Internet Appendix to

The only constant is change: Non-constant volatility and implied volatility spreads

Internet Appendix

This appendix provides additional details and tests as describe in "The Only Constant is Change: Non-constant Volatility and Implied Volatility Spreads."

A. Zero vs. Non-Zero VS - Additional Discussion

To generate a non-zero VS in our model, it is crucial that the put option is optimally exercised early in some future state. Otherwise, both the call and put will behave as if they are European options with put-call parity in Eq. (1) holding with equality. Figure A1 replicates the same calculations as described in Figure 1 in the body of the paper, except now the options have a strike price of $6.5. With the lower strike price, the put option is never exercised early either in the time-varying volatility case or the constant volatility case when the implied volatility is computed. So, the put's valuation collapses to a European put valuation. Subsequently, the VS between the call and the put collapses to zero. This illustrates the importance of early exercise for generating non-zero VS without violating put-call parity when volatility is time-varying.

Our theoretical results also suggest a v-shaped relation between the expected down-state change in volatility and VS. Direct empirical testing of this prediction is problematic, as it requires a measure of expected down-state volatility that is not based on the VS. However, it is straightforward to argue that firms with greater baseline volatility would have the potential for larger swings in volatility on average. This would suggest that firms with greater volatility would experience larger (more negative) VS on average. Although not included in a figure, we also examine the impact that baseline firm volatility has on VS in our model. On average, we find that VS becomes more negative, with greater potential for large negative spreads, as baseline firm volatility increases, consistent with this intuition. Similar to the effect of the down-state change in volatility, increasing baseline volatility decreases VS (drives VS to be more negative) at a decreasing rate. This suggests that VS should decrease (become more negative) as firm-level volatility increases over a substantial range of volatility, but this effect is non-monotonic; VS can also increase as firm-level volatility increases at sufficiently high levels of volatility. In other words, we expect VS to be negatively related to firm-level volatility and positively related to the squared value of volatility. In untabulated analysis, we regress firm-level volatility spreads on firm-level volatility and volatility squared in pooled OLS regressions with robust standard errors. The results confirm our predictions: volatility spreads are negatively related to volatility (-0.231, t-statistic = -28.62) and positively related to volatility squared (0.219, t-statistic = 3.75).

B. Contemporaneous Returns

The results presented in the body of the paper are consistent with a risk-based explanation for the VS-stock return predictability. If our interpretation is correct, firms in VS5 require higher future returns to compensate investors for aggregate volatility risk. To earn the higher required return in the month following portfolio formation, the same stocks would need to experience a price decrease during the month of portfolio formation. In other words, we would expect a *negative* contemporaneous relation between VS and the underlying stock returns. To test this prediction, we calculate the returns to the VS hedge portfolio during the month of portfolio formation. The results of this test is presented in Table A1.

[Insert Table A1 here]

Consistent with our predictions, we document a negative and significant contemporaneous relation between VS and underlying stock returns. In particular, VS5 firms experience a price decrease, and the hedge portfolio returns are negative and significant throughout our sample period. This supports the prediction that a higher return is required for VS5 firms in the following month, necessitating a price decrease in the current month. Additional untabulated results show that the VS hedge portfolio returns during the holding period month load negatively on the contemporaneous (holding-period month) innovations in expected aggregate volatility. This is consistent with VS5 firms requiring higher future returns when volatility is expected to increase, and experiencing a contemporaneous price decrease to generate the higher required returns in the future.

