Online Appendix for
Anomaly Discovery and Arbitrage Trading

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October 27, 2022

This note includes the following. Section 1 presents a model of risk-based anomaly. Section 2 analyzes a mispricing-based anomaly. Section 3 contrasts risk- and mispricing-based anomalies. The numerical algorithm and proofs are in Section 4.

1 A model of the discovery of a risk-based anomaly

Consider a two-period model, with time $t = 0, 1, 2$. Trading takes place at $t = 0, 1$, and consumption occurs at $t = 2$. There is one risk-free asset, and its interest rate is normalized to 0. There are two risky assets, asset 1 and asset 2, each of which is a claim to a single cash flow at $t = 2$. There is a continuum of identical investors, with a population size of one. At $t = 0$, investors are endowed with one unit of each asset and k dollars cash.

Asset i, for $i = 1, 2$, is a claim to a cash flow D_i at time $t = 2$. Moreover, D_1 and D_2 are independent from each other and have the same *ex ante* distribution at $t = 0$. Specifically, for $i = 1, 2$, we have

$$D_i = \mu_{i,1} \times \mu_{i,2}, \tag{1}$$

where $\mu_{i,1}$ and $\mu_{i,2}$ are random variables that will be realized at time $t = 1$ and $t = 2$, respectively. Moreover, $\mu_{i,t}$, for $i = 1, 2$ and $t = 1, 2$, are independent across i and t, and have the same binary distribution:

$$\mu_{i,t} = \begin{cases} \mu + \sigma & \text{with probability } p, \\ \mu - \sigma & \text{with probability } 1 - p, \end{cases} \tag{2}$$

where $\mu > \sigma > 0$, and $0 < p < 1$.

For $i = 1, 2$, and $t = 0, 1, 2$, we use $P_{i,t}$ to denote the price of asset i at time t, which will be determined endogenously in equilibrium. At $t = 2$, asset prices are pinned down by the final cash flow: $P_{i,2} = D_i$. We denote the gross return of asset i at time t, for $t = 1, 2$, as

$$r_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}}.$$

1.1 Hedging demand

Investors are endowed with a nontradable asset (e.g., labor income), which is a claim to a cash flow ρD_1 at $t = 2$, with $\rho \geq 0$. That is, this endowment is perfectly correlated with the payoff from asset 1. Denote investors' wealth, excluding their nontradable endowment, at time t as W_t for $t = 0, 1, 2$. If investors allocate a fraction $\theta_{i,t}$ of W_t to asset i at time t, for $i = 1, 2$ and $t = 0, 1$, their wealth dynamic is given by

$$W_{t+1} = W_t \left[\sum_{i \in \{1,2\}} \theta_{i,t} r_{i,t+1} + \left(1 - \sum_{i \in \{1,2\}} \theta_{i,t} \right) \right], \tag{3}$$

with $W_0 = k + P_{1,0} + P_{2,0}$. Investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$, to

$$\max_{\theta_{i,t}} E_0 \left[\log \left(W_2 + \rho D_1 \right) \right], \tag{4}$$

subject to (3). In a reduced form, the above formulation captures the essence of risk-based anomalies: Investors find asset 1 riskier because its return is correlated with their endowment. As we will see later, due to this hedging demand, asset 1 has a lower price and a higher expected return in equilibrium. We will label this return pattern as an "anomaly," because when an econometrician observes the return data alone, he would not be able to explain it by CAPM.

1.2 Arbitrageurs

Traditional risk-based explanations of anomalies abstract away from the *discovery aspect*. Let us use the value premium as an example. By definition, the "discovery" of the value premium in Basu (1983) should make at least *some* market participants aware of the return pattern for the first time, unless one believes Basu was actually the last person to find out about the return pattern. In traditional risk-based models of the value premium, however, *all* investors knew about the value premium even before the discovery in Basu (1983). That is, this traditional approach does not take into account the effect of discovery, which is exactly the focus of our paper. That is, we analyze the fact that the discovery of the anomaly informs some agents about the return pattern for the first time. For convenience, we call those agents "arbitrageurs," to highlight that their risk exposure is different from that of the previously described "investors."

Specifically, there is a continuum of identical arbitrageurs, with a population size of one. Their aggregate endowment at $t = 0$ is $W_0^a \geq 0$ dollars in cash. Importantly, they do not have the hedging demand that investors have, perhaps because arbitrageurs have a different labor income profile. To analyze the discovery effect across anomalies, we assume that arbitrageurs have access to another investment opportunity, which presumably exploits existing anomalies (say, e.g., currency carry trade). This opportunity is not available to the investors described earlier, perhaps because those investors do not have the expertise to analyze and implement the strategy. We call this existing anomaly "asset e," and assume its gross return at $t = 1, 2$ is

$$r_{e,t} = \begin{cases} \mu_e + \sigma_e, & \text{with probability } p_e, \\ \mu_e - \sigma_e, & \text{with probability } 1 - p_e, \end{cases}$$

where $\mu_e > \sigma_e > 0$, and $0 < p_e < 1$. Moreover, $r_{e,t}$ is assumed to be independent from $\mu_{i,t}$. That is, the fundamentals of assets 1 and 2 are independent from the existing anomaly—asset e.

For simplicity, we assume that the return of the existing anomaly $r_{e,t}$ is exogenously given. This simplification shuts down the effect of the discovery on the returns of existing anomalies. This effect, however, is going to be small if the amount of the capital attracted by this new anomaly is small relative to the aggregate arbitrage capital attracted by all existing anomalies.[1]

1.3 Discovery effect

To analyze the discovery effect, we compare the equilibria across the following two economies. In the first (pre-discovery) economy, arbitrageurs are not aware of the anomaly (i.e., that assets 1 and 2 have the same fundamentals but different prices at $t = 0$). Hence, they invest in asset e, but not in assets 1 or 2. In the second (post-discovery) economy, arbitrageurs become aware of the anomaly and start exploiting it, as well as investing in the existing anomaly—asset e. To capture this, we assume that arbitrageurs take a long-short strategy in the two assets so that they can exploit the anomaly and stay "market neutral."[2] Specifically, we use $\theta^a_{i,t}$ to denote the fraction of arbitrageurs' wealth that is invested in asset $i = 1, 2$, at time $t = 0, 1$. A market-neutral strategy is such that, for $t = 0, 1$,

$$\theta^a_{1,t} + \theta^a_{2,t} = 0. \tag{5}$$

Let us use $\theta^a_{e,t}$ to denote the fraction of arbitrageurs' wealth that is invested in asset e at time

[1]Of course, our simplification may miss some subtle dynamics. For example, one might conjecture that investors may substitute between major anomalies, and generate negative correlation among them. Asness, Moskowitz, and Pedersen (2013) document a negative correlation between value and momentum returns. These specific dynamics are beyond the scope of this paper.

[2]This assumption is made so that arbitrageurs focus on exploiting the anomaly. Alternatively, we can simply assume that after the discovery, arbitrageurs become aware of the existence of assets 1 and 2. Under this alternative assumption, however, arbitrageurs will not only take a long-short position in the two assets, but also start investing in both assets. The latter will simply push up the prices of both assets. We are not interested in analyzing this latter effect. Moreover, in the value premium example, for instance, it seems more natural to think that, after the discovery of the value premium, hedge funds start buying value stocks and shorting growth stocks, rather than hedge funds becoming aware of the existence of both value and growth stocks and starting to buy both of them.

