Spectral analogues of the law of the wall, the defect law, and the log law: Supplementary Material

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1. Relative discrepancy

The figures of the paper (that is, Figs. 1 to 6) consist of panels. In each panel we include two types of plots: 1) plots of the turbulent-energy spectra scaled (and rendered dimensionless) in accord with one of the spectral analogues and 2) plots of the same turbulent-energy spectra in unscaled, dimensional form. To test the spectral analogues, we verify that the scaled turbulent-energy spectra collapse on a single master curve in the pertinent range of dimensionless wavenumbers.

Figs. S-1, S-2 and S-3 are modified versions of Fig. 1, Fig. 5 and Fig. 6, respectively. In each panel of Figs. S-1, S-2 and S-3 we include two types of plots: 1) plots of the turbulent-energy spectra scaled (and rendered dimensionless) in accord with one of the spectral analogues and 2) plots of the attendant relative discrepancy $\delta Y/Y$. For an explanation of the relative discrepancy, we refer the reader to the caption of Fig. 1.

From Fig. S-1, Fig. S-2 and Fig. S-3, we can verify that the values of $\delta Y/Y$ remain relatively low where the scaled turbulent-energy spectra appear to collapse on a single master curve, and become progressively higher where the scaled turbulent-energy spectra are seen to fan out.