C. Alternate Explanations: Informed Trading, Transaction Costs, Liquidity, and Short-Sale Constraints

As described in the body of the paper, we take additional steps to address and rule out alternate explanations for our results. We begin this additional analysis by analyzing the ability of firm-level measures of informed trading and liquidity to explain the volatility spread-stock return relationship using Fama and MacBeth (1973) characteristics regressions estimated on each monthly cross-section. We regress future monthly excess returns $R_{i,t+1}$ on the firm's implied volatility spread $VS_{i,t}$, measures of informed trading or liquidity, and other characteristics representing firm risk measured at time t. These include market beta from a 48-month rolling regression (β^{MKT}), market value of equity (MVE), market-to-book ratio (MB), and cumulative stock returns over the last 6 months (MOM). Specifically:

$$\text{XRET}_{i,t+1} = \alpha_0 + \alpha_1 \text{VS}_{i,t} + \alpha_2 \text{A}_{i,t} + \alpha_3 \text{VS}_{i,t} \times \text{A}_{i,t} + \mathbf{C}_t \Gamma' + \epsilon_{i,t+1} \tag{1}$$

where $A_{i,t}$ captures the alternative explanations. As noted in the paper, we consider eight alternate measures that may potentially explain our results: (1) Stock PIN, (2) Analyst Coverage, (3) Option Volume, (4) Stock Volume, (5) Relative Option-to-Stock Volume, (6) Option Illiquidity, (7) Stock Illiquidity, and (8) Relative Option-to-Stock Illiquidity. In each case, we interact the measure of informed trading with the firm's implied volatility spread to attempt to determine whether the observed VS-stock return relationship is explained by informed trading. Each of the measures of informed trading and liquidity is described below.

PIN is the probability of informed trading based on Easley et al. (2002) and Duarte and Young (2008).[1] An additional measure of the likelihood of informed trading is analyst coverage, and is calculated as the total number of analysts in the I/B/E/S database providing earnings estimates for a particular firm each month, following Hong et al. (2000).

[1] PIN is obtained from Jefferson Duarte's website (http://www.owlnet.rice.edu/ jd10/publications.htm).

We also consider stock/option volume as a potential explanation for our results, as this may be correlated with informed trading (Easley et al., 1998) or transaction costs. Stock volume is defined as the total number of shares traded. Following Chakravarty et al. (2004), option volume is measured as the average time-series sum of the daily total option contract volume for all options for each underlying security. Relative Volume is the ratio of Option Volume to Stock Volume (Roll et al., 2010).

Finally, we analyze the impact of liquidity, which may also be related to the likelihood of informed trading (Easley et al., 1996). As such, we include Amihud's illiquidity (Amihud, 2002), which we multiply by 10^6 for reporting purposes. Option Illiquidity is calculated similarly to Amihud's illiquidity, with two exceptions. Following Cao and Wei (2010), we adjust the change in option price for the change in stock price times the beginning-of-the-day option delta, and we calculate aggregate illiquidity across options for each firm by taking a volume-weighted-average of individual option illiquidities. Relative Illiquidity is then calculated as the ratio of Option Illiquidity to Stock Illiquidity. We then analyze the ability of each of these measures to explain the VS-stock return predictability. Table A2 presents the results from this analysis.

[Insert Table A2 here]

If firm-specific informed trading or liquidity is the primary driver of this relationship, we would expect that the direct relation between VS and subsequent firm stock returns to be insignificant, and the VS-stock return relation should only be found for the interaction terms. Our results do not support this. In each specification, VS remains positive and significant, and no interaction term is significant. This does not support informed trading as a driver of the VS-stock return predictability.

To further support our results linking the VS-return predictability to aggregate volatility, we investigate the VS-stock return predictability conditional on the level of volatility in FM regressions. We follow a similar procedure to Mashruwala et al. (2006) and estimate the FM regressions with interactions between VS and (separately) the decile rank of macro variables VIX, RD, MVAR, and EVRP. This allows us to examine whether the impact of VS differs across periods of high and low volatility. We perform this test using only interactions as the volatility measures are macro-variables, and thus cannot be included in the firm characteristic regressions. Table A3 presents the results of this analysis.

[Insert Table A3 here]

We find that the interaction coefficients are positive and significant in each case. Moreover, VS does not appear to have the significant predictive ability during low volatility periods when VIX, RD, or MVAR is used to proxy for aggregate volatility. This provides further evidence that VS captures some aspect of aggregate volatility, leading to the observed predictability.[2] Taken together, these results are consistent with our interpretation of differing sensitivities to aggregate volatility risk driving differences in returns to stocks with high and low option implied volatility spreads. On the other hand, the VS-return relation does not appear to be explained by informed trading, liquidity, or potential transaction costs.

