Supplemental Materials for:

"Microstructure and Rheology Relationships for Shear Thickening Colloidal Dispersions"

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Summary of integrals dependencies (superscript indicates neutron beam direction):

Radial geometry: (velocity – vorticity)

$$W_0^{gradient}(q)\left(B_{0,0}^{+}(q),-B_{2,0}^{+}(q),B_{4,0}^{+}(q)\right)$$
$$W_2^{gradient}(q)\left(B_{2,2}^{+}(q),B_{4,2}^{+}(q)\right)$$
$$W_4^{gradient}(q)\left(B_{4,4}^{+}(q)\right)$$
$$W_{12}^{gradient}(q)=0$$

1-2 flow plane geometry: (velocity – velocity gradient)

$$W_0^{vorticity}(q)\left(B_{0,0}^{+}(q),B_{2,0}^{+}(q),B_{2,2}^{+}(q)B_{4,0}^{+}(q),B_{4,2}^{+}(q),B_{4,4}^{+}(q)\right)$$
$$W_2^{vorticity}(q)\left(B_{2,0}^{+}(q),-B_{2,2}^{+}(q),B_{4,0}^{+}(q),B_{4,2}^{+}(q),-B_{4,4}^{+}(q)\right)$$
$$W_4^{vorticity}(q)\left(B_{4,0}^{+}(q),-B_{4,2}^{+}(q),B_{4,4}^{+}(q)\right)$$
$$W_{12}^{vorticity}(q)\left(B_{2,1}^{+}(q),B_{4,1}^{+}(q)\right)$$

Tangential geometry: (velocity gradient – vorticity)

$$W_0^{velocity}(q)\left(B_{0,0}^{+}(q),B_{2,0}^{+}(q),B_{2,2}^{+}(q)B_{4,0}^{+}(q),-B_{4,2}^{+}(q),-B_{4,4}^{+}(q)\right)$$
$$W_2^{velocity}(q)\left(B_{2,0}^{+}(q),-B_{2,2}^{+}(q),-B_{4,0}^{+}(q),-B_{4,2}^{+}(q),B_{4,4}^{+}(q)\right)$$
$$W_4^{velocity}(q)\left(-B_{4,0}^{+}(q),B_{4,2}^{+}(q),-B_{4,4}^{+}(q)\right)$$
$$W_{12}^{velocity}(q)=0$$

From Theory the hydrodynamic and thermodynamic shear stresses are: (from Ackerson and Wagner eqns 52 and 62)

$$\tau^{\text{thermodynamic}}(\dot{\gamma})=-\frac{\rho}{\pi\sqrt{30\pi}}\int\theta^{*}(q)B_{2,1}^{+}(q;Pe)q^{2}\,dq$$

$$\tau^{\text{hydrodynamic}}(\dot{\gamma})\Big/2\dot{\gamma}\mu=1+\frac{5}{2}\phi(1+\phi)+2.7\phi^{2}+\frac{5}{2}\phi\sqrt{\pi}\int\left(\begin{array}{l}B_{0,0}^{+}(q;Pe)\left(2\alpha(q)+\frac{4}{15}\zeta_0(q)\right)\\+B_{2,0}^{+}(q;Pe)\left(\frac{2}{3\sqrt{5}}\beta(q)-\frac{4\sqrt{2}}{21\sqrt{7}}\zeta_2(q)\right)\\+B_{2,2}^{+}(q;Pe)\left(\frac{\sqrt{2}}{\sqrt{15}}\beta(q)+\frac{2\sqrt{2}}{105}\zeta_2(q)\right)\\+B_{4,0}^{+}(q;Pe)\left(\frac{-16}{105}\zeta_4(q)\right)\\+B_{4,2}^{+}(q;Pe)\left(\frac{4\sqrt{2}}{21\sqrt{5}}\frac{-16}{105}\zeta_4(q)\right)\end{array}\right)q^{2}dq$$

And for the thermodynamic and hydrodynamic first and second normal stresses are:

$$N_1^{\text{thermodynamic}}(\dot{\gamma}) = \frac{\rho}{\pi\sqrt{30\pi}} \int \theta^*(q) B_{2,2}^+(q;Pe) - \sqrt{6} B_{2,0}^+(q;Pe) q^2 \, dq$$

$$N_2^{\text{thermodynamic}}(\dot{\gamma}) = \frac{\rho}{\pi\sqrt{30\pi}} \int \theta^*(q) B_{2,2}^+(q;Pe) + \sqrt{6} B_{2,0}^+(q;Pe) q^2 \, dq$$

$$N_1^{\text{hydrodynamic}}(\dot{\gamma}) \Big/ 2\dot{\gamma}\mu = -5\phi\sqrt{\pi} \int \begin{pmatrix} B_{4,1}^+(q;Pe)\left(\dfrac{5\sqrt{6}+24\sqrt{5}}{105\sqrt{30}} \zeta_4(q) \right) \\ +B_{4,3}^+(q;Pe)\left(\dfrac{1}{3\sqrt{35}} \zeta_4(q) \right) \end{pmatrix} q^2 dq$$

$$N_2^{\text{hydrodynamic}}(\dot{\gamma}) \Big/ 2\dot{\gamma}\mu = -5\phi\sqrt{\pi} \int \begin{pmatrix} B_{2,1}^+(q;Pe)\left(\dfrac{2\sqrt{2}}{\sqrt{15}} \beta(q) + \dfrac{2\sqrt{2}}{7\sqrt{15}} \zeta_2(q) \right) \\ +B_{4,1}^+(q;Pe)\left(\dfrac{5\sqrt{6}-24\sqrt{5}}{105\sqrt{30}} \zeta_4(q) \right) \\ +B_{4,3}^+(q;Pe)\left(\dfrac{1}{3\sqrt{35}} \zeta_4(q) \right) \end{pmatrix} q^2 dq$$

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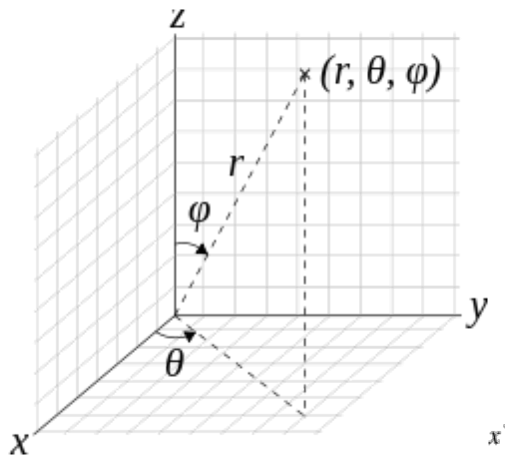
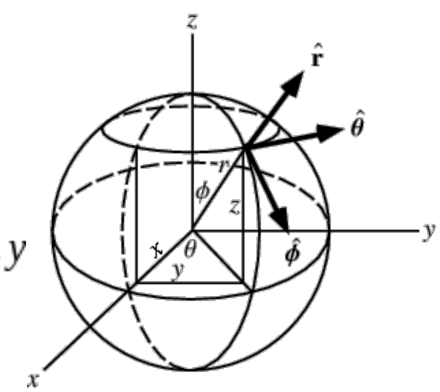
For SANS decomposition in 3 planes of shear:

