Capital Structure with Information about the Upside and the Downside

Internet Appendix

Pierre Chaigneau

1 Optimal securities

This section considers general securities, without restricting attention to debt and equity. Securities are such that the payoff of security h at $t = 1$ can be a function of asset output y, and is denoted by $s_h(y)$. The payoff vector of security h is $(s_h(L), s_h(M), s_h(H))$. For any number $N \in \{1, 2, \dots\}$ of securities used, securities' payoffs must satisfy:

$$s_h(y) \geq 0 \qquad \text{for } y \in \{L, M, H\} \tag{35}$$

$$\sum_{h=1}^{N} s_h(y) = y \qquad \text{for } y \in \{L, M, H\} \tag{36}$$

First, any security must have a nonnegative payoff in any state (equation (35)). Second, as in seminal security design papers (Allen and Gale (1988) and Boot and Thakor (1993)), designing securities is equivalent to splitting claims on the asset's output (equation (36)). Securities with linearly dependent payoffs (a payoff vector of (s^L, s^M, s^H) for one security and of (ks^L, ks^M, ks^H) with $k > 0$ for another) are counted as one security.

In principle, there is an infinity of optimal security designs, including security designs with redundant securities, and security designs that are equivalent for asset valuation when trading is costless. Assumptions 2 and 3 allow us to narrow down the set of potentially optimal security designs by postulating small costs of security issuance and trading.

Assumption 2. *A security design with n securities, with $n > 2$, involves an incremental cost $c(n) > 0$ for the market maker.*

Assumption 3. *The market maker incurs an additional cost for any market transaction above the second.*

If the cost of two market transactions were equal to $C > 0$, the fair pricing condition would need to be modified to incorporate that cost, and so would market pricing and firm valuation. To simplify the exposition, since there will be at minimum two transactions, and since it is the relative cost of additional transactions that matters for security design, we set this cost to zero.

We now specify how these costs enter the market maker's objective function. In this section, we assume that the market maker has lexicographic preferences whereby it first maximizes the probability that the firm accepts the offer, and second it minimizes the administrative costs in Assumptions 2 and 3. This captures the notion that these costs are small compared to the adverse selection problem. Assumption 2 allows to rule out security designs with redundant securities. Assumption 3 allows to rule out security designs which result in unnecessary transactions.

The next Lemma uses Assumptions 2 and 3 to reduce the set of potentially optimal security designs to the ones described in Propositions 1 and 5 (the latter, in section 2 of this Internet Appendix, is a generalization of the security design described in Proposition 3).

Lemma 2. *The market maker will only offer security designs of the following types:*

- *The securities have payoff vectors of the following form, with* $\gamma \in [0, L]$*:*

$$(\gamma, \gamma, \gamma) \tag{37}$$

$$(L-\gamma, M-\gamma, H-\gamma) = (L, M, H) - (\gamma, \gamma, \gamma). \tag{38}$$

- *The securities have payoff vectors of the following form, with* $\Gamma \in [0, L]$*:*

$$(L-\Gamma, M-\Gamma, M-\Gamma) = (L, M, M) - (\Gamma, \Gamma, \Gamma) \tag{39}$$

$$(\Gamma, \Gamma, H-M+\Gamma) = (0, 0, H-M) + (\Gamma, \Gamma, \Gamma). \tag{40}$$

We now explain the logic behind Lemma 2. In the first case, security issuance is such that markets are incomplete. Then there is only one information-sensitive security, so that the security design is either unleveraged equity (for $\gamma = 0$) or riskfree debt and leveraged equity (for $\gamma \in (0, L]$). In the second case of Lemma 2, since a riskfree asset already exists, the markets are complete with two assets with nonconstant and linearly independent payoff vectors. In the absence of arbitrage opportunities, the overall value of the asset (the sum of securities' values) is independent of security design as long as markets are complete. Furthermore, payoff vectors in equations (39) and (40) are the only ones such that only two securities are used, and a hedger optimally trades only one of these two securities, which minimizes the security issuance costs and transaction costs borne by the market maker.

Proof of Lemma 2:

First step. Consider for now a security design offered by the market maker that does not involve two or more securities with nonconstant and linearly independent payoffs. There are two possible cases. First, suppose that the security design involves only one security.

Because of the assumption in equation (36), this security must be unleveraged equity with a payoff vector (L, M, H). Second, suppose that the security design involves two or more securities – but not two securities with nonconstant and linearly independent payoffs (this case is studied below in the second step). Unless the security design involves a riskfree security with constant payoff vector and another security, this is a contradiction, since by assumption two securities with linearly dependent payoffs are counted as one security. Thus, the only remaining possibility is that the security design involves a riskfree security and another security which must be such that the assumption in equation (36) holds. These two securities therefore have payoff vectors: (γ, γ, γ) and $(L - \gamma, M - \gamma, H - \gamma)$, with $\gamma \in (0, L]$ ($\gamma = 0$ corresponds to the first case mentioned in this paragraph). This concludes the first step.

Second step. Consider for now a security design offered by the market maker that involves two or more securities with nonconstant and linearly independent payoffs. Consider a postulated equilibrium with a security design different from the one described in equations (39) and (40) in Lemma 2. The proof will show that there exists another equilibrium with a security design as described in equations (39) and (40) instead which is strictly preferred by the market maker (MM).

With a security design with two securities with nonconstant and linearly independent payoffs, the assumed existence of the riskfree asset then implies that there are three assets with linearly independent payoffs. Given that the economy has three possible states of the world (L, M, and H), this in turn implies that markets are complete. When markets are complete, portfolio allocations and prices can equivalently be analyzed with existing securities or with contingent claims. We now use the latter approach.

First, consider market orders. Let $\tilde{w}_H^-(\mathbf{x})$ denote the stochastic wealth of a hedger with negative exposure δ_H to $y = H$ with portfolio $\mathbf{x} \equiv \{x_L, x_M, x_H\}$. The problem of this hedger is to choose a portfolio of contingent claims such that:

$$\min_{\mathbf{x}} var\left(\tilde{w}_H^-(\mathbf{x})\right), \text{ where } w_H^-(\mathbf{x}) = \begin{cases} x_L - \sum_{k=L,M,H} x_k p_k & \text{for } y = L \\ x_M - \sum_{k=L,M,H} x_k p_k & \text{for } y = M \\ -\delta_H + x_H - \sum_{k=L,M,H} x_k p_k & \text{for } y = H \end{cases}$$

The solution to this optimization problem is simply: $\mathbf{x} = \{0, 0, \delta_H\}$, which achieves a zero variance. Likewise, a hedger with positive exposure δ_H to $y = H$ chooses the portfolio of contingent claims $\mathbf{x} = \{0, 0, -\delta_H\}$, and a hedger with positive (negative) exposure δ_L to $y = L$ chooses the portfolio of contingent claims $\mathbf{x} = \{-\delta_L, 0, 0\}$ ($\mathbf{x} = \{\delta_L, 0, 0\}$).

Second, consider asset valuation. When markets are complete, in the absence of arbitrage the vector of state prices exists and is unique. Denote state prices by $\{p_L, p_M, p_H\}$. The price of any security h with payoff vector $(s_h(L), s_h(M), s_h(H))$ is then: $P_h \equiv \sum_{y=L,M,H} p_y s_h(y)$.

Firm value is the sum of security prices:

$$(41) \qquad \sum_{h=1}^{N} P_h = \sum_{h=1}^{N} \sum_{y=L,M,H} p_y s_h(y) = \sum_{y=L,M,H} p_y \sum_{h=1}^{N} s_h(y) = \sum_{y=L,M,H} p_y y,$$

where the last equality uses equation (36). Thus, asset valuation only depends on state prices, which are independent of security design as long as markets are complete.

In sum, any security design with strictly more than two securities with nonconstant and linearly independent payoffs yields equivalent market orders and asset valuation as a security design as in equations (39) and (40), but the latter is strictly preferred by the MM due to the incremental security design cost that would be incurred by the MM with the former.