$t = 0, 1$. Then, arbitrageurs' wealth dynamic is given by

$$W_{t+1}^a = W_t^a \left[\sum_{i \in \{1,2,e\}} \theta_{i,t}^a r_{i,t+1} + \left(1 - \sum_{i \in \{1,2,e\}} \theta_{i,t}^a \right) \right], \tag{6}$$

for $t = 0, 1$. Their objective is to choose $\theta_{i,t}^a$ for $i = 1, 2, e$, and $t = 0, 1$, to

$$\max_{\theta_{i,t}^a} E_0 \left[\log \left(W_2^a \right) \right], \tag{7}$$

subject to (5) and (6).

In the pre-discovery economy, arbitrageurs are on the sidelines and have no impact on the markets for assets 1 and 2.[3] Hence, the equilibrium can be defined as follows. The pre-discovery competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and investors' portfolios ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$), such that investors' portfolios optimize (4), and markets clear, i.e., for $i = 1, 2$ and $t = 0, 1$,

$$W_t \theta_{i,t} = P_{i,t}. \tag{8}$$

Similarly, the post-discovery competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and portfolios of investors and arbitrageurs ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$; and $\theta_{i,t}^a$ for $t = 0, 1$, $i = 1, 2, e$), such that investors' portfolios optimize (4), arbitrageurs' portfolios optimize (7), and markets clear, i.e., for $i = 1, 2$ and $t = 0, 1$,

$$W_t \theta_{i,t} + W_t^a \theta_{i,t}^a = P_{i,t}. \tag{9}$$

The implicit assumption is that arbitrageurs do not have any hedging demand in asset 1 or 2. Moreover, after the discovery, they know that the cause of the anomaly is investors' hedging demand. These are simplifying assumptions. What is necessary is that arbitrageurs have less hedging demand in asset 1 than investors. Finally, even if arbitrageurs do not know the true cause of the anomaly, they will still invest in it, and the main implications in this alternative model remain similar to those in our current setup.[4]

1.4 Equilibrium

Proposition 1 (Pre-discovery) *The pre-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ can be characterized by equation (8) and*

$$E_t \left[\frac{r_{i,t+1} - 1}{W_{t+1} + \rho P_{1,t+1}} \right] = 0, \text{ for } i = 1, 2,\ t = 0, 1. \tag{10}$$

Moreover, in this equilibrium, we have $P_{1,0} < P_{2,0}$.

[3] This assumption perhaps resembles the preference of hedge funds, who attempt to deliver market-neutral returns, and so have little interest in assets 1 and 2 before the discovery. Another reason is that hedge funds may choose to self-impose restrictions on their investment opportunity set (He and Xiong (2013)).

[4] See Brennan and Xia (2001) for an analysis of this intuition in the portfolio choice context.

The above proposition illustrates the anomaly: Although both assets have the same fundamentals *ex ante*, they have different prices and hence different future expected returns. Due to their endowment, investors find asset 1 riskier than asset 2, leading to a lower price for asset 1. We label this as an anomaly because if econometricians had only the price data, they would find the return pattern puzzling. This is similar to the anomalies we see in the literature. For example, the value premium is a puzzle if one looks at the return data alone. Risk-based models try to explore the idea that value stocks have a higher exposure to certain risk factors, which is similar to the reduced-formulation of the hedging demand in our model. While traditional risk-based models focus on the detailed analysis of the exact mechanism through which the hedging demand arises, they assume away the discovery aspect since all investors know the return pattern all along. In contrast, we are not interested in the details of the hedging demand, but focus on the analysis of the consequences of the discovery.

The following proposition characterizes the post-discovery equilibrium.

Proposition 2 (Post-discovery) *The post-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ and $\theta^a_{i,1}$ can be characterized by equations (5), (9), (10), and for $t = 0, 1$,*

$$\begin{aligned} E_t\left[\frac{r_{1,t+1} - r_{2,t+1}}{W^a_{t+1}}\right] &= 0, \\ E_t\left[\frac{r_{e,t+1} - 1}{W^a_{t+1}}\right] &= 0. \end{aligned}$$

Since arbitrageurs are not exposed to the endowment risk that investors have, they find the anomaly an attractive investment opportunity, and buy asset 1 and short asset 2. For convenience, we call the return from this long-short portfolio, $r_{1,1} - r_{2,1}$, the "anomaly return."

To analyze the discovery effect, we will compare the post-discovery equilibrium in Proposition 2 with the pre-discovery equilibrium in Proposition 1.[5] In particular, following the algorithm in Section 4.4, we solve both equilibria numerically. The baseline parameter values are summarized in Table 1. In the following numerical analysis, we vary only one parameter at a time to examine the effects of the discovery. We have also repeated our numerical analyses for other parameter values, and none of the following qualitative results are specific to the chosen parameters.

[5] The equation system in Proposition 2 is highly nonlinear and we have not been able to establish the existence and uniqueness of their solutions. However, we have always been able to solve the equation system numerically, and the solution appears to be unique. One might be somewhat surprised that the simple two-period structure in our model does not allow for a closed-form solution. In fact, the wealth effect in our model has similar complexity as that in the continuous-time model in Xiong (2001), which also heavily relies on numerical analysis. As noted in Gromb and Vayanos (2002), a two-period model of arbitrageurs and investors with a wealth effect is not as tractable as its appearance suggests (page 381). In a recent study, Kondor and Vayanos (2019) gain more tractability by simplifying investors' decisions.

1.5 Anomaly returns

Figure 1 illustrates the effects of discovery on the expected anomaly returns. The dashed line represents the size of the anomaly (i.e., the expected anomaly return $E_0[r_{1,1} - r_{2,1}]$) before the discovery. Since arbitrageurs have no influence on the markets for assets 1 and 2 before the discovery, the dashed line is flat: The expected anomaly return is around 5.5% regardless of arbitrageurs' wealth.

After the discovery, arbitrageurs start exploiting the opportunity, reducing the expected anomaly return. As shown by the solid line in Panel A, the post-discovery expected anomaly return is lower than that in the pre-discovery case (i.e., the solid line is below the dashed line). In the case $W_0^a = 2$, for example, the discovery reduces the expected anomaly return from 5.5% to 5%.

The plot also shows that the effect of discovery is stronger when arbitrageurs have more wealth. For example, in the case $W_0^a = 5$, the discovery reduces the expected anomaly return from 5.5% to 4%. The discovery effect disappears when $W_0^a = 0$. One can think of this $W_0^a = 0$ case as representing the traditional modeling approach, where discovery does not change the set of investors who are aware of the anomaly.

Panels B and C demonstrate the effects of arbitrageurs' existing investment opportunity (i.e., asset e). If arbitrageurs' existing strategy is more attractive (i.e., μ_e is higher, or σ_e is lower), they will allocate less capital to exploit the new anomaly and so its expected return will drop less. As shown in Panels B and C, after the discovery of an anomaly, its expected return is increasing in μ_e and decreasing in σ_e.

1.6 Correlation among anomaly returns

By the construction of our model, before the discovery, the anomaly return $r_{1,1} - r_{2,1}$ is independent of the return of the existing anomaly $r_{e,1}$. How does the discovery affect the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$?

Intuitively, after the discovery of an anomaly, arbitrageurs start exploiting it, as well as the existing anomaly, asset e. This creates a correlation through the wealth effect. Suppose the return from asset e is unexpectedly high one period. This increases the wealth of these arbitrageurs. Everything else being equal, they will allocate more investment to the newly discovered anomaly. This higher investment pushes up the price of asset 1 and pushes down the price of asset 2, leading to a high anomaly return $r_{1,1} - r_{2,1}$. Similarly, an unexpectedly low return from asset e leads to a low anomaly return. That is, the wealth effect increases the correlation between the newly discovered anomaly return and the return from the existing anomaly.