Starting with the spherical harmonic expansion in terms of the spherical coordinate system shown here:





$$\hat{h}(q)=\sum_{l,m} B_{l,m}^{+}(q)\left(Y_{l,-m}(\Omega_k)+(-1)^m Y_{l,m}(\Omega_k)\right),\ \ 1\geq m\geq 0$$

$$\hat{h}(q)=\sum_{l,m} B_{l,m}^{+}(q)\left(Y_{l,-m}(\Omega_k)+(-1)^m Y_{l,m}(\Omega_k)\right)$$

$$[l=0,m=0]=B_{0,0}^{+}(q)\left(Y_{0,-0}(\Omega_k)+(-1)^0 Y_{0,0}(\Omega_k)\right)$$

$$[l=0,m=0]=B_{0,0}^{+}(q)\left(\frac{1}{2\sqrt{\pi}}+\frac{1}{2\sqrt{\pi}}\right)$$

$$[l=0,m=0]=B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(Y_{2,-0}(\Omega_k)+(-1)^0\,Y_{2,0}(\Omega_k)\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\cos^2\theta-1\right)\right)$$

$$[l=2,m=1]=B_{2,1}^{+}(q)\left(Y_{2,-1}(\Omega_k)+(-1)^1\,Y_{2,1}(\Omega_k)\right)$$

$$[l=2,m=1]=B_{2,1}^{+}(q)\left(\frac{1}{2}e^{-i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta)-\frac{-1}{2}e^{i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(Y_{2,-2}(\Omega_k)+(-1)^2\,Y_{2,2}(\Omega_k)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(\frac{1}{4}e^{-2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)+\frac{1}{4}e^{2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(Y_{4,-0}(\Omega_k)+(-1)^0\,Y_{4,0}(\Omega_k)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3\left(3-30\cos^2(\theta)+35\cos^4(\theta)\right)}{8\sqrt{\pi}}\right)$$

$$[l=4,m=1]=B_{4,1}^{+}(q)\left(Y_{4,-1}(\Omega_k)+(-1)^1 Y_{4,1}(\Omega_k)\right)$$

$$[l=4,m=1]=B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}e^{-i\phi}\cos(\theta)\sin(\theta)\left(-3+7\cos^2(\theta)\right)-\frac{-3\sqrt{5}}{8\sqrt{\pi}}e^{i\phi}\cos(\theta)\sin(\theta)\left(-3+7\cos^2(\theta)\right)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(Y_{4,-2}(\Omega_k)+(-1)^2 Y_{4,2}(\Omega_k)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3}{8}e^{-2i\phi}\sqrt{\frac{5}{2\pi}}\sin^2(\theta)\left(-1+7\cos^2(\theta)\right)+\frac{3}{8}e^{2i\phi}\sqrt{\frac{5}{2\pi}}\sin^2(\theta)\left(-1+7\cos^2(\theta)\right)\right)$$

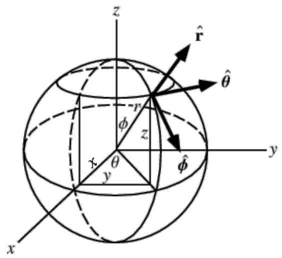
$$[l=4,m=3]=B_{4,3}^{+}(q)\left(Y_{4,-3}(\Omega_k)+(-1)^3 Y_{4,3}(\Omega_k)\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{8}e^{-3i\phi}\sqrt{\frac{35}{\pi}}\cos(\theta)\sin^3(\theta)-\frac{-3}{8}e^{3i\phi}\sqrt{\frac{35}{\pi}}\cos(\theta)\sin^3(\theta)\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(Y_{4,-4}(\Omega_k)+(-1)^4 Y_{4,4}(\Omega_k)\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{16\sqrt{2\pi}}e^{-4i\phi}\sin^4(\theta)+\frac{3\sqrt{35}}{16\sqrt{2\pi}}e^{4i\phi}\sin^4(\theta)\right)$$

To change from spherical harmonics to Cartesian coordinate frame as defined below we use the following definitions:



$$e^{i\phi} = \frac{x+iy}{\sqrt{x^2+y^2}} \text{ and } e^{-i\phi} = \frac{x-iy}{\sqrt{x^2+y^2}}$$

$$e^{2i\phi} = \frac{(x+iy)^2}{x^2+y^2} = \frac{x^2+2ixy-y^2}{x^2+y^2} \text{ and } e^{-2i\phi} = \frac{(x+iy)^2}{x^2+y^2} = \frac{x^2-2ixy-y^2}{x^2+y^2}$$

$$e^{3i\phi} = \frac{(x+iy)^3}{(x^2+y^2)^{3/2}} = \frac{x^3+3ix^2y-3xy^2-iy^3}{(x^2+y^2)^{3/2}} \text{ and } e^{-3i\phi} = \frac{(x-iy)^3}{(x^2+y^2)^{3/2}} = \frac{x^3-3ix^2y-3xy^2+iy^3}{(x^2+y^2)^{3/2}}$$

$$e^{4i\phi} = \frac{(x+iy)^4}{(x^2+y^2)^2} = \frac{x^4+4ix^3y-6x^2y^2-4xy^3+y^4}{(x^2+y^2)^2} \text{ and } e^{-4i\phi} = \frac{(x-iy)^4}{(x^2+y^2)^2} = \frac{x^4-4ix^3y-6x^2y^2+4xy^3+y^4}{(x^2+y^2)^2}$$

$$\sin(\theta) = \sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}$$

$$\cos(\theta) = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

In Cartesian coordinate system (x, y, z)

$$[l=0, m=0] = B^{+}_{0,0}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\cos^2\theta - 1\right)\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2 - 1\right)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(Y_{2,-1}(\Omega_k) + (-1)^1 Y_{2,1}(\Omega_k)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(\frac{1}{2}e^{-i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta) - \frac{-1}{2}e^{i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(\frac{1}{2}e^{-i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta) + \frac{1}{2}e^{i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(\cos(\phi)\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{x}{\sqrt{x^2+y^2}}\right)\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)\left(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\right)\right)$$

$$\left[l=2, m=1\right] = B^{+}_{2,1}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{xz}{\left(x^2+y^2+z^2\right)}\right)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(Y_{2,-2}(\Omega_k)+(-1)^2 Y_{2,2}(\Omega_k)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(\frac{1}{4}e^{-2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)+\frac{1}{4}e^{2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(\sqrt{\frac{15}{8\pi}}e^{2i\phi}\sin^2(\theta)\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(\sqrt{\frac{15}{8\pi}}\left(\frac{x^2-y^2}{x^2+y^2}\right)\left(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\right)^2\right)$$

