Internet Appendix for

"The Design of a Central Counterparty"

IA.1. Contract with binding resource constraint

We relax Assumption 3 to analyze the situation in which the resource constraint (4) may bind. We assume that monitoring is costless ($\psi = 0$), which means it is optimal and (bilaterally) incentive-compatible. This implies that we can set $r_s(d) = r_f(d)$ for all $d \in \{1, ..., N-1\}$ without loss of generality (see the discussion following Lemma 1).

We first define $\bar{r}_N(d,x)$ as the maximum receiver transfer given a collateral amount x and a state $d \in \{0, 1, ..N-1\}$. Using budget constraint (6) and the resource constraints (4) and (5), we have

$$\bar{r}_N(d,x) \equiv 2x + \frac{N-d}{N}(1-x)2R. \tag{IA1}$$

Assumption 3 is equivalent to $\bar{r}_N(N-1,0) \geq \hat{c}$. We also note that $\bar{r}_N(N-1,0) \geq \hat{c}$ implies that $\bar{r}_N(d,0) \geq \hat{c}$ for all $d \in \{0,1,..N-1\}$ because $\bar{r}_N(d,x)$ is decreasing with d. When Assumption 3 does not hold, that is, when $\bar{r}_N(N-1,0) < \hat{c}$, define $\hat{x}_N(N-1) \in \left(0, \frac{\hat{c}}{2}\right)$ such that $\bar{r}(N-1, \hat{x}_N(N-1)) = \hat{c}$. This threshold exists because $\bar{r}_N(d,x)$ is increasing with x and $\bar{r}_N(d,1) = 2 > \hat{c}$ by Assumption 1.

Observe next that Assumption 3 is only sufficient for resource constraint (4) to be slack at the optimal contract of Proposition 3. In fact, in Cases 1 and 3, the resource constraint (4) holds even without Assumption 3. In Case 2, constraint (4) still holds for $d = N-1$, even when Assumption 3 fails if $\hat{x}_N(N-1) < x^{OM}$, with x^{OM} being the optimal collateral requirement in equation (10). Hence, our analysis will differ from that in the main text only if both Assumption 3 and this latter condition are relaxed.

In what follows, we consider the case $N = 3$, which is the smallest value of N such that the resource constraint may bind at the optimal contract. We thus impose $\bar{r}_3(2,0) < \hat{c}$ and $\hat{x}_3(2) \geq x^{OM}_{|N=3}$, which can be written in a compact form as

$$R < \frac{3}{2}\min\left\{\hat{c}, \frac{\beta q}{1-(1-q)^3}\right\}. \tag{IA2}$$

We now derive the optimal contract for $N = 3$ when equation (IA2) holds. The possibility that resource constraint (4) binds has two effects. First, as the maximum receiver transfer $\bar{r}_3(2,x)$ increases with x, collateral has an additional hedging value in the state of the world with two payers defaults. By the pledgeability constraint, however, if transfers from payers are reduced due to a lack of resources, less collateral is needed for incentives. The result below shows how these two effects interact.

Proposition IA1. *Let $N = 3$, $\psi = 0$, and $\kappa > k$. Under Assumption* (IA2)*, there exists a threshold*

$$\underline{k}_3(2) = \underline{k}_3 + (\nu - 1)q(1-q)^2(3-R) \in (\underline{k}_3, \bar{k}), \tag{IA3}$$

such that the optimal contract is

1. *the contract of Proposition 3, if $k < \underline{k}_3$ or $k > \bar{k}$,*
2. *if $k \in [\underline{k}_3, \underline{k}_3(2)]$, the optimal amount of collateral is given by $\tilde{x}^{OM} = \hat{x}_3(2) > x^{OM}$, and if $k \in [\underline{k}_3(2), \bar{k}]$, it is given by*

$$\tilde{x}^{OM} = \frac{\left[q^3 + 3q^2(1-q)\right]\hat{c} - q\beta + 2q(1-q)^2 R}{2\left[1-(1-q)^3\right] - 2q(1-q)^2(3-R) - q\beta} < x^{OM}. \tag{IA4}$$

The proof is in Internet Appendix IA.2. Case 2 of Proposition IA1 shows the effect of the resource constraint on the optimal contract. When Assumption 3 does not hold, a single payer cannot cover the hedging needs of three receivers if no collateral is pledged. Hence, collateral has a hedging value in the states where all three payers default and two out of three payers default. By contrast, when Assumption 3 holds, this insurance value is only enjoyed in the worst default state. This explains why investors optimally post more collateral than in the optimal contract of Proposition 3 when collateral is relatively cheap.

When the collateral cost is higher, however, that is, when $k \in [\underline{k}_3(2), \