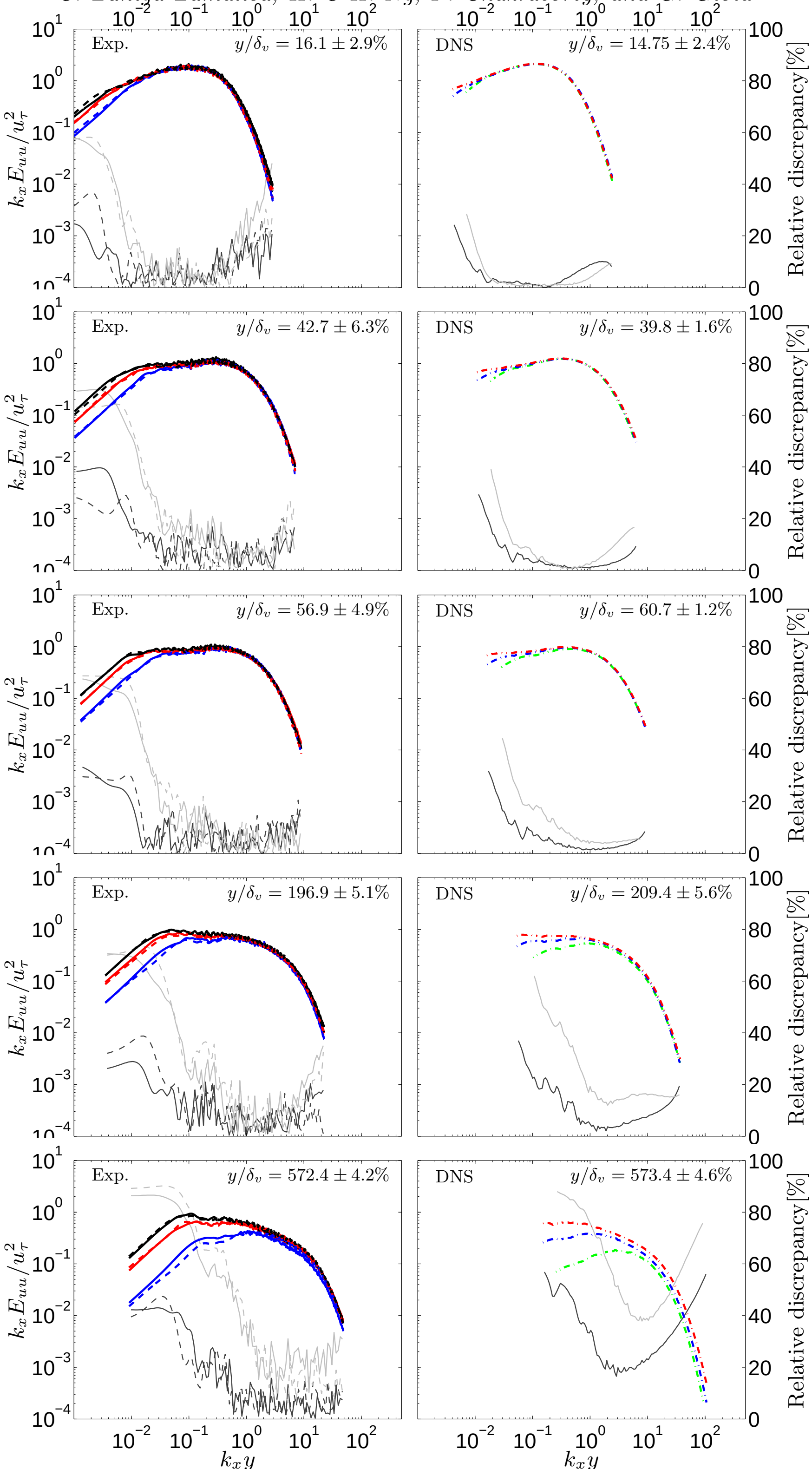

FIGURE S-1. Spectral analogue of the law of the wall and the corresponding relative discrepancy Left: Experimental data on channel flow $(-)$ and pipe flow $(--)$ for $Re_\tau = 1000$(blue), 2000(red) and 3000(black) (from: Ng *et al.* 2011). Right: DNS data on channel flow for $Re_\tau = 550$(green), 934(blue) and 2003(red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). The curves in dark grey represent the relative discrepancy between the highest Reynolds number and the intermediate Reynolds number data (e.g., for the experimental data: $\left|\left(k_x E_{uu}/u_\tau^2\right)_{Re_\tau=3000} - \left(k_x E_{uu}/u_\tau^2\right)_{Re_\tau=2000}\right| / \left|\left(k_x E_{uu}/u_\tau^2\right)_{Re_\tau=3000}\right|$). The curves in light grey represent the relative discrepancy between the highest Reynolds number and the lowest Reynolds number data. Value of y/δ_v as indicated.