$$[l=2,m=2]=B_{2,2}^{+}(q)\left(\sqrt{\frac{15}{8\pi}}\left(\frac{x^2-y^2}{x^2+y^2+z^2}\right)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(Y_{4,-0}(\Omega_k)+(-1)^0 Y_{4,0}(\Omega_k)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3\left(3-30\cos^2(\theta)+35\cos^4(\theta)\right)}{8\sqrt{\pi}}\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\frac{3}{8\sqrt{\pi}}\left(3-30\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2+35\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^4\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(Y_{4,-1}\left(\Omega_k\right)+\left(-1\right)^1 Y_{4,1}\left(\Omega_k\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}e^{-i\phi}\cos\left(\theta\right)\sin\left(\theta\right)\left(-3+7\cos^2\left(\theta\right)\right)-\frac{-3\sqrt{5}}{8\sqrt{\pi}}e^{i\phi}\cos\left(\theta\right)\sin\left(\theta\right)\left(-3+7\cos^2\left(\theta\right)\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}e^{-i\phi}\cos\left(\theta\right)\sin\left(\theta\right)\left(-3+7\cos^2\left(\theta\right)\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\frac{x}{\sqrt{x^2+y^2}}\frac{z}{\sqrt{x^2+y^2+z^2}}\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\left(-3+7\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\frac{xz}{x^2+y^2+z^2}\left(-3+7\left(\frac{z^2}{x^2+y^2+z^2}\right)\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\frac{xz}{x^2+y^2+z^2}\left(7\left(\frac{z^2}{x^2+y^2+z^2}\right)-3\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(Y_{4,-2}\left(\Omega_k\right)+\left(-1\right)^2 Y_{4,2}\left(\Omega_k\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(\frac{3}{8}e^{-2i\phi}\sqrt{\frac{5}{2\pi}}\sin^2\left(\theta\right)\left(-1+7\cos^2\left(\theta\right)\right)+\frac{3}{8}e^{2i\phi}\sqrt{\frac{5}{2\pi}}\sin^2\left(\theta\right)\left(-1+7\cos^2\left(\theta\right)\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{2\pi}}\left(\frac{x^2-y^2}{x^2+y^2}\right)\left(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\right)^2\left(-1+7\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{2\pi}}\left(\frac{x^2-y^2}{x^2+y^2+z^2}\right)\left(-1+7\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2\right)\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(Y_{4,-3}(\Omega_k)+(-1)^3 Y_{4,3}(\Omega_k)\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{8}e^{-3i\phi}\sqrt{\frac{35}{\pi}}\cos(\theta)\sin^3(\theta)-\frac{-3}{8}e^{-3i\phi}\sqrt{\frac{35}{\pi}}\cos(\theta)\sin^3(\theta)\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(e^{-3i\phi}\right)\left(\cos(\theta)\right)\left(\sin^3(\theta)\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{x\left(x^2-3y^2\right)}{\left(x^2+y^2\right)^{3/2}}\right)\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)\left(\frac{x^2+y^2}{x^2+y^2+z^2}\right)^{3/2}$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{zx\left(x^2-3y^2\right)}{\left(x^2+y^2+z^2\right)^2}\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(Y_{4,-4}(\Omega_k)+(-1)^4 Y_{4,4}(\Omega_k)\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{16\sqrt{2\pi}}e^{-4i\phi}\sin^4(\theta)+\frac{3\sqrt{35}}{16\sqrt{2\pi}}e^{-4i\phi}\sin^4(\theta)\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(e^{-4i\phi}\right)\left(\sin^4(\theta)\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{x^4-6x^2y^2+y^4}{\left(x^2+y^2\right)^2}\right)\left(\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}\right)^4$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{x^4-6x^2y^2+y^4}{\left(x^2+y^2+z^2\right)^2}\right)$$

Change in notation from x, y and z to q's where:

$x = q_x$

$y = q_y$

$z = q_z$

and $q = \sqrt{x^2 + y^2 + z^2}$ and $q^2 = x^2 + y^2 + z^2$

$$[l = 0, m = 0] = B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l = 2, m = 0] = B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)^2 - 1\right)\right)$$

$$[l = 2, m = 0] = B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\frac{q_z^2}{q^2} - 1\right)\right)$$

$$[l = 2, m = 1] = B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{xz}{\left(x^2 + y^2 + z^2\right)}\right)\right)$$

$$[l = 2, m = 1] = B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{q_x q_z}{q^2}\right)\right)$$

$$[l = 2, m = 2] = B_{2,2}^{+}(q)\left(\sqrt{\frac{15}{8\pi}}\left(\frac{x^2 - y^2}{x^2 + y^2 + z^2}\right)\right)$$

$$[l = 2, m = 2] = B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\left(\frac{q_x^2 - q_y^2}{q^2}\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\frac{3}{8\sqrt{\pi}}\left(3-30\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2+35\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^4\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\frac{3}{8\sqrt{\pi}}\left(3-30\left(\frac{q_z^2}{q^2}\right)+35\left(\frac{q_z^4}{q^4}\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\frac{xz}{x^2+y^2+z^2}\left(7\left(\frac{z^2}{x^2+y^2+z^2}\right)-3\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\frac{3\sqrt{5}}{4\sqrt{\pi}}\left(\frac{q_x q_z}{q^2}\right)\left(7\left(\frac{q_z^2}{q^2}\right)-3\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{2\pi}}\left(\frac{x^2-y^2}{x^2+y^2+z^2}\right)\left(-1+7\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{2\pi}}\left(\frac{q_x^2-q_y^2}{q^2}\right)\left(-1+7\left(\frac{q_z^2}{q^2}\right)\right)\right)$$

$$\left[l=4,m=3\right]=B_{4,3}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{zx\left(x^2-3y^2\right)}{\left(x^2+y^2+z^2\right)^2}\right)$$

$$\left[l=4,m=3\right]=B_{4,3}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{q_x q_z\left(q_x^2-3q_y^2\right)}{q^4}\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{x^4-6xy+y^4}{\left(x^2+y^2+z^2\right)^2}\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{\left(q_x^4-6q_xq_y-q_y^4\right)}{q^4}\right)$$

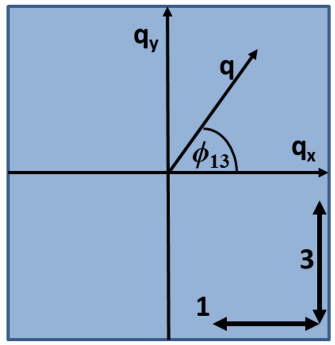
Now define each set of spherical harmonics for each experiment (dependent upon geometry)