The above intuition is illustrated in Figure 2. Panel A plots the correlation coefficient between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. Before the discovery, as illustrated by the dashed line, the correlation is 0. In contrast, the post-discovery correlation, shown by the solid line, is positive. The only exception is the case $W_0^a = 0$, where the correlation is zero, the same as in the pre-discovery case. Again, one can view this special case as the traditional approach that abstracts away from discovery.

This discovery effect (i.e., the change in the correlation across the pre- and post-discovery cases) is initially increasing in the size of arbitrage capital W_0^a, and is not monotonic. This is because arbitrageurs have two effects on the correlation. The first is the aforementioned wealth effect, which increases the correlation. The second is that as arbitrage capital increases, the prices of assets 1 and 2 are more driven by their fundamentals. This reduces the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. When the size of arbitrage capital is sufficiently large, the second effect dominates, and hence a further increase in arbitrage capital reduces the correlation.

The above intuition is further illustrated in Panels B and C. In particular, when arbitrageurs have a larger position in asset e (due to a higher μ_e or a lower σ_e), their wealth becomes more sensitive to its realized return $r_{e,1}$. This leads to a stronger wealth effect, i.e., the discovery has a stronger effect in generating the correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. In Panel B, for example, as the expected return from asset e increases (i.e., a higher μ_e), it leads to a higher correlation between $r_{1,1} - r_{2,1}$ and $r_{e,1}$. Similarly, in Panel C, as the volatility of asset e increases (i.e., a higher σ_e), it leads to a weaker wealth effect and a lower correlation.

1.7 Correlation between assets 1 and 2

Our model shows that the discovery of an anomaly reduces the correlation coefficient between the returns of assets 1 and 2. The intuition is as follows. After the discovery, arbitrageurs long asset 1 and short asset 2 to exploit the anomaly. Now, suppose arbitrageurs' wealth increases due to, say, a high return from their investment in asset e. They will buy more of asset 1 and sell more of asset 2. This increases asset 1's return but decreases asset 2's return. Similarly, when arbitrageurs' wealth decreases, they will unwind some of their positions in the long-short portfolio. That is, they will sell asset 1 and buy asset 2, decreasing asset 1's return but increasing asset 2's return. In both cases, arbitrageurs' wealth shocks push the returns of the two assets in opposite directions, which reduces the correlation between the returns of assets 1 and 2.

This intuition is illustrated in Figure 3. The dashed line in Panel A is for the pre-discovery correlation between assets 1 and 2. Since arbitrageurs are on the sidelines before the discovery, their wealth level W_0^a does not affect the correlation. Hence, the dashed line is flat. The post-discovery case is represented by the solid line. It is below the dashed line, suggesting that the discovery reduces the correlation between assets 1 and 2. It also shows that the larger the size of arbitrage capital, the larger the reduction in the correlation.

The above intuition further suggests that the discovery effect is stronger when arbitrageurs' wealth is more volatile. To illustrate this intuition, we plot the correlation between assets 1 and 2 against arbitrageurs' wealth volatility, which is an endogenous variable. Specifically, we vary arbitrageurs' wealth volatility by changing μ_e from 1.1 to 1.46.[6] The solid line in Panel B shows that after the discovery, the correlation between assets 1 and 2 is decreasing in arbitrageurs' wealth volatility. In contrast, this relation does not hold before the discovery, as shown by the

[6] Qualitatively similar results can be generated by varying σ_e instead.

dashed line.

1.8 The transition period

In our analysis so far, we have focused on the pre- and post-discovery equilibria and left out the transition period between the two equilibria. We now briefly analyze the price behavior during this transition period. To do that, we need to analyze the discovery "within the model," rather than comparing equilibria across "two models." By definition, a discovery is a surprise to arbitrageurs. Hence, one simple formulation is as follows. Let us consider the economy analyzed before. At $t = 0$, arbitrageurs are not aware of the anomaly, and hence the equilibrium prices and investors' portfolio holdings are the same as those in Proposition 1. At $t = 1$, arbitrageurs recognize the anomaly and start exploiting it. This is a surprise to investors. When they planned their dynamic portfolio strategy at $t = 0$, they didn't expect the arrival of arbitrageurs. Now that arbitrageurs are taking long-short positions in assets 1 and 2, investors have to adjust their portfolios accordingly. Hence, the new equilibrium prices at $t = 1$ are not what investors expected at $t = 0$.

We solve this version of the model numerically, and the asset price behavior during the discovery process is captured in Figure 4. It plots the expected anomaly return, conditional on the discovery at $t = 1$, $E[r_{1,1} - r_{2,1}|Discovery]$, against arbitrageurs' wealth at $t = 1$. It shows that, the greater the arbitrageurs' wealth, the higher the average anomaly return during the transition period when it is discovered. This is because when the anomaly is discovered, arbitrageurs long asset 1 and short asset 2, causing a higher anomaly return during the transition period. Afterward, however, as shown earlier in Figure 1, the expected anomaly return is going to be lower.

The above formulation is of course stylized. The transition period is likely to be a process rather than instantaneous. Moreover, instead of treating the discovery as a complete surprise, one can assume that investors have a prior about the probability for a discovery in the next period and follow the Bayes rule to update their belief. The intuition captured in Figure 4 is likely to be robust in these two alternative formulations. We leave more elaborate formulations to future studies.

2 Mispricing-based anomaly

To compare mispricing- and risk-based anomalies, we now analyze a model in which the anomaly is caused by investors' behavioral bias. Specifically, we modify the previous model by setting $\rho = 0$; that is, there is no hedging demand. The fundamentals of the two assets are still given by (1) and (2). However, investors are biased about asset 1 and believe that for $t = 1, 2$,

$$\mu_{1,t} = \begin{cases} \mu + \sigma & \text{with probability } p - b, \\ \mu - \sigma & \text{with probability } p + b, \end{cases} \tag{11}$$

where $0 \leq b < p$. That is, investors underestimate asset 1's expected cash flow, and b measures the degree of the bias. In contrast, their belief about asset 2 is correct.

Investors' objective is to choose $\theta_{i,t}$, for $i = 1, 2$, and $t = 0, 1$, to

$$\max_{\theta_{i,t}} E_0^* \left[\log \left(W_2\right)\right], \tag{12}$$

subject to (3), where $E_0^* [\cdot]$ indicates that the expectation is taken under the biased belief in (11). Arbitrageurs have correct beliefs, and their objective is given by (7), as in the previous section.

This formulation is meant to capture the essence of mispricing-based interpretations of anomalies in a reduced form. For instance, in the value premium example, Lakonishok, Shleifer, and Vishny (1994) argue that investors are overly enthusiastic about glamorous growth stocks and have a low demand for value stocks. Similarly, in our model, investors underestimate the payoff from asset 1 and so have a low demand.

Similar to the case of the risk-based anomaly, in the pre-discovery case, arbitrageurs have no influence on the markets for assets 1 and 2. The competitive equilibrium for this case is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and investors' portfolios ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$), such that investors' portfolios optimize (12), and markets clear as in (8).

The post-discovery competitive equilibrium is defined as asset prices ($P_{i,t}$ for $i = 1, 2$, and $t = 0, 1$) and portfolios of investors and arbitrageurs ($\theta_{i,t}$ for $t = 0, 1$ and $i = 1, 2$; and $\theta_{i,t}^a$ for $t = 0, 1$, $i = 1, 2, e$), such that investors' portfolios optimize (12), arbitrageurs' portfolios optimize (7), and markets clear as in (9).