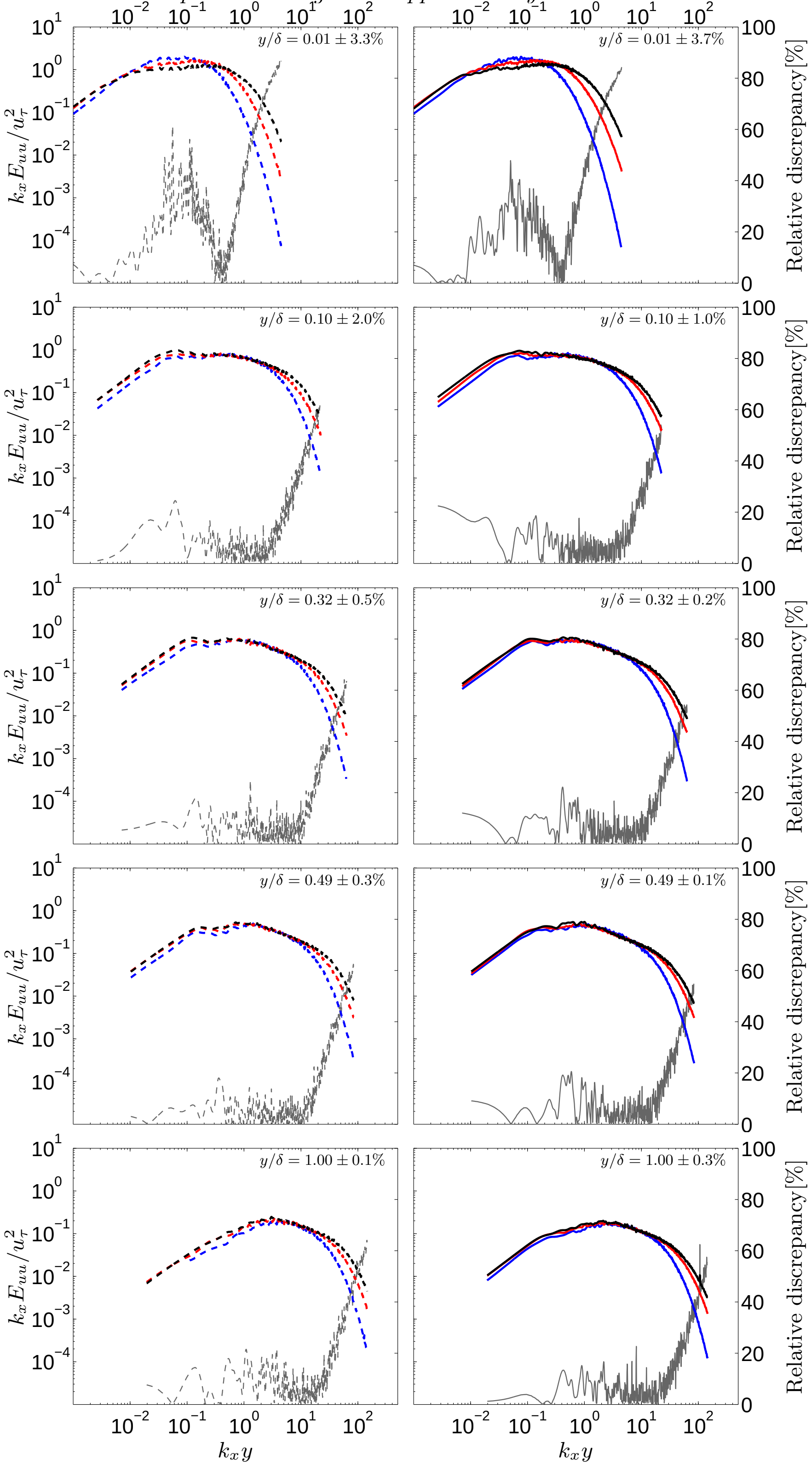

FIGURE S-2. Spectral analogue of the law of the wall and the corresponding relative discrepancy Left and right: Experimental data on pipe flow (−−) and channel flow (−), respectively, for Re_τ = 1000(blue), 2000(red) and 3000(black) (from: Ng *et al.* 2011). The curves in dark grey represent the relative discrepancy between the highest Reynolds number and the intermediate Reynolds number data. Value of y/δ as indicated.

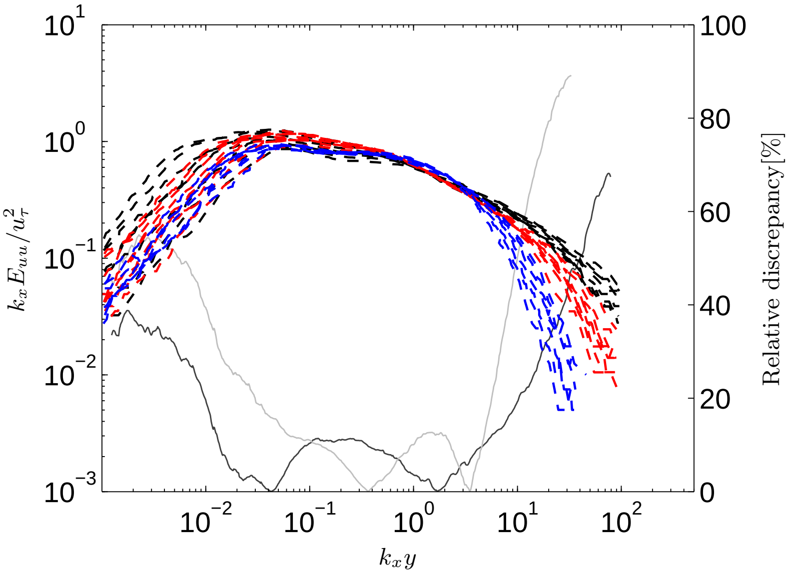

Figure S-3. Spectral analogue of the law of the wall and the corresponding relative discrepancy. Experimental data on pipe flow for $Re_\tau = 3334$ (blue), 20250 (red) and 98190 (black) (from: Rosenberg *et al.* 2013). The curves in dark grey represent the relative discrepancy between the highest Reynolds number and the intermediate Reynolds number data ($\left|\left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=98190} - \left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=20250}\right| / \left|\left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=98190}\right|$, where the overbar denotes averaging over y positions in the overlap layer). The curves in light grey represent the relative discrepancy between the highest Reynolds number and the lowest Reynolds number data ($\left|\left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=98190)} - \left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=3334}\right| / \left|\left(\overline{k_x E_{uu}/u_\tau^2}\right)_{Re_\tau=98190}\right|$, where the overbar denotes averaging over y positions in the overlap layer).

2. Further test of the spectral analogue of the law of the wall

Using DNS data on channel flow, here we show the plots for the spectral analogue of the law of the wall for all possible realizations of $E(k)$. As noted in the manuscript, the spectral analogue of the law of the wall holds for all realizations of $E(k)$.

