$$
\begin{aligned}
\mathbf{G}_w(\mathbf{R}'', x_3''; \alpha) &= \frac{1}{4\pi\eta}\left(\frac{1}{2\pi^2\alpha^{7/2}R''^9}\right) \qquad (1)\\
&\times \Big\{\mathbf{I}\Big[2\alpha^3 R''^3\left(\alpha^2 R''^2 - \pi^2 R''^6 + \alpha R_3''\left[2\pi x_3'' R''^2 - \left(\pi R''^2 + 4\alpha\right)R_3''\right]\right)\tilde{\phi}_{-1/2}\\
&- \alpha^2 R''^3\left(6\alpha^3 R''^2 - 2\pi\alpha^2 R''^4 + 9\pi^2\alpha R''^6 - 2\pi^3 R''^6\left[R''^2 - 10x_3''^2\right]\right.\\
&\left.+4\pi x_3'' R_3'' R''^2\left[\pi^2 R''^4 - \pi\alpha R''^2 + 3\alpha^2\right] + 2\alpha R_3''^2\left[\pi^2 R''^4 + \pi\alpha R''^2 - 12\alpha^2\right]\right)\tilde{\phi}_{1/2}\\
&+ \pi\alpha R''^5\left(-6\alpha^3 R''^2 + 21\pi\alpha^2 R''^4 + 16\pi^2\alpha R''^6 + 36\pi^3 x_3''^2 R''^6\right.\\
&\left.+2\alpha R_3''\left[2\pi x_3'' R''^2\left(23\pi R''^2 - 3\alpha\right) + R_3''\left(2\pi^2 R''^4 + \pi\alpha R''^2 + 12\alpha^2\right)\right]\right)\tilde{\phi}_{3/2}\\
&- 2\pi^2 R''^7\left(5\alpha^3 R''^2 + 12\pi\alpha^2 R''^4 + 2\pi^2\alpha R''^6 + 4\pi^3 x_3''^2\right.\\
&\left.+2\alpha R_3''\left[\pi R''^2\left(24\pi R''^2 + 5\alpha\right) + \alpha R_3''\left(20\pi R''^2 - 3\alpha\right)\right]\right)\tilde{\phi}_{5/2}\\
&+ 4\pi^3 R''^9\left(\alpha R''^2\left[\pi R''^2 + \alpha\right] + 2\pi x_3'' R_3'' R''^2\left[2\pi R''^2 + \alpha\right]\alpha R_3''\left[15\pi R''^2 + 7\alpha\right]\right)\tilde{\phi}_{7/2}\\
&-8\pi^4 R_3''^2 R''^{11}\left(\pi R''^2 + \alpha\right)\tilde{\phi}_{9/2}\Big]\\
&+ \mathbf{e}_3\mathbf{e}_3\Big[-\alpha^4 R''^3\left(10\alpha R''^2 + \pi R''^4 + 20\pi x_3'' R_3'' R''^2 - 8R_3''^2\left[\pi R''^2 + 8\alpha\right]\right)\tilde{\phi}_{-1/2}\\
&+ \alpha^2 R''^3\left(30\alpha^3 R''^2 - 7\pi\alpha^2 R''^4 + 14\pi^2\alpha R''^6 + 40\pi^3 R_3''^2 R''^6\right.\\
&\left.-4\pi x_3'' R_3'' R''^2\left[6\pi^2 R''^4 + 5\pi\alpha R''^2 - 16\alpha^2\right] + 8\alpha R_3''^2\left[\pi^2 R''^4 + 5\pi\alpha R''^2 - 24\alpha^2\right]\right)\tilde{\phi}_{1/2}\\
&+ 2\pi\alpha R''^5\left(15\alpha^3 R''^2 - 11\pi\alpha^2 R''^4 - 14\pi^2\alpha R''^6 - 36\pi^3 R_3'' R''^6\right.\\
&\left.+2\pi R''^2 x_3'' R_3'' R''^2\left[4\pi^2 R''^4 - 30\pi\alpha R''^2 + 16\alpha^2\right] + 4\alpha R_3''^2\left[3\pi R''^4 + \pi\alpha R''^2 - 24\alpha^2\right]\right)\tilde{\phi}_{3/2}\\