Lastly, we consider whether short-sale constraints could explain our results (Muravyev et al., 2018). First, we examine whether short interest can explain the VS-return relation in FM regressions, similar to the analysis for informed trading detailed above. Specifically, we repeat the FM regressions, but include short interest (SIR) and an interaction between VS and SIR. If short-sale constraints can explain the VS-return relation in our sample, we would expect VS to have an insignificant direct effect, as it should be subsumed by the interaction. Similar to the results for informed trading, we find that VS remains significant, and while SIR is negative and significant as expected, the interaction is not significant. These results are presented in Column 9 of Table A2. As an additional test, we exclude all stocks in the lowest decile of short interest, similar to Muravyev et al. (2018)'s exclusion of difficult-to-short stocks, and repeat the tests in Tables 3 and Table 5. These results are presented in Tables A4 and A5, respectively. We find that VS continues to predict

[2] As an alternative, we also estimate the FM regressions separately for sample periods sorted into quintiles based on the contemporaneous change in the implied volatility of S&P500 index options (Δ VIX). We perform this test as a sort because ΔVIX is a macro-variable, and thus cannot be included in the firm characteristic regressions. The (untabulated) results support the same conclusion.

stock returns across firm-level volatility quintiles, but only when aggregate volatility is high, the VS hedge portfolio loads significant on FVIX and CFVIX, and the hedge portfolio's alpha is meaningfully reduced after these factors are included. This helps to rule out short-sale constraints as the primary driver of our results.

[Insert Table A4 here]

[Insert Table A5 here]

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TABLE A1
Contemporaneous Sorting Results

Table A1 presents the contemporaneous performance to the five monthly-rebalanced volatility spread quintile portfolios and the hedge portfolio (High - Low) for the full sample and five sample subperiods: (1) 1996-1999, (2) 2000-2003, (3) 2004-2007, (4) 2008-2012, and (5) 2013-2017. The implied volatility spread is calculated as the difference in implied volatilities for each firm's matched call and put options, averaged over the month. We then calculate the raw (Panel A) and abnormal Daniel et al. (1998) (Panel B) returns over the following month for each quintile portfolio sorted on the volatility spread. T-statistics are in parentheses below. The sample period is from January 1996 to December 2017. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Panel A. Raw Returns

	Low VS	2	3	4	High VS	High - Low
1996-1999	2.11	2.54	1.94	1.33	0.30	-1.81***
						(-5.73)
2000-2003	1.65	2.18	1.16	0.33	-0.57	-2.23***
						(-5.62)
2004-2007	2.27	2.00	1.36	0.59	-0.74	-3.01***
						(-19.38)
2008-2012	2.00	1.55	0.93	0.20	-1.62	-3.62***
						(-11.57)
2013-2017	2.82	1.93	1.23	0.56	-0.47	-3.29***
						(-16.31)
Full Sample	2.23	2.03	0.56	0.59	-0.61	-2.84***
						(-21.84)

Panel B. Benchmark-adjusted Returns

	Low VS	2	3	4	High VS	High - Low
1996-1999	0.59	0.92	0.42	-0.12	-0.82	-1.41***
						(-4.82)
2000-2003	1.15	1.73	0.79	0.06	-1.13	-2.28***
						(-6.11)
2004-2007	1.52	1.02	0.43	-0.22	-1.44	-2.96***
						(-14.99)
2008-2012	1.78	1.14	0.49	-0.03	-1.90	-3.68***
						(-11.53)
2013-2017	1.99	0.97	0.18	-0.40	-1.30	-3.29***
						(-17.07)
Full Sample	1.46	1.14	0.44	-0.17	-1.32	-2.78***
						(-21.37)