In the last part of the proof, we will consider the subset of security designs with two securities with nonconstant and linearly independent payoffs. We will describe the structure of security payoffs such that a hedger can achieve a variance of zero by trading only one security. This type of security design, if it exists, will be strictly preferred by the MM to other security designs in the aforementioned subset, since it minimizes the transaction costs incurred by the MM.

Consider the optimization problem of a trader with negative exposure δ_H to $y = H$, and consider a given security with payoff vector $(s_F(L), s_F(M), s_F(H))$, denoted as security F with price p_F:

$$\min_{x_F} var\left(\tilde{w}_H^-(x_F)\right), \text{ where } w_H^-(x_F) = \begin{cases} x_F s_F(L) - x_F p_F & \text{for } y = L \\ x_F s_F(M) - x_F p_F & \text{for } y = M \\ -\delta_H + x_F s_F(H) - x_F p_F & \text{for } y = H \end{cases}$$

A variance of zero is achieved by trading only this security if and only if:

$$\begin{aligned} & x_F s_F(L) - x_F p_F = x_F s_F(M) - x_F p_F = -\delta_H + x_F s_F(H) - x_F p_F \\ \Leftrightarrow \quad & s_F(L) = s_F(M), \quad s_F(H) \neq s_F(M), \quad \text{and} \quad x_F = \frac{\delta_H}{s_F(H) - s_F(M)}, \end{aligned}$$

i.e., a variance of zero can be achieved if and only if the payoff vector of the security takes the form (Γ, Γ, F), with $F \neq \Gamma$. In this case, and only in this case, the hedger with negative exposure to $y = H$ can achieve a variance of zero by trading only one security, with $x_F = \frac{\delta_H}{F - \Gamma}$. The demonstration for the case of a hedger with positive exposure to $y = H$ is similar (he achieves a variance of zero with $x_F = -\frac{\delta_H}{F-\Gamma}$).

Now consider the optimization problem of a trader with negative exposure δ_L to $y = L$, and consider a given security with payoff vector $(s_f(L), s_f(M), s_f(H))$, denoted as security

f with price p_f:

$$\min_{x_f} var\left(\tilde{w}_L^-(x_f)\right), \text{ where } w_L^-(x_f) = \begin{cases} -\delta_L + x_f s_f(L) - x_f p_f & \text{for } y = L \\ x_f s_f(M) - x_f p_f & \text{for } y = M \\ x_f s_f(H) - x_f p_f & \text{for } y = H \end{cases}$$

A variance of zero is achieved by trading only this security if and only if:

$$\begin{aligned} & -\delta_L + x_f s_f(L) - x_f p_f = x_f s_f(M) - x_f p_f = x_f s_f(H) - x_f p_f \\ \Leftrightarrow \quad & s_F(H) = s_F(M), \quad s_F(L) \neq s_F(M), \text{ and } \quad x_f = -\frac{\delta_L}{s_f(M) - s_f(L)}, \end{aligned}$$

i.e., a variance of zero can be achieved if and only if the payoff vector of the security takes the form (f, k, k), with $f \neq k$. In this case, and only in this case, the hedger with negative exposure to $y = L$ can achieve a variance of zero by trading only one security, with $x_f = -\frac{\delta_L}{k-f}$. The demonstration for the case of a hedger with positive exposure to $y = L$ is similar (he achieves a variance of zero with $x_f = \frac{\delta_L}{k-f}$).

We have shown that a hedger with exposure to $y = H$ can achieve a variance of zero by trading only one security if and only if this security has a payoff vector of the form (Γ, Γ, F) with $\Gamma \neq F$. Likewise, a hedger with exposure to $y = L$ can achieve a variance of zero by trading only one security if and only if this security has a payoff vector of the form (f, k, k) with $f \neq k$.

Now consider a security with payoff vector (Γ, Γ, F), and another security with payoff vector (f, k, k). These parameter values must be such that assumption (36) on security design is satisfied:

$$\begin{cases} \Gamma + f = L \\ \Gamma + k = M \\ k + F = H \end{cases}$$

This is a system of three linear equations with four unknowns. This system has an infinity of solutions, which must satisfy:

$$\begin{cases} f = L - \Gamma \\ k = M - \Gamma \\ F = H - M + \Gamma \end{cases}$$

where, according to assumption (35), the free parameter Γ is such that $\Gamma \in [0, L]$. That is, for $\Gamma \in [0, L]$, the securities with payoffs (Γ, Γ, F) and (f, k, k) have payoff vectors such that:

$$(f, k, k) = (L - \Gamma, M - \Gamma, M - \Gamma) = (L, M, M) - (\Gamma, \Gamma, \Gamma) \tag{42}$$

$$(\Gamma, \Gamma, F) = (\Gamma, \Gamma, H - M + \Gamma) = (0, 0, H - M) + (\Gamma, \Gamma, \Gamma) \tag{43}$$

In sum, the securities described in equations (42) and (43), for $\Gamma \in [0, L]$, make up the only security design consisting of two securities with nonconstant and linearly independent payoffs such that a hedger trades only one security. As is standard, an informed trader will submit the same trades as a hedger, otherwise he would be identified as an informed trader and make zero profit. This concludes the second step. ■

2 Composite securities

In this subsection, we revisit the trading phase equilibrium in section III.B by considering securities with payoff vectors $(L - \Gamma, M - \Gamma, M - \Gamma)$ and $(\Gamma, \Gamma, H - M + \Gamma)$ for $\Gamma \in [0, L]$. These are composite securities which were derived in section 1 of this Internet Appendix. The first security has the payoff of debt with face value M, with payoff vector (L, M, M), minus a riskfree security with payoff vector (Γ, Γ, Γ) where $\Gamma \in [0, L]$. The second security has the payoff of leveraged equity, with payoff vector $(0, 0, H - M)$, plus the same riskfree security with payoff vector (Γ, Γ, Γ). For example, in the polar cases $\Gamma = 0$ and $\Gamma = L$, the payoff vectors of the securities used are:

$$\text{With } \Gamma = 0 : \begin{cases} (L, M, M) \\ (0, 0, H - M) \end{cases} \qquad \text{With } \Gamma = L : \begin{cases} (0, M - L, M - L) \\ (L, L, H - M + L) \end{cases}$$

With $\Gamma = 0$, the securities are debt with face value M and leveraged equity, as in Proposition 3. With $\Gamma = L$, there is a composite security which combines senior debt with face value L and equity, and the other security is junior debt with face value $M - L$. Intuitively, the asset generates an output of L in the worst case scenario, and this baseline level of output can be allocated either to the risky security exposed to the downside of the distribution ("risky debt"), with $\Gamma = 0$, or to the risky security exposed to the upside of the distribution ("leveraged equity"), with $\Gamma = L$, or it can be allocated partially to both, with $\Gamma \in (0, L)$.

The expected payoffs conditional on asset type $\{\theta_1, \theta_2\}$ of "leveraged equity" (indexed by E) and "risky debt" (indexed by D), respectively, are:

$$\begin{aligned} v_E(\theta_1 = 1) &= \tfrac{H-M}{2} + \Gamma, \quad & v_E(\theta_1 = 0) &= \Gamma, \\ v_D(\theta_2 = 1) &= M - \Gamma, \quad & v_D(\theta_2 = 0) &= \tfrac{L+M}{2} - \Gamma. \end{aligned}$$

The value of leveraged equity only depends on θ_1, and the value of risky debt only depends on θ_2.

To focus on properties of this equilibrium when the firm accepts the market maker's offer and sells its asset, we simply assume that $\overline{\beta} = 0$, i.e. the firm is highly "impatient". Proposition 3 generalizes as follows.

Proposition 5. *When securities offered by the market maker have payoff vectors $(\Gamma, \Gamma, H - M + \Gamma)$ and $(L - \Gamma, M - \Gamma, M - \Gamma)$ for $\Gamma \in [0, L]$:*

- *Trader 1 trades leveraged equity by submitting a market order for a quantity $\frac{\delta_H}{H-M}$. A hedger with a negative (positive) exposure to $y = H$ buys (sells) this amount; an informed trader who observes $\theta_1 = 1$ buys this amount; an informed trader who observes $\theta_1 = 0$ sells this amount.*
  Trader 2 trades risky debt by submitting a market order for a quantity $\frac{\delta_L}{M-L}$. A hedger with a negative (positive) exposure to $y = L$ sells (buys) this amount; an informed trader who observes $\theta_2 = 1$ buys this amount; an informed trader who observes $\theta_2 = 0$ sells this amount.