&+ 4\pi^2 R''^7\left(2\alpha^3 R''^2 + 11\pi\alpha^2 R''^4 + 2\pi^2\alpha R''^6 + 4\pi^3 x_3'' R''^6\right.\\
&\left.+4\alpha R_3''\left[\pi x_3'' R''^2\left(11\pi R''^2 + \alpha\right) - R_3''\left(\pi^2 R''^4 - 4\pi\alpha R''^2 + 6\alpha^2\right)\right]\right)\tilde{\phi}_{5/2}\\
&- 8\pi^3 R''^9\left(\alpha R''^2\left[\pi R''^2 + \alpha\right] + 2\pi x_3'' R_3''\left[2\pi R''^2 + \alpha\right] + \alpha R_3''^2\left[13\pi R''^2 + \alpha\right]\right)\tilde{\phi}_{7/2}\\
&+ 16\pi^4 R_3''^2 R''^{11}\left(\pi R''^2 + \alpha\right)\tilde{\phi}_{9/2}\,]
\end{aligned}
$$

$$
\begin{aligned}
&+ \mathbf{R}''\mathbf{e}_3 \left[2\alpha^4 R'' \left(-3\pi x_3'' R''^4 + R_3'' \left[24\alpha R''^2 + \pi R''^4 + 24\pi x_3'' R_3'' R''^2 - 8R_3''^2 \left(\pi R''^2 + 9\alpha\right)\right]\right) \tilde{\phi}_{-1/2}\right. \\
&+ 2\alpha^2 R'' \left(\pi x_3'' R''^4 \left[2\pi^2 R''^4 - 3\pi\alpha R''^2 + 9\alpha^2\right] + \alpha R_3'' \left[\pi R''^2 - 3\alpha\right]\right. \\
&\quad \left.\times \left[24\alpha R''^2 + \pi R''^4 + 24\pi x_3'' R_3'' R''^2 - 8R_3'' \left(\pi R''^2 + 9\alpha\right)\right]\right) \tilde{\phi}_{1/2} \\
&- 2\pi\alpha R''^3 \left(-\pi\alpha x_3'' R''^4 \left[16\pi R''^2 + 9\alpha\right] + R_3'' \left[72\alpha^3 R''^2 - 9\pi\alpha^2 R''^4 - 8\pi^2\alpha R''^6 - 28\pi^3 x_3'' R''^6\right.\right. \\
&\left.\left.+4x_3'' R_3'' \left(2\pi^2 R''^4 - 3\pi\alpha R''^2 + 18\alpha^2\right) + \alpha R_3'' \left(\pi^2 R''^4 + 3\pi\alpha R''^2 - 54\alpha^2\right)\right]\right) \tilde{\phi}_{3/2} \\
&- 4\pi R''^5 \left(\pi\alpha x_3'' R''^4 \left[2\pi R''^2 + 3\alpha\right] + R_3'' \left[18\alpha^3 R''^2 + 25\pi\alpha^2 R''^4 + 2\pi^2\alpha R''^6 + 4\pi^3 x_3'' R''^6\right.\right. \\
&\left.\left.+2\alpha R_3'' \left(\pi x_3'' \left[20\pi R''^2 + 9\alpha\right] - R_3'' \left[2\pi^2 R''^4 + 27\alpha^2\right]\right)\right]\right) \tilde{\phi}_{5/2} \\
&+ 8\pi^3 R_3'' R''^7 \left(2\alpha R''^2 \left[\pi R''^2 + 3\alpha\right] + 2\pi x_3'' R_3'' \left[2\pi R''^2 + 3\alpha\right] + \alpha R_3'' \left[13\pi R''^2 + 9\alpha\right]\right) \tilde{\phi}_{7/2} \\
&\left.-16\pi^4 x_3'' R''^9 \left(\pi R''^2 + 3\alpha\right) \tilde{\phi}_{9/2}\right] \\