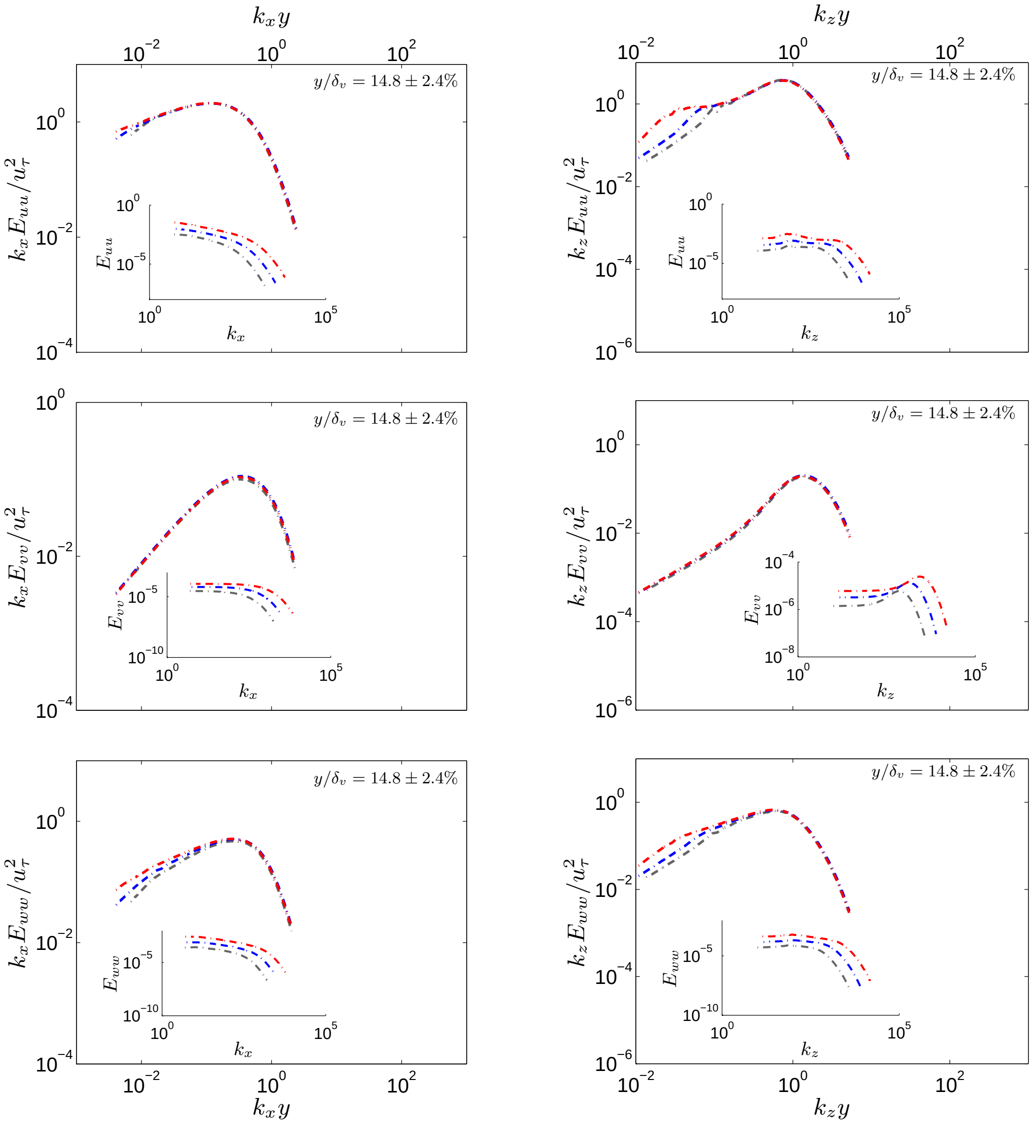

FIGURE S-4. Test of the spectral analogue of the law of the wall at moderate Re_τ for all possible realizations of $E(k)$. The unscaled, dimensional spectra in SI units (insets) are scaled (and rendered dimensionless) in accord with equation 4.2. Computational data on channel flow for $Re_\tau = 550$ (grey), 934 (blue) and 2003 (red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). Value of y/δ_v as indicated.

REFERENCES

DEL ÁLAMO, J., JIMÉNEZ, J., ZANDONADE, P. & MOSER, R. D. 2004 Scaling of energy spectra in turbulent channels. *Journal of Fluid Mechanics* **500**, 135–144.

HOYAS, S. & JIMÉNEZ, J. 2006 Scaling of velocity fluctuations in turbulent channel flows up to $Re_\tau = 2003$. *Physics of Fluids* **18** (011702).

