Appendix to "Global mode interaction and pattern selection in the wake of a disk : a weakly nonlinear expansion"

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Journal of Fluid Mechanics, vol. 633 (2009), pp. 159-189

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The objective of this material is to provide the exhaustive expression of all nonlinear coefficients, as has been done in the main paper of the $\tilde{\chi}_A$ and $\tilde{\chi}_B$ coefficients. The $\tilde{\lambda}_A$ coefficient arises from a forcing term of amplitude A, generated by a single $q^1 - q^2$ interaction and by the Reynolds number variation acting here on the second-order solution \hat{u}_{δ}^2 :

$$egin{aligned} \hat{F}_{A}^{3} &= -\delta \mathcal{C}_{1,\,0}\left(\hat{u}_{A}^{1},\hat{u}_{\delta}^{2}
ight) - \delta oldsymbol{
abla}^{2} \hat{u}_{\delta}^{2} \,, \ & \tilde{\lambda}_{A} &= \left\langle \hat{q}_{A}^{1\dagger} \,, \, \hat{F}_{A}^{3}
ight
angle \,. \end{aligned}$$

The $\tilde{\mu}_A$ coefficient arises from a forcing term of amplitude $A|A|^2$, generated by two different $q^1 - q^2$ interactions:

The $\tilde{\nu}_A$ coefficient arises from a forcing term of amplitude $A|B^+|^2$, generated by three different $q^1 - q^2$ interactions:

$$egin{aligned} \hat{F}^{3}_{A|B^{+}|^{2}} &= -\mathcal{C}_{1,\,0}\left(\hat{u}^{1}_{A},\hat{u}^{2}_{B^{+}B^{+*}}
ight) - \mathcal{C}_{-1,\,2}\left(\hat{u}^{1*}_{B^{+}},\hat{u}^{2}_{AB^{+}}
ight) - \mathcal{C}_{1,\,0}\left(\hat{u}^{1}_{B^{+}},\hat{u}^{2*}_{B^{+}A^{*}}
ight)\,, \ & ilde{
u}_{A} &= \left\langle \hat{q}^{1\dagger}_{A} \ , \ \hat{F}^{3}_{A|B^{+}|^{2}}
ight
angle \,. \end{aligned}$$

The $\tilde{\lambda}_B$ coefficient arises from a forcing term of amplitude B^+ , generated by a single $q^1 \cdot q^2$ interaction and by the Reynolds number variation acting here on the second-order solution \hat{u}_{δ}^2 :

$$\begin{split} \hat{F}_{B^+}^3 &= -\delta \mathcal{C}_{1,0} \left(\hat{u}_{B^+}^1, \hat{u}_{\delta}^2 \right) - \delta \nabla^2 \hat{u}_{\delta}^2, \\ \tilde{\lambda}_{\scriptscriptstyle B} &= \left\langle \hat{q}_{B^+}^{1\dagger}, \; \hat{F}_{B^+}^3 \right\rangle. \end{split}$$

The $\tilde{\mu}_B$ coefficient arises from a forcing term of amplitude $B^+|B^+|^2$, generated by two different q^1-q^2 interactions:

$$\begin{split} \hat{F}^{3}_{B^{+}|B^{+}|^{2}} &= -\mathcal{C}_{-1,\,2}\left(\hat{u}^{1}_{B^{+*}},\hat{u}^{2}_{B^{+}B^{+}}\right) - \mathcal{C}_{1,\,0}\left(\hat{u}^{1}_{B^{+}},\hat{u}^{2}_{B^{+}B^{+*}}\right) \,,\\ \tilde{\mu}_{B} &= \left\langle \hat{q}^{1\dagger}_{B^{+}} \,, \; \hat{F}^{3}_{B^{+}|B^{+}|^{2}} \right\rangle \,. \end{split}$$

The $\tilde{\nu}_B$ coefficient arises from a forcing term of amplitude $B^+|B^-|^2$, generated by three different $q^1 \cdot q^2$ interactions:

$$\begin{split} \hat{F}^{3}_{B^{+}|B^{-}|}{}^{2} &= -\mathcal{C}_{1,\,0}\left(\hat{u}^{1^{*}}_{B^{-}},\hat{u}^{2}_{B^{+}B^{-}}\right) - \mathcal{C}_{1,\,0}\left(\hat{u}^{1}_{B^{+}},\hat{u}^{2}_{B^{-}B^{-*}}\right) - \mathcal{C}_{-1,\,2}\left(\hat{u}^{1}_{B^{-}},\hat{u}^{2}_{B^{+}B^{-*}}\right),\\ \tilde{\nu}_{B} &= \left\langle \hat{q}^{1\dagger}_{B^{+}} , \ \hat{F}^{3}_{B^{+}|B^{-}|}{}^{2} \right\rangle. \end{split}$$

The $\tilde{\eta}_B$ coefficient arises from a forcing term of amplitude $B^+|A|^2$, generated by three different $q^1 - q^2$ interactions:

$$egin{aligned} \hat{F}^{\mathbf{3}}_{B^+|m{A}|^{\mathbf{2}}} &= -\mathcal{C}_{-1,\,2}\left(\hat{u}^{\mathbf{1}^*}_{m{A}},\hat{u}^{\mathbf{2}}_{m{A}B^+}
ight) - \mathcal{C}_{1,\,0}\left(\hat{u}^{\mathbf{1}}_{m{A}},\hat{u}^{\mathbf{2}}_{B^+m{A}^*}
ight) - \mathcal{C}_{1,\,0}\left(\hat{u}^{\mathbf{1}}_{B^+},\hat{u}^{\mathbf{2}}_{m{A}A^*}
ight) \ , \ & ilde{
u}_B &= \left\langle \hat{q}^{\mathbf{1}\dagger}_{B^+} \ , \ \hat{F}^{\mathbf{3}}_{B^+|m{A}|^2}
ight
angle \ . \end{aligned}$$