What is implicitly assumed here is that the discovery does *not* affect investors' bias b. That is, the bias is systematic and deeply rooted, and investors do not adjust their behavior after the discovery of the anomaly. This assumption is made for simplicity. Alternatively, if the bias is partially reduced after the discovery, the results remain qualitatively similar.

Proposition 3 *The pre-discovery equilibrium prices $P_{i,t}$ and portfolio choices $\theta_{i,t}$ can be characterized by (8), and for $i = 1, 2$, $t = 0, 1$,*

$$E_t^* \left[\frac{r_{i,t+1} - 1}{W_{t+1}}\right] = 0. \tag{13}$$

The post-discovery equilibrium prices $P_{i,t}$ and portfolio choices ($\theta_{i,t}$, and $\theta_{i,1}^a$) can be characterized by equations (5), (9), (13), and for $t = 0, 1$,

$$E_t \left[\frac{r_{1,t+1} - r_{2,t+1}}{W_{t+1}^a}\right] = 0, \tag{14}$$

$$E_t \left[\frac{r_{e,t+1} - 1}{W_{t+1}^a}\right] = 0. \tag{15}$$

Similar to the risk-based case in the previous section, investors have a lower demand for asset 1 than for asset 2. The only difference is the motivation. In the risk-based case, the motivation is to hedge, while in the mispricing-based case, the motivation is investors' wrong belief. Can we distinguish a risk-based anomaly from a mispricing-based one by examining asset prices? We will examine this in the next section.

3 Comparing risk- and mispricing-based anomalies

3.1 Welfare

How does the discovery of an anomaly affect investor welfare? To address this question, we first need to clarify our welfare measures. For the risk-based case, we simply use investors' expected utility at $t = 0$. For the mispricing-based case, we use "subjective welfare" to refer to investors' subjective expected utility at $t = 0$, and use "objective welfare" to refer to investors' utility evaluated under the objective belief at $t = 0$.

Proposition 4 *The discovery of a risk-based anomaly increases investors' welfare. The discovery of a mispricing-based anomaly increases investors' subjective welfare, but reduces their objective welfare.*

In the case of a risk-based anomaly, arbitrageurs essentially offer better risk sharing to investors. Before the discovery, the endowment risk is shared only among investors (i.e., arbitrageurs are not involved). After the discovery, this endowment risk is shared between investors and arbitrageurs: Investors unload asset 1 to arbitrageurs to hedge against their endowment risk. Arbitrageurs' trading makes the hedging cheaper. For the mispricing-based case, however, when arbitrageurs start exploiting the anomaly, naive investors think they are better off, since they can offload some of asset 1, which they are pessimistic about. That is, investors' *subjective* expected utility increases after the discovery. However, the discovery reduces naive investors' objective welfare. For instance, suppose the value premium was caused by investors' overly optimistic perception about growth stocks. The discovery of this anomaly attracts arbitrageurs to buy value and sell growth stocks. Consequently, investors end up holding more over-priced growth stocks and fewer under-priced value stocks, and they will suffer from worse performance in the future.

3.2 Distinguishing risk- and mispricing-based anomalies

We now compare the risk-based anomaly (Propositions 1 and 2) with the mispricing-based one (Proposition 3). In particular, we set $b = 0.055$ and adopt all other parameters from Table 1. We choose this value for b so that, before the discovery, the expected anomaly returns are the same across the risk-based case and the mispricing-based case. We now compare the post-discovery return dynamic across the two cases.

Panel A of Figure 5 shows that it is difficult to distinguish a risk-based anomaly from a

Table 1: **Parameter values**

Parameter	W_0^a	k	ρ	μ	σ	p	μ_e	σ_e	p_e
Value	1	1	1	1.2	0.6	0.5	1.4	0.5	0.5

mispricing-based one by examining the post-discovery performance. The solid and dashed lines represent the post-discovery expected anomaly return for the risk- and mispricing-based cases, respectively. The pre-discovery expected anomaly return for both cases is flat at around 5.5% (we omitted this flat line). The plot shows that the discovery of an anomaly reduces its expected return regardless of whether the anomaly is caused by risk or mispricing. Moreover, both lines are downward sloping, implying that the more arbitrage capital (W_0^a), the stronger the effect. Panel B shows that, for both the risk- and mispricing-based cases, the discovery of an anomaly increases the correlation between its return and the existing anomaly return. Even the non-monotonic pattern is similar across the two cases. Finally, Panel C shows that the discovery of the anomaly reduces the correlation between assets 1 and 2 for both risk- and mispricing-based cases. Moreover, this correlation is decreasing in arbitrageurs' wealth level W_0^a in both cases.

3.3 One possible solution

The above results highlight the difficulty in distinguishing between risk- and mispricing-based anomalies by examining asset prices.[7] What is the solution then? We argue that it is more promising to analyze investors' *portfolios*. The idea is that investors' holdings might offer direct evidence on *why* they overweight one asset and underweight another.

In a risk-based anomaly, investors recognize the fact that asset 1's expected return is higher than asset 2's, and so they have a *higher* total exposure to asset 1 than to asset 2, once we include the exposure implied by their nontradable endowment. That is, in this case, although investors underweight asset 1 in the stock market, their total exposure to asset 1 is actually higher than to asset 2. In a mispricing-based anomaly, however, investors have a lower exposure to asset 1, because they mistakenly believe that it has a lower future payoff and underweight it.

Therefore, investors' portfolio holdings can help separate risk- and mispricing-based anomalies. For example, Fama and French (1993, 1996) interpret the value premium as value stocks exposing investors to risks associated with economy-wide financial distress. To evaluate this risk-based explanation, one can examine whether the investors who underweight value stocks are those who are more exposed to risk of financial distress (e.g., their labor income or other assets are more exposed to financial distress).[8]

[7]This is parallel to the result in Brav and Heaton (2002), which emphasizes the difficulty in distinguishing a biased belief from a rational belief with structural uncertainty.

[8]This idea can be applied more broadly to the measurement of many other hard-to-measure variables. For example, Choi, Jin, and Yan (2014) try to measure the degree of information asymmetry at an individual stock level by tracking the activities of all investors in the stock market in China. The detailed transaction data of the

To be fair, while examining portfolio holdings is a direct approach, it is very demanding on the dataset. It requires detailed information on investors' positions, including their nontradable assets. Nevertheless, this test may not be completely infeasible. For example, Betermier, Calvet, and Sodini (2017) have recently analyzed the characteristics of investors of value and growth stocks, and potentially shedding light on why investors hold value or growth stocks.

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whole population reveals the degree of information asymmetry for each stock.

4 Proofs and Numerical Procedure

4.1 Proof of Propositions 1 and 2

Due to the logarithmic preference, the maximization problem (4) is equivalent to maximizing the log wealth growth for each period. Hence, investors' first-order conditions are given by

$$E_t\left[\frac{r_{i,t+1}-1}{W_{t+1}+\rho P_{1,t+1}}\right]=0,$$

for $i=1,2, t=0,1$. Similarly, the arbitrageurs' optimization problem (7) can also be decomposed into a period-by-period optimization problem, and the first-order conditions are given by

$$\begin{aligned} E_t\left[\frac{r_{1,t+1}-r_{2,t+1}}{W^a_{t+1}}\right] &= 0, \\ E_t\left[\frac{r_{a,t+1}-1}{W^a_{t+1}}\right] &= 0. \end{aligned}$$

Combining the above first-order conditions with the market-clearing conditions, we can characterize the equilibria in Propositions 1 and 2.

