeta bc_{n,\,m},\, \eta b_{n,\,m} = \eta bc_{n,\,n},\, \psi b_{n,\,m}=\psi bc_{n,\,m},\, \zeta b_{-n-1,\,m}=\zeta bc_{-n-1,\,m},\, \eta b_{-n-1,\,m}=\eta bc_{-n-1,\,n},\, \psi b_{-n-1,\,m} = \psi bc_{-n-1,\,m}, E_{-2,\,1}\Big)\Big):$

> $rel4 := E_{-2,\,0}=0:$

> $rel5 := E_{-2,\,1}=0:$

> $rel6 := Ec_{-2,\,1}=0:$

> $sol2 := solve\big(\{rel1, rel2, rel3, rel4, rel5, rel6\},\, \{U2_{dx}, U2_{dy}, U2_{dz}, U3_{dx}, U3_{dy}, U3_{dz}\}\big):$

>

> ***#Droplet velocity***

> $U2_{dx} := rhs(sol2[1]):$

> $U2_{dy} := rhs(sol2[2]):$

> $U2_{dz} := rhs(sol2[3]):$

> $U3_{dx} := rhs(sol2[4]):$

> $U3_{dy} := rhs(sol2[5]):$

> $U3_{dz} := rhs(sol2[6]):$

>

> ***# Inner drop shape***

> *restart* :

> $p_{n,\,3} := \lambda_3 \cdot A_{n,\,m} \cdot R^n:$

> $\Phi_{n,\,3} := B_{n,\,m} \cdot R^n:$

> $\chi_{n,\,3} := C_{n,\,m} \cdot R^n:$

> $p_{n,\,2} := \lambda_2 \cdot E_{n,\,m} \cdot R^n:$

> $\Phi_{n,\,2} := F_{n,\,m} \cdot R^n:$

> $\chi_{n,2} := G_{n,m} \cdot R^n :$

> $p_{-n-1,2} := \lambda_2 \cdot E_{-n-1,m} \cdot R^{-n-1} :$

> $\Phi_{-n-1,2} := F_{-n-1,m} \cdot R^{-n-1} :$

> $\chi_{-n-1,2} := G_{-n-1,m} \cdot R^{-n-1} :$

>

> $$ns := \frac{2\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\Phi_{-n-1,2}}{R^2} - \frac{(n^2+3\cdot n-1)}{(2\cdot n-1)}\cdot p_{-n-1,2} + \frac{2\cdot n\cdot(n-1)\cdot\lambda_2\cdot\Phi_{n,2}}{R^2} + \frac{(n^2-n-3)}{(2\cdot n+3)}\cdot p_{n,2} - \frac{\lambda_3\cdot 2\cdot n\cdot(n-1)\cdot\Phi_{n,3}}{R^2} - \frac{(n^2-n-3)}{(2\cdot n+3)}\cdot p_{n,3} = \omega_1\cdot\left(\frac{n\cdot(n+1)}{R} - \frac{2}{R}\right)\cdot f_{23,n,m} - \frac{2\cdot \mathrm{alpha}\cdot\omega_2}{R}\cdot \mathit{\Gamma 23}_{n,m} :$$

> $f_{23,n,m} := solve(ns, f_{23,n,m}) :$

>

> # ***Outer drop shape***

> $restart :$

> $p_{-n-1,1} := A_{-n-1,m} :$

> $\Phi_{-n-1,1} := B_{-n-1,m} :$

> $\chi_{-n-1,1} := C_{-n-1,m} :$

> $p_{n,inf} := \frac{2\cdot(2\cdot n+3)}{n}\cdot \zeta b_{n,m} :$

> $\Phi_{n,inf} := \frac{1}{n}\cdot \eta b_{n,m} :$

> $\chi_{n,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{n,m} :$

> $p_{-n-1,inf} := \frac{2\cdot(2\cdot n-1)}{n+1}\cdot \zeta b_{-n-1,m} :$

> $\Phi_{-n-1,inf} := -\frac{1}{n+1}\cdot \eta b_{-n-1,m} :$

> $\chi_{-n-1,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{-n-1,m} :$

> $p_{n,2} := \lambda_2 \cdot E_{n,m} :$

> $\Phi_{n,2} := F_{n,m} :$

> $\chi_{n,2} := G_{n,m} :$

> $p_{-n-1,2} := \lambda_2 \cdot E_{-n-1,m} :$

> $\Phi_{-n-1,2} := F_{-n-1,m} :$

> $\chi_{-n-1,2} := G_{-n-1,m} :$

> $ns := 2\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,1} - \frac{(n^2+3\cdot n-1)}{2\cdot n-1}\cdot p_{-n-1,1} + 2\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,inf} - \frac{(n^2+3\cdot n-1)}{2\cdot n-1}\cdot p_{-n-1,inf} + 2\cdot n\cdot(n-1)\cdot\Phi_{n,inf} + \frac{(n^2-n-3)}{(2\cdot n+3)}\cdot p_{n,inf} - \lambda_2\cdot 2\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,2} + \frac{(n^2+3\cdot n-1)}{(2\cdot n-1)}\cdot p_{-n-1,2} - \lambda_2\cdot 2\cdot n\cdot(n-1)\cdot\Phi_{n,2} - \frac{(n^2-n-3)}{(2\cdot n+3)}\cdot p_{n,2} = (n\cdot(n+1)-2)\cdot f_{12,n,m} - 2\cdot\omega_2\cdot \Gamma 12_{n,m}:$

> $f_{12,n,m} := solve(ns, f_{12,n,m}):$

>

> ***# S1.1. Representative example : Uniaxial extensional flow***

> *restart* :

> $p_{-n-1,1} := A_{-n-1,m}:$

> $\Phi_{-n-1,1} := B_{-n-1,m}:$

> $\chi_{-n-1,1} := C_{-n-1,m}:$

> $p_{n,inf} := \frac{2\cdot(2\cdot n+3)}{n}\cdot \zeta b_{n,m}:$

> $\Phi_{n,inf} := \frac{1}{n}\cdot \eta b_{n,m}:$

> $\chi_{n,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{n,m}:$

> $p_{-n-1,inf} := \frac{2\cdot(2\cdot n-1)}{n+1}\cdot \zeta b_{-n-1,m}:$

> $\Phi_{-n-1,inf} := -\frac{1}{n+1}\cdot \eta b_{-n-1,m}:$

> $\chi_{-n-1,inf} := \frac{1}{n\cdot(n+1)}\cdot \psi b_{-n-1,m}:$

> $p_{n,2} := \lambda_2\cdot E_{n,m}:$

> $\Phi_{n,2} := F_{n,m}:$

> $\chi_{n,2} := G_{n,m}:$

> $p_{-n-1,2} := \lambda_2\cdot E_{-n-1,m}:$

> $\Phi_{-n-1,2} := F_{-n-1,m}:$

> $\chi_{-n-1,2} := G_{-n-1,m}:$

> *# equations 1 - 7 at the inner interface r=1*

> $eq1 := \frac{n}{2\cdot(2\cdot n+3)}\cdot p_{n,inf} + n\cdot\Phi_{n,inf} + \frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,inf} - (n+1)\cdot\Phi_{-n-1,inf} + \frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,1} - (n+1)\cdot\Phi_{-n-1,1} = 0:$

> $eq2 := \frac{n}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,2} + n\cdot\Phi_{n,2} + \frac{(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,2} - (n+1)\cdot\Phi_{-n-1,2}$

$= 0 :$

> $eq3 := -\left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,1} + (n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1} + \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)}\cdot p_{n,\,inf} + n\cdot(n-1)\cdot\Phi_{n,\,inf} - \left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,inf} + (n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf} = \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2} + n\cdot(n-1)\cdot\Phi_{n,\,2} - \left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\right)\cdot p_{-n-1,\,2} + (n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,2} :$

> $eq4 := n\cdot(n+1)\cdot\chi_{-n-1,\,1} + n\cdot(n+1)\cdot\chi_{n,\,inf} + n\cdot(n+1)\cdot\chi_{-n-1,\,inf} = n\cdot(n+1)\cdot\chi_{n,\,2} + n\cdot(n+1)\cdot\chi_{-n-1,\,2} :$