- *The market price P_E of leveraged equity conditional on the observed order is:*

$$P_E(buy) = \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma \tag{44}$$

$$P_E(sell) = \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma \tag{45}$$

  The market price P_D of risky debt conditional on the observed order is:

$$P_D(buy) = \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \tag{46}$$

$$P_D(sell) = \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \tag{47}$$

- *The expected market value of the asset conditional on asset type is:*

$$\mathbb{E}\left[P_E + P_D | \theta_1 = 1, \theta_2 = 1\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \tag{48}$$

$$\mathbb{E}\left[P_E + P_D | \theta_1 = 1, \theta_2 = 0\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \tag{49}$$

$$\mathbb{E}\left[P_E + P_D | \theta_1 = 0, \theta_2 = 1\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \tag{50}$$

$$\mathbb{E}\left[P_E + P_D | \theta_1 = 0, \theta_2 = 0\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \tag{51}$$

Even though the prices of each of the two securities depend on the parameter Γ, the market value of the asset does not, since changes in Γ merely reallocate a constant payoff across securities. Proposition 5 and its proof below are therefore very similar to Proposition 3 and its proof, and some parts are abbreviated.

Proof of Proposition 5:

Suppose that the market offers offers securities with payoff vectors $(\Gamma, \Gamma, H - M + \Gamma)$ and $(L - \Gamma, M - \Gamma, M - \Gamma)$ for $\Gamma \in [0, L]$, for $\Gamma \in [0, L]$, associated with subscripts D and E, respectively. Consider the optimization problem of a hedger with negative exposure to $y = H$:

$$\min_{x_D, x_E} var\left(\tilde{w}_H^-(x_D, x_E)\right), \text{ where } w_H^-(x_D, x_E) = \begin{cases} x_D(L - \Gamma - p_D) + x_E(\Gamma - p_E) & \text{for } y = L \\ x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) & \text{for } y = M \\ -\delta_H + x_D(M - \Gamma - p_D) + x_E(H - M + \Gamma - p_E) & \text{for } y = H \end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} x_D(L - \Gamma - p_D) + x_E(\Gamma - p_E) = x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) \\ x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) = -\delta_H + x_D(M - \Gamma - p_D) + x_E(H - M + \Gamma - p_E) \end{cases}$$

The first equality gives $x_D = 0$, and the second equality gives $x_E = \frac{\delta_H}{H-M}$. Likewise, for a hedger with positive exposure to $y = H$, $x_D = 0$ and $x_E = -\frac{\delta_H}{H-M}$.

Consider the optimization problem of a hedger with negative exposure to $y = L$:

$$\min_{x_D, x_E} var\left(\tilde{w}_L^-(x_D, x_E)\right), \text{ where } w_L^-(x_D, x_E) = \begin{cases} -\delta_L + x_D(L - \Gamma - p_D) + x_E(\Gamma - p_E) & \text{for } y = L \\ x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) & \text{for } y = M \\ x_D(M - \Gamma - p_D) + x_E(H - M + \Gamma - p_E) & \text{for } y = H \end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} -\delta_L + x_D(L - \Gamma - p_D) + x_E(\Gamma - p_E) = x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) \\ x_D(M - \Gamma - p_D) + x_E(\Gamma - p_E) = x_D(M - \Gamma - p_D) + x_E(H - M + \Gamma - p_E) \end{cases}$$

The second equality gives $x_E = 0$, and the second equality gives $x_D = -\frac{\delta_L}{M-L}$. Likewise, for a hedger with positive exposure to $y = L$, $x_E = 0$ and $x_D = \frac{\delta_L}{M-L}$.

Using standard arguments, if trader 1 is informed, he will buy or sell the same quantity of leveraged equity as a hedger – either 'buy', corresponding to $x_E = \frac{\delta_H}{H-M}$, or 'sell', corresponding to $x_E = -\frac{\delta_H}{H-M}$ – otherwise the order would reveal his information to the market maker. Likewise, if trader 2 is informed, he will buy or sell the same quantity of risky debt as a hedger – either 'buy', corresponding to $x_D = \frac{\delta_L}{M-L}$, or 'sell', corresponding to $x_D = -\frac{\delta_L}{M-L}$

– otherwise the order would reveal his information to the market maker.

We now take the market maker's perspective. The market price of leveraged equity conditional on a buy or sell order is:

$$
\begin{aligned}
P_E(\text{buy}) &= Pr\left(\theta_1 = 1|\text{buy}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{buy}\right) v_E(\theta_1 = 0) = \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma \\
P_E(\text{sell}) &= Pr\left(\theta_1 = 1|\text{sell}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{sell}\right) v_E(\theta_1 = 0) = \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma
\end{aligned}
$$

The market price of risky debt conditional on a buy or sell order is:

$$
\begin{aligned}
P_D(\text{buy}) &= Pr\left(\theta_2 = 1|\text{buy}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{buy}\right) v_D(\theta_2 = 0) \\
&= \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\
P_D(\text{sell}) &= Pr\left(\theta_2 = 1|\text{sell}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{sell}\right) v_D(\theta_2 = 0) \\
&= \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma
\end{aligned}
$$

Total expected asset value conditional on market orders is:

$$
\begin{aligned}
P_E(\text{buy}) + P_D(\text{buy}) &= \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\
&= \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \\
P_E(\text{buy}) + P_D(\text{sell}) &= \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \\
&= \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \\
P_E(\text{sell}) + P_D(\text{buy}) &= \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\
&= \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \\
P_E(\text{sell}) + P_D(\text{sell}) &= \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \\
&= \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2}
\end{aligned}
$$

Finally, we take the firm's perspective. Expected security prices depend on the asset type as follows:

$$
\begin{aligned}
\mathbb{E}\left[P_E|\theta_1=1\right] &= \frac{1+\mu_1^2}{2}\frac{H-M}{2}+\Gamma \\
\mathbb{E}\left[P_E|\theta_1=0\right] &= \frac{1-\mu_1^2}{2}\frac{H-M}{2}+\Gamma \\
\mathbb{E}\left[P_D|\theta_2=1\right] &= \frac{3+\mu_2^2}{2}\frac{M}{2}+\frac{1-\mu_2^2}{2}\frac{L}{2}-\Gamma \\
\mathbb{E}\left[P_D|\theta_2=0\right] &= \frac{3-\mu_2^2}{2}\frac{M}{2}+\frac{1+\mu_2^2}{2}\frac{L}{2}-\Gamma
\end{aligned}
$$

Total expected asset value for different asset types are:

$$
\begin{aligned}
\mathbb{E}\left[P_E+P_D|\theta_1=1,\theta_2=1\right] &= \frac{1+\mu_1^2}{4}H+\frac{2-\mu_1^2+\mu_2^2}{4}M+\frac{1-\mu_2^2}{4}L \\
\mathbb{E}\left[P_E+P_D|\theta_1=1,\theta_2=0\right] &= \frac{1+\mu_1^2}{4}H+\frac{2-\mu_1^2-\mu_2^2}{4}M+\frac{1+\mu_2^2}{4}L \\
\mathbb{E}\left[P_E+P_D|\theta_1=0,\theta_2=1\right] &= \frac{1-\mu_1^2}{4}H+\frac{2+\mu_1^2+\mu_2^2}{4}M+\frac{1-\mu_2^2}{4}L \\
\mathbb{E}\left[P_E+P_D|\theta_1=0,\theta_2=0\right] &= \frac{1-\mu_1^2}{4}H+\frac{2+\mu_1^2-\mu_2^2}{4}M+\frac{1+\mu_2^2}{4}L
\end{aligned}
$$