Radial (velocity-vorticity) where the neutrons propagate along the gradient direction such that

$$\frac{q_x}{q} = \cos(\phi_{13})$$

$$\frac{q_y}{q} = \sin(\phi_{13})$$

$$q_z = 0$$



$$[l=0, m=0] = B^{+}_{0,0}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\frac{q_z^2}{q^2}-1\right)\right)$$

$$[l=2, m=0] = -B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\right)$$

$$[l=2, m=1] = B^{+}_{2,1}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{q_x q_z}{q^2}\right)\right)$$

$$[l=2, m=1] = 0$$

$$[l=2, m=2] = B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\left(\frac{q_x^2 - q_y^2}{q^2}\right)$$

$$[l=2, m=2] = B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\left(\cos^2(\phi_{13}) - \sin^2(\phi_{13})\right)$$

$$[l=2, m=2] = B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\cos(2\phi_{13})$$

$$[l=4, m=0] = B_{4,0}^{+}(q)\frac{3}{8\sqrt{\pi}}\left(3 - 30\left(\frac{q_z^2}{q^2}\right) + 35\left(\frac{q_z^4}{q^4}\right)\right)$$

$$[l=4, m=0] = B_{4,0}^{+}(q)\left(\frac{9}{8\sqrt{\pi}}\right)$$

$$[l=4, m=1] = B_{4,1}^{+}(q)\frac{3\sqrt{5}}{4\sqrt{\pi}}\left(\frac{q_x q_z}{q^2}\right)\left(7\left(\frac{q_z^2}{q^2}\right) - 3\right)$$

$$[l=4, m=1] = 0$$

$$[l=4, m=2] = B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{q_x^2 - q_y^2}{q^2}\right)\left(-1 + 7\left(\frac{q_z^2}{q^2}\right)\right)$$

$$[l=4, m=2] = B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{q_x^2 - q_y^2}{q^2}\right)$$

$$[l=4, m=2] = B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\cos^2(\phi_{13}) - \sin^2(\phi_{13})\right)$$

$$[l=4, m=2] = B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos(2\phi_{13})$$

$$\left[l=4,m=3\right]=B_{4,3}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{q_x q_z\left(q_x^2-3q_y^2\right)}{q^4}\right)$$

$$\left[l=4,m=3\right]=0$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{q_x^4-6q_x^2q_y^2+q_y^4}{q^4}\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\cos^4\left(\phi_{13}\right)-6\cos^2\left(\phi_{13}\right)\sin^2\left(\phi_{13}\right)+\sin^4\left(\phi_{13}\right)\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{3}{8}+\frac{1}{2}\cos\left(2\phi_{13}\right)+\frac{1}{8}\cos\left(4\phi_{13}\right)-6\frac{1}{8}\left(1-\cos\left(4\phi_{13}\right)\right)+\frac{3}{8}-\frac{1}{2}\cos\left(2\theta\right)+\frac{1}{8}\cos\left(4\phi_{13}\right)\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos\left(4\phi_{13}\right)$$

The structure factor for the neutrons propagate along the gradient direction

$$S^{gradient}\left(q\right)=1+B_{0,0}^{+}\left(q\right)\left(\frac{1}{\sqrt{\pi}}\right)-B_{2,0}^{+}\left(q\right)\left(\sqrt{\frac{5}{4\pi}}\right)+$$

$$B_{2,2}^{+}\left(q\right)\sqrt{\frac{15}{8\pi}}\cos\left(2\phi_{13}\right)+B_{4,0}^{+}\left(q\right)\left(\frac{9}{8\sqrt{\pi}}\right)+B_{4,2}^{+}\left(q\right)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos\left(2\phi_{13}\right)+B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos\left(4\phi_{13}\right)$$

Therefore we define the W_n's for the neutrons in the gradient direction as:

$$W_n^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}Cos(n\phi_{13})S(q)\,d\phi_{13}$$

$$W_0^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(0\phi_{13})S(q)\,d\phi_{13}$$

$$W_0^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{matrix}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)-B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)+B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\cos(2\phi_{13})+B_{4,0}^{+}(q)\left(\frac{9}{8\sqrt{\pi}}\right)\\+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos(2\phi_{13})+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos(4\phi_{13})+\ldots(l>4)\end{matrix}\cos(0\phi_{13})\,d\phi_{13}$$

$$W_0^{gradient}(q)S^{eq}(q)=1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)-B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)+B_{4,0}^{+}(q)\left(\frac{9}{8\sqrt{\pi}}\right)+\ldots(l>4)$$

$$W_2^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(2\phi_{13})S(q)\,d\phi_{13}$$

$$W_2^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{matrix}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)-B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)+B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\cos(2\phi_{13})+B_{4,0}^{+}(q)\left(\frac{9}{8\sqrt{\pi}}\right)\\+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos(2\phi_{13})+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos(4\phi_{13})+\ldots(l>4)\end{matrix}\cos(2\phi_{13})\,d\phi_{13}$$

$$W_2^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left(B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\cos(2\phi_{13})+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos(2\phi_{13})+\ldots(l>4)\right)\cos(2\phi_{13})\,d\phi_{13}$$

$$W_2^{gradient}(q)S^{eq}(q)=B_{2,2}^{+}(q)\left(\sqrt{\frac{15}{8\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)+\ldots(l>4)$$

$$W_4^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(4\phi_{13})S(q)\,d\phi_{13}$$

$$W_4^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{matrix}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)-B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)+B_{2,2}^{+}(q)\sqrt{\frac{15}{8\pi}}\cos(2\phi_{13})+B_{4,0}^{+}(q)\left(\frac{9}{8\sqrt{\pi}}\right)\\+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\cos(2\phi_{13})+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos(4\phi_{13})+\ldots(l>4)\end{matrix}\cos(4\phi_{13})\,d\phi_{13}$$

$$W_4^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left(B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)\cos(4\phi_{13})+\ldots(l>4)\right)\cos(4\phi_{13})\,d\phi_{13}$$

$$W_4^{gradient}(q)S^{eq}(q)=4\left(B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)+\ldots(l>4)\right)$$

$$W_6^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(6\phi_{13})S(q)d\phi_{13}=0$$

$$W_{12}^{gradient}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(\phi_{13})\sin(\phi_{13})S(q)d\phi_{13}=0$$

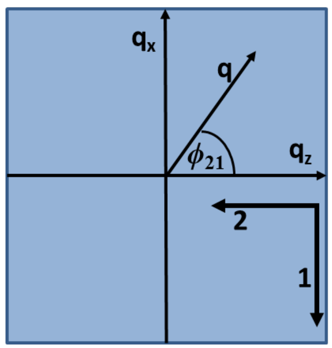
1-2 plane (velocity- velocity gradient)