> $eq5 := -\frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,1} + 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1} - \frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,inf} + 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf} - \frac{n^2\cdot(n+2)}{2\cdot n+3}\cdot p_{n,\,inf} - 2\cdot(n-1)\cdot n\cdot(n+1)\cdot\Phi_{n,\,inf} + \frac{(n+1)^2\cdot(n-1)}{(2\cdot n-1)}\cdot p_{-n-1,\,2} - 2\cdot n\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\Phi_{-n-1,\,2} + \frac{n^2\cdot(n+2)}{(2\cdot n+3)}\cdot p_{n,\,2} + 2\cdot(n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\Phi_{n,\,2} = -\omega_2\cdot n\cdot(n+1)\cdot\Gamma_{12,\,n,\,m} :$

> $eq6 := (n-1)\cdot n\cdot(n+1)\cdot\chi_{n,\,inf} - n\cdot(n+1)\cdot(n+2)\cdot\chi_{-n-1,\,inf} - n\cdot(n+1)\cdot(n+2)\cdot\chi_{-n-1,\,1} - (n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\chi_{n,\,2} + n\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\chi_{-n-1,\,2} = 0 :$

> $eq7 := \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2} + n\cdot(n-1)\cdot\Phi_{n,\,2} - \frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2} + (n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,2} = n\cdot(n+1)\cdot\Gamma_{12,\,n,\,m} :$

> *# equations 8 - 14 at the outer interface r=R*

> *# redefining solid harmonics at r=R*

> $p_{n,\,3} := \lambda_3\cdot A_{n,\,m}\cdot R^n :$

> $\Phi_{n,\,3} := B_{n,\,m}\cdot R^n :$

> $\chi_{n,\,3} := C_{n,\,m}\cdot R^n :$

> $p_{n,\,2} := \lambda_2\cdot E_{n,\,m}\cdot R^n :$

> $\Phi_{n,\,2} := F_{n,\,m}\cdot R^n :$

> $\chi_{n,\,2} := G_{n,\,m}\cdot R^n :$

> $p_{-n-1,\,2} := \lambda_2\cdot E_{-n-1,\,m}\cdot R^{-n-1} :$

> $\Phi_{-n-1,\,2} := F_{-n-1,\,m}\cdot R^{-n-1} :$

> $\chi_{-n-1,\,2} := G_{-n-1,\,m}\cdot R^{-n-1} :$

$$> eq8 := \frac{n \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_2} \cdot p_{n,2} + \frac{n}{R} \cdot \Phi_{n,2} + \frac{(n+1) \cdot R}{2 \cdot (2 \cdot n-1) \cdot \lambda_2} \cdot p_{-n-1,2} - \frac{(n+1)}{R} \cdot \Phi_{-n-1,2} = \left(U3_{dz} - U2_{dz}\right) \cdot f_{1,0} + \left(U3_{dx} - U2_{dx}\right) \cdot f_{1,1} :$$

$$> eq9 := \frac{n \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n}{R} \cdot \Phi_{n,3} = \left(U3_{dz} - U2_{dz}\right) \cdot f_{1,0} + \left(U3_{dx} - U2_{dx}\right) \cdot f_{1,1} :$$

$$> eq10 := \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_2} \cdot p_{n,2} + \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,2} - \left(\frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n-1) \cdot \lambda_2}\right) \cdot p_{-n-1,2} + \frac{(n+1) \cdot (n+2)}{R} \cdot \Phi_{-n-1,2} - \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} - \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,3} = 0 :$$

$$> eq11 := n \cdot (n+1) \cdot \left(\chi_{n,2} + \chi_{-n-1,2}\right) - n \cdot (n+1) \cdot \chi_{n,3} = 0 :$$

$$> eq12 := -\frac{(n+1)^2 \cdot (n-1) \cdot R}{(2 \cdot n-1)} \cdot p_{-n-1,2} + 2 \cdot n \cdot (n+1) \cdot \frac{(n+2)}{R} \cdot \lambda_2 \cdot \Phi_{-n-1,2} - \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n+3)} \cdot p_{n,2} - \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_2 \cdot \Phi_{n,2} + \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n+3)} \cdot p_{n,3} + \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_3 \cdot \Phi_{n,3} = -\alpha \cdot \delta_2 \cdot n \cdot (n+1) \cdot \Gamma_{23,n,m} :$$

$$> eq13 := \lambda_2 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,2} - \lambda_2 \cdot n \cdot (n+1) \cdot (n+2) \cdot \chi_{-n-1,2} - \lambda_3 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,3} = 0 :$$

$$> eq14 := \frac{1}{d} \cdot \left(\frac{n \cdot (n+1)}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n \cdot (n-1)}{R^2} \cdot \Phi_{n,3}\right) = n \cdot (n+1) \cdot \Gamma_{23,n,m} :$$

$$> sol1 := solve\left(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13, eq14\}, \left\{A_{n,m}, B_{n,m}, C_{n,m}, A_{-n-1,m}, B_{-n-1,m}, C_{-n-1,m}, E_{n,m}, F_{n,m}, G_{n,m}, E_{-n-1,m}, F_{-n-1,m}, G_{-n-1,m}, \Gamma_{12,n,m}, \Gamma_{23,n,m}\right\}\right) :$$

$> A_{n,m} := simplify(rhs(sol1[1])) :$

$> A_{-n-1,m} := simplify(rhs(sol1[2])) :$

$> B_{n,m} := simplify(rhs(sol1[3])) :$

$> B_{-n-1,m} := simplify(rhs(sol1[4])) :$

$> C_{n,m} := simplify(rhs(sol1[5])) :$

$> C_{-n-1,m} := simplify(rhs(sol1[6])) :$

$> E_{n,m} := simplify(rhs(sol1[7])) :$

$> E_{-n-1,m} := simplify(rhs(sol1[8])) :$

$> F_{n,m} := simplify(rhs(sol1[9])) :$

$> F_{-n-1,m} := simplify(rhs(sol1[10])) :$

$> G_{n,m} := simplify(rhs(sol1[11])) :$

$> G_{-n-1,m} := simplify(rhs(sol1[12])) :$

$> \Gamma 12_{n,m} := simplify(rhs(sol1[13])) :$

> $\Gamma23_{n,m} := simplify(rhs(sol1[14])):$

> $A_{-2,0} := simplify\left(subs\left(f_{1,0}=1, f_{1,1}=0, n=1, m=0, A_{-n-1,m}\right)\right):$

> $A_{-2,1} := simplify\left(subs\left(f_{1,0}=0, f_{1,1}=1, n=1, m=1, A_{-n-1,m}\right)\right):$

> $Ac_{-2,1} := simplify\Big(subs\Big(f_{1,0}=0, f_{1,1}=1, U2_{dx}=U2_{dy}, U3_{dx}=U3_{dy}, \zeta b_{n,m}=\zeta bc_{n,m}, \eta b_{n,m} = \eta bc_{n,n}, \psi b_{n,m}=\psi bc_{n,m}, \zeta b_{-n-1,m}=\zeta bc_{-n-1,m}, \eta b_{-n-1,m}=\eta bc_{-n-1,n}, \psi b_{-n-1,m} = \psi bc_{-n-1,m}, A_{-2,1}\Big)\Big):$

> $\eta b_{1,0} := \eta t_{1,0} - U2_{dz}:$

> $\eta b_{1,1} := \eta t_{1,1} - U2_{dx}:$

> $\eta bc_{1,1} := \eta ct_{1,1} - U2_{dy}:$

> $rel1 := A_{-2,0}=0:$

> $rel2 := A_{-2,1}=0:$

> $rel3 := Ac_{-2,1}=0:$

> $E_{-2,0} := simplify\left(subs\left(f_{1,0}=1, f_{1,1}=0, n=1, m=0, E_{-n-1,m}\right)\right):$

> $E_{-2,1} := simplify\left(subs\left(f_{1,0}=0, f_{1,1}=1, n=1, m=1, E_{-n-1,m}\right)\right):$