NG, H. C. H., MONTY, J. P., HUTCHINS, N., CHONG, M. S. & MARUSIC, I. 2011 Comparison of turbulent channel and pipe flows with varying reynolds number. *Experiments in Fluids* **51**, 1261–1281.

ROSENBERG, B. J., HULTMARK, M., VALLIKIVI, M., BAILEY, S. C. C. & SMITS, A. J. 2013

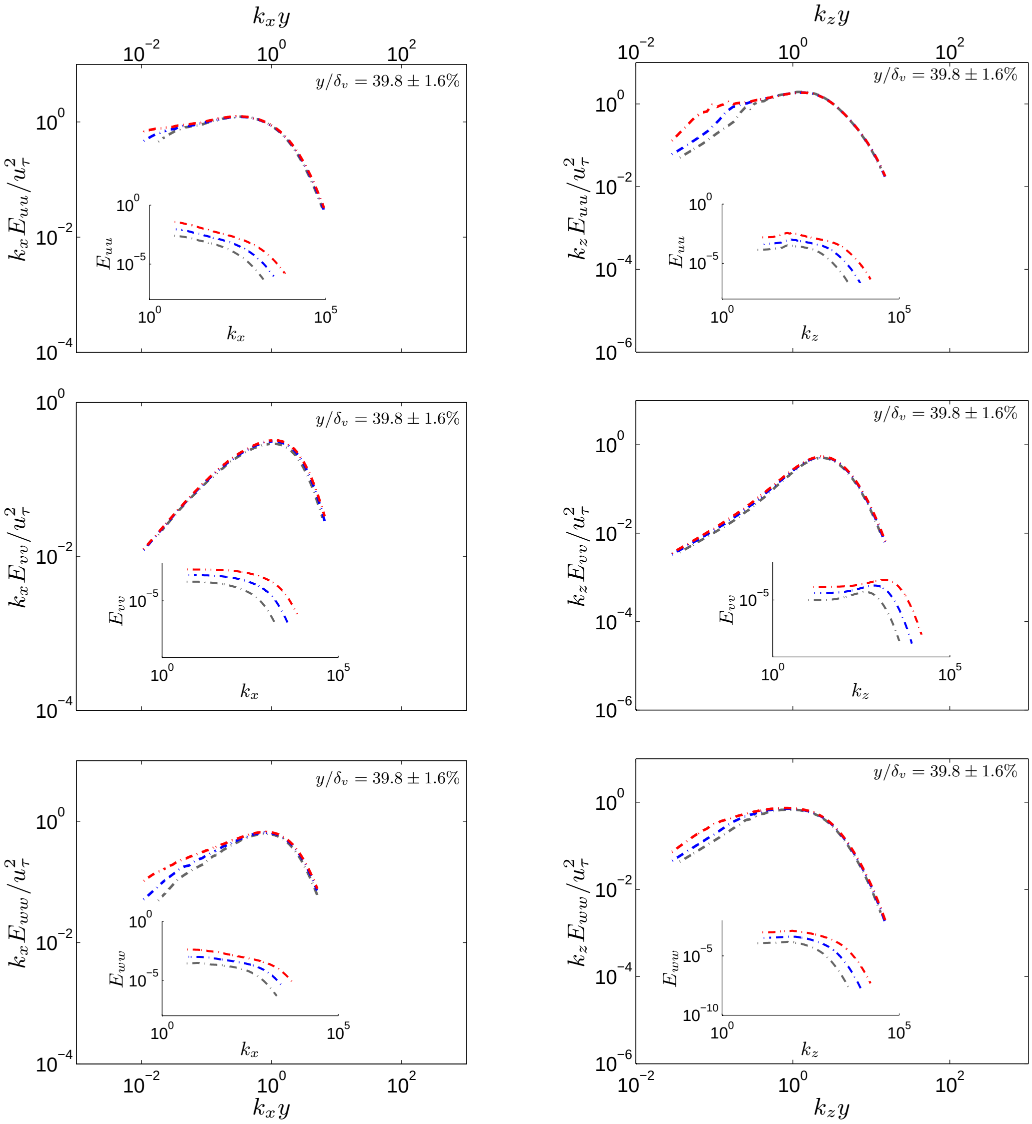

FIGURE S-5. Test of the spectral analogue of the law of the wall at moderate Re_τ for all possible realizations of $E(k)$. The unscaled, dimensional spectra in SI units (insets) are scaled (and rendered dimensionless) in accord with equation 4.2. Computational data on channel flow for $\mathrm{Re}_\tau = 550$ (grey), 934 (blue) and 2003 (red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). Value of y/δ_v as indicated.

