Supporting material for the paper titled:

Gas Flow in Ultra-Tight Shale Gas Strata

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Nomenclature

A	Cross-sectional area of cylindrical core, m^2
D_k	Knudsen diffusion coefficient, m^2/sec
$D_{k,pm}$	Knudsen diffusion coefficient in porous media, m^2/sec
F	Slip coefficient
K	General permeability function, m^2
L	Length of cylindrical sample, m
M	molar mass, $kg.mol^{-1}$
Q	Flow rate, $m^3.s^{-1}$
R	Universal gas constant, 8.314 $J.K^{-1}.mol^{-1}$
R_{avg}	Average pore radius, m
R_{nt}	Nano tube pore radius, m
T	Temperature, K
V_d	Downstream reservoir volume, m^3
V_p	Effective sample pore volume, m^3
V_u	Upstream reservoir volume, m^3
V_s	Sample cell volume, m^3
V_{std}	Gas molar volume at standard temperature and pressure, 22.413×10^3 $m^3.mol^{-1}$
a	Ratio of sample storage capacity to upstream reservoir
b	Ratio of sample storage capacity to downstream reservoir
c_g	Gas compressibility, Pa^{-1}
k	Permeability, m^2
k_D	Darcy permeability, m^2
k_a	Partial derivative of adsorbate density with respect to gas density
k_{app}	Apparent permeability in nano-tube, m^2
$k_{app,pm}$	Apparent permeability in porous media, m^2
k_n	Knudsen number
m	Pseudo-pressure function, $Pa.m^2.s^{-1}$
p	Pressure, Pa
p_{avg}	Average core pressure, Pa
p_d	Pressure in downstream reservoir, Pa
p_{d0}	Initial equilibrium pressure, Pa
p_i	Initial reservoir pressure, Pa
p_L	Langmuir pressure, Pa
p_u	Pressure in upstream reservoir, Pa
p_w	Bottomhole pressure, Pa
q	Adsorbate density, $mol.m^{-3}$
q_L	Langmuir volume, $cm^3.g^{-1}$
r	Distance from wellbore, m
r_w	Wellbore radius, m
s_1	Slope of the straight line part of $\ln(\Delta p_D)$ versus time at late-time
t	Time, s
t_D	Dimensionless time

z	Gas compressibility factor (=1.0 for ideal gas)
Δp	Pressure difference between upstream and downstream reservoirs, Pa
Δp_D	Dimensionless differential pressure
Δm_D	Dimensionless pseudo-pressure function
Φ	Effective porosity contributed by adsorption
α	Tangential momentum accommodation coefficient
μ	Viscosity, Pa.s^{-1}
ρ	Gas density, kg.m^{-3}
ρ_s	Core sample density, kg.m^{-3}
φ	Porosity
τ	Tortousity

Appendix A – Finite-Difference Numerical Solution

The general material balance equation for one dimensional core sample is:

$$c_g \rho \Phi \frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left(T\frac{\partial P}{\partial x}\right). \quad \text{(A-1)}$$

Equation A-1 is subjected to one initial condition and two boundary conditions,

$$P(x,0) = P_0, 0 < x \le L, \quad \text{(A-2)}$$

$$\rho c_{c_g} \frac{\partial P}{\partial t} = \frac{TA}{V_u}\frac{\partial P}{\partial x}, x = 0, t > 0, \quad \text{(A-3)}$$

$$\rho c_g \frac{\partial P}{\partial t} = -\frac{TA}{V_d}\frac{\partial P}{\partial x}, x = L, t > 0, \quad \text{(A-4)}$$

where $\Phi = [\phi + (1-\phi)K_a]$, $P=p^2$, $T=K/(\mu z)$. K is a general permeability function, which may represent either Darcy permeability or apparent permeability function. Figure A-1 shows the schematic gridding of a one-dimensional core sample.

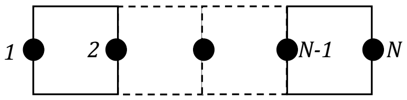

Figure A-1 Gridding in a one-dimensional core sample

Central approximation is used in space and backward approximation is used in time to discretize Eq. A-1,

$$\frac{T_{i-1}^{n}\Delta t}{\Delta x^2}P_{i-1}^{n+1} - \left(\frac{c_{g,i}^{n}\Phi_i^n}{z_i^n} + \frac{T_{i-1}^{n}\Delta t}{\Delta x^2} + \frac{T_i^{n}\Delta t}{\Delta x^2}\right)P_i^{n+1} + \frac{T_i^{n}\Delta t}{\Delta x^2}P_{i+1}^{n+1} = -\frac{c_{g,i}^{n}\Phi_i^n}{z_i^n}P_i^n \quad i = 2,...,N-1, \quad \text{(A-5)}$$

where n+1 refers to current time and n refers to the previous time level. Eq. A-5 generates N-2 equations and N unknowns. By discretizing the boundary conditions, two independent equations are generated,

$$\left(\frac{c_{g1}^n}{z_1^n} + \frac{AT_1^n\Delta t}{V_u\Delta x}\right)P_1^{n+1} - \frac{AT_1^n\Delta t}{V_u\Delta x}P_2^{n+1} = \frac{c_{g1}^n}{z_1^n}P_1^n, \quad \text{(A-6)}$$

$$-\frac{AT_{N-2}^{n}\Delta t}{V_d \Delta x}P_{N-1}^{n+1}+(\frac{c_{gN}^{n}}{z_N^{n}}+\frac{AT_{N-1}^{n}\Delta t}{V_d \Delta x})P_N^{n+1}=\frac{c_{gN}^{n}}{z_N^{n}}P_N^{n}. \quad \text{(A-7)}$$

Equations A-5 to A-7 are iteratively solved to find the pressure distribution over time. After finding the pressure distribution at each time step, all properties (ρ, μ, z, K and c_g) are updated and used for next time step.