> $Ec_{-2,1} := simplify\Big(subs\Big(f_{1,0}=0, f_{1,1}=1, U2_{dx}=U2_{dy}, U3_{dx}=U3_{dy}, \zeta b_{n,m}=\zeta bc_{n,m}, \eta b_{n,m} = \eta bc_{n,n}, \psi b_{n,m}=\psi bc_{n,m}, \zeta b_{-n-1,m}=\zeta bc_{-n-1,m}, \eta b_{-n-1,m}=\eta bc_{-n-1,n}, \psi b_{-n-1,m} = \psi bc_{-n-1,m}, E_{-2,1}\Big)\Big):$

> $rel4 := E_{-2,0}=0:$

> $rel5 := E_{-2,1}=0:$

> $rel6 := Ec_{-2,1}=0:$

> $sol2 := solve\left(\{rel1, rel2, rel3, rel4, rel5, rel6\}, \left\{U2_{dx}, U2_{dy}, U2_{dz}, U3_{dx}, U3_{dy}, U3_{dz}\right\}\right):$

> #Droplet velocity

> $U2_{dx} := rhs(sol2[1]):$

> $U2_{dy} := rhs(sol2[2]):$

> $U2_{dz} := rhs(sol2[3]):$

> $U3_{dx} := rhs(sol2[4]):$

> $U3_{dy} := rhs(sol2[5]):$

> $U3_{dz} := rhs(sol2[6]):$

> # Uniaxial Extensional Flow

> $\eta b_{2,0} := 1 : \zeta b_{2,0} := 0 : \psi b_{2,0} := 0 : \eta b_{-3,0} := 0 : \zeta b_{-3,0} := 0 : \psi b_{-3,0} := 0 :$

> $U2_{dx} := subs\left(\zeta b_{-2,1}=0, \zeta b_{1,1}=0, \eta t_{1,1}=0, U2_{dx}\right):$

> $U2_{dy} := subs\left(\zeta b_{-2,1}=0, \zeta b_{1,1}=0, \eta t_{1,1}=0, U2_{dy}\right):$

> $U2_{dz} := subs\left(\zeta b_{-2,0}=0, \zeta b_{1,0}=0, \zeta b_{-2,1}=0, \zeta b_{1,1}=0, \eta t_{1,0}=0, U2_{dz}\right):$

> $U3_{dx} := subs\left(\eta t_{1,1}=0, \zeta b_{-2,0}=0, \zeta b_{1,0}=0, \zeta b_{-2,1}=0, \zeta b_{1,1}=0, \eta t_{1,0}=0, U3_{dx}\right):$

> $U3_{dy} := subs\left(\eta t_{1,\,1}=0,\ \zeta b_{-2,\,0}=0,\ \zeta b_{1,\,0}=0,\ \zeta b_{-2,\,1}=0,\ \zeta b_{1,\,1}=0,\ \eta t_{1,\,0}=0,\ U3_{dy}\right):$

> $U3_{dz} := subs\left(\eta t_{1,\,1}=0,\ \zeta b_{-2,\,0}=0,\ \zeta b_{1,\,0}=0,\ \zeta b_{-2,\,1}=0,\ \zeta b_{1,\,1}=0,\ \eta t_{1,\,0}=0,\ U3_{dz}\right):$

> *# Drop shape calculation*

> $A_{-3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ A_{-n-1,\,m}\right)\right):$

> $A_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ A_{n,\,m}\right)\right):$

> $E_{-3,\,0} := simplify\left(\left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ E_{-n-1,\,m}\right)\right)\right):$

> $E_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ E_{n,\,m}\right)\right):$

> $B_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ B_{n,\,m}\right)\right):$

> $B_{-3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ B_{-n-1,\,m}\right)\right):$

> $F_{-3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ F_{-n-1,\,m}\right)\right):$

> $F_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ F_{n,\,m}\right)\right):$

> $\Gamma 12_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ \Gamma 12_{n,\,m}\right)\right):$

> $\Gamma 23_{2,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0,\ n=2,\ m=0,\ \Gamma 23_{n,\,m}\right)\right):$

>

> $$f_{12,\,2,\,0} := \left(-6\,\lambda_2\,F_{-3,\,0} + \frac{3}{4}\,\lambda_2\,E_{-3,\,0} - \lambda_2\,F_{2,\,0} + \frac{1}{28}\,\lambda_2\,E_{2,\,0} + \frac{1}{2}\,\omega_2\,\Gamma 12_{2,\,0} - \frac{1}{4}\,\zeta b_{2,\,0} - 2\,\eta b_{-3,\,0} + 6\,B_{-3,\,0} + \frac{1}{2}\,\eta b_{2,\,0} - \frac{3}{4}\,A_{-3,\,0} - \frac{3}{2}\,\zeta b_{-3,\,0}\right):$$

> $$f_{23,\,2,\,0} := \left(-\frac{1}{84}\,\frac{1}{R\,\omega_1}\left(-42\,\alpha\,\omega_2\,\Gamma 23_{2,\,0}\,R + \frac{63\,\lambda_2\,E_{-3,\,0}}{R} + 3\,\lambda_2\,E_{2,\,0}\,R^4 - 3\,\lambda_3\,A_{2,\,0}\,R^4 - 84\,\lambda_2\,F_{2,\,0}\,R^2 + 84\,\lambda_3\,B_{2,\,0}\,R^2 - \frac{504\,\lambda_2\,F_{-3,\,0}}{R^3}\right)\right):$$

>

> ***# S1.2. Representative example : Poiseuille flow***

> *restart* :

> $p_{-n-1,\,1} := A_{-n-1,\,m}:$

> $\Phi_{-n-1,\,1} := B_{-n-1,\,m}:$

> $\chi_{-n-1,\,1} := C_{-n-1,\,m}:$

> $p_{n,\,inf} := \dfrac{2\cdot(2\cdot n+3)}{n}\cdot\zeta b_{n,\,m}:$

> $\Phi_{n,\,inf} := \dfrac{1}{n}\cdot\eta b_{n,\,m}:$

> $\chi_{n,\,inf} := \dfrac{1}{n\cdot(n+1)}\cdot\psi b_{n,\,m}:$

> $p_{-n-1,\,inf} := \dfrac{2\cdot(2\cdot n-1)}{n+1}\cdot\zeta b_{-n-1,\,m}:$

> $\Phi_{-n-1,\,inf} := -\dfrac{1}{n+1}\cdot\eta b_{-n-1,\,m}:$

> $\chi_{-n-1,\,inf} := \frac{1}{n\cdot(n+1)}\cdot\psi b_{-n-1,\,m}:$

> $p_{n,\,2} := \lambda_2\cdot E_{n,\,m}:$

> $\Phi_{n,\,2} := F_{n,\,m}:$

> $\chi_{n,\,2} := G_{n,\,m}:$

> $p_{-n-1,\,2} := \lambda_2\cdot E_{-n-1,\,m}:$

> $\Phi_{-n-1,\,2} := F_{-n-1,\,m}:$

> $\chi_{-n-1,\,2} := G_{-n-1,\,m}:$

> *# equations 1 - 7 at the inner interface r=1*

> $$eq1 := \frac{n}{2\cdot(2\cdot n+3)}\cdot p_{n,\,inf}+n\cdot\Phi_{n,\,inf}+\frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,\,inf}-(n+1)\cdot\Phi_{-n-1,\,inf}+\frac{(n+1)}{2\cdot(2\cdot n-1)}\cdot p_{-n-1,\,1}-(n+1)\cdot\Phi_{-n-1,\,1}=0:$$

> $$eq2 := \frac{n}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2}+n\cdot\Phi_{n,\,2}+\frac{(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2}-(n+1)\cdot\Phi_{-n-1,\,2}=0:$$

> $$eq3 := -\left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,1}+(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1}+\frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)}\cdot p_{n,\,inf}+n\cdot(n-1)\cdot\Phi_{n,\,inf}-\left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)}\right)\cdot p_{-n-1,\,inf}+(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf}=\frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2}+n\cdot(n-1)\cdot\Phi_{n,\,2}-\left(\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\right)\cdot p_{-n-1,\,2}+(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,2}:$$