Turbulence spectra in smooth- and rough-wall pipe flow at extreme Reynolds numbers. *Journal of Fluid Mechanics* **731**, 46–63.

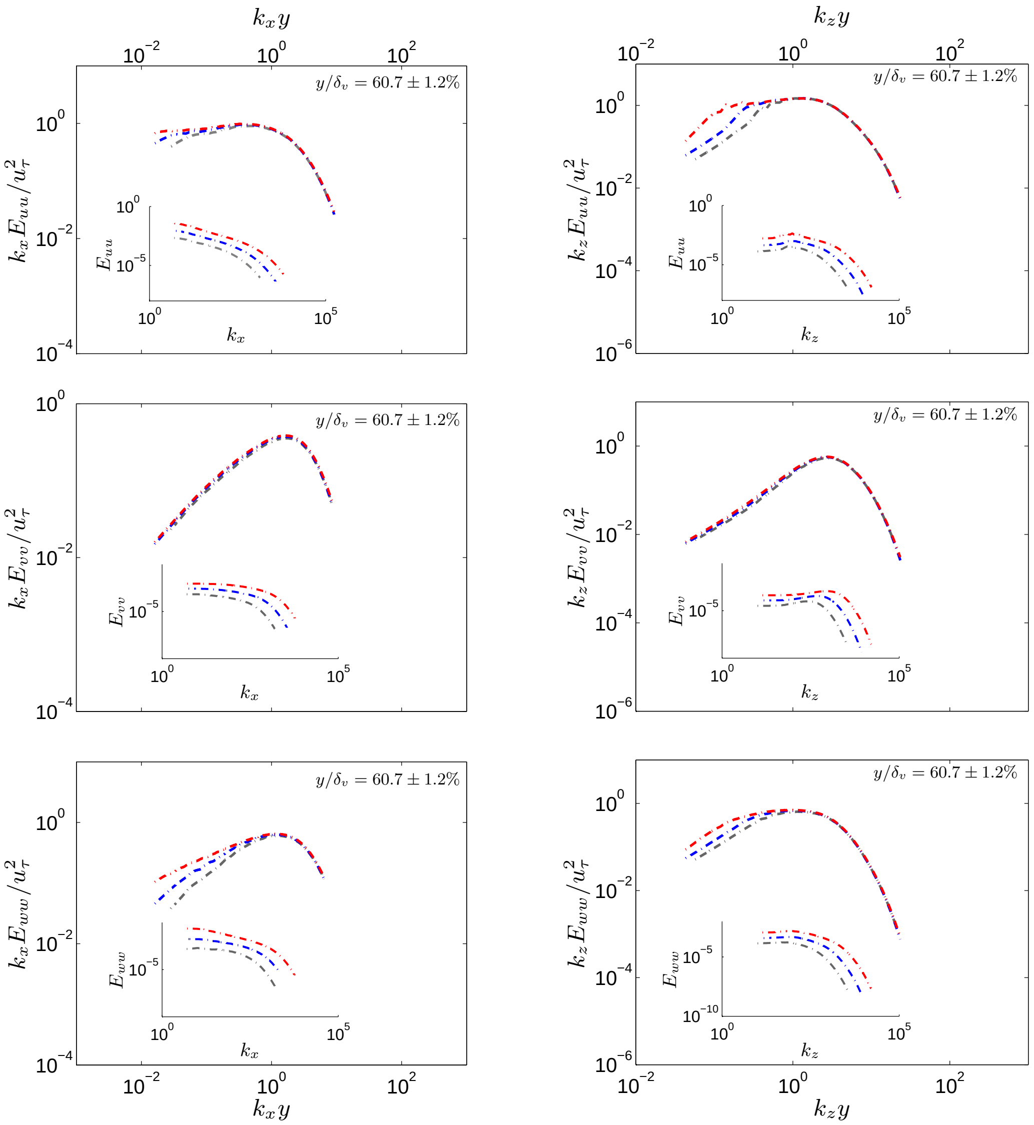

FIGURE S-6. Test of the spectral analogue of the law of the wall at moderate Re_τ for all possible realizations of $E(k)$. The unscaled, dimensional spectra in SI units (insets) are scaled (and rendered dimensionless) in accord with equation 4.2. Computational data on channel flow for $\mathrm{Re}_\tau = 550$ (grey), 934 (blue) and 2003 (red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). Value of y/δ_v as indicated.