TABLE A2

VS interacted with Informed Trading and Short Interest

Table A2 presents the results of Fama and MacBeth (1973) characteristics regressions estimated on each monthly cross-section of stock returns. We analyze the ability of measures of informed trading and shorting costs to explain the volatility spread-stock return predictability. We consider eight measures of informed trading: PIN (column 2), Analyst Coverage (column 3), Option Illiquidity (column 4), Stock Illiquidity (column 5), Option Volume (column 6), Stock Volume (column 7), and Relative Option-to-Stock Volume (column 8) as well as Short Interest as a measure of shorting costs (column 9). Controls for the firm's market beta, the market value of equity, market to book, and momentum are also included. Finally, we include the firm's option implied volatility spread and interaction of the volatility spread with each measure of informed trading. T-statistics in parentheses are corrected for heteroskedasticity and auto-correlation using the Newey-West estimator with three lags. The sample period is from January 1996 to December 2017. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1) XRET	(2) XRET	(3) XRET	(4) XRET	(5) XRET	(6) XRET	(7) XRET	(8) XRET	(9) XRET
VS	0.02***	0.04**	0.03**	0.02***	0.01***	0.02***	0.01***	0.02***	0.05***
	(5.57)	(2.47)	(2.09)	(5.15)	(5.04)	(5.40)	(4.92)	(5.12)	(4.76)
PIN		0.00							
		(0.18)							
VS×PIN		-0.03							
		(-1.46)							
ANLST			0.02**						
			(2.49)						
VS×ANLST			-0.01						
			(-0.37)						
OILLIQ				-0.01					
				(-1.03)					
VS×OILLIQ				-0.02					
				(-1.07)					
ILLIQ					0.01				
					(1.47)				
VS×ILLIQ					0.00				
					(0.49)				
OVOL						0.03**			
						(2.12)			
VS×OVOL						0.00			
						(0.15)			
SVOL							0.01**		
							(2.45)		
VS×SVOL							0.00		
							(0.51)		
O/S								-0.00	
								(-0.43)	
VS×O/S								-0.01	
								(-0.68)	
SIR									-4.81***
									(-4.32)
VS×SIR									-0.18
									(-1.31)
N	393083	79555	119691	392314	368268	387226	387226	387226	350874
R2	0.06	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06
CNTRLS	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

TABLE A3
VS and Aggregate Volatility

Table A3 presents the results of Fama and MacBeth (1973) characteristics regressions of next period excess return on volatility spreads and its interaction with measures of aggregate volatility. We analyze the ability of aggregate volatility to explain the volatility spread-stock return predictability. We consider four measures of aggregate volatility: The implied volatility of S&P 500 options (VIX, column 2), cross-sectional return dispersion (RD, column 3), market variance (MVAR, column 4), and the expected variance risk premium (EVRP, column 5). We conduct this analysis using interactions between VS and the decile ranks of the the aggregate volatility measures. The standardized coefficients are reported in each column. T-statistics in parentheses are corrected for heteroskedasticity and auto-correlation using the Newey-West estimator with three lags. The sample period is from January 1996 to December 2017. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1) XRET	(2) XRET	(3) XRET	(4) XRET	(5) XRET
VS	0.02***	0.00	0.00	0.00	0.00**
	(5.57)	(0.32)	(0.92)	(1.42)	(2.05)
VS×VIX Rank		0.02***			
		(5.02)			
VS×RD Rank			0.02***		
			(5.59)		
VS×MVAR Rank				0.02***	
				(4.81)	
VS×EVRP Rank					0.01***
					(3.84)
N	393083	393083	393083	393083	393083
R2	0.06	0.06	0.06	0.06	0.06
CNTRLS	Yes	Yes	Yes	Yes	Yes

TABLE A4
Sensitivity to Firm-level and Aggregate Volatility - Excluding Difficult-to-Short Stocks