> $$eq4 := n\cdot(n+1)\cdot\chi_{-n-1,\,1}+n\cdot(n+1)\cdot\chi_{n,\,inf}+n\cdot(n+1)\cdot\chi_{-n-1,\,inf}=n\cdot(n+1)\cdot\chi_{n,\,2}+n\cdot(n+1)\cdot\chi_{-n-1,\,2}:$$

> $$eq5 := -\frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,1}+2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,1}-\frac{(n+1)^2\cdot(n-1)}{2\cdot n-1}\cdot p_{-n-1,\,inf}+2\cdot n\cdot(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,inf}-\frac{n^2\cdot(n+2)}{2\cdot n+3}\cdot p_{n,\,inf}-2\cdot(n-1)\cdot n\cdot(n+1)\cdot\Phi_{n,\,inf}+\frac{(n+1)^2\cdot(n-1)}{(2\cdot n-1)}\cdot p_{-n-1,\,2}-2\cdot n\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\Phi_{-n-1,\,2}+\frac{n^2\cdot(n+2)}{(2\cdot n+3)}\cdot p_{n,\,2}+2\cdot(n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\Phi_{n,\,2}=-\omega_2\cdot n\cdot(n+1)\cdot\Gamma_{12,\,n,\,m}:$$

> $$eq6 := (n-1)\cdot n\cdot(n+1)\cdot\chi_{n,\,inf}-n\cdot(n+1)\cdot(n+2)\cdot\chi_{-n-1,\,inf}-n\cdot(n+1)\cdot(n+2)\cdot\chi_{-n-1,\,1}-(n-1)\cdot n\cdot(n+1)\cdot\lambda_2\cdot\chi_{n,\,2}+n\cdot(n+1)\cdot(n+2)\cdot\lambda_2\cdot\chi_{-n-1,\,2}=0:$$

> $$eq7 := \frac{n\cdot(n+1)}{2\cdot(2\cdot n+3)\cdot\lambda_2}\cdot p_{n,\,2}+n\cdot(n-1)\cdot\Phi_{n,\,2}-\frac{n\cdot(n+1)}{2\cdot(2\cdot n-1)\cdot\lambda_2}\cdot p_{-n-1,\,2}+(n+1)\cdot(n+2)\cdot\Phi_{-n-1,\,2}=n\cdot(n+1)\cdot\Gamma_{12,\,n,\,m}:$$

> # equations 8 - 14 at the outer interface r=R

> # redefining solid harmonics at r=R

> $p_{n,3} := \lambda_3 \cdot A_{n,m} \cdot R^n :$

> $\Phi_{n,3} := B_{n,m} \cdot R^n :$

> $\chi_{n,3} := C_{n,m} \cdot R^n :$

> $p_{n,2} := \lambda_2 \cdot E_{n,m} \cdot R^n :$

> $\Phi_{n,2} := F_{n,m} \cdot R^n :$

> $\chi_{n,2} := G_{n,m} \cdot R^n :$

> $p_{-n-1,2} := \lambda_2 \cdot E_{-n-1,m} \cdot R^{-n-1} :$

> $\Phi_{-n-1,2} := F_{-n-1,m} \cdot R^{-n-1} :$

> $\chi_{-n-1,2} := G_{-n-1,m} \cdot R^{-n-1} :$

> $$eq8 := \frac{n \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_2} \cdot p_{n,2} + \frac{n}{R} \cdot \Phi_{n,2} + \frac{(n+1) \cdot R}{2 \cdot (2 \cdot n-1) \cdot \lambda_2} \cdot p_{-n-1,2} - \frac{(n+1)}{R} \cdot \Phi_{-n-1,2} = \left(U3_{dz} - U2_{dz}\right) \cdot f_{1,0} + \left(U3_{dx} - U2_{dx}\right) \cdot f_{1,1} :$$

> $$eq9 := \frac{n \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n}{R} \cdot \Phi_{n,3} = \left(U3_{dz} - U2_{dz}\right) \cdot f_{1,0} + \left(U3_{dx} - U2_{dx}\right) \cdot f_{1,1} :$$

> $$eq10 := \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_2} \cdot p_{n,2} + \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,2} - \left(\frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n-1) \cdot \lambda_2} \right) \cdot p_{-n-1,2} + \frac{(n+1) \cdot (n+2)}{R} \cdot \Phi_{-n-1,2} - \frac{n \cdot (n+1) \cdot R}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} - \frac{n \cdot (n-1)}{R} \cdot \Phi_{n,3} = 0 :$$

> $$eq11 := n \cdot (n+1) \cdot \left(\chi_{n,2} + \chi_{-n-1,2}\right) - n \cdot (n+1) \cdot \chi_{n,3} = 0 :$$

> $$eq12 := -\frac{(n+1)^2 \cdot (n-1) \cdot R}{(2 \cdot n-1)} \cdot p_{-n-1,2} + 2 \cdot n \cdot (n+1) \cdot \frac{(n+2)}{R} \cdot \lambda_2 \cdot \Phi_{-n-1,2} - \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n+3)} \cdot p_{n,2} - \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_2 \cdot \Phi_{n,2} + \frac{n^2 \cdot (n+2) \cdot R}{(2 \cdot n+3)} \cdot p_{n,3} + \frac{2 \cdot (n-1) \cdot n \cdot (n+1)}{R} \cdot \lambda_3 \cdot \Phi_{n,3} = -\alpha \cdot \delta_2 \cdot n \cdot (n+1) \cdot \Gamma_{23,n,m} :$$

> $$eq13 := \lambda_2 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,2} - \lambda_2 \cdot n \cdot (n+1) \cdot (n+2) \cdot \chi_{-n-1,2} - \lambda_3 \cdot n \cdot (n+1) \cdot (n-1) \cdot \chi_{n,3} = 0 :$$

> $$eq14 := \frac{1}{d} \cdot \left(\frac{n \cdot (n+1)}{2 \cdot (2 \cdot n+3) \cdot \lambda_3} \cdot p_{n,3} + \frac{n \cdot (n-1)}{R^2} \cdot \Phi_{n,3} \right) = n \cdot (n+1) \cdot \Gamma_{23,n,m} :$$

> $$sol1 := solve\left(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13, eq14\}, \left\{A_{n,m}, B_{n,m}, C_{n,m}, A_{-n-1,m}, B_{-n-1,m}, C_{-n-1,m}, E_{n,m}, F_{n,m}, G_{n,m}, E_{-n-1,m}, F_{-n-1,m}, G_{-n-1,m}, \Gamma_{12,n,m}, \Gamma_{23,n,m}\right\}\right) :$$

> $A_{n,m} := simplify(rhs(sol1[1])) :$

> $A_{-n-1,\,m} := simplify(rhs(sol1[2]))$:

> $B_{n,\,m} := simplify(rhs(sol1[3]))$:

> $B_{-n-1,\,m} := simplify(rhs(sol1[4]))$:

> $C_{n,\,m} := simplify(rhs(sol1[5]))$:

> $C_{-n-1,\,m} := simplify(rhs(sol1[6]))$:

> $E_{n,\,m} := simplify(rhs(sol1[7]))$:

> $E_{-n-1,\,m} := simplify(rhs(sol1[8]))$:

> $F_{n,\,m} := simplify(rhs(sol1[9]))$:

> $F_{-n-1,\,m} := simplify(rhs(sol1[10]))$:

> $G_{n,\,m} := simplify(rhs(sol1[11]))$:

> $G_{-n-1,\,m} := simplify(rhs(sol1[12]))$:

> $\Gamma12_{n,\,m} := simplify(rhs(sol1[13]))$:

> $\Gamma23_{n,\,m} := simplify(rhs(sol1[14]))$:

> $A_{-2,\,0} := simplify\big(subs\big(f_{1,\,0}=1, f_{1,\,1}=0, n=1, m=0, A_{-n-1,\,m}\big)\big)$:

> $A_{-2,\,1} := simplify\big(subs\big(f_{1,\,0}=0, f_{1,\,1}=1, n=1, m=1, A_{-n-1,\,m}\big)\big)$:

