Derivations of A_g, the air gap area between the drop and bath.

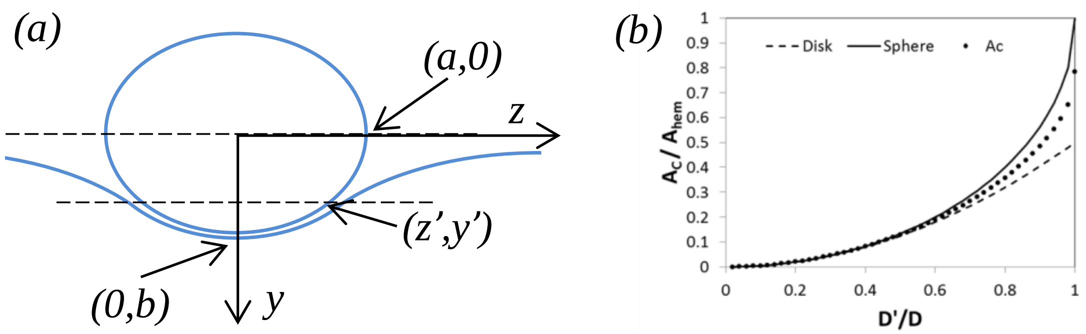

Figure 1: (a) Geometry and variables defined. (b) The calculated area of a rigid oblate ellipsoid as it is increasingly submerged grows less than a sphere and more than a disk, as expected.

To begin, we choose a coordinate system shown in figure 1a. With this system the surface area for a surface of revolution is,

$$A = 2\pi \int_{y'}^{b} z(y)\sqrt{1 + (z_y(y))^2}dy. \tag{1}$$

The ellipsoidal geometry defines $z(y)$ as

$$z(y) = a\sqrt{1 - \frac{y^2}{b^2}},\ \epsilon^2 = 1 - \frac{b^2}{a^2}, \tag{2}$$

with ellipticity ϵ and a, b and y' defined in figure 1a. Utilizing the relationships in equation 2 we get

$$A = \frac{2\pi\epsilon}{1-\epsilon^2}\int_{z'}^{b}\sqrt{g^2 + z^2}dz,\ g = \frac{a(1-\epsilon^2)}{\epsilon}. \tag{3}$$

Completing the definite integral we arrive at,

$$\begin{aligned} A_g &= \pi a^2 - \frac{\pi}{a^2}\sqrt{(a^2 - z^2)(a^2 - \epsilon^2 z^2)} \\ &+ \pi a^2 \left(\frac{1-\epsilon^2}{\epsilon}\right) \ln\left|\frac{a(1+\epsilon)}{\epsilon\sqrt{a^2 - z^2} + \sqrt{a^2 - \epsilon^2 z^2}}\right|, \end{aligned} \tag{4}$$

which, for $z = a$, reduces to the familiar form for the surface area of a hemi-ellipsoid.

$$A_g = \pi a^2 \left[1 + \left(\frac{1-\epsilon^2}{\epsilon}\right)\tanh^{-1}\epsilon\right]. \tag{5}$$

Equation 4 is the air gap area for an elliptical cap. The plane at $z = z'$ is the plane intersecting the periphery of the air gap.

Derivation of $\beta \propto D^{-1}$

The gravitational energy for a spherical droplet is,

$$E_{g0} = \frac{4}{3}\pi\rho g(r^3)r, \tag{6}$$

where the vertical coordinate is measured from the lowest point of the droplet. When the droplet flattens we have,

$$E_g = \frac{4}{3}\pi\rho g(a^2 b)b. \tag{7}$$

Since the volume remains constant we have $r^3 = a^2 b$. In terms of the semi-major (a) and semi-minor axes (b), the reduction in gravitational energy is,

$$\Delta E_g = \frac{4}{3}\pi\rho g a^2 b\left[a^{2/3}b^{1/3} - b\right]. \tag{8}$$

The surface energy of the sphere is,

$$E_\sigma = 4\pi\sigma r^2. \tag{9}$$

For the flattened droplet this increases to,

$$E_\sigma = 2\pi\sigma a^2\left[1 + \left(\frac{1-\epsilon^2}{\epsilon}\right)\tanh^{-1}\epsilon\right], \tag{10}$$

where we have used twice the area from equation 5. The increase in surface energy, in terms of a and b is then,

$$\Delta E_\sigma = 2\pi\sigma a^2\left\{\left[1 + \left(\frac{1-\epsilon^2}{\epsilon}\right)\tanh^{-1}\epsilon\right] - 2b^{2/3}a^{-2/3}\right\}. \tag{11}$$

Additional flattening of the drop would incur a larger increase in surface energy then the energy lost from further lowering the center-of-mass of the droplet. Therefore, a and b define the shape of the stable oblate spheroid when $\Delta E_g = \Delta E_\sigma$. After some simplification, this equality can be written in a somewhat cleaner form as $F(a,b) = G(\epsilon)$, where

$$F(a,b) = \left(\frac{2\rho g}{2\sigma}\right)b^{4/3}\left[a^{2/3} - b^{2/3}\right] + 2a^{-2/3}b^{2/3}, \tag{12}$$

and

$$G(\epsilon) = 1 + \left(\frac{1-\epsilon^2}{\epsilon}\right)\tanh^{-1}\epsilon. \tag{13}$$

Numerically solving for values of a and b for which $F(a,b) = G(\epsilon)$ allows us to calculate $\beta = (2/5)(2 + b/D)$ as a function of droplet diameter, D. Doing so for the range of droplet diameters explored gives an inverse relationship as seen in figure .

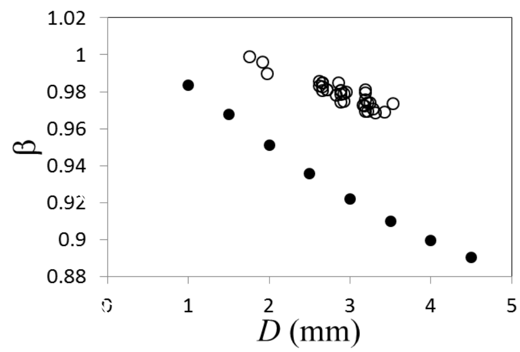

Figure 2: Numerical (closed circles and measured (open circles) results for $\beta(D)$.