Table A4 presents the abnormal performance (Daniel et al., 1998) for the implied volatility spreads (VS) portfolios formed within quintiles of stocks sorted on idiosyncratic volatility, after excluding difficult-to-short stocks. Panel A presents the results of a dependent double-sorting procedure with stocks sorted first on idiosyncratic volatility and then implied volatility spreads. The benchmark-adjusted returns to each of the five VS portfolios as well as the hedge portfolio (High - Low) are presented for firms within each quintile of firm volatility. Panel B presents the results of a triple-sorting procedure that first sorts the sample into high and low VIX periods, and then repeats the double-sorting procedure from Panel A within high and low VIX periods. For brevity, we restrict the presentation to the VS hedge portfolio returns (High - Low) for this triple-sort. Also included are the hedge portfolio (High - Low) performance formed within high and low VIX periods, respectively, but unconditional on firm volatility (All). T-statistics are in parentheses below. The sample period is from January 1996 to December 2017. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Pavel A: Volatility Spread Premium Conditional on Idiosyncratic Volatility

Idiosyncratic Volatility Rank	Low VS	2	3	4	High VS	High - Low
Low	-0.04	0.16	0.03	0.22*	0.25**	0.29**
	(-0.31)	(1.31)	(0.30)	(1.91)	(2.04)	(2.82)
2	-0.07	0.24**	0.25***	0.13	0.27***	0.33***
	(-0.54)	(2.50)	(2.78)	(1.37)	(2.60)	(2.86)
3	0.12	0.17*	0.26***	0.34***	0.50***	0.38**
	(1.06)	(1.84)	(2.74)	(3.74)	(4.54)	(2.52)
4	0.19	0.24*	0.19	0.43***	0.49***	0.29*
	(1.42)	(1.71)	(1.61)	(3.43)	(3.40)	(1.65)
High	0.36	0.31	0.23	0.34*	1.00***	0.64***
	(1.43)	(1.43)	(1.12)	(1.68)	(4.66)	(3.26)

Pavel B: Volatility Spread Premium Conditional on Aggregate-level Volatility

VIX	All	Low IV	2	3	3	High IV	High - Low
Low VIX	-0.08	0.19	0.16	-0.08	-0.07	0.22	-0.03
	(-0.86)	(1.44)	(1.16)	(-0.48)	(-0.37)	(1.01)	(-0.12)
High VIX	0.73***	0.38**	0.48***	0.76**	0.60**	0.99***	0.61*
	(5.52)	(2.43)	(2.66)	(3.18)	(2.13)	(3.22)	(1.67)

TABLE A5

Factor Model Regressions - Excluding Difficult-to-Short Stocks

Table A5 presents the results of VS hedge portfolios returns (HP) regressed on benchmark return models, after excluding difficult-to-short stocks. Column (1) presents the results from the benchmark model used by Cremers and Weinbaum (2010), based on a Fama and French (1993) three-factor model plus momentum and coskewness (Harvey and Siddique, 2000). Columns (2) through (6) include the standard benchmark model plus factors designed to capture aggregate volatility risk and jump risk. FVIX is the aggregate volatility risk factor following Ang et al. (2006), CFVIX is a factor defined to capture any non-linear impact of FVIX between high and low FVIX states, and JUMP and VOL are jump and volatility factors based on option straddles following Cremers et al. (2015). T-statistics in parentheses are corrected for heteroskedasticity and auto-correlation using the Newey-West estimator with three lags. The sample period is from January 1996 to December 2017. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1) HP	(2) HP	(3) HP
α	0.48***	0.41***	0.21*
	(5.87)	(5.70)	(1.92)
MKTRF	-0.03	-0.32**	-0.33***
	(-1.20)	(-2.23)	(-2.69)
SMB	-0.00	0.02	0.03
	(-0.09)	(0.83)	(1.09)
HML	0.04	-0.01	-0.00
	(1.24)	(-0.15)	(-0.11)
MOM	-0.01	-0.01	-0.00
	(-0.50)	(-0.28)	(-0.08)
CSK	-0.06*	-0.07**	-0.08**
	(-1.74)	(-2.13)	(-2.29)
FVIX		-0.27**	-0.24**
		(-2.18)	(-2.18)
CFVIX			-0.11*
			(-1.92)
N	263	263	263
R2	0.03	0.07	0.09