For the 2D detector when the neutrons propagate along the vorticity direction:

$$\frac{q_x}{q}=-\sin(\phi_{21})$$

$$\frac{q_y}{q}=0$$

$$q_z=-\cos(\phi_{21})$$



$$[l=0,m=0]=B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\frac{q_z^2}{q^2}-1\right)\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)\left(3\cos^2(\phi_{21})-1\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)\left(3\left(\frac{1}{2}\left(1+\cos(2\phi_{21})\right)\right)-1\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\sqrt{\frac{5}{4\pi}}\right)\left(\frac{3}{2}\left(1+\cos(2\phi_{21})\right)-\frac{2}{2}\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\frac{1}{2}\sqrt{\frac{5}{4\pi}}\right)\left(\frac{1}{2}+\frac{3}{2}\cos(2\phi_{21})\right)$$

$$[l=2,m=0]=B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)\left(1+3\cos(2\phi_{21})\right)$$

$$[l=2,m=1]=B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{q_x q_z}{q^2}\right)\right)$$

$$[l=2,m=1]=B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\sqrt{\frac{15}{8\pi}}\left(\frac{q_x^2-q_y^2}{q^2}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\sqrt{\frac{15}{8\pi}}\left(\frac{q_x^2}{q^2}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\sqrt{\frac{15}{8\pi}}\right)\sin^2\left(\phi_{21}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\frac{1}{2}\sqrt{\frac{15}{2\pi}}\right)\left(\frac{1}{2}\left(1-\cos\left(2\phi_{21}\right)\right)\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos\left(2\phi_{21}\right)\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{8\sqrt{\pi}}\right)\left(3-30\left(\frac{q_z^2}{q^2}\right)+35\left(\frac{q_z^4}{q^4}\right)\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{8\sqrt{\pi}}\right)\left(3-30\cos^2\left(\phi_{21}\right)+35\cos^4\left(\phi_{21}\right)\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{8\sqrt{\pi}}\right)\left(3-15\left(1+\cos\left(2\phi_{21}\right)\right)+\frac{35}{8}\left(3+4\cos\left(2\phi_{21}\right)+\cos\left(4\phi_{21}\right)\right)\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{8\sqrt{\pi}}\right)\left(-\frac{96}{8}-\frac{120}{8}\cos\left(2\phi_{21}\right)+\frac{105}{8}+\frac{140}{8}\cos\left(2\phi_{21}\right)+\frac{35}{8}\cos\left(4\phi_{21}\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\right)\left(\frac{q_x}{q}\right)\left(\frac{q_z}{q}\right)\left(7\left(\frac{q_z^2}{q^2}\right)-3\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{8\sqrt{\pi}}\right)\left(\frac{9}{8}+\frac{20}{8}\cos\left(2\phi_{21}\right)+\frac{35}{8}\cos\left(4\phi_{21}\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\right)\sin\left(\phi_{21}\right)\cos\left(\phi_{21}\right)\left(7\cos^2\left(\phi_{21}\right)-3\right)$$

$$\left[l=4,m=0\right]=B_{4,0}^{+}\left(q\right)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9+20\cos\left(2\phi_{21}\right)+35\cos\left(4\phi_{21}\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{4\sqrt{\pi}}\right)\sin\left(\phi_{21}\right)\cos\left(\phi_{21}\right)\left(\frac{7}{2}\left(1+\cos\left(2\phi_{21}\right)\right)-3\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\sin\left(\phi_{21}\right)\cos\left(\phi_{21}\right)\left(1+7\cos\left(2\phi_{21}\right)\right)$$

$$\left[l=4,m=1\right]=B_{4,1}^{+}\left(q\right)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin\left(\phi_{21}\right)\cos\left(\phi_{21}\right)+\frac{7}{4}\sin\left(4\phi_{21}\right)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{q_x^2-q_y^2}{q^2}\right)\left(-1+7\left(\frac{q_z^2}{q^2}\right)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{q_x^2}{q^2}\right)\left(-1+7\left(\frac{q_z^2}{q^2}\right)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\sin^2(\phi_{21})\left(-1+7\cos^2(\phi_{21})\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{1}{2}\left(1-\cos(2\phi_{21})\right)\right)\left(-1+7\frac{1}{2}\left(1+\cos(2\phi_{21})\right)\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(1-\cos(2\phi_{21})\right)\left(5+7\cos(2\phi_{21})\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(5+2\cos(2\phi_{21})-\frac{7}{2}-\frac{7}{2}\cos(4\phi_{21})\right)$$

$$[l=4,m=2]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{q_x^3 q_z}{q^4}\right)$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\sin^3(\phi_{21})\cos(\phi_{21})$$

$$[l=4,m=3]=B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)\left(2\sin(2\phi_{21})+\sin(4\phi_{21})\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{\left(q_x^2-2q_xq_y-q_y^2\right)\left(q_x^2+2q_xq_y-q_y^2\right)}{q^4}\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{q_x^4}{q^4}\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\sin^4(\phi_{21})$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{3}{8}-\frac{1}{2}\cos(2\phi_{21})+\frac{1}{8}\cos(4\phi_{21})\right)$$

$$[l=4,m=4]=B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{21})+\cos(4\phi_{21})\right)$$

Structure factor for the 2D detector when the neutrons propagate along the vorticity direction:

$$\begin{aligned}S^{vorticity}(q)=1&+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)\left(1+3\cos(2\phi_{21})\right)+B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{21})\right)\\&+B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9+20\cos(2\phi_{21})+35\cos(4\phi_{21})\right)+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin(\phi_{21})\cos(\phi_{21})+\frac{7}{4}\sin(4\phi_{21})\right)\\&+B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)+B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)\left(2\sin(2\phi_{21})+\sin(4\phi_{21})\right)\\&+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{21})+\cos(4\phi_{21})\right)+\ldots(l>4)\end{aligned}$$

W_n's for the 2D detector when the neutrons propagate along the vorticity direction:

$$W_n^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(n\phi_{21})S(q)d\phi_{21}$$

$$W_0^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left[\begin{array}{l}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)\left(1+3\cos(2\phi_{21})\right)+B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{21})\right)\\ +B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9+20\cos(2\phi_{21})+35\cos(4\phi_{21})\right)+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin(\phi_{21})\cos(\phi_{21})+\frac{7}{4}\sin(4\phi_{21})\right)\\ +B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)+B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)\left(2\sin(2\phi_{21})+\sin(4\phi_{21})\right)\\ +B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{21})+\cos(4\phi_{21})\right)+\ldots(l>4)\end{array}\right]\cos(0\phi_{21})d\phi_{21}$$

$$W_0^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left[\begin{array}{l}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\\ +B_{4,0}^{+}(q)\frac{27}{64\sqrt{\pi}}+B_{4,2}^{+}(q)\left(\frac{9\sqrt{5}}{32\sqrt{2\pi}}\right)+B_{4,4}^{+}(q)\left(\frac{9\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\end{array}\right]\cos(0\phi_{21})d\phi_{21}$$

$$W_0^{vorticity}(q)S^{eq}(q)=1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)+B_{4,0}^{+}(q)\left(\frac{27}{64\sqrt{\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{9\sqrt{5}}{32\sqrt{2\pi}}\right)+B_{4,4}^{+}(q)\left(\frac{9\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)$$

$$W_2^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(2\phi_{21})S(q)\,d\phi_{21}$$

$$W_2^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left[\begin{aligned}&1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)\left(1+3\cos(2\phi_{21})\right)+B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{21})\right)\\&+B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9+20\cos(2\phi_{21})+35\cos(4\phi_{21})\right)+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin(\phi_{21})\cos(\phi_{21})+\frac{7}{4}\sin(4\phi_{21})\right)\\&+B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)+B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)\left(2\sin(2\phi_{21})+\sin(4\phi_{21})\right)\\&+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{21})+\cos(4\phi_{21})\right)+\ldots(l>4)\end{aligned}\right]\cos(2\phi_{21})\,d\phi_{21}$$