The adverse selection discount $\mathrm{ASD}^2_{\theta_1,\theta_2}$, which for any given asset type $\{\theta_1,\theta_2\}$ is equal to $v_{\theta_1,\theta_2}-\mathbb{E}\left[P_E+P_D|\theta_1,\theta_2\right]$, is:

$$
\mathrm{ASD}^2_{1,1} = \frac{1-\mu_1^2}{4}H+\frac{\mu_1^2-\mu_2^2}{4}M+\frac{-1+\mu_2^2}{4}L \tag{52}
$$

$$
\mathrm{ASD}^2_{1,0} = \frac{1-\mu_1^2}{4}H+\frac{-2+\mu_1^2+\mu_2^2}{4}M+\frac{1-\mu_2^2}{4}L \tag{53}
$$

$$
\mathrm{ASD}^2_{0,1} = \frac{-1+\mu_1^2}{4}H+\frac{2-\mu_1^2-\mu_2^2}{4}M+\frac{-1+\mu_2^2}{4}L \tag{54}
$$

$$
\mathrm{ASD}^2_{0,0} = \frac{-1+\mu_1^2}{4}H-\frac{\mu_1^2-\mu_2^2}{4}M+\frac{1-\mu_2^2}{4}L \tag{55}
$$

With $\mu_1\in(0,1)$, $\mu_2\in(0,1)$, and $H>M>L$, the adverse selection discount for an asset type $\{1,1\}$ in equation (52) is the highest. ■

3 High discount factor

Let $\beta\to 1$. By construction, in any equilibrium that involves the participation of more than one asset type, the adverse selection discount will be strictly positive for some asset types, and strictly negative for other asset types (according to the zero profit condition for the market maker, the adverse selection discount must by construction be zero on average across participating asset types in any potential equilibrium). As $\beta \to 1$, this is a contradiction, because an asset type with a strictly positive adverse selection discount will optimally not participate.

Consequently, the only possible equilibria involves the participation of either zero or one asset type, whose adverse selection discount is by construction zero (since its type is known). Since the market maker maximizes the probability of asset type participation, the selected equilibrium involves the participation of one asset type. Suppose that this asset type is other than the low type on both dimensions ($\{0, 0\}$), with intrinsic value v_{θ_1,θ_2} as defined in equation (1), with $\{\theta_1, \theta_2\} \neq \{0, 0\}$. Then by fair pricing, market valuation is v_{θ_1,θ_2}, which is strictly higher than the intrinsic value $v_{0,0}$ of a $\{0, 0\}$ type. This implies that asset type $\{0, 0\}$ will participate as well, a contradiction. In sum, the only possible equilibrium involves the participation of a $\{0, 0\}$ type which is priced as $v_{0,0}$ (see equation (1)) regardless of market orders.

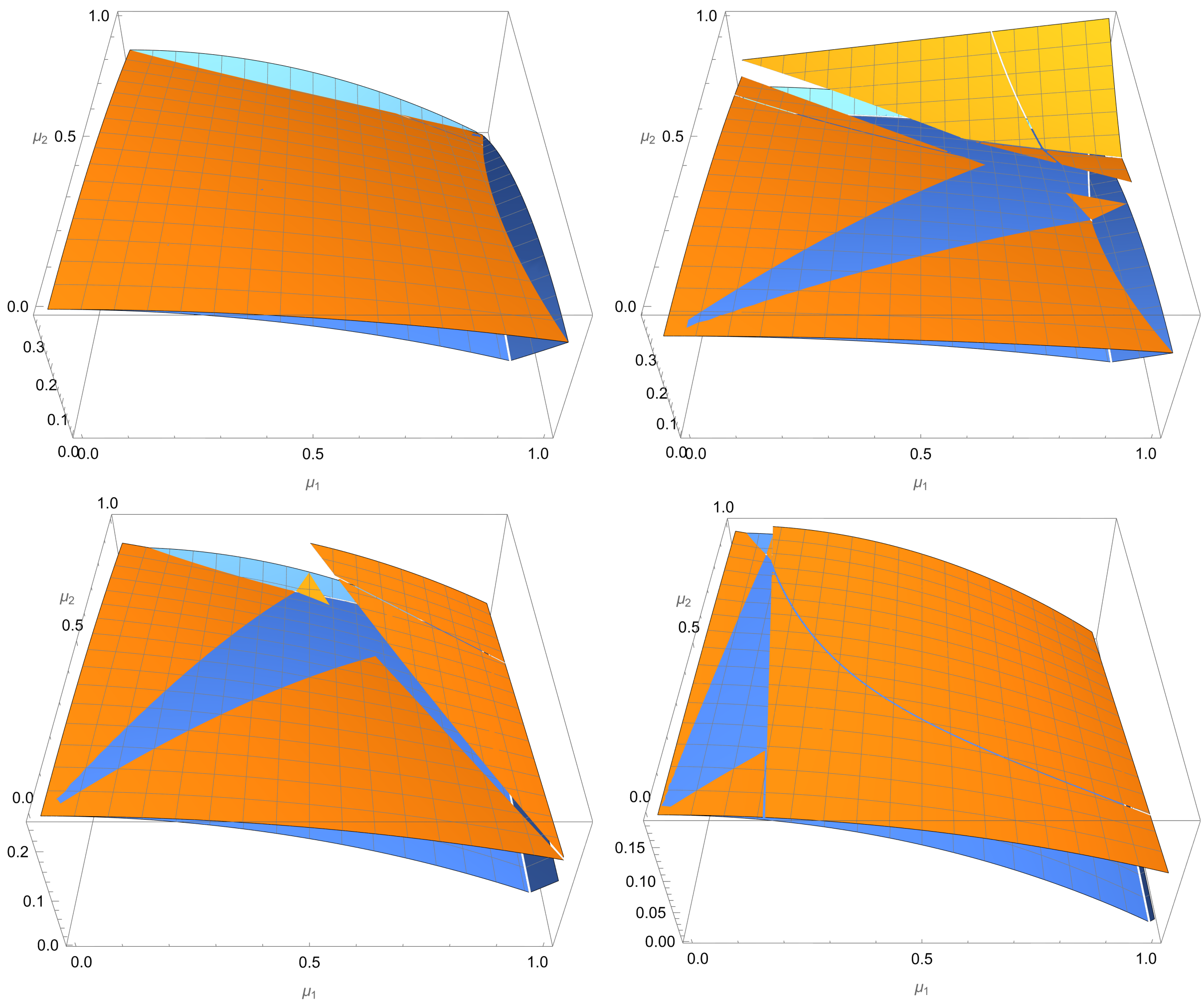
4 Plots of the highest relative adverse selection discount

Figure 11 depicts the ratio of the highest adverse selection discount to intrinsic asset value (HRASD) across asset types as a function of the probabilities of informed trading μ_1 and μ_2 for the security designs described in sections III.A and III.B and several values of the upside $H - M$ and downside $M - L$. Demarcations in the surfaces correspond to changes in the expression for the highest relative ASD, due either to changes in the asset type with the highest relative ASD or to changes in the relevant set of parameter values (cf. the condition that differs in Propositions 1 and 2). The discontinuities in the orange surface correspond to points such that highest ASD with the security design described in section III.A switches from the one described in Proposition 1 to the one described in Proposition 2.

Figure 11

HRASD for two capital structures.

In all graphs, as a function of μ_1 and μ_2, the orange surface is the highest value of $\mathrm{ASD}^1_{i,j}/v_{i,j}$ across asset types, and the blue surface is the highest value of $\mathrm{ASD}^2_{i,j}/v_{i,j}$ across asset types. Top row left: $H - M = 1$, $M - L = 1$. Top row right: $H - M = 0.5$, $M - L = 1$. Bottom row left: $H - M = 1$, $M - L = 0.5$. Bottom row right: $H - M = 1$, $M - L = 0.1$.



5 Probability of informed trading and value relevance

This section studies numerically the outcome when the probability of informed trading is increasing in the value-relevance of each dimension of uncertainty. We assume that the greater the upside $H - M$ (respectively downside $M - L$), the higher the probability of informed trading on the upside (resp. downside). Specifically, letting X be the upside $H - M$ or downside $M - L$, we let the probability μ of informed trading on this dimension be $\mu = a \times X^b$, for $a \in (0, 1)$, $b \in (0, 2)$, and several values of $X \in (0, 1]$, which ensures that $\mu_1 \in (0, 1)$ and $\mu_2 \in (0, 1)$. The parameter a parameterizes the sensitivity of the probability of informed trading to the value-relevance of each dimension of uncertainty. The parameter b parameterizes the curvature of this probability with respect to the value-relevance of each dimension of uncertainty: it is concave for $b < 1$, linear for $b = 1$, and convex for $b > 1$.