We now prove $P_{1,0}<P_{2,0}$ by contradiction. Suppose $P_{1,0}\geq P_{2,0}$. Note that investors' optimal portfolio in equilibrium is to hold one unit of both assets. Suppose an investor sells ϵ unit of asset 1 and buys ϵ unit of asset 2. Define his expected utility as

$$U(\epsilon)\equiv E_0[\log(k+(1+\rho-\epsilon)D_1+(1+\epsilon)D_2)].$$

It is easy to see that $\frac{dU}{d\epsilon}|_{\epsilon=0}>0$. That is, he can strictly improve his portfolio by selling ϵ unit of asset 1 and buying ϵ unit of asset 2. This leads to a contradiction.

4.2 Proof of Proposition 3

The first-order condition to the maximization problem (12) is given by (13). The first-order conditions for arbitrageurs are still given by (14) and (15). These optimality and market-clearing conditions lead to the results in the proposition.

4.3 Proof of Proposition 4

In both the risk-based and mispricing-based cases, investors have the option not to trade. The participation constraint implies that the investors' expected utility cannot be lower than that in the pre-discovery case. Moreover, investors' concave utility function and convex budget constraint imply that the investors' optimization problem has a unique solution. It is easy to see that in the case of $W_0^a > 0$, the portfolio characterized in Proposition 2 is strictly different from the non-participation portfolio. Hence, discovery strictly increases investor welfare. Similarly, in the mispricing-based case, discovery strictly increases investors' subjective welfare.

To analyze naive investors' objective welfare, we note that naive investors' portfolio in the economy in Section 2 can be decomposed into one unit in assets 1 and 2, and a position x_t (for $t = 0, 1$) in the long-short strategy (long asset 1 and short asset 2). It is easy to show that naive investors' objective welfare $E[\log(W_2)]$ is concave in x_0 and x_1. In the pre-discovery case, $x_0 = x_1 = 0$. In the post-discovery case, however, x_t is "further away" from the optimum point for maximizing $E[\log(W_2)]$. For example, an naive investor's choice is $x_0 < 0$ although $\partial E[\log(W_2)]/\partial x_0|_{x_0=0} > 0$. Therefore, an naive investor's objective welfare is lower in the post-discovery case.

4.4 Numerical procedure

We follow the procedure described below to solve the model:

1. Take initial guesses for the total wealth for investors and arbitrageurs at $t = 1$: W_1 and W_1^a for the eight states at date 1.

2. For each of the eight states, take W_1 and W_1^a as given, solve for the portfolios ($\theta_{i,1}$ for $i = 1, 2$, and $\theta_{i,1}^a$ for $i = 1, 2, e$) and prices $P_{1,1}$ and $P_{2,1}$.

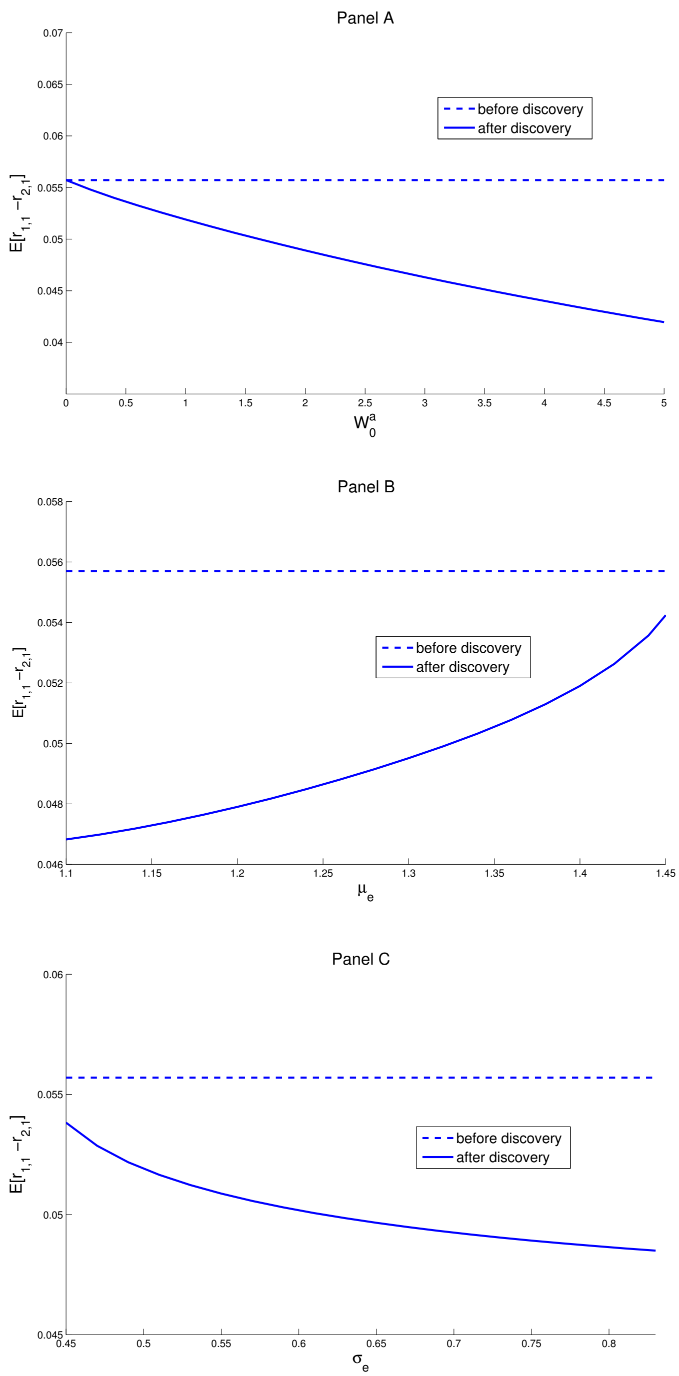
3. Take the prices $P_{1,1}$ and $P_{2,1}$ for the eight states in step two as given, solve for the $t = 0$ portfolios ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and prices $P_{1,0}$ and $P_{2,0}$.

4. Based on the portfolios in step three ($\theta_{i,0}$ for $i = 1, 2$, and $\theta_{i,0}^a$ for $i = 1, 2, e$) and the prices

in steps two and three ($P_{1,0}$, $P_{2,0}$, and $P_{1,1}$, $P_{2,1}$ for all eight states at $t = 1$), calculate the investors' and arbitrageurs' updated wealth, W_1 and W_1^a, in the eight cases at $t = 1$.