$$W_2^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left[\begin{aligned}&B_{2,0}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{5}{\pi}}\right)\cos(2\phi_{21})+B_{2,2}^{+}(q)\left(-\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\cos(2\phi_{21})\\&+B_{4,0}^{+}(q)\frac{60}{64\sqrt{\pi}}\cos(2\phi_{21})+B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{2\pi}}\right)\cos(2\phi_{21})+B_{4,4}^{+}(q)\left(\frac{-3\sqrt{35}}{16\sqrt{2\pi}}\right)\cos(2\phi_{21})+\ldots(l>4)\end{aligned}\right]\cos(2\phi_{21})\,d\phi_{21}$$

$$W_2^{vorticity}(q)S^{eq}(q)=\frac{1}{2}\left(B_{2,0}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{5}{\pi}}\right)-B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)+B_{4,0}^{+}(q)\left(\frac{60}{64\sqrt{\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{2\pi}}\right)-B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{16\sqrt{2\pi}}\right)+\ldots(l>4)\right)$$

$$W_2^{vorticity}(q)S^{eq}(q)=B_{2,0}^{+}(q)\left(\frac{3}{8}\sqrt{\frac{5}{\pi}}\right)-B_{2,2}^{+}(q)\left(\frac{1}{8}\sqrt{\frac{15}{2\pi}}\right)+B_{4,0}^{+}(q)\left(\frac{60}{128\sqrt{\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)-B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{32\sqrt{2\pi}}\right)+\ldots(l>4)$$

$$W_4^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(4\phi_{21})S(q)\,d\phi_{21}$$

$$W_4^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{array}{l} 1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)\left(1+3\cos(2\phi_{21})\right)+B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{21})\right) \\ +B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9+20\cos(2\phi_{21})+35\cos(4\phi_{21})\right)+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin(\phi_{21})\cos(\phi_{21})+\frac{7}{4}\sin(4\phi_{21})\right) \\ +B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)+B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)\left(2\sin(2\phi_{21})+\sin(4\phi_{21})\right) \\ +B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{21})+\cos(4\phi_{21})\right)+\ldots(l>4) \end{array}\cos(4\phi_{21})d\phi$$

$$W_4^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}B_{4,0}^{+}(q)\left(\frac{105}{64\sqrt{\pi}}\right)\cos(4\phi_{21})+B_{4,2}^{+}(q)\left(\frac{-21\sqrt{5}}{32\sqrt{2\pi}}\right)\cos(4\phi_{21})+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)\cos(4\phi_{21})+\ldots(l>4)\cos(4\phi_{21})d\phi_{21}$$

$$W_4^{vorticity}(q)S^{eq}(q)=2\left(B_{4,0}^{+}(q)\left(\frac{105}{64\sqrt{\pi}}\right)-B_{4,2}^{+}(q)\left(\frac{21\sqrt{5}}{16\sqrt{2\pi}}\right)+B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\right)$$

$$W_6^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(6\phi_{21})S(q)\,d\phi_{21}$$

$$W_6^{vorticity}(q)S^{eq}(q)=0$$

$$W_{12}^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(\phi_{21})\sin(\phi_{21})S(q)\,d\phi_{21}$$

$$W_{12}^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{pmatrix}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)(1+3\cos(2\phi_{21}))+B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)(1-\cos(2\phi_{21})) \\ +B_{4,0}^{+}(q)\frac{3}{64\sqrt{\pi}}(17+260\cos(2\phi_{21})+35\cos(4\phi_{21}))+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\left(\sin(\phi_{21})\cos(\phi_{21})+\frac{7}{4}\sin(4\phi_{21})\right) \\ +B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{16\sqrt{2\pi}}\right)\left(\frac{3}{2}+2\cos(2\phi_{21})-\frac{7}{2}\cos(4\phi_{21})\right)+B_{4,3}^{+}(q)\left(\frac{3}{32}\sqrt{\frac{35}{\pi}}\right)(2\sin(2\phi_{21})+\sin(4\phi_{21})) \\ +B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)(3-4\cos(2\phi_{21})+\cos(4\phi_{21}))+\ldots(l>4)\end{pmatrix}\cos(\phi_{21})\sin(\phi_{21})\,d\phi_{21}$$

$$W_{12}^{vorticity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\left(B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)\sin(\phi_{21})\cos(\phi_{21})+\ldots(l>4)\right)\cos(\phi_{21})\sin(\phi_{21})\,d\phi_{21}$$

$$W_{12}^{vorticity}(q)S^{eq}(q)=\frac{1}{8}\left(B_{2,1}^{+}(q)\left(\sqrt{\frac{15}{2\pi}}\right)+B_{4,1}^{+}(q)\left(\frac{3\sqrt{5}}{8\sqrt{\pi}}\right)+\ldots(l>4)\right)$$

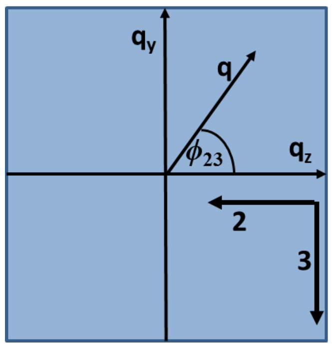
Tangential (velocity gradient-vorticity)

For the 2D detector when the neutrons propagate along the flow direction:

$$\frac{q_x}{q} = 0$$

$$\frac{q_y}{q} = -\sin(\phi_{23})$$

$$q_z = -\cos(\phi_{23})$$



$$[l=0, m=0] = B^{+}_{0,0}(q)\left(\frac{1}{\sqrt{\pi}}\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(3\frac{q_z^2}{q^2}-1\right)\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(-1+3\cos^2(\phi_{23})\right)\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\sqrt{\frac{5}{4\pi}}\left(-1+3\left(\frac{1}{2}\left(1+\cos(2\phi_{23})\right)\right)\right)\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\frac{1}{2}\sqrt{\frac{5}{4\pi}}\left(1+3\cos(2\phi_{23})\right)\right)$$

$$[l=2, m=0] = B^{+}_{2,0}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(1+3\cos(2\phi_{23})\right)\right)$$

$$\left[l=2,m=1\right]=B_{2,1}^{+}\left(q\right)\left(\sqrt{\frac{15}{2\pi}}\left(\frac{q_x q_z}{q^2}\right)\right)$$

$$\left[l=2,m=1\right]=0$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\sqrt{\frac{15}{8\pi}}\left(\frac{q_x^2-q_y^2}{q^2}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\sqrt{\frac{15}{8\pi}}\left(\frac{-q_y^2}{q^2}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\sqrt{\frac{15}{8\pi}}\right)\sin^2\left(\phi_{23}\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\sqrt{\frac{15}{8\pi}}\right)\left(\frac{1}{2}\left(1-\cos\left(2\phi_{23}\right)\right)\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\frac{1}{2}\sqrt{\frac{15}{8\pi}}\right)\left(1-\cos\left(2\phi_{23}\right)\right)$$