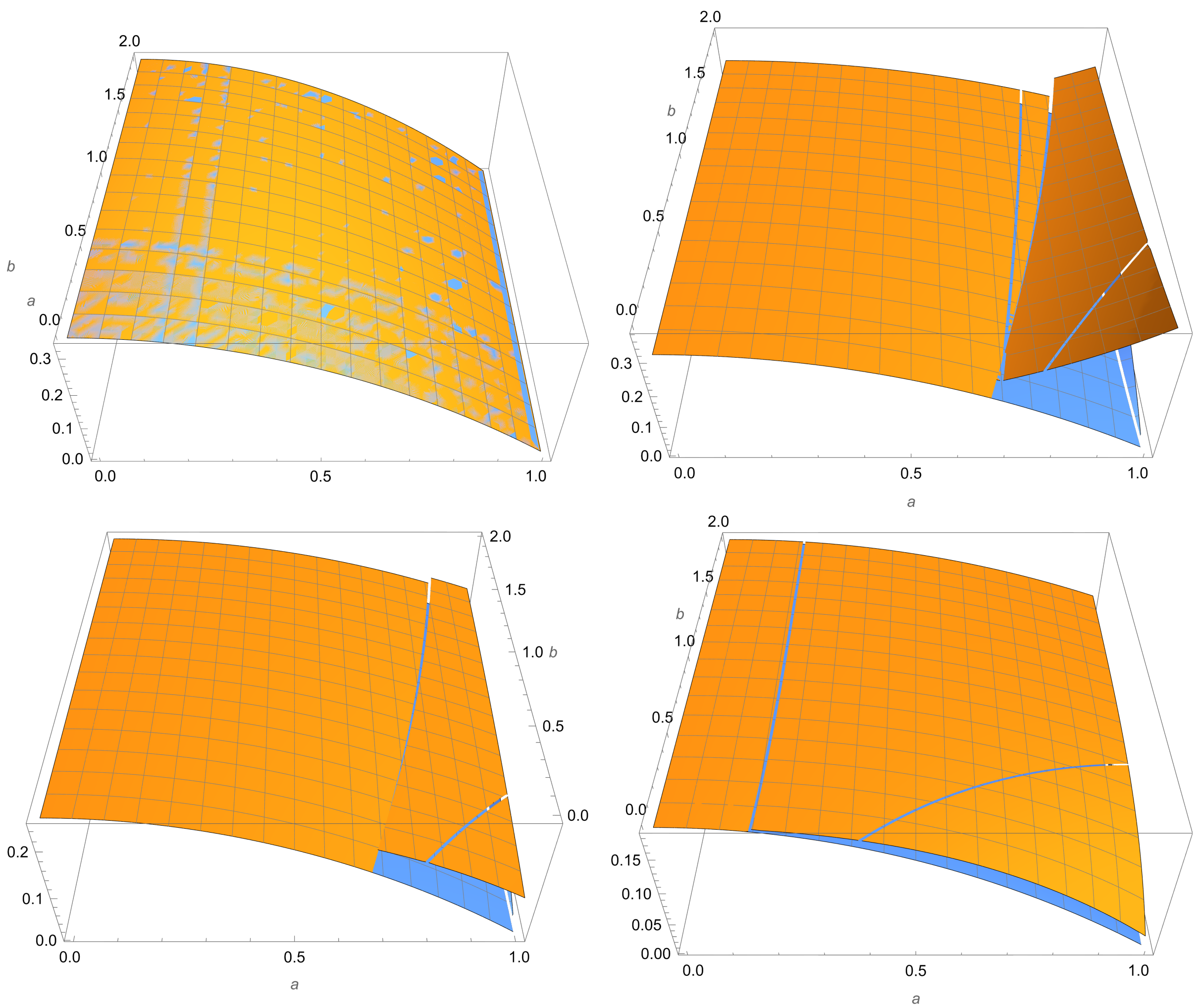
Numerical results in Figure 12 have the following implications. First, if both dimensions of uncertainty have the same value relevance ($H - M = M - L$), the highest ASD is the same for both types of capital structures, so that the market maker is indifferent with regards to the capital structure offered. Second, if the probability of informed trading is higher for the more value relevant dimension of uncertainty, then the equilibrium involves two information-sensitive securities – leveraged equity and risky debt. Indeed, in this case, the intra-subsidy effect does not exist.

Demarcations in the surfaces correspond to changes in the expression for the highest relative ASD, due either to changes in the asset type with the highest relative ASD or to changes in the relevant set of parameter values (cf. the condition that differs in Propositions 1 and 2). Discontinuities in the orange surface correspond to points such that the highest relative adverse selection discount with the security design described in section III.A switches from the one described in Proposition 1 to the one described in Proposition 2.

Figure 12

HRASD for two capital structures.

In all graphs, the orange surface is the highest value of $\mathrm{ASD}^1_{i,j}/v_{i,j}$ across asset types, and the blue surface is the highest value of $\mathrm{ASD}^2_{i,j}/v_{i,j}$ across asset types. In all cases, we have $\mu_1 = a \times (H - M)^b$, and $\mu_2 = a \times (M - L)^b$. Top row left: $H = 2$, $M = 1$, $L = 0$. Top row right: $H = 1.5$, $M = 1$, $L = 0$. Bottom row left: $H = 2$, $M = 1$, $L = 0.5$. Bottom row right: $H = 2$, $M = 1$, $L = 0.9$.



6 Bayesian updating: additional calculations

A Additional calculations for the proof of Proposition 1:

The probability of buy or sell orders (non-ordered) conditional on asset type is:

$$
\begin{aligned}
Pr\left(\text{buy},\text{buy}|\theta_1=1,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\text{buy},\text{sell}|\theta_1=1,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\text{sell},\text{sell}|\theta_1=1,\theta_2=1\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{buy},\text{buy}|\theta_1=1,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{buy},\text{sell}|\theta_1=1,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{sell},\text{sell}|\theta_1=1,\theta_2=0\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\text{buy},\text{buy}|\theta_1=0,\theta_2=1\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\text{buy},\text{sell}|\theta_1=0,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{sell},\text{sell}|\theta_1=0,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{buy},\text{buy}|\theta_1=0,\theta_2=0\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\text{buy},\text{sell}|\theta_1=0,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\text{sell},\text{sell}|\theta_1=0,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}
\end{aligned}
$$

Using Bayesian updating, the distribution of $\tilde{\theta}_1, \tilde{\theta}_2$ conditional on observed orders is given by:

$$\begin{aligned}
Pr\left(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{buy}\right) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{1}{4}}{\frac{1}{4}} = \frac{1+\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{sell}\right) &= \frac{\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right) \\
Pr\left(\theta_1 = 1, \theta_2 = 1|\text{sell}, \text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\theta_1 = 1, \theta_2 = 0|\text{buy}, \text{buy}\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\theta_1 = 1, \theta_2 = 0|\text{buy}, \text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right) \\
Pr\left(\theta_1 = 1, \theta_2 = 0|\text{sell}, \text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\theta_1 = 0, \theta_2 = 1|\text{buy}, \text{buy}\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} \\
Pr\left(\theta_1 = 0, \theta_2 = 1|\text{buy}, \text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right) \\
Pr\left(\theta_1 = 0, \theta_2 = 1|\text{sell}, \text{sell}\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{buy}\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2} \\
Pr\left(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right) \\
Pr\left(\theta_1 = 0, \theta_2 = 0|\text{sell}, \text{sell}\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}
\end{aligned}$$