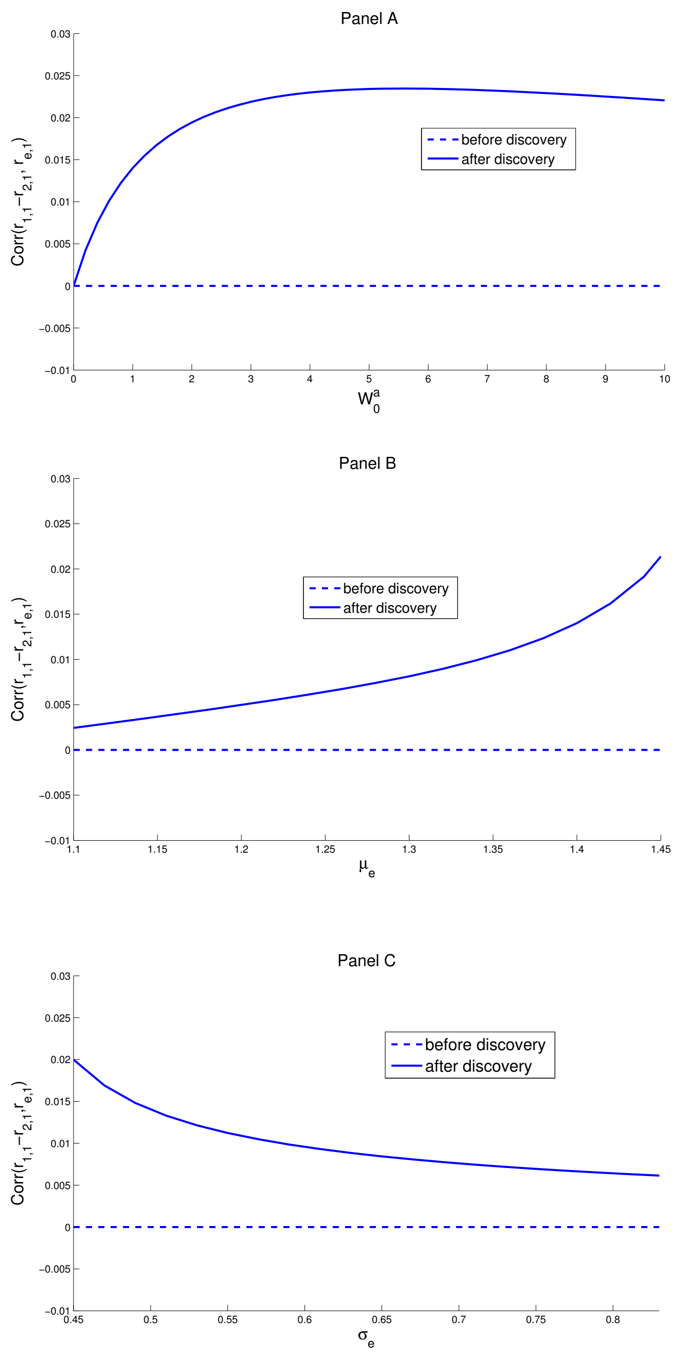
5. Repeat steps two to three until the wealth, portfolios, and prices converge, i.e., for each variable, the difference between two iterations is no greater than 0.00005.

Figure 1: **Anomaly Return**



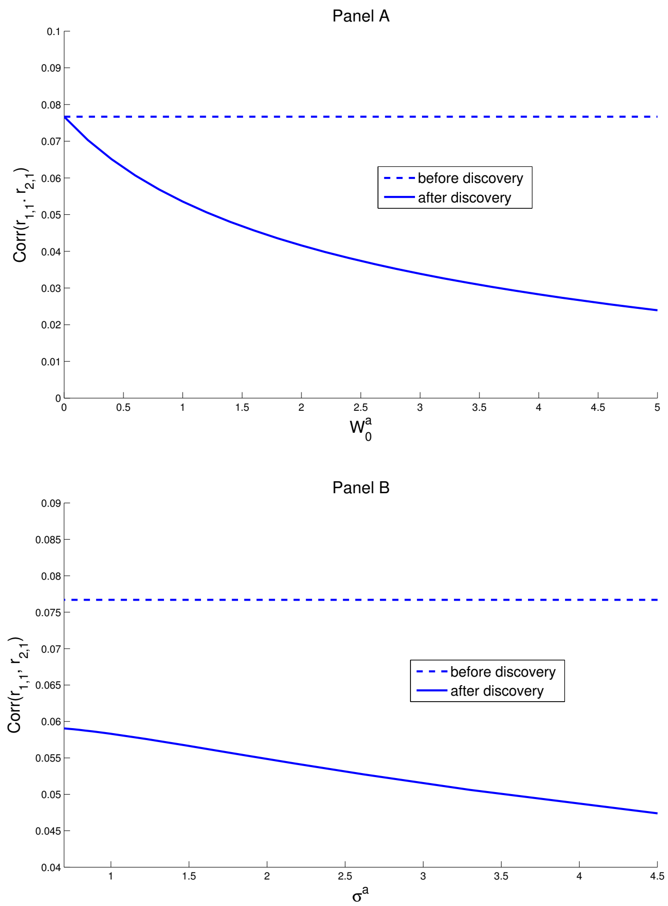
Panels A–C plot the expected anomaly return, $E[r_{1,1} - r_{2,1}]$, on arbitrageurs' initial wealth W_0^a, asset e's expected return μ_e and volatility σ_e, respectively. The parameter values are given by Table 1.

Figure 2: **Correlation Among Anomaly Returns**



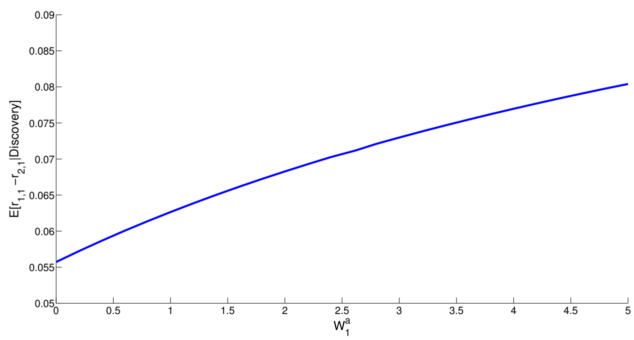
Panels A–C plot the correlation coefficient between the anomaly return and asset e's return, $Corr(r_{1,1} - r_{2,1}, r_{e,1})$, on arbitrageurs' initial wealth W_0^a, asset e's expected return μ_e, and its volatility σ_e, respectively. The parameter values are given by Table 1.

Figure 3: **Correlation Between Assets 1 and 2**



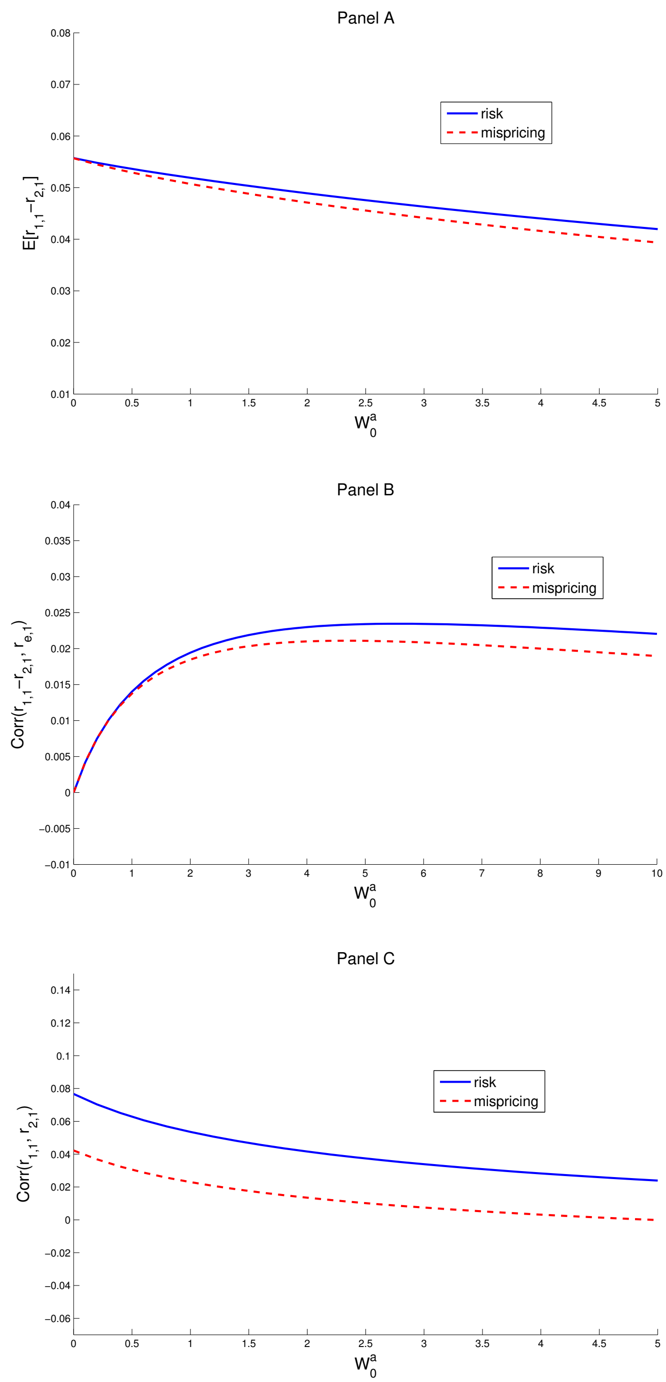
Panels A and B plot the correlation coefficient between assets 1 and 2, $Corr(r_{1,1}, r_{2,1})$, on arbitrageurs' initial wealth W_0^a, and their wealth volatility σ^a, respectively. Arbitrageurs' wealth volatility σ^a is an endogenous variable. We generate its variation by varying μ_e from 1.1 to 1.46. All other parameter values are given by Table 1.

