Appendix to "Global mode interaction and pattern selection in the wake of a disk : a weakly nonlinear expansion"

By P. Meliga, J.-M. Chomaz & D. Sipp

Journal of Fluid Mechanics, vol. 633 (2009), pp. 159-189

The objective of this material is to provide the exhaustive expression of all nonlinear coefficients, as has been done in the main paper of the $\tilde{\chi}_A$ and $\tilde{\chi}_B$ coefficients. The $\tilde{\lambda}_A$ coefficient arises from a forcing term of amplitude A, generated by a single $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interaction and by the Reynolds number variation acting here on the second-order solution $\hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A}} = -\delta\mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{A}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}\right) - \delta\boldsymbol{\nabla}^2\hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}\,,$$
$$\tilde{\lambda}_A = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{A}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A}} \right\rangle\,.$$

The $\tilde{\mu}_A$ coefficient arises from a forcing term of amplitude $A|A|^2$, generated by two different $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interactions:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A|A|^2}} = -\mathcal{C}_{-1,\,2}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{A^*}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{AA}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{A}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{AA^*}}\right)\,,$$
$$\tilde{\mu}_A = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{A}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A|A|^2}} \right\rangle\,.$$

The $\tilde{\nu}_A$ coefficient arises from a forcing term of amplitude $A|B^+|^2$, generated by three different $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interactions:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A|B^+|^2}} = -\mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{A}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+B^{+*}}}\right) - \mathcal{C}_{-1,\,2}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1^*}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{AB^+}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2^*}}_{\boldsymbol{B^+A^*}}\right)\,,$$
$$\tilde{\nu}_A = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{A}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{A|B^+|^2}} \right\rangle\,.$$

The $\tilde{\lambda}_B$ coefficient arises from a forcing term of amplitude B^+, generated by a single $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interaction and by the Reynolds number variation acting here on the second-order solution $\hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+}} = -\delta\mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}\right) - \delta\boldsymbol{\nabla}^2\hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{\delta}}\,,$$
$$\tilde{\lambda}_B = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{B^+}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+}} \right\rangle\,.$$

The $\tilde{\mu}_B$ coefficient arises from a forcing term of amplitude $B^+|B^+|^2$, generated by two different $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interactions:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|B^+|^2}} = -\mathcal{C}_{-1,\,2}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^{+*}}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+B^+}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+B^{+*}}}\right)\,,$$
$$\tilde{\mu}_B = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{B^+}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|B^+|^2}} \right\rangle\,.$$

The $\tilde{\nu}_B$ coefficient arises from a forcing term of amplitude $B^+|B^-|^2$, generated by three different $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interactions:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|B^-|^2}} = -\mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1^*}}_{\boldsymbol{B^-}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+B^-}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^-B^{-*}}}\right) - \mathcal{C}_{-1,\,2}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^-}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+B^{-*}}}\right)\,,$$
$$\tilde{\nu}_B = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1}\dagger}_{\boldsymbol{B^+}}\ ,\ \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|B^-|^2}} \right\rangle\,.$$

The $\tilde{\eta}_B$ coefficient arises from a forcing term of amplitude $B^+|A|^2$, generated by three different $\boldsymbol{q^1}$-$\boldsymbol{q^2}$ interactions:

$$\hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|A|^2}} = -\mathcal{C}_{-1,\,2}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}^*}_{\boldsymbol{A}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{AB^+}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{A}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{B^+A^*}}\right) - \mathcal{C}_{1,\,0}\left(\hat{\boldsymbol{u}}^{\boldsymbol{1}}_{\boldsymbol{B^+}}, \hat{\boldsymbol{u}}^{\boldsymbol{2}}_{\boldsymbol{AA^*}}\right)\,,$$

$$\tilde{\nu}_B = \left\langle \hat{\boldsymbol{q}}^{\boldsymbol{1\dagger}}_{\boldsymbol{B^+}}\,,\; \hat{\boldsymbol{F}}^{\boldsymbol{3}}_{\boldsymbol{B^+|A|^2}} \right\rangle\,.$$