The asset's market price (the sum of security prices) conditional on observed orders is:

$$
\begin{aligned}
P_U\,(\text{buy},\text{buy}) &= Pr\,(\theta_1=1,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=1)\\
&\quad +Pr\,(\theta_1=1,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=0)\\
&\quad +Pr\,(\theta_1=0,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=1)\\
&\quad +Pr\,(\theta_1=0,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=0)\\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{H+M}{2}+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\frac{H+L}{2}\\
&\quad +\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}M+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\frac{L+M}{2}\\
&= \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1+\mu_2}{2}\frac{M}{2}+\frac{1-\mu_2}{2}\frac{L}{2}\\
P_U\,(\text{buy},\text{sell}) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{H+M}{2}+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)\frac{H+L}{2}\\
&\quad +\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)M+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{L+M}{2}\\
&= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2}\\
P_U\,(\text{sell},\text{sell}) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\frac{H+M}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\frac{H+L}{2}\\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}M+\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{L+M}{2}\\
&= \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1-\mu_2}{2}\frac{M}{2}+\frac{1+\mu_2}{2}\frac{L}{2}
\end{aligned}
$$

B Additional calculations for the proof of Proposition 2:

Suppose that equation (18) does not hold, so that trader 1 does not trade when informed. The market maker is aware that one trade emanates from trader 2 (which occurs with probability μ_1), and that two trades emanate from trader 2 and hedger 1 (which occurs with probability $1-\mu_1$). Thus:

$$\begin{aligned}
P_U\,(\text{buy}) &= \frac{H-M}{4}+\frac{1+\mu_2}{2}M+\frac{1-\mu_2}{2}\frac{L+M}{2} \\
P_U\,(\text{sell}) &= \frac{H-M}{4}+\frac{1-\mu_2}{2}M+\frac{1+\mu_2}{2}\frac{L+M}{2} \\
P_U\,(\text{buy},\text{buy}) &= Pr\,(\theta_1=1,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=1) \\
&\quad +Pr\,(\theta_1=1,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=0) \\
&\quad +Pr\,(\theta_1=0,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=1) \\
&\quad +Pr\,(\theta_1=0,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=0) \\
&= \frac{1}{2}\frac{1+\mu_2}{2}\frac{H+M}{2}+\frac{1}{2}\frac{1-\mu_2}{2}\frac{H+L}{2}+\frac{1}{2}\frac{1+\mu_2}{2}M+\frac{1}{2}\frac{1-\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{2+\mu_2}{2}\frac{M}{2}+\frac{1-\mu_2}{2}\frac{L}{2} \\
P_U\,(\text{buy},\text{sell}) &= \frac{1}{2}\left(\frac{1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1}{2}\right)\frac{H+M}{2}+\frac{1}{2}\left(\frac{1}{2}\frac{1+\mu_2}{2}+\frac{1}{2}\frac{1-\mu_2}{2}\right)\frac{H+L}{2} \\
&\quad +\frac{1}{2}\left(\frac{1}{2}\frac{1+\mu_2}{2}+\frac{1}{2}\frac{1-\mu_2}{2}\right)M+\frac{1}{2}\left(\frac{1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1}{2}\right)\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \\
P_U\,(\text{sell},\text{sell}) &= \frac{1}{2}\frac{1-\mu_2}{2}\frac{H+M}{2}+\frac{1}{2}\frac{1+\mu_2}{2}\frac{H+L}{2}+\frac{1}{2}\frac{1-\mu_2}{2}M+\frac{1}{2}\frac{1+\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{2-\mu_2}{2}\frac{M}{2}+\frac{1+\mu_2}{2}\frac{L}{2}
\end{aligned}$$

Suppose that equation (19) does not hold, so that trader 2 does not trade when informed. The market maker is aware that one trade emanates from trader 1 (which occurs with probability μ_2), and that two trades emanate from trader 1 and hedger 2 (which occurs with probability $1-\mu_2$). Thus:

$$\begin{aligned}
P_U\left(\text{buy}\right) &= \frac{1+\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4} \\
P_U\left(\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4} \\
P_U\left(\text{buy},\text{buy}\right) &= Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=1) \\
&\quad +Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=0) \\
&\quad +Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=1) \\
&\quad +Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=0) \\
&= \frac{1+\mu_1}{2}\frac{1}{2}\frac{H+M}{2}+\frac{1+\mu_1}{2}\frac{1}{2}\frac{H+L}{2} \\
&\quad +\frac{1-\mu_1}{2}\frac{1}{2}M+\frac{1-\mu_1}{2}\frac{1}{2}\frac{L+M}{2} \\
&= \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2} \\
P_U\left(\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1}{2}\frac{1-\mu_1}{2}\right)\frac{H+M}{2}+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\right)\frac{H+L}{2} \\
&\quad +\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\right)M+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1}{2}\frac{1-\mu_1}{2}\right)\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \\
P_U\left(\text{sell},\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1}{2}\frac{H+M}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\frac{H+L}{2} \\
&\quad +\frac{1+\mu_1}{2}\frac{1}{2}M+\frac{1+\mu_1}{2}\frac{1}{2}\frac{L+M}{2} \\
&= \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2}
\end{aligned}$$

C Additional calculations for the proof of Proposition 3:

The probability of buy or sell orders conditional on asset type for leveraged equity is:

$$\begin{aligned}
Pr\left(\text{buy}|\theta_1=1\right) &= \mu_1+(1-\mu_1)\frac{1}{2}=\frac{1+\mu_1}{2} \\
Pr\left(\text{sell}|\theta_1=1\right) &= \frac{1-\mu_1}{2}
\end{aligned}$$

Using Bayesian updating, the distribution of $\tilde{\theta}_1$ conditional on observed orders is given by:

$$\begin{aligned}
Pr\left(\theta_1 = 1|\text{order}\right) &= \frac{Pr\left(\theta_1 \cap \text{order}\right)}{Pr\left(\text{order}\right)} = \frac{Pr\left(\text{order}|\theta_1\right) Pr\left(\theta_1 = 1\right)}{Pr\left(\text{order}\right)} \\
Pr\left(\theta_1 = 1|\text{buy}\right) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1}{2}} = \frac{1+\mu_1}{2} \\
Pr\left(\theta_1 = 0|\text{buy}\right) &= \frac{1-\mu_1}{2} \\
Pr\left(\theta_1 = 1|\text{sell}\right) &= \frac{1-\mu_1}{2} \\
Pr\left(\theta_1 = 0|\text{sell}\right) &= \frac{1+\mu_1}{2}
\end{aligned}$$

The probability of a buy or sell order conditional on asset type for risky debt is:

$$\begin{aligned}
Pr\left(\text{buy}|\theta_2 = 1\right) &= \mu_2 + (1-\mu_2)\frac{1}{2} = \frac{1+\mu_2}{2} \\
Pr\left(\text{sell}|\theta_2 = 1\right) &= \frac{1-\mu_2}{2}
\end{aligned}$$

Using Bayesian updating, the distribution of $\tilde{\theta}_2$ conditional on observed orders is given by:

$$\begin{aligned}
Pr\left(\theta_2 = 1|\text{order}\right) &= \frac{Pr\left(\theta_2 \cap \text{order}\right)}{Pr\left(\text{order}\right)} = \frac{Pr\left(\text{order}|\theta_2\right) Pr\left(\theta_2 = 1\right)}{Pr\left(\text{order}\right)} \\
Pr\left(\theta_2 = 1|\text{buy}\right) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1}{2}} = \frac{1+\mu_2}{2} \\
Pr\left(\theta_2 = 0|\text{buy}\right) &= \frac{1-\mu_2}{2} \\
Pr\left(\theta_2 = 1|\text{sell}\right) &= \frac{1-\mu_2}{2} \\
Pr\left(\theta_2 = 0|\text{sell}\right) &= \frac{1+\mu_2}{2}
\end{aligned}$$

D Proof of Lemma 1

We rule out "separating" and "semi-separating" equilibria such that a market maker (MM) offers a menu of security designs (instead of only one security design) such that firms with different asset types make different choices. For brevity, below we refer to "asset types" making choices.

The MM sets market prices after observing the asset type's choice of security design, by using its beliefs about asset types' choices and the fair pricing condition. For any postulated separating or semi-separating equilibrium, we will show that there is a profitable deviation for at least one asset type: at least one asset type will not choose the security design that the MM believes. This in turn implies that the MM's beliefs are not verified, which invalidates the postulated equilibrium.