> $Ac_{-2,\,1} := simplify\big(subs\big(f_{1,\,0}=0, f_{1,\,1}=1, U2_{dx}=U2_{dy}, U3_{dx}=U3_{dy}, \zeta b_{n,\,m}=\zeta bc_{n,\,m}, \eta b_{n,\,m} = \eta bc_{n,\,n}, \psi b_{n,\,m}=\psi bc_{n,\,m}, \zeta b_{-n-1,\,m}=\zeta bc_{-n-1,\,m}, \eta b_{-n-1,\,m}=\eta bc_{-n-1,\,n}, \psi b_{-n-1,\,m} = \psi bc_{-n-1,\,m}, A_{-2,\,1}\big)\big)$:

> $\eta b_{1,\,0} := \eta t_{1,\,0} - U2_{dz}$:

> $\eta b_{1,\,1} := \eta t_{1,\,1} - U2_{dx}$:

> $\eta bc_{1,\,1} := \eta ct_{1,\,1} - U2_{dy}$:

> $rel1 := A_{-2,\,0}=0$:

> $rel2 := A_{-2,\,1}=0$:

> $rel3 := Ac_{-2,\,1}=0$:

> $E_{-2,\,0} := simplify\big(subs\big(f_{1,\,0}=1, f_{1,\,1}=0, n=1, m=0, E_{-n-1,\,m}\big)\big)$:

> $E_{-2,\,1} := simplify\big(subs\big(f_{1,\,0}=0, f_{1,\,1}=1, n=1, m=1, E_{-n-1,\,m}\big)\big)$:

> $Ec_{-2,\,1} := simplify\big(subs\big(f_{1,\,0}=0, f_{1,\,1}=1, U2_{dx}=U2_{dy}, U3_{dx}=U3_{dy}, \zeta b_{n,\,m}=\zeta bc_{n,\,m}, \eta b_{n,\,m} = \eta bc_{n,\,n}, \psi b_{n,\,m}=\psi bc_{n,\,m}, \zeta b_{-n-1,\,m}=\zeta bc_{-n-1,\,m}, \eta b_{-n-1,\,m}=\eta bc_{-n-1,\,n}, \psi b_{-n-1,\,m} = \psi bc_{-n-1,\,m}, E_{-2,\,1}\big)\big)$:

> $rel4 := E_{-2,\,0}=0$:

> $rel5 := E_{-2,\,1}=0$:

> $rel6 := Ec_{-2,\,1}=0$:

> $sol2 := solve\big(\{rel1, rel2, rel3, rel4, rel5, rel6\}, \{U2_{dx}, U2_{dy}, U2_{dz}, U3_{dx}, U3_{dy}, U3_{dz}\}\big)$:

> *#Droplet velocity*

```
> U2[dx] := rhs(sol2[1]) :
> U2[dy] := rhs(sol2[2]) :
> U2[dz] := rhs(sol2[3]) :
> U3[dx] := rhs(sol2[4]) :
> U3[dy] := rhs(sol2[5]) :
> U3[dz] := rhs(sol2[6]) :
> # special flow: Poiseuille flow
```

$$> \zeta b_{1,\,0} := -\frac{2}{5}\cdot\frac{1}{R0^2} : \eta t_{1,\,0} := 1-\left(\frac{b}{R0}\right)^2 : \eta b_{2,\,1} := -\frac{2}{3}\cdot\frac{b}{(R0)^2} : \eta b_{3,\,0} := \frac{2}{5\cdot(R0)^2} :$$

$$> \eta t_{1,\,1} := 0 : \eta tc_{1,\,1} := 0 : \zeta b_{1,\,1} := 0 : \zeta b_{-2,\,1} := 0 : \zeta b_{-2,\,0} := 0 : \zeta b_{2,\,0} := 0 : \eta b_{2,\,0} := 0 : \zeta b_{2,\,1} := 0 : \eta b_{-4,\,0} := 0 : \zeta b_{3,\,0} := 0 :$$

$$> U2_{dx} := simplify(U2_{dx}) :$$
$$> U2_{dy} := simplify(U2_{dy}) :$$
$$> U2_{dz} := simplify(U2_{dz}) :$$
$$> U3_{dx} := simplify(U3_{dx}) :$$
$$> U3_{dy} := simplify(U3_{dy}) :$$
$$> U3_{dz} := simplify(subs(\ \ U3_{dz})) :$$

\>

\> # Calculation of drop shape

$$> A_{-4,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, A_{-n-1,\,m}\right)\right) :$$
$$> A_{-3,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, A_{-n-1,\,m}\right)\right) :$$
$$> A_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, A_{n,\,m}\right)\right) :$$
$$> A_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, A_{n,\,m}\right)\right) :$$
$$> E_{-4,\,0} := simplify\left(\left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, E_{-n-1,\,m}\right)\right)\right) :$$
$$> E_{-3,\,1} := simplify\left(\left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, E_{-n-1,\,m}\right)\right)\right) :$$
$$> E_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, E_{n,\,m}\right)\right) :$$
$$> E_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, E_{n,\,m}\right)\right) :$$
$$> B_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, B_{n,\,m}\right)\right) :$$
$$> B_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, B_{n,\,m}\right)\right) :$$
$$> B_{-4,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, B_{-n-1,\,m}\right)\right) :$$
$$> B_{-3,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, B_{-n-1,\,m}\right)\right) :$$
$$> F_{-4,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, F_{-n-1,\,m}\right)\right) :$$
$$> F_{-3,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, F_{-n-1,\,m}\right)\right) :$$
$$> F_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, F_{n,\,m}\right)\right) :$$
$$> F_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, F_{n,\,m}\right)\right) :$$
$$> \Gamma 12_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, \Gamma 12_{n,\,m}\right)\right) :$$

> $\Gamma 12_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, \Gamma 12_{n,\,m}\right)\right):$

> $\Gamma 23_{3,\,0} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=3, m=0, \Gamma 23_{n,\,m}\right)\right):$

> $\Gamma 23_{2,\,1} := \left(subs\left(f_{1,\,0}=0, f_{1,\,1}=0, n=2, m=1, \Gamma 23_{n,\,m}\right)\right):$

>

> $$f_{23,\,3,\,0} := \left(-\frac{1}{450}\,\frac{1}{R\,\omega_1}\left(-90\,\alpha\,\omega_2\,\Gamma 23_{3,\,0}\,R + \frac{153\,\lambda_2\,E_{-4,\,0}}{R^2} - 15\,\lambda_2\,E_{3,\,0}\,R^5 + 15\,\lambda_3\,A_{3,\,0}\,R^5 - \frac{1800\,\lambda_2\,F_{-4,\,0}}{R^4} - 540\,\lambda_2\,F_{3,\,0}\,R^3 + 540\,\lambda_3\,B_{3,\,0}\,R^3\right)\right):$$

> $$f_{23,\,2,\,1} := \left(-\frac{1}{84}\,\frac{1}{R\,\omega_1}\left(-42\,\alpha\,\omega_2\,\Gamma 23_{2,\,1}\,R + \frac{63\,\lambda_2\,E_{-3,\,1}}{R} + 3\,\lambda_2\,E_{2,\,1}\,R^4 - 3\,\lambda_3\,A_{2,\,1}\,R^4 - 84\,\lambda_2\,F_{2,\,1}\,R^2 - \frac{504\,\lambda_2\,F_{-3,\,1}}{R^3} + 84\,\lambda_3\,B_{2,\,1}\,R^2\right)\right):$$

> $$f_{12,\,3,\,0} := \left(\frac{1}{5}\,\omega_2\,\Gamma 12_{3,\,0} + \frac{17}{50}\,\lambda_2\,E_{-4,\,0} - \frac{1}{30}\,\lambda_2\,E_{3,\,0} - 4\,\lambda_2\,F_{-4,\,0} - \frac{6}{5}\,\lambda_2\,F_{3,\,0} + \frac{1}{5}\,\zeta b_{3,\,0} - \eta b_{-4,\,0} + 4\,B_{-4,\,0} + \frac{2}{5}\,\eta b_{3,\,0} - \frac{17}{50}\,A_{-4,\,0} - \frac{17}{20}\,\zeta b_{-4,\,0}\right):$$