$$\left[l=2,m=2\right]=B_{2,2}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos\left(2\phi_{23}\right)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\frac{3}{8\sqrt{\pi}}\left(3-30\left(\frac{q_z^2}{q^2}\right)+35\left(\frac{q_z^4}{q^4}\right)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3}{8\sqrt{\pi}}\right)\left(3-30\cos^2(\phi_{23})+35\cos^4(\phi_{23})\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3}{8\sqrt{\pi}}\right)\left(3-15\left(1+\cos(2\phi_{23})\right)+\frac{35}{8}\left(3+4\cos(2\phi_{23})+\cos(4\phi_{23})\right)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3}{8\sqrt{\pi}}\right)\left(-12-15\cos(2\phi_{23})+\frac{35}{8}\left(3+4\cos(2\phi_{23})+\cos(4\phi_{23})\right)\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3}{8\sqrt{\pi}}\right)\left(-\frac{96}{8}-\frac{120}{8}\cos(2\phi_{23})+\frac{105}{8}+\frac{140}{8}\cos(2\phi_{23})+\frac{35}{8}\cos(4\phi_{23})\right)$$

$$[l=4,m=0]=B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9-20\cos(2\phi_{23})-35\cos(4\phi_{23})\right)$$

$$[l=4,m=1]=B_{4,1}^{+}(q)\frac{3\sqrt{5}}{4\sqrt{\pi}}\left(\frac{q_x q_z}{q^2}\right)\left(7\left(\frac{q_z^2}{q^2}\right)-3\right)$$

$$[l=4,m=1]=0$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{q_x^2-q_y^2}{q^2}\right)\left(-1+7\left(\frac{q_z^2}{q^2}\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{-q_y^2}{q^2}\right)\left(-1+7\left(\frac{q_z^2}{q^2}\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\sin^2(\phi_{23})\left(-1+7\cos^2(\phi_{23})\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{4\sqrt{2\pi}}\right)\left(\frac{1}{2}\left(1-\cos(2\phi_{23})\right)\right)\left(-1+7\left(\frac{1}{2}\left(1+\cos(2\phi_{23})\right)\right)\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{8\sqrt{2\pi}}\right)\left(1-\cos(2\phi_{23})\right)\left(5+7\cos(2\phi_{23})\right)$$

$$\left[l=4,m=2\right]=B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{32\sqrt{2\pi}}\right)\left(5+2\cos(2\phi_{23})-\frac{7}{2}\cos(4\phi_{23})\right)$$

$$\left[l=4,m=3\right]=B_{4,3}^{+}(q)\left(\frac{3}{4}\sqrt{\frac{35}{\pi}}\right)\left(\frac{q_x q_z\left(q_x^2-3q_y^2\right)}{q^4}\right)$$

$$\left[l=4,m=3\right]=0$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{\left(q_x^2-2q_xq_y-q_y^2\right)\left(q_x^2+2q_xq_y-q_y^2\right)}{q^4}\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\frac{q_y^4}{q^4}\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{-3\sqrt{35}}{8\sqrt{2\pi}}\right)\left(\sin^4\left(\phi_{23}\right)\right)$$

$$\left[l=4,m=4\right]=B_{4,4}^{+}\left(q\right)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos\left(2\phi_{23}\right)+\cos\left(4\phi_{23}\right)\right)$$

Structure factor for the 2D detector when the neutrons propagate along the flow direction:

$$\begin{aligned}S^{velocity}\left(q\right)=1+B_{0,0}^{+}\left(q\right)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(1+3\cos\left(2\phi_{23}\right)\right)\right)+B_{2,2}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos\left(2\phi_{23}\right)\right)\\+B_{4,0}^{+}\left(q\right)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9-20\cos\left(2\phi_{23}\right)-35\cos\left(4\phi_{23}\right)\right)+B_{4,2}^{+}\left(q\right)\left(\frac{-3\sqrt{5}}{32\sqrt{2\pi}}\right)\left(5+2\cos\left(2\phi_{23}\right)-\frac{7}{2}\cos\left(4\phi_{23}\right)\right)\\+B_{4,4}^{+}\left(q\right)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos\left(2\phi_{23}\right)+\cos\left(4\phi_{23}\right)\right)+\ldots\left(l>4\right)\end{aligned}$$

W_n's for the 2D detector when the neutrons propagate along the flow direction:

$$W_n^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(n\phi_{23})S(q)\,d\phi_{23}$$

$$W_0^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{array}{l}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(1+3\cos(2\phi_{23})\right)\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{23})\right)\\ +B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9-20\cos(2\phi_{23})-35\cos(4\phi_{23})\right)+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{32\sqrt{2\pi}}\right)\left(5+2\cos(2\phi_{23})-\frac{7}{2}\cos(4\phi_{23})\right)\\ +B_{4,4}^{+}(q)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{23})+\cos(4\phi_{23})\right)+\ldots(l>4)\end{array}\cos(0\phi_{23})\,d\phi_{23}$$

$$W_0^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\begin{array}{l}1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\\ +B_{4,0}^{+}(q)\left(\frac{27}{64\sqrt{\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{-15\sqrt{5}}{32\sqrt{2\pi}}\right)+B_{4,4}^{+}(q)\left(\frac{-9\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\end{array}\cos(0\phi_{23})\,d\phi_{23}$$

$$W_0^{velocity}(q)S^{eq}(q)=1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)+B_{4,0}^{+}(q)\left(\frac{27}{64\sqrt{\pi}}\right)-B_{4,2}^{+}(q)\left(\frac{15\sqrt{5}}{32\sqrt{2\pi}}\right)-B_{4,4}^{+}(q)\left(\frac{9\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)$$

$$W_2^{velocity}\left(q\right)S^{eq}\left(q\right)=\frac{1}{2\pi}\int_0^{2\pi}\cos(2\phi_{23})S\left(q\right)d\phi_{23}$$

$$W_2^{velocity}\left(q\right)S^{eq}\left(q\right)=\frac{1}{2\pi}\int_0^{2\pi}\begin{matrix}1+B_{0,0}^{+}\left(q\right)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(1+3\cos\left(2\phi_{23}\right)\right)\right)+B_{2,2}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos\left(2\phi_{23}\right)\right)\\ +B_{4,0}^{+}\left(q\right)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9-20\cos\left(2\phi_{23}\right)-35\cos\left(4\phi_{23}\right)\right)+B_{4,2}^{+}\left(q\right)\left(\frac{-3\sqrt{5}}{32\sqrt{2\pi}}\right)\left(5+2\cos\left(2\phi_{23}\right)-\frac{7}{2}\cos\left(4\phi_{23}\right)\right)\\ +B_{4,4}^{+}\left(q\right)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos\left(2\phi_{23}\right)+\cos\left(4\phi_{23}\right)\right)+\ldots(l>4)\end{matrix}\cos(2\phi_{23})d\phi_{23}$$