Appendix B – Modified Analytical Solution to the Pulse-Decay Diffusivity Equation

The material balance equation for gas flow in one dimensional core sample with adsorption and considering Knudsen diffusion and slip flow is

$$\phi\frac{\partial \rho}{\partial t}+(1-\phi)\frac{\partial q}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\rho K}{\mu}\frac{\partial p}{\partial x}\right), \quad \text{(B-1)}$$

where q is adsorbate density per unit sample volume, K is a general permeability function, which may represent either Darcy permeability or apparent permeability function, and μ is gas viscosity. Using the chain rule to substitute the adsorbate density with density, we have

$$\frac{\partial q}{\partial t}=\frac{dq}{d\rho}\frac{\partial \rho}{\partial t}. \quad \text{(B-2)}$$

Using a Langmuir adsorption function (Cui *et al.* 2009):

$$q_a=\frac{q_L p}{p_L+p}, \quad \text{(B-3)}$$

$$q=\frac{\rho_s q_a}{V_{std}}, \quad \text{(B-4)}$$

$$\frac{dq}{d\rho}=\frac{\rho_s q_L}{V_{std}}\frac{p_L}{(p_L+p)^2}\frac{1}{\rho c_g}\equiv k_a. \quad \text{(B-5)}$$

Substituting Eq. B-5 into Eq. B-1, we have

$$(\phi+(1-\phi)k_a)\frac{\partial \rho}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\rho K}{\mu}\frac{\partial p}{\partial x}\right). \quad \text{(B-6)}$$

Using the real-gas law, gas molar density is

$$\rho=\frac{pM}{zRT}. \quad \text{(B-7)}$$

Assuming gas viscosity μ, compressibility factor z, gas compressibility c_g and apparent permeability to be constant, we substitute Eq. B-7 into Eq. B-6 and simplify to get

$$(\phi+(1-\phi)k_a)pc_g\frac{\partial p}{\partial t}=\frac{K}{\mu}\frac{\partial}{\partial x}\left(p\frac{\partial p}{\partial x}\right). \quad \text{(B-8)}$$

We define a general pseudo-pressure function $m(p)=2\int_0^p \frac{K}{\mu z}p'dp'$ and reform Eq. B-8,

$$\frac{\partial m}{\partial t}=\frac{K}{\mu c_g(\phi+(1-\phi)k_a)}\frac{\partial^2 m}{\partial x^2}, 0<x<L, t>0. \quad \text{(B-9)}$$

Equation B-9 is subjected to two boundary conditions and one initial condition,

$$m(x,0)=m_d(0),\ 0<x\le L, \quad \text{(B-10)}$$

$$m(0,t) = m_u(t)\,, t \geq 0, \tag{B-11}$$

$$m(L,t) = m_d(t)\,, t \geq 0. \tag{B-12}$$

The boundary conditions in Eqs. B-10 and B-11 vary over time and are determined by material balance at the inlet and outlet of core sample:

$$\frac{\partial m_u}{\partial t} = \frac{KV_p}{\mu c_g \phi L V_u} \frac{\partial m}{\partial x}\Big|_{x=0}\,, t > 0 \tag{B-13}$$

$$\frac{\partial m_d}{\partial t} = -\frac{KV_p}{\mu c_g \phi L V_d} \frac{\partial m}{\partial x}\Big|_{x=L}\,, t > 0. \tag{B-14}$$

We define three dimensionless groups $\Delta m_D=(m_u-m_d)/(m_{u0}-m_{d0})$, $t_D=k_{app,pm}t/(\mu c_g \varphi L^2)$, $x_D=x/L$. With these dimensionless groups, the analytical solution can be found similar to Hsieh *et al.* (1981):

$$\Delta m_D = \frac{m_u(t) - m_d(t)}{m_u(0) - m_d(0)} = 2\sum_{n=1}^{\infty} \frac{a(b^2+\theta_n^2) + (-1)^n b\sqrt{(a^2+\theta_n^2)(b^2+\theta_n^2)}}{\theta_n^2(\theta_n^2 + a + a^2 + b + b^2) + ab(a+b+ab)} \times e^{(-\theta_n^2 t_D)}. \tag{B-15}$$

As discussed by Jones (1997), if the ratios of core sample pore volume to upstream and downstream reservoir volumes is one (a=b=1), then even terms of Eq. B-7 cancel; if t_D>0.1, then the contribution of remaining terms to solution is less than 0.16% of the first term. The late transient solution of the Eq. B-9 is

$$\ln(\Delta m_D) = \ln(f_0) + s_1 t, \tag{B-16}$$

where $f_0 = 2\left(a(b^2+\theta_1^2) + b\sqrt{(a^2+\theta_1^2)(b^2+\theta_1^2)}\right) / \left(\theta_1^2(\theta_1^2 + a + a^2 + b + b^2) + ab(a+b+ab)\right)$, $s_1 = -Kf_1 A(1/V_u + 1/V_d)/\left(\mu L c_g\right)$, $f_1 = \theta_1^2/(a+b)$ and θ_1 is the first solution to equation $\tan\theta = \left((a+b)\theta\right)/\left(\theta^2 - ab\right)$. With the experimental pulse-decay data, the plot of Δm_D on a log scale versus time yields a straight line at late transient time. The slope of the line, s_1, is related to apparent permeability:

$$K = -\frac{s_1 \mu L c_g}{f_1 A(1/V_u + 1/V_d)}. \tag{B-17}$$