> $$f_{12,\,2,\,1} := \left(\frac{1}{2}\,\omega_2\,\Gamma 12_{2,\,1} + \frac{3}{4}\,\lambda_2\,E_{-3,\,1} + \frac{1}{28}\,\lambda_2\,E_{2,\,1} - 6\,\lambda_2\,F_{-3,\,1} - \lambda_2\,F_{2,\,1} - \frac{1}{4}\,\zeta b_{2,\,1} - 2\,\eta b_{-3,\,1} + 6\,B_{-3,\,1} + \frac{1}{2}\,\eta b_{2,\,1} - \frac{3}{4}\,A_{-3,\,1} - \frac{3}{2}\,\zeta b_{-3,\,1}\right):$$

> $f_{23,\,3,\,0} := simplify\left(f_{23,\,3,\,0}\right):$

> $f_{23,\,2,\,1} := simplify\left(f_{23,\,2,\,1}\right):$

> $f_{12,\,3,\,0} := simplify\left(f_{12,\,3,\,0}\right):$

> $f_{12,\,2,\,1} := simplify\left(f_{12,\,2,\,1}\right):$

>

> ***# S1.3. Calculation of effective viscosity for linear flow***

> *restart* : *with*(*VectorCalculus*) : *SetCoordinates*($'spherical'_{r,\ \mathrm{theta},\ \mathrm{phi}}$) :

> $e_x := VectorField(\langle \sin(\mathrm{theta})\cdot\cos(\mathrm{phi}), \cos(\mathrm{theta})\cdot\cos(\mathrm{phi}), (-\sin(\mathrm{phi}))\rangle):$

> $e_y := VectorField(\langle \sin(\mathrm{theta})\cdot\sin(\mathrm{phi}), \cos(\mathrm{theta})\cdot\sin(\mathrm{phi}), \cos(\mathrm{phi})\rangle):$

> $e_z := VectorField(\langle \cos(\mathrm{theta}), (-\sin(\mathrm{theta})), 0\rangle):$

> $P_{2,\,0} := \frac{1}{2}\cdot\left(3\cdot(\cos(\mathrm{theta}))^2 - 1\right):$

>

> $$A_{-3,\,0} := \Big(-\big(-1500\,R^5\,d\,\lambda_2^2 - 750\,\lambda_3\,R^4\,d\,\lambda_2 + 750\,\lambda_3\,R^5\,d\,\omega_2 - 300\,\alpha\,\omega_2\,\lambda_2\,R^2 - 100\,R\,\omega_2\,\lambda_2\,d - 100\,\lambda_3\,R\,d\,\omega_2 + 150\,R^5\,\alpha\,\omega_2\,\lambda_2 + 225\,R^6\,\lambda_3\,d\,\omega_2 - 300\,\lambda_3\,d\,\omega_2\,R^3 - 300\,\lambda_2\,d\,\omega_2\,R^3 + 150\,R^6\,d\,\omega_2\,\lambda_2 + 200\,\alpha\,\omega_2\,R^7\,\lambda_2 + 200\,\lambda_2\,R^8\,d\,\omega_2 - 200\,\lambda_3\,R^8\,d\,\omega_2$$

$$+ 300 R^7 d \omega_2 \lambda_2 + 300 \alpha \omega_2 R^6 \lambda_2 - 300 \lambda_3 R^7 d \omega_2 - 150 \alpha \omega_2 R^3 \lambda_2 + 225 \lambda_3 R^4 d \omega_2$$
$$- 200 \alpha \omega_2 \lambda_2 R - 200 R^2 \omega_2 \lambda_2 d + 750 R^6 \lambda_3 d \lambda_2 + 1000 \lambda_3 R^8 d \lambda_2 + 1500 \lambda_3 R^7 d \lambda_2$$
$$- 1000 \lambda_3 R^2 d \lambda_2 + 500 \lambda_3 R^9 d \lambda_2 - 1500 R^6 \lambda_2^2 d - 120 \alpha \omega_2 R^6 + 150 \alpha \omega_2^2 R^4$$
$$+ 450 R^6 \lambda_3 d - 60 \alpha \omega_2^2 R^6 + 300 R^6 \lambda_2 d + 300 \alpha \omega_2 R^4 - 60 \alpha \omega_2^2 R^2 - 100 \alpha \omega_2 \lambda_2$$
$$+ 400 \lambda_2 R^8 d - 200 R \lambda_2 d - 500 \lambda_3 R d \lambda_2 - 400 \lambda_3 R^8 d - 600 \lambda_3 d R^3 - 600 \lambda_3 R^7 d$$
$$- 1000 \lambda_2^2 R^8 d - 1500 R^7 \lambda_2^2 d - 200 \lambda_3 R d + 600 R^7 \lambda_2 d - 1000 R^2 \lambda_2^2 d - 400 R^2 \lambda_2 d$$
$$+ 450 \lambda_3 R^4 d - 1500 \lambda_2^2 d R^3 - 300 R^4 \lambda_2 d - 1500 R^4 \lambda_2^2 d - 600 \lambda_2 d R^3 - 500 R \lambda_2^2 d$$
$$+ 1500 \lambda_3 R^5 d - 200 \lambda_3 R^9 d + 200 R^9 \lambda_2 d - 500 R^9 \lambda_2^2 d - 400 \lambda_3 R^2 d - 1500 \lambda_3 d \lambda_2 R^3$$
$$- 40 \alpha \omega_2^2 R^7 + 100 \alpha \omega_2 R^8 \lambda_2 + 100 R^9 d \omega_2 \lambda_2 - 100 \lambda_3 R^9 d \omega_2 - 150 R^4 \lambda_2 d \omega_2$$
$$- 200 \lambda_3 R^2 d \omega_2 - 40 \alpha \omega_2 - 20 \alpha \omega_2^2 - 40 \alpha \omega_2^2 R - 80 \alpha \omega_2 R - 40 \alpha \omega_2 R^8 - 20 \alpha$$
$$\omega_2^2 R^8 + 45 \alpha \omega_2^2 R^3 - 80 \alpha \omega_2 R^7 - 120 \alpha \omega_2 R^2 + 90 R^5 \alpha \omega_2 + 90 \alpha \omega_2 R^3 + 45 \alpha \omega_2^2 R^5)$$
$$/ (-300 R^5 d \lambda_2^2 - 150 \lambda_3 R^4 d \lambda_2 + 150 \lambda_3 R^5 d \omega_2 - 60 \alpha \omega_2 \lambda_2 R^2 - 20 R \omega_2 \lambda_2 d$$
$$- 20 \lambda_3 R d \omega_2 + 30 R^5 \alpha \omega_2 \lambda_2 + 45 R^6 \lambda_3 d \omega_2 - 60 \lambda_3 d \omega_2 R^3 - 60 \lambda_2 d \omega_2 R^3$$
$$+ 30 R^6 d \omega_2 \lambda_2 + 40 \alpha \omega_2 R^7 \lambda_2 + 40 \lambda_2 R^8 d \omega_2 - 40 \lambda_3 R^8 d \omega_2 + 60 R^7 d \omega_2 \lambda_2$$
$$+ 60 \alpha \omega_2 R^6 \lambda_2 - 60 \lambda_3 R^7 d \omega_2 - 30 \alpha \omega_2 R^3 \lambda_2 + 45 \lambda_3 R^4 d \omega_2 - 40 \alpha \omega_2 \lambda_2 R$$
$$- 40 R^2 \omega_2 \lambda_2 d + 150 R^6 \lambda_3 d \lambda_2 + 200 \lambda_3 R^8 d \lambda_2 + 300 \lambda_3 R^7 d \lambda_2 - 200 \lambda_3 R^2 d \lambda_2$$
$$+ 100 \lambda_3 R^9 d \lambda_2 - 300 R^6 \lambda_2^2 d - 60 \alpha \omega_2 R^6 + 30 \alpha \omega_2^2 R^4 + 225 R^6 \lambda_3 d - 12 \alpha \omega_2^2 R^6$$
$$+ 150 R^6 \lambda_2 d + 150 \alpha \omega_2 R^4 - 12 \alpha \omega_2^2 R^2 - 20 \alpha \omega_2 \lambda_2 + 200 \lambda_2 R^8 d - 100 R \lambda_2 d$$
$$- 100 \lambda_3 R d \lambda_2 - 200 \lambda_3 R^8 d - 300 \lambda_3 d R^3 - 300 \lambda_3 R^7 d - 200 \lambda_2^2 R^8 d - 300 R^7 \lambda_2^2 d$$
$$- 100 \lambda_3 R d + 300 R^7 \lambda_2 d - 200 R^2 \lambda_2^2 d - 200 R^2 \lambda_2 d + 225 \lambda_3 R^4 d - 300 \lambda_2^2 d R^3$$
$$- 150 R^4 \lambda_2 d - 300 R^4 \lambda_2^2 d - 300 \lambda_2 d R^3 - 100 R \lambda_2^2 d + 750 \lambda_3 R^5 d - 100 \lambda_3 R^9 d$$
$$+ 100 R^9 \lambda_2 d - 100 R^9 \lambda_2^2 d - 200 \lambda_3 R^2 d - 300 \lambda_3 d \lambda_2 R^3 - 8 \alpha \omega_2^2 R^7 + 20 \alpha \omega_2 R^8 \lambda_2$$
$$+ 20 R^9 d \omega_2 \lambda_2 - 20 \lambda_3 R^9 d \omega_2 - 30 R^4 \lambda_2 d \omega_2 - 40 \lambda_3 R^2 d \omega_2 - 20 \alpha \omega_2 - 4 \alpha \omega_2^2$$
$$- 8 \alpha \omega_2^2 R - 40 \alpha \omega_2 R - 20 \alpha \omega_2 R^8 - 4 \alpha \omega_2^2 R^8 + 9 \alpha \omega_2^2 R^3 - 40 \alpha \omega_2 R^7 - 60 \alpha \omega_2 R^2$$
$$+ 45 R^5 \alpha \omega_2 + 45 \alpha \omega_2 R^3 + 9 \alpha \omega_2^2 R^5)) :$$