&+ \mathbf{e}_3\mathbf{R}'' \left[2\alpha^4 R''^3 \left(3\pi x_3'' R''^2 - R_3'' \left[\pi R''^2 + 12\alpha\right]\right) \tilde{\phi}_{-1/2}\right. \\
&+ 2\alpha^2 R''^3 \left(\pi x_3'' R''^2 \left[8\pi^2 R''^4 + 3\pi\alpha R''^2 - 9\alpha^2\right] - \alpha R_3'' \left[\pi R''^2 - 3\alpha\right] \left[\pi R''^2 + 12\alpha\right]\right) \tilde{\phi}_{1/2} \\
&- 2\pi\alpha R''^5 \left(\pi x_3'' R''^2 \left[4\pi^2 R''^4 + 16\pi\alpha R''^2 + 9\alpha^2\right] + \alpha R_3'' \left[8\pi^2 R''^4 + 3\pi\alpha R''^2 - 36\alpha^2\right]\right) \tilde{\phi}_{3/2} \\
&+ 4\pi^2\alpha R''^7 \left(\pi x_3'' R''^2 \left[2\pi R''^2 + 3\alpha\right] + R_3'' \left[2\pi^2 R''^4 + 12\pi\alpha R''^2 + 9\alpha^2\right]\right) \tilde{\phi}_{5/2} \\
&\left.-8\pi^2\alpha R''^9 \left(\pi R''^2 + 3\alpha\right) \tilde{\phi}_{7/2}\right] \\
&+ \mathbf{R}''\mathbf{R}'' \left[-\alpha^4 R'' \left(12\alpha R''^2 - \pi R''^4 + 8R_3'' \left[3\pi x_3'' R''^2 - R_3'' \left(\pi R''^2 + 9\alpha\right)\right]\right) \tilde{\phi}_{-1/2}\right. \\
&+ \alpha R'' \left(36\alpha^3 R''^2 - 16\pi\alpha^2 R''^4 + \pi^2\alpha R''^6 - 2\pi^3 R''^8\right. \\
&\left.-8\alpha R_3'' \left[\pi R''^2 - 3\alpha\right] \left[3\pi x_3'' R''^2 - R_3'' \left(\pi R''^2 + 9\alpha\right)\right]\right) \tilde{\phi}_{1/2} \\
&+ \pi\alpha R''^3 \left(36\alpha^3 R''^2 - 9\pi\alpha^2 R''^4 - 12\pi^2\alpha R''^6 - 28\pi^3 x_3''^2 R''^6\right. \\
&\left.+4R_3'' \left[\pi x_3'' R''^2 \left(2\pi^2 R''^4 - 3\pi\alpha R''^2 + 18\alpha^2\right) + \alpha R_3'' \left(\pi^2 R''^4 + 3\pi\alpha R''^2 - 54\alpha^2\right)\right]\right) \tilde{\phi}_{3/2} \\
&+ 2\pi^2 R''^5 \left(9\alpha^3 R''^2 + 14\pi\alpha^2 R''^4 + 2\pi^2\alpha R''^6 + 4\pi^3 x_3''^2 R''^6\right. \\
&\left.+2\alpha R_3'' \left[\pi x_3'' R''^2 \left(20\pi R''^2 + 9\alpha\right) - R_3'' \left(2\pi^2 R''^4 + 27\alpha^2\right)\right]\right) \tilde{\phi}_{5/2} \\
&- 4\pi^3 R''^7 \left(\alpha R''^2 \left[\pi R''^2 + 3\alpha\right] + 2\pi x_3'' R_3'' R''^2 \left[2\pi R''^2 + 3\alpha\right] + \alpha R_3''^2 \left[13\pi R''^2 + 9\alpha\right]\right) \tilde{\phi}_{7/2} \\
&\left.+\ 8\pi^4 R_3''^2 R''^9 \left(\pi R''^2 + 3\alpha\right) \tilde{\phi}_{9/2}\right]\Big\},
\end{aligned}
$$