FIGURE A1

Lower Strike Price Volatility Spread Example. This figure presents a three-period binomial option pricing model for a call and a put that expires in one year. In Panel A, the stock's volatility is time-varying with an annual volatility of 25% if the stock rises and an annual volatility of 45% if the stock falls. The initial stock price is $S(0) = 10$. Panel B and Panel C compute implied volatilities using the call's price and the put's price respectively from Panel A. A riskless bond also exists with an annual simple compounded interest rate of 5%. The strike price for the options is $X = 6.5$.

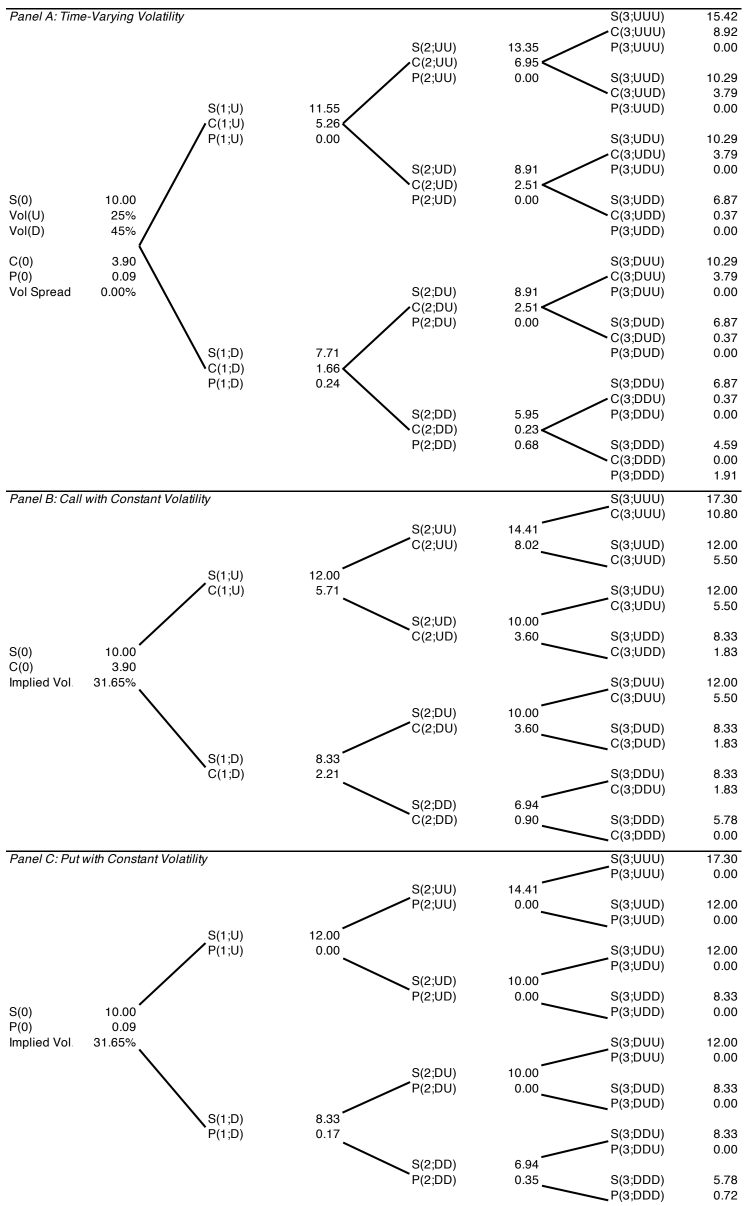

FIGURE A2

VS and moneyness. Figure A2 presents average implied volatility spreads across different moneyness categories. In particular, we sort all matched option pairs in the sample into deciles based on their moneyness (X/S) and report the average implied volatility spread for each portfolio.