> $$B_{-3,\,0} := \Big(-\frac{1}{2} \, (-300 R^5 d \lambda_2^2 - 150 \lambda_3 R^4 d \lambda_2 + 150 \lambda_3 R^5 d \omega_2 - 60 \alpha \omega_2 \lambda_2 R^2$$
$$- 20 R \omega_2 \lambda_2 d - 20 \lambda_3 R d \omega_2 + 30 R^5 \alpha \omega_2 \lambda_2 + 45 R^6 \lambda_3 d \omega_2 - 60 \lambda_3 d \omega_2 R^3$$
$$- 60 \lambda_2 d \omega_2 R^3 + 30 R^6 d \omega_2 \lambda_2 + 40 \alpha \omega_2 R^7 \lambda_2 + 40 \lambda_2 R^8 d \omega_2 - 40 \lambda_3 R^8 d \omega_2$$
$$+ 60 R^7 d \omega_2 \lambda_2 + 60 \alpha \omega_2 R^6 \lambda_2 - 60 \lambda_3 R^7 d \omega_2 - 30 \alpha \omega_2 R^3 \lambda_2 + 45 \lambda_3 R^4 d \omega_2$$

$$- 40\, \alpha\, \omega_2\, \lambda_2\, R - 40\, R^2\, \omega_2\, \lambda_2\, d + 150\, R^6\, \lambda_3\, d\, \lambda_2 + 200\, \lambda_3\, R^8\, d\, \lambda_2 + 300\, \lambda_3\, R^7\, d\, \lambda_2$$
$$- 200\, \lambda_3\, R^2\, d\, \lambda_2 + 100\, \lambda_3\, R^9\, d\, \lambda_2 - 300\, R^6\, \lambda_2^2\, d + 30\, \alpha\, \omega_2^2\, R^4 - 12\, \alpha\, \omega_2^2\, R^6 - 12\, \alpha\, \omega_2^2\, R^2$$
$$- 20\, \alpha\, \omega_2\, \lambda_2 - 100\, \lambda_3\, R\, d\, \lambda_2 - 200\, \lambda_2^2\, R^8\, d - 300\, R^7\, \lambda_2^2\, d - 200\, R^2\, \lambda_2^2\, d - 300\, \lambda_2^2\, d\, R^3$$
$$- 300\, R^4\, \lambda_2^2\, d - 100\, R\, \lambda_2^2\, d - 100\, R^9\, \lambda_2^2\, d - 300\, \lambda_3\, d\, \lambda_2\, R^3 - 8\, \alpha\, \omega_2^2\, R^7 + 20\, \alpha\, \omega_2\, R^8\, \lambda_2$$
$$+ 20\, R^9\, d\, \omega_2\, \lambda_2 - 20\, \lambda_3\, R^9\, d\, \omega_2 - 30\, R^4\, \lambda_2\, d\, \omega_2 - 40\, \lambda_3\, R^2\, d\, \omega_2 - 4\, \alpha\, \omega_2^2 - 8\, \alpha\, \omega_2^2\, R$$
$$- 4\, \alpha\, \omega_2^2\, R^8 + 9\, \alpha\, \omega_2^2\, R^3 + 9\, \alpha\, \omega_2^2\, R^5 \Big) \Big/ \Big(-300\, R^5\, d\, \lambda_2^2 - 150\, \lambda_3\, R^4\, d\, \lambda_2 + 150\, \lambda_3\, R^5\, d\, \omega_2$$
$$- 60\, \alpha\, \omega_2\, \lambda_2\, R^2 - 20\, R\, \omega_2\, \lambda_2\, d - 20\, \lambda_3\, R\, d\, \omega_2 + 30\, R^5\, \alpha\, \omega_2\, \lambda_2 + 45\, R^6\, \lambda_3\, d\, \omega_2$$
$$- 60\, \lambda_3\, d\, \omega_2\, R^3 - 60\, \lambda_2\, d\, \omega_2\, R^3 + 30\, R^6\, d\, \omega_2\, \lambda_2 + 40\, \alpha\, \omega_2\, R^7\, \lambda_2 + 40\, \lambda_2\, R^8\, d\, \omega_2$$
$$- 40\, \lambda_3\, R^8\, d\, \omega_2 + 60\, R^7\, d\, \omega_2\, \lambda_2 + 60\, \alpha\, \omega_2\, R^6\, \lambda_2 - 60\, \lambda_3\, R^7\, d\, \omega_2 - 30\, \alpha\, \omega_2\, R^3\, \lambda_2$$
$$+ 45\, \lambda_3\, R^4\, d\, \omega_2 - 40\, \alpha\, \omega_2\, \lambda_2\, R - 40\, R^2\, \omega_2\, \lambda_2\, d + 150\, R^6\, \lambda_3\, d\, \lambda_2 + 200\, \lambda_3\, R^8\, d\, \lambda_2$$
$$+ 300\, \lambda_3\, R^7\, d\, \lambda_2 - 200\, \lambda_3\, R^2\, d\, \lambda_2 + 100\, \lambda_3\, R^9\, d\, \lambda_2 - 300\, R^6\, \lambda_2^2\, d - 60\, \alpha\, \omega_2\, R^6 + 30\, \alpha$$
$$\omega_2^2\, R^4 + 225\, R^6\, \lambda_3\, d - 12\, \alpha\, \omega_2^2\, R^6 + 150\, R^6\, \lambda_2\, d + 150\, \alpha\, \omega_2\, R^4 - 12\, \alpha\, \omega_2^2\, R^2 - 20\, \alpha\, \omega_2\, \lambda_2$$
$$+ 200\, \lambda_2\, R^8\, d - 100\, R\, \lambda_2\, d - 100\, \lambda_3\, R\, d\, \lambda_2 - 200\, \lambda_3\, R^8\, d - 300\, \lambda_3\, d\, R^3 - 300\, \lambda_3\, R^7\, d$$
$$- 200\, \lambda_2^2\, R^8\, d - 300\, R^7\, \lambda_2^2\, d - 100\, \lambda_3\, R\, d + 300\, R^7\, \lambda_2\, d - 200\, R^2\, \lambda_2^2\, d - 200\, R^2\, \lambda_2\, d$$
$$+ 225\, \lambda_3\, R^4\, d - 300\, \lambda_2^2\, d\, R^3 - 150\, R^4\, \lambda_2\, d - 300\, R^4\, \lambda_2^2\, d - 300\, \lambda_2\, d\, R^3 - 100\, R\, \lambda_2^2\, d$$
$$+ 750\, \lambda_3\, R^5\, d - 100\, \lambda_3\, R^9\, d + 100\, R^9\, \lambda_2\, d - 100\, R^9\, \lambda_2^2\, d - 200\, \lambda_3\, R^2\, d - 300\, \lambda_3\, d\, \lambda_2\, R^3$$
$$- 8\, \alpha\, \omega_2^2\, R^7 + 20\, \alpha\, \omega_2\, R^8\, \lambda_2 + 20\, R^9\, d\, \omega_2\, \lambda_2 - 20\, \lambda_3\, R^9\, d\, \omega_2 - 30\, R^4\, \lambda_2\, d\, \omega_2$$
$$- 40\, \lambda_3\, R^2\, d\, \omega_2 - 20\, \alpha\, \omega_2 - 4\, \alpha\, \omega_2^2 - 8\, \alpha\, \omega_2^2\, R - 40\, \alpha\, \omega_2\, R - 20\, \alpha\, \omega_2\, R^8 - 4\, \alpha\, \omega_2^2\, R^8$$
$$+ 9\, \alpha\, \omega_2^2\, R^3 - 40\, \alpha\, \omega_2\, R^7 - 60\, \alpha\, \omega_2\, R^2 + 45\, R^5\, \alpha\, \omega_2 + 45\, \alpha\, \omega_2\, R^3 + 9\, \alpha\, \omega_2^2\, R^5 \Big) \Big) :$$