The proof is in four parts. The first part rules out separating equilibria, the next three parts rule out semi-separating equilibria.

Part (i). We rule out equilibria in which each asset type chooses a different set of securities (in principle, there can be more than two sets of securities on the menu offered by the MM; for example, let γ take different values in the type of security design described in Proposition 1). If this were an equilibrium, then an asset would be valued according to its type $\{\theta_1, \theta_2\}$, which would be revealed by security issuance decisions, so that asset market valuations would be as in equation (1) in the absence of arbitrage. In particular, with securities associated with type $\{0, 0\}$, the asset would be assigned a valuation $\frac{L+M}{2}$, and could achieve a strictly higher valuation by issuing a set of securities associated with another asset type, which contradicts that asset type $\{0, 0\}$ will issue a different set of securities (relative to other asset types) in equilibrium.

Part (ii). We rule out equilibria in which two asset types issue the same set of securities, a third type chooses another set of securities, and the fourth type chooses yet another set of securities. There are several possibilities:

1. If $\{0, 0\}$ is the only asset type issuing a given set of securities, then in equilibrium its type is revealed by security issuance decisions. With this choice of securities, the asset will be assigned a valuation $\frac{L+M}{2}$ according to equation (1) in the absence of arbitrage. However, it could achieve a strictly higher valuation by issuing a set of securities associated with another asset type, which contradicts that an asset type $\{0, 0\}$ will be the only asset type issuing a given set of securities in equilibrium.

2. If $\{1, 1\}$ is the only asset type issuing a given set of securities, then in equilibrium its type is revealed by security issuance decisions. With this choice of securities, the asset will be assigned a valuation $\frac{H+M}{2}$ according to equation (1) in the absence of arbitrage. However, any other asset type could then achieve a strictly higher valuation compared to its equilibrium valuation by issuing this set of securities, which contradicts that an asset type $\{1, 1\}$ will be the only asset type issuing a given set of securities in equilibrium.

3. If $\{1, 1\}$ and $\{0, 0\}$ are issuing the same set of securities, and $\{1, 0\}$ and $\{0, 1\}$ are each issuing a different set of securities, then issuing a set of securities that is in equilibrium issued by asset type $\{1, 0\}$ (asset type $\{0, 1\}$) will give the asset a valuation of $\frac{H+L}{2}$ (M) according to equation (1) in the absence of arbitrage. There are two non-trivial

cases (the case when all asset types choose the same type of security design – either of the type described in Proposition 1 or of the type described in Proposition 3 – is trivial). First, suppose that asset types $\{1,1\}$ and $\{0,0\}$ issue only one security with nonconstant payoff. In this equilibrium, when an asset type chooses one security with nonconstant payoff:

$$
\begin{aligned}
Pr(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 1, \theta_2 = 1|\text{sell}, \text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{sell}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}
\end{aligned}
$$

The price schedule for one security with nonconstant payoff is:

$$
\begin{aligned}
P_U(\text{buy}, \text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2} \\
P_U(\text{buy}, \text{sell}) &= \frac{1}{2}\frac{H+M}{2} + \frac{1}{2}\frac{L+M}{2} = \frac{H}{4} + \frac{M}{2} + \frac{L}{4} \\
P_U(\text{sell}, \text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}
\end{aligned}
$$

In this equilibrium, expected asset value conditional on asset type is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&\quad + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=0\right] &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad + \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&\quad + \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right)
\end{aligned}$$

If an asset type $\{1,0\}$ deviates and chooses one security, its expected value is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=0\right] &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&\quad + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L+\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L]
\end{aligned}$$

If an asset type $\{0,1\}$ deviates and chooses one security, its expected value is:

$$
\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L-\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L]
\end{aligned}
$$

An asset type $\{1,0\}$ had rather deviate if and only if $\mathbb{E}\left[P_U|\theta_1=1,\theta_2=0\right]$ is larger than $\frac{H+L}{2}$, that is:

$$
\frac{H+L}{2}-\frac{\mu_1^2-\mu_2^2}{4+4\mu_1\mu_2}[H-L]<M \tag{56}
$$

An asset type $\{0,1\}$ had rather deviate if and only if $\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right]$ is larger than M, that is:

$$
\frac{H+L}{2}H-\frac{\mu_1^2-\mu_2^2}{4+4\mu_1\mu_2}[H-L]>M \tag{57}
$$

Generically, either equation (56) holds or equation (57) holds. That is, either type $\{1,0\}$ or type $\{0,1\}$ deviates, which invalidates the equilibrium. Second, suppose that an asset types $\{1,0\}$ and $\{0,1\}$ each issue only one security with nonconstant payoff, and asset types $\{1,1\}$ or $\{0,0\}$ choose the same security design with at least two securities with nonconstant and linearly independent payoffs. The proof that such an equilibrium does not exist is similar to point 3. above.

Part (iii). We rule out equilibria in which one asset type chooses a given set of securities, and the other three types choose another set of securities.

1. Suppose that asset type $\{0,0\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{0,0\}$ and assigned a valuation $\frac{L+M}{2}$. An asset type $\{0,0\}$ could

achieve a strictly higher valuation by issuing the same set of securities as other asset types, which contradicts that an asset type $\{0,0\}$ chooses the former set of securities in equilibrium.

2. Suppose that asset type $\{1,1\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{1,1\}$ and assigned a valuation $\frac{H+M}{2}$. An asset type $\{1,0\}$, $\{0,1\}$, or $\{0,0\}$ could achieve a strictly higher valuation by issuing the same set of securities as a $\{1,1\}$ asset type and achieve the same high valuation, which contradicts that only $\{1,1\}$ chooses the former set of securities in equilibrium.

3. Suppose that asset type $\{1,0\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{1,0\}$ and assigned a valuation $\frac{H+L}{2}$. First, suppose that asset types other than $\{1,0\}$ choose only one security with nonconstant payoff whereas asset type $\{1,0\}$ chooses another security design. In this equilibrium, when an asset type chooses one security with nonconstant payoff, Bayesian updating gives:

$$
\begin{aligned}
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1+\mu_2-\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2+2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=1,\theta_2=1|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1+\mu_1-\mu_2-\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}
\end{aligned}
$$

The price schedule for an asset type issuing one security with nonconstant payoff is:

$$\begin{aligned}
P_U(\text{buy}, \text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\frac{H+M}{2} + \frac{1-\mu_1+\mu_2-\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}M + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\frac{L+M}{2} \\
P_U(\text{buy}, \text{sell}) &= \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\frac{H+M}{2} + \frac{2+2\mu_1\mu_2}{6-2\mu_1\mu_2}M + \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\frac{L+M}{2} \\
P_U(\text{sell}, \text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\frac{H+M}{2} + \frac{1+\mu_1-\mu_2-\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}M + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\frac{L+M}{2}
\end{aligned}$$

Consider the problem of an asset type $\{0,0\}$. If it deviates and chooses two securities with nonconstant and linearly independent payoffs as does asset type $\{1,0\}$, then its assigned value is $\frac{H+L}{2}$. If it chooses one security with nonconstant payoff, its expected value is:

$$(58)\quad \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy}, \text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy}, \text{sell}) + \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell}, \text{sell})$$

If the expression in equation (58) is smaller than $\frac{H+L}{2}$, then asset type $\{0,0\}$ deviates, which is inconsistent with the postulated equilibrium. If the expression in equation (58) is larger than $\frac{H+L}{2}$, then asset type $\{0,0\}$ does not deviate. However, this implies that the following expression:

$$\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy}, \text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy}, \text{sell}) + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell}, \text{sell})$$

is larger than $\frac{H+L}{2}$, so that it is in the interests of asset type $\{1,0\}$ to deviate by issuing only one security with nonconstant payoff, which is again inconsistent with the postulated equilibrium. The proof that there is no equilibrium in which asset types other than $\{1,0\}$ choose two securities with nonconstant and linearly independent payoffs whereas asset type $\{1,0\}$ chooses another security design is similar and is omitted.