Figure 4: **Anomaly Return During the Discovery Process**



This figure plots the conditional average anomaly return, $E[r_{1,1} - r_{2,1}|Discovery]$, from $t = 0$ to $t = 1$, when the anomaly is unexpectedly discovered, against arbitrageurs' wealth at $t = 1$, W_1^a. All parameter values are given by Table 1.

Figure 5: **Comparison: Asset Prices**



Panels A–C plot the expected anomaly return, $E[r_{1,1} - r_{2,1}]$, its correlation with asset e's return, $Corr(r_{1,1} - r_{2,1}, r_{e,1})$, and the correlation between assets 1 and 2, $Corr(r_{1,1}, r_{2,1})$, on arbitrageurs' initial wealth W_0^a, respectively. The solid line is for the risk-based case, and the dashed line the mispricing-based case. Parameter values: $b = 0.055$, and other parameter values are given by Table 1.

Online Appendix for:

Anomaly Discovery and Arbitrage Trading

Table O1. Detailed Information of Anomalies

This table provides the name, authors, and publication information of the 99 anomalies used in our paper. Journal title abbreviations: FAJ=Financial Analysts Journal; JAE= Journal of Accounting and Economics; JAR=Journal of Accounting Review; JBFA=Journal of Business, Finance & Accounting; JF=Journal of Finance; JEF=Journal of Empirical Finance; JFE=Journal of Financial Economics; JFM=Journal of Financial Markets; JFQA=Journal of Financial and Quantitative Analysis; JPE=Journal of Political Economy; RAS=Review of Accounting Studies; TAR=The Accounting Review; WP=Working Paper.

No.	**Anomaly Name**	**Author(s)**	**Date, Journal**
1	Beta	Fama & MacBeth	1973, JPE
2	Beta squared	Fama & MacBeth	1973, JPE
3	Earnings-to-price	Basu	1977, JF
4	O-score	Ohlson	1980, JAR
5	Dividends-to-price	Litzenberger & Ramaswamy	1982, JF
6	Unexpected quarterly earnings	Rendelman, Jones & Latane	1982, JFE
7	Change in forecasted annual EPS	Hawkins, Chamberlin & Daniel	1984, FAJ
8	36-month reversal	De Bondt & Thaler	1985, JF
9	Forecasted growth in 5-year EPS	Bauman & Dowen	1988, FAJ
10	Leverage	Bhandari	1988, JF
11	% change in current ratio	Ou & Penman	1989, JAE
12	% change in quick ratio	Ou & Penman	1989, JAE
13	% change in sales-to-inventory	Ou & Penman	1989, JAE
14	Cash flow-to-debt	Ou & Penman	1989, JAE
15	Current ratio	Ou & Penman	1989, JAE
16	Quick ratio	Ou & Penman	1989, JAE
17	Sales-to-cash	Ou & Penman	1989, JAE
18	Sales-to-inventory	Ou & Penman	1989, JAE
19	Sales-to-receivables	Ou & Penman	1989, JAE
20	Amihud illiquidity	Amihud & Mendelson	1989, JF
21	Bid-ask spread	Amihud & Mendelson	1989, JF
22	12-month momentum	Jegadeesh	1990, JF
23	1-month reversal	Jegadeesh	1990, JF
24	6-month momentum	Jegadeesh & Titman	1990, JF

25	Net stock issue	Ritter	1991, JF
26	% change in depreciation-to-gross PP&E	Holthausen & Larcker	1992, JAE
27	Depreciation-to-gross PP&E	Holthausen & Larcker	1992, JAE
28	Book-to-market	Fama & French	1992, JF
29	Size (market cap)	Fama & French	1992, JF
30	Annual sales growth	Lakonishok, Shleifer & Vishny	1994, JF
31	Industry-adjusted change in employees	Asness, Porter & Stevens	1994, WP
32	New equity issue	Loughran, Ritter & Ritter	1995, JF
33	Sales-to-price	Barbee, Mukherji & Raines	1996, FAJ
34	Working capital accruals	Sloan	1996, TAR
35	Share turnover	Datar, Naik & Radcliffe	1998, JFM
36	% change in CAPEX - industry % change in CAPEX	Abarbanell & Bushee	1998, TAR
37	% change in gross margin - % change in sales	Abarbanell & Bushee	1998, TAR
38	% change in sales - % change in accounts receivable	Abarbanell & Bushee	1998, TAR
39	% change in sales - % change in inventory	Abarbanell & Bushee	1998, TAR
40	% change in sales - % change in SG&A	Abarbanell & Bushee	1998, TAR
41	# of consecutive earnings increases	Barth, Elliott & Finn	1999, JAR
42	Industry momentum	Moskowitz & Grinblatt	1999, JF
43	Financial statements score	Piotroski	2000, JAR
44	Industry-adjusted book-to-market	Asness, Porter & Stevens	2000, WP
45	Industry-adjusted cash flow-to-price ratio	Asness, Porter & Stevens	2000, WP
46	Industry-adjusted firm size	Asness, Porter & Stevens	2000, WP
47	Abnormal volume	Gervais, Kaniel & Mingelgrin	2001, JF
48	Dollar trading volume in month t-2	Chordia, Subrahmanyam & Anshuman	2001, JFE
49	Volatility of dollar trading volume	Chordia, Subrahmanyam & Anshuman	2001, JFE
50	Volatility of share turnover	Chordia, Subrahmanyam & Anshuman	2001, JFE