>

> $p_{-3} := \left(\frac{1}{r^3} \cdot A_{-3,\,0} \cdot P_{2,\,0} \right) :$

> $p_e := p_{-3} :$

> $\chi_e := 0 :$

> $\Phi_{-3} := \left(\frac{1}{r^3} \cdot B_{-3,\,0} \cdot P_{2,\,0} \right) :$

> $\Phi_e := \Phi_{-3} :$

> $T5 := Curl\big(\chi_e \cdot VectorField(\langle r, 0, 0 \rangle) \big) :$

> $T6 := Gradient\big(\Phi_e \big) :$

> $fn3 := subs\left(n = -n - 1,\ \frac{(n+3)}{2 \cdot (n+1) \cdot (2 \cdot n + 3)} \right) : fn4 := subs\Big(n = -n - 1,$
$- \frac{n}{(n+1) \cdot (2 \cdot n + 3)} \Big) :$

$$> T7 := subs(n=2, fn3)\cdot r^2\cdot Gradient(p_{-3}) :$$

$$> T8 := subs(n=2, fn4)\cdot VectorField(\langle r, 0, 0\rangle)\cdot (p_{-3}) :$$

$$>$$

$$> V_{inf} := -\frac{1}{2}\cdot r\cdot \sin(\text{theta})\cdot \cos(\text{phi})\cdot e_x - \frac{1}{2}\cdot r\cdot \sin(\text{theta})\cdot \sin(\text{phi})\cdot e_y + r\cdot \cos(\text{theta})\cdot e_z :$$

$$> u_e := V_{inf} + simplify(T5 + T6 + T7 + T8) :$$

$$>$$

$$> u_{ex} := DotProduct(u_e, e_x) :$$

$$> u_{ey} := DotProduct(u_e, e_y) :$$

$$> u_{ez} := DotProduct(u_e, e_z) :$$

$$> n_x := DotProduct(VectorField(\langle 1, 0, 0\rangle), e_x) :$$

$$> n_y := DotProduct(VectorField(\langle 1, 0, 0\rangle), e_y) :$$

$$> n_z := DotProduct(VectorField(\langle 1, 0, 0\rangle), e_z) :$$

$$> E_{xx} := 2\cdot u_{ex}\cdot n_x : E_{yy} := 2\cdot u_{ey}\cdot n_y : E_{zz} := 2\cdot u_{ez}\cdot n_z :$$

$$> E_{xy} := u_{ex}\cdot n_y + u_{ey}\cdot n_x : E_{xz} := u_{ex}\cdot n_z + u_{ez}\cdot n_x : E_{yz} := u_{ey}\cdot n_z + u_{ez}\cdot n_y :$$

$$> E_{xz} := u_{ex}\cdot n_z + u_{ez}\cdot n_x :$$

$$> i_{11} := int\left(int\left(subs\left(r=1, \left(E_{xx}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> i_{12} := int\left(int\left(subs\left(r=1, \left(E_{xy}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> i_{13} := int\left(int\left(subs\left(r=1, \left(E_{xz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> i_{22} := int\left(int\left(subs\left(r=1, \left(E_{yy}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> i_{23} := int\left(int\left(subs\left(r=1, \left(E_{yz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> i_{33} := int\left(int\left(subs\left(r=1, \left(E_{zz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta}=0..\text{Pi}\right), \text{phi}=0..2\cdot\text{Pi}\right) :$$

$$> \beta_{2, 0} := 1 :$$

$$> \Phi_{2, inf} := \frac{1}{2}\cdot r^2\cdot \beta_{2, 0}\cdot P_{2, 0} :$$

$$> \tau_{inf} := \frac{1}{r}\cdot 2\cdot Gradient(\Phi_{2, inf}) :$$

$$> \tau_e := \tau_{inf} + \frac{1}{r}\cdot\left((-8)\cdot Gradient(\Phi_{-3}) + \left(-\frac{3}{2}\right)\cdot(VectorField(\langle r, 0, 0\rangle)\cdot p_{-3}) + \left(\frac{1}{2}\right)\cdot(r^2 \cdot Gradient(p_{-3}))\right) :$$

$$> \tau_{ex} := DotProduct(\tau_e, e_x) : \tau_{ey} := DotProduct(\tau_e, e_y) : \tau_{ez} := DotProduct(\tau_e, e_z) :$$

$$>$$

$$> F_{xx} := \frac{1}{2}\cdot\left(2\cdot \tau_{ex}\cdot n_x - \frac{2}{3}\cdot DotProduct(\tau_e, VectorField(\langle 1, 0, 0\rangle))\right) : F_{yy} := \frac{1}{2}\cdot\left(2\cdot \tau_{ey}\cdot n_y - \frac{2}{3}\cdot DotProduct(\tau_e, VectorField(\langle 1, 0, 0\rangle))\right) : F_{zz} := \frac{1}{2}\cdot\left(2\cdot \tau_{ez}\cdot n_z - \frac{2}{3}\right.$$

$\cdot DotProduct\left(\tau_e, VectorField(\langle 1, 0, 0\rangle)\right)\Big)$:

> $F_{xy} := \frac{1}{2}\cdot\left(\tau_{ex}\cdot n_y + \tau_{ey}\cdot n_x\right) : F_{xz} := \frac{1}{2}\cdot\left(\tau_{ex}\cdot n_z + \tau_{ez}\cdot n_x\right) : F_{yz} := \frac{1}{2}\cdot\left(\tau_{ey}\cdot n_z + \tau_{ez}\cdot n_y\right)$:

> $j_{11} := int\left(int\left(subs\left(r=1, \left(F_{xx}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $j_{12} := int\left(int\left(subs\left(r=1, \left(F_{xy}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $j_{13} := int\left(int\left(subs\left(r=1, \left(F_{xz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $j_{23} := int\left(int\left(subs\left(r=1, \left(F_{yz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $j_{22} := int\left(int\left(subs\left(r=1, \left(F_{yy}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $j_{33} := int\left(int\left(subs\left(r=1, \left(F_{zz}\cdot r^2\cdot \sin(\text{theta})\right)\right), \text{theta} = 0..\text{Pi}\right), \text{phi} = 0..2\cdot\text{Pi}\right)$:

> $\lambda_{eff} := 1 + simplify\left(\frac{3\cdot v}{4\cdot\text{Pi}}\cdot\frac{(j_{33} - i_{33})}{2}\right)$:

> *# where v is the volume fraction of the compound droplet*

>