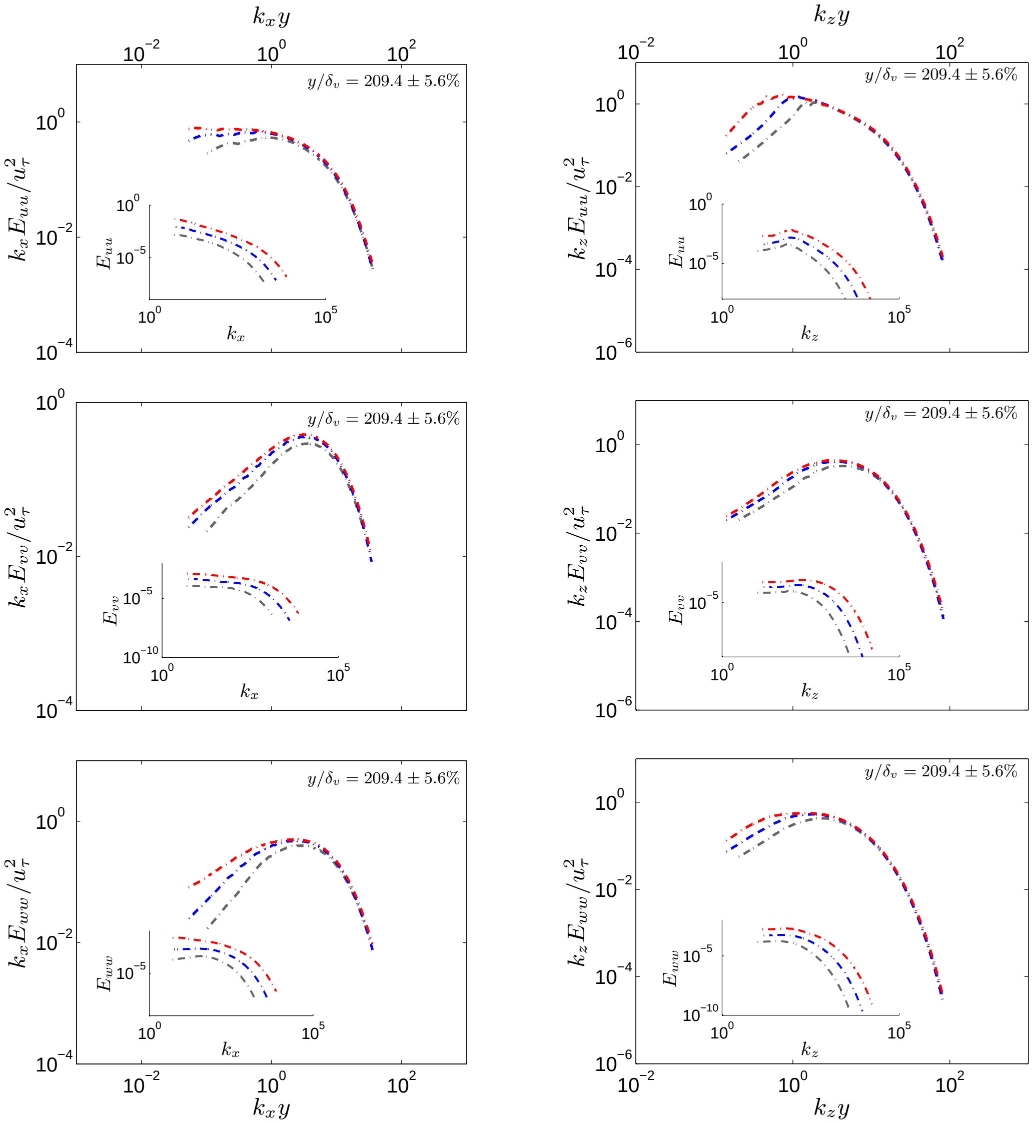

FIGURE S-7. Test of the spectral analogue of the law of the wall at moderate Re_τ for all possible realizations of $E(k)$. The unscaled, dimensional spectra in SI units (insets) are scaled (and rendered dimensionless) in accord with equation 4.2. Computational data on channel flow for $\mathrm{Re}_\tau = 550$ (grey), 934 (blue) and 2003 (red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). Value of y/δ_v as indicated.