4. Suppose that asset type $\{0,1\}$ chooses a given set of securities, and other asset types choose another set of securities. The proof that such an equilibrium does not exist is similar to point 3. above.

Part (iv). We rule out equilibria in which two asset types choose one set of securities, and the other two asset types choose another set of securities.

1. Suppose that asset types $\{0,1\}$ and $\{0,0\}$ choose a given set of securities, and asset types $\{1,1\}$ and $\{1,0\}$ choose another set of securities. It is then easy to show that an

asset type $\{0,0\}$ would be better off deviating and issuing the latter set of security(ies) to pool with types $\{1,1\}$ and $\{1,0\}$ (indeed, on the market for the latter set of securities, prices are a weighted average of the market values of asset types $\{1,1\}$ and $\{1,0\}$, whereas on the market for the former set of securities, prices are a weighted average of the values of asset types $\{0,1\}$ and $\{0,0\}$), a contradiction.

2. Suppose that asset types $\{1,0\}$ and $\{0,0\}$ choose a given set of securities, and asset types $\{1,1\}$ and $\{0,1\}$ choose another set of securities. It is then easy to show that asset type $\{0,0\}$ would be better off deviating and issuing the latter set of securities to pool with types $\{1,1\}$ and $\{0,1\}$ (indeed, on the market for the latter set of securities, prices are a weighted average of the market values of asset types $\{1,1\}$ and $\{0,1\}$, whereas on the market for the former set of securities, prices are a weighted average of the values of asset types $\{1,0\}$ and $\{0,0\}$), a contradiction.

3. There are two non-trivial cases (the case when all asset types choose the same type of security design – either of the type described in Proposition 1 or of the type described in Proposition 3 – is trivial). First, suppose that asset types $\{1,0\}$ and $\{0,1\}$ choose two securities with nonconstant and linearly independent payoffs, and asset types $\{1,1\}$ and $\{0,0\}$ choose only one security with nonconstant payoff. In this equilibrium, when an asset type chooses one security with nonconstant payoff:

$$\begin{aligned}
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1=1,\theta_2=1|\text{sell},\text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1=0,\theta_2=0|\text{sell},\text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}
\end{aligned}$$

The price schedule for an asset type issuing one security with nonconstant payoff is:

$$
\begin{aligned}
P_U(\text{buy}, \text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2} \\
P_U(\text{buy}, \text{sell}) &= \frac{1}{2}\frac{H+M}{2} + \frac{1}{2}\frac{L+M}{2} = \frac{H}{4} + \frac{M}{2} + \frac{L}{4} \\
P_U(\text{sell}, \text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}
\end{aligned}
$$

In this equilibrium, expected asset value conditional on asset type is:

$$
\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy}, \text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy}, \text{sell}) \\
&\quad + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell}, \text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4} + \frac{M}{2} + \frac{L}{4}\right) \\
&\quad + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy}, \text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy}, \text{sell}) \\
&\quad + \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell}, \text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4} + \frac{M}{2} + \frac{L}{4}\right) \\
&\quad + \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right)
\end{aligned}
$$

In this equilibrium, when an asset type chooses two securities with nonconstant and linearly independent payoffs, on the market for the security with payoff vector of the type (Γ, Γ, F):

$$\begin{aligned}
Pr(\theta_1 = 1|\text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2} + \frac{1-\mu_1}{2}\frac{1}{2}} = \frac{1+\mu_1}{2} \\
Pr(\theta_1 = 1|\text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2} + \frac{1-\mu_1}{2}\frac{1}{2}} = \frac{1-\mu_1}{2} \\
Pr(\theta_1 = 0|\text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2} + \frac{1-\mu_1}{2}\frac{1}{2}} = \frac{1-\mu_1}{2} \\
Pr(\theta_1 = 0|\text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2} + \frac{1-\mu_1}{2}\frac{1}{2}} = \frac{1+\mu_1}{2}
\end{aligned}$$

The price schedule for the security with payoff vector of the type (Γ, Γ, F) is:

$$\begin{aligned}
P_E(\text{buy}) &= \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma \\
P_E(\text{sell}) &= \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma
\end{aligned}$$

In this equilibrium, when an asset type chooses two securities with nonconstant and linearly independent payoffs, on the market for the security with payoff vector of the type (f, k, k):

$$\begin{aligned}
Pr(\theta_2 = 1|\text{buy}) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1+\mu_2}{2} \\
Pr(\theta_2 = 1|\text{sell}) &= \frac{\frac{1-\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1-\mu_2}{2} \\
Pr(\theta_2 = 0|\text{buy}) &= \frac{\frac{1-\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1-\mu_2}{2} \\
Pr(\theta_2 = 0|\text{sell}) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1+\mu_2}{2}
\end{aligned}$$

The price schedule for the security with payoff vector of the type (f, k, k) is:

$$\begin{aligned}
P_D(\text{buy}) &= \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma = \frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} - \Gamma \\
P_D(\text{sell}) &= \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma = \frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} - \Gamma
\end{aligned}$$

In this equilibrium, expected asset value conditional on asset type is:

$$
\begin{aligned}
\mathbb{E}\left[P_E + P_D | \theta_1 = 1, \theta_2 = 0\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_1}{2}\frac{H-M}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_1}{2}\frac{H-M}{2} \\
&\quad + \frac{1-\mu_2}{2}\left[\frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2}\right] + \frac{1+\mu_2}{2}\left[\frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2}\right] \\
&= \frac{1+\mu_1^2}{2}\frac{H}{2} + \frac{2-\mu_1^2-\mu_2^2}{2}\frac{M}{2} + \frac{1+\mu_2^2}{2}\frac{L}{2} \\
&= \frac{1}{4}H + \frac{1}{2}M + \frac{1}{4}L + \frac{\mu_1^2}{4}[H-M] - \frac{\mu_2^2}{4}[M-L] \\
\mathbb{E}\left[P_E + P_D | \theta_1 = 0, \theta_2 = 1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_1}{2}\frac{H-M}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_1}{2}\frac{H-M}{2} \\
&\quad + \frac{1+\mu_2}{2}\left[\frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2}\right] + \frac{1-\mu_2}{2}\left[\frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2}\right] \\
&= \frac{1-\mu_1^2}{2}\frac{H}{2} + \frac{2+\mu_1^2+\mu_2^2}{2}\frac{M}{2} + \frac{1-\mu_2^2}{2}\frac{L}{2} \\
&= \frac{1}{4}H + \frac{1}{2}M + \frac{1}{4}L - \frac{\mu_1^2}{4}[H-M] + \frac{\mu_2^2}{4}[M-L]
\end{aligned}
$$

If an asset type $\{1,0\}$ deviates and chooses only one security with nonconstant payoff, its expected value is:

$$
\begin{aligned}
\mathbb{E}\left[P_U | \theta_1 = 1, \theta_2 = 0\right] &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy}) + \left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad + \left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4} + \frac{M}{2} + \frac{L}{4}\right] \\
&\quad + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8} + \frac{M}{2} + \frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H + \frac{1}{2}M + \frac{1}{4}L + \frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L]
\end{aligned}
$$

If an asset type $\{0,1\}$ deviates and chooses only one security with nonconstant payoff, its expected value is:

$$
\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L-\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L]
\end{aligned}
$$

An asset type $\{1,0\}$ had rather deviate if and only if $\mathbb{E}\left[P_U|\theta_1=1,\theta_2=0\right]$ is larger than $\mathbb{E}\left[P_E+P_D|\theta_1=1,\theta_2=0\right]$. An asset type $\{0,1\}$ had rather deviate if and only if $\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right]$ is larger than $\mathbb{E}\left[P_E+P_D|\theta_1=0,\theta_2=1\right]$. Generically, one of these two asset types deviates, which invalidates the equilibrium.

Second, suppose that asset types $\{1,0\}$ and $\{0,1\}$ choose only one security with nonconstant payoff, whereas asset types $\{1,1\}$ and $\{0,0\}$ choose at least two securities with nonconstant and linearly independent payoffs. The proof that such an equilibrium does not exist is similar to point 3. above.

This concludes the proof. ■