51	# of analysts covering stock	Elgers, Lo & Pfeiffer	2001, TAR
52	Scaled analyst forecast of one year ahead earnings	Elgers, Lo & Pfeiffer	2001, TAR
53	Dispersion in forecasted eps	Diether, Malloy & Scherbina	2002, JF
54	Changes in inventory	Thomas & Zhang	2002, RAS
55	Idiosyncratic return volatility	Ali, Hwang & Trombley	2003, JFE
56	Growth in long-term net operating assets	Fairfield, Whisenant & Yohn	2003, TAR
57	Net operating assets	Hirshleifer et al.	2004, JAE
58	RD_increase	Eberhart, Maxwell & Siddique	2004, JF
59	Investment to assets	Titman, Wei & Xie	2004, JFQA
60	Cash flow-to-price	Desai, Rajgopal & Venkatachalam	2004, TAR
61	Earnings volatility	Francis, LaFond, Olsson & Schipper	2004, TAR
62	Taxable income to book income	Lev & Nissim	2004, TAR
63	Change in common shareholder equity	Richardson, Sloan, Soliman & Tuna	2005, JAE
64	Change in long-term debt	Richardson, Sloan, Soliman & Tuna	2005, JAE
65	# of years since first Compustat coverage	Jiang, Lee & Zhang	2005, RAS
66	Financial statements score	Mohanram	2005, RAS
67	Price delay	Hou & Moskowitz	2005, RFS
68	R&D-to-market cap	Guo, Lev & Shi	2006, JBFA
69	R&D-to-sales	Guo, Lev & Shi	2006, JBFA
70	% change over two years in CAPEX	Anderson & Garcia-Feijoo	2006, JF
71	Composite equity issue	Daniel & Titman	2006, JF
72	Industry sales concentration	Hou & Robinson	2006, JF
73	Return volatility	Ang, Hodrick, Xing & Zhang	2006, JF
74	Return on assets	Fama & French	2006, JFE
75	Zero-trading days	Liu	2006, JFE
76	Abnormal volume in earnings announcement month	Lerman, Livnat & Mendenhall	2007, WP
77	Change in # analysts	Scherbina	2007, WP
78	Return on invested capital	Brown & Rowe	2007, WP
79	Asset growth	Cooper, Gulen & Schill	2008, JF
80	Financial distress	Campbell, et al.	2008, JF

81	Industry-adjusted change in asset turnover	Soliman	2008, TAR
82	Industry-adjusted change in profit margin	Soliman	2008, TAR
83	3-day return around earnings announcement	Brandt, Kishore, Santa-Clara & Venkatachalam	2008, WP
84	Revenue surprise	Kama	2009, JBFA
85	Cash flow volatility	Huang	2009, JEF
86	Debt capacity-to-firm tangibility	Hahn & Lee	2009, JF
87	Cash productivity	Chandrashekar & Rao	2009, WP
88	Employee growth rate	Bazdresch, Belo & Lin	2009, WP
89	Real estate holdings	Tuzel	2010, RFS
90	Absolute accruals	Bandyopadhyay, Huang & Wirjanto	2010, WP
91	Accrual volatility	Bandyopadhyay, Huang & Wirjanto	2010, WP
92	Change in tax expense	Thomas & Zhang	2011, JAR
93	Maximum daily return in prior month	Bali, Cakici & Whitelaw	2011, JFE
94	Percent accruals	Hafzalla, Lundholm & Van Winkle	2011, TAR
95	Cash holdings	Palazzo	2012, JFE
96	Organizational capital	Eisfeldt & Papanikolaou	2013, JF
97	Asset turnover	Novy-Marx	2013, JFE
98	Gross profitability	Novy-Marx	2013, JFE
99	Secured debt-to-total debt	Valta	2015, JFQA

Table O2. Simulated p-values

This table is based on Panel A of Table 2 in the paper. The only modification is the replacement of the *t*-values in the original table by the simulated *p*-values. We randomly assign a pseudo discovery year for each anomaly from the distribution of anomaly publication years in our sample. We then rerun the regressions in Panel A of Table 2 and keep the coefficient estimate of $Discovery_{i,t}$. This procedure is repeated 10,000 times to obtain a simulated distribution of the coefficient estimates. The simulated *p*-values, reported in brackets, are calculated as the portion of the simulated coefficient estimates that are smaller (i.e., more negative) than their corresponding coefficient estimates in Table 2. *, **, and *** indicate that the coefficients are statistically significant at the 10%, 5%, and 1% level, respectively.

Dep.Var.= $X_{i,t}$	(1)	(2)	(3)	(4)	(5)	(6)
	Equal-weighted anomaly portfolios			Value-weighted anomaly portfolios		
	NoCite Weight	RawCite Weight	Cite PerYear Weight	NoCite Weight	RawCite Weight	Cite PerYear Weight
$Discovery_{i,t}$	-0.04**	-0.06**	-0.05***	-0.05**	-0.1**	-0.08***
	{0.046}	{0.018}	{0.000}	{0.048}	{0.027}	{0.000}
Anomaly FEs	Yes	Yes	Yes	Yes	Yes	Yes
Observations	80,309	80,309	80,309	80,309	80,309	80,309
R^2	0.37	0.71	0.64	0.40	0.77	0.65

Table O3. Discovery and Correlation (Excluding Small Stocks)

This table reports the results from the regressions in Panel A of Table 2, based on the sample after excluding stocks that are smaller than the 20th NYSE size percentile.

Dep.Var.= $X_{i,t}$	(1)	(2)	(3)	(4)	(5)	(6)
	Equal-weighted anomaly portfolios			Value-weighted anomaly portfolios		
	NoCite Weight	RawCite Weight	Cite PerYear Weight	NoCite Weight	RawCite Weight	Cite PerYear Weight
$\text{Discovery}_{i,t}$	-0.02***	-0.06***	-0.05***	-0.03***	-0.1***	-0.08***
	(-3.86)	(-6.53)	(-4.86)	(-2.99)	(-4.33)	(-4.07)
Anomaly FEs	Yes	Yes	Yes	Yes	Yes	Yes
Observations	79,925	79,925	79,925	79,925	79,925	79,925
R^2	0.44	0.71	0.68	0.43	0.75	0.70

Table O4. Discovery, Wealth Volatility, and Correlation (Excluding Small Stocks)

This table reports the results from the regressions in Table 3, based on the sample after excluding stocks that are smaller than the 20th NYSE size percentile.

Dep.Var.= $X_{i,t}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Equal-weighted anomaly portfolios						Value-weighted anomaly portfolios					
	No Cite Weight	NoCite Weight	RawCite Weight	RawCite Weight	CitePerYear Weight	CitePerYear Weight	NoCite Weight	NoCite Weight	RawCite Weight	RawCite Weight	CitePerYear Weight	CitePerYear Weight
$Wealth\_Vol_t$	0.006***	0.02***	0.02***	0.05***	0.05**	0.1***	0.007**	0.02***	0.01***	0.04***	0.05**	0.1***
	(3.03)	(3.55)	(3.01)	(4.41)	(3.10)	(3.15)	(2.47)	(4.01)	(2.70)	(3.85)	(2.76)	(3.10)
$Discovery_{i,t}$ $\times Wealth\_Vol_t$		-0.02***		-0.04***		-0.1**		-0.03***		-0.04***		-0.1**
		(-2.69)		(-3.56)		(-2.47)		(-3.11)		(-3.21)		(-2.48)
$Discovery_{i,t}$		0.003		-0.06***		-0.05***		0.001		-0.07***		-0.06***
		(0.45)		(-4.28)		(-3.60)		(0.17)		(-6.16)		(-4.30)
Anomaly FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	37,409	37,409	37,409	37,409	37,409	37,409	37,409	37,409	37,409	37,409	37,409	37,409
R^2	0.57	0.57	0.73	0.73	0.72	0.73	0.50	0.50	0.72	0.74	0.72	0.73