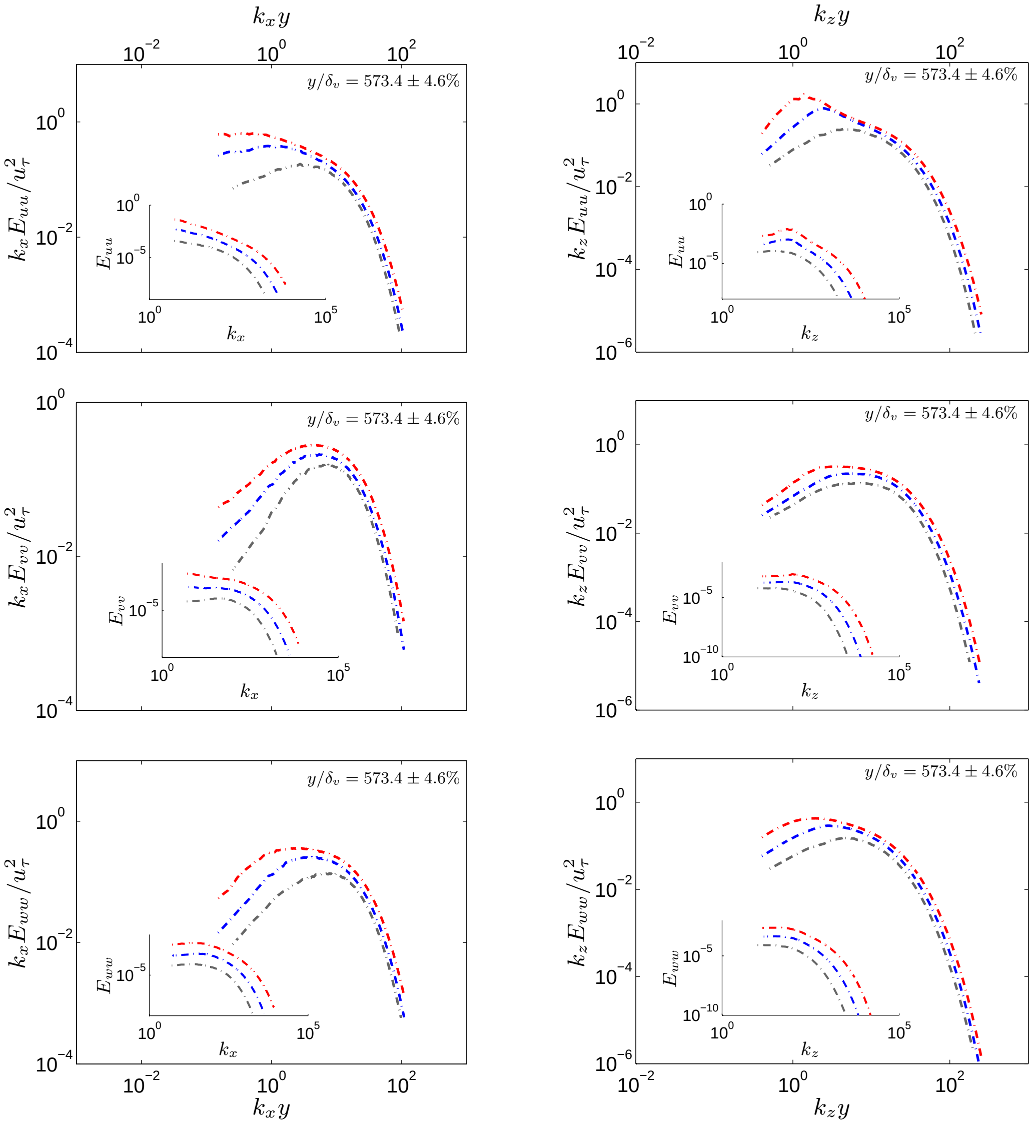

FIGURE S-8. Test of the spectral analogue of the law of the wall at moderate Re_τ for all possible realizations of $E(k)$. The unscaled, dimensional spectra in SI units (insets) are scaled (and rendered dimensionless) in accord with equation 4.2. Computational data on channel flow for $\mathrm{Re}_\tau = 550$ (grey), 934 (blue) and 2003 (red) ((from: del Álamo *et al.* 2004; Hoyas & Jiménez 2006, available at http://torroja.dmt.upm.es/channels/data/)). Value of y/δ_v as indicated.