$$W_2^{velocity}\left(q\right)S^{eq}\left(q\right)=\frac{1}{2\pi}\int_0^{2\pi}\begin{matrix}B_{2,0}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{\pi}}\right)\cos\left(2\phi_{23}\right)+B_{2,2}^{+}\left(q\right)\left(\frac{-1}{4}\sqrt{\frac{15}{2\pi}}\right)\cos\left(2\phi_{23}\right)\\ +B_{4,0}^{+}\left(q\right)\left(\frac{-60}{64\sqrt{\pi}}\right)\cos\left(2\phi_{23}\right)+B_{4,2}^{+}\left(q\right)\left(\frac{-6\sqrt{5}}{32\sqrt{2\pi}}\right)\cos\left(2\phi_{23}\right)+B_{4,4}^{+}\left(q\right)\left(\frac{12\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\end{matrix}\cos(2\phi_{23})d\phi_{23}$$

$$W_2^{velocity}\left(q\right)S^{eq}\left(q\right)=\frac{1}{2}\left(B_{2,0}^{+}\left(q\right)\left(\frac{3}{4}\sqrt{\frac{5}{\pi}}\right)-B_{2,2}^{+}\left(q\right)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)-B_{4,0}^{+}\left(q\right)\left(\frac{60}{64\sqrt{\pi}}\right)\cos\left(2\phi_{23}\right)-B_{4,2}^{+}\left(q\right)\left(\frac{6\sqrt{5}}{32\sqrt{2\pi}}\right)\cos\left(2\phi_{23}\right)+B_{4,4}^{+}\left(q\right)\left(\frac{12\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\right)$$

$$W_4^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(4\phi_{23})S(q)\,d\phi_{23}$$

$$1+B_{0,0}^{+}(q)\left(\frac{1}{\sqrt{\pi}}\right)+B_{2,0}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(1+3\cos(2\phi_{23})\right)\right)+B_{2,2}^{+}(q)\left(\frac{1}{4}\sqrt{\frac{15}{2\pi}}\right)\left(1-\cos(2\phi_{23})\right)$$

$$W_4^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\quad +B_{4,0}^{+}(q)\left(\frac{3}{64\sqrt{\pi}}\right)\left(9-20\cos(2\phi_{23})-35\cos(4\phi_{23})\right)+B_{4,2}^{+}(q)\left(\frac{-3\sqrt{5}}{32\sqrt{2\pi}}\right)\left(5+2\cos(2\phi_{23})-\frac{7}{2}\cos(4\phi_{23})\right)\cos(4\phi_{23})\,d\phi_{23}$$

$$W_6^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}\cos(6\phi_{23})S(q)\,d\phi_{23}$$

$$+B_{4,4}^{+}(q)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\left(3-4\cos(2\phi_{23})+\cos(4\phi_{23})\right)+\ldots(l>4)$$

$$W_6^{velocity}(q)S^{eq}(q)=0$$

$$W_4^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}B_{4,0}^{+}(q)\left(\frac{-105}{64\sqrt{\pi}}\right)\cos(4\phi_{23})+B_{4,2}^{+}(q)\left(\frac{21\sqrt{5}}{64\sqrt{2\pi}}\right)\cos(4\phi_{23})+B_{4,4}^{+}(q)\left(\frac{-3\sqrt{35}}{64\sqrt{2\pi}}\right)\cos(4\phi_{23})+\ldots(l>4)\cos(4\phi_{23})\,d\phi_{23}$$

$$W_{12}^{velocity}(q)S^{eq}(q)=\frac{1}{2\pi}\int_0^{2\pi}2\cos(\phi_{23})\sin(\phi_{23})S(q)\,d\phi_{23}$$

$$W_4^{velocity}(q)S^{eq}(q)=2\left(-B_{4,0}^{+}(q)\left(\frac{105}{64\sqrt{\pi}}\right)+B_{4,2}^{+}(q)\left(\frac{21\sqrt{5}}{64\sqrt{2\pi}}\right)-B_{4,4}^{+}(q)\left(\frac{3\sqrt{35}}{64\sqrt{2\pi}}\right)+\ldots(l>4)\right)$$

$$W_{12}^{velocity}(q)S^{eq}(q)=0$$

Identities:

$$\sin^2(\theta) = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\sin^4(\theta) = \frac{3}{8} - \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta)$$

$$\cos^4(\theta) = \frac{3}{8} + \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta)$$

$$\cos^2(\theta)\sin^2(\theta) = \frac{1}{8}\left(1 - \cos(4\theta)\right)$$

$$\cos(\theta)\sin^3(\theta) = \frac{1}{4}\sin(2\theta) + \frac{1}{8}\sin(4\theta)$$

$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$

$$\cos(-a) = \cos(a)$$

$$\sin(-a) = -\sin(a)$$

$$\int_0^{2\pi} Sin\varphi d\varphi = -Cos\varphi\Big|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Sin^2\varphi d\varphi = \left(\frac{\varphi}{2} - \frac{1}{4}Sin2\varphi\right)\Bigg|_0^{2\pi} = \pi$$

$$\int_0^{2\pi} Sin^3\varphi d\varphi = \left(-Cos\varphi - \frac{1}{3}Cos^3\varphi\right)\Bigg|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Sin^4\varphi d\varphi = \left(\frac{3\varphi}{8} - \frac{1}{4}Sin2\varphi + -\frac{1}{32}Sin4\varphi\right)\Bigg|_0^{2\pi} = \frac{3\pi}{4}$$

$$\int_0^{2\pi} Cos\varphi d\varphi = Sin\varphi\Big|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Cos^2\varphi d\varphi = \left(\frac{\varphi}{2} + \frac{1}{4}Sin2\varphi\right)\Bigg|_0^{2\pi} = \pi$$

$$\int_0^{2\pi} Cos^3\varphi d\varphi = \left(Sin\varphi - \frac{1}{3}Sin^3\varphi\right)\Bigg|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Cos^4\varphi d\varphi = \left(\frac{3\varphi}{8} + \frac{1}{4}Sin2\varphi + \frac{1}{32}Sin4\varphi\right)\Bigg|_0^{2\pi} = \frac{3\pi}{4}$$

$$\int_0^{2\pi} Sin\varphi Cos\varphi d\varphi = \frac{1}{2}Sin^2\varphi\Bigg|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Sin\varphi Cos^2\varphi d\varphi = -\frac{1}{3}Cos^3\varphi\Bigg|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Sin^2\varphi Cos\varphi d\varphi = \frac{1}{3}Sin^3\varphi\Bigg|_0^{2\pi} = 0$$

$$\int_0^{2\pi} Sin^2\varphi Cos^2\varphi d\varphi = \left(\frac{\varphi}{8} - \frac{1}{32}Sin4\varphi\right)\Bigg|_0^{2\pi} = \frac{\pi}{4}$$

$$\int_0^{2\pi} Sin\varphi Cos^3\varphi d\varphi = -\frac{1}{4}Cos^4\varphi\Bigg|_0^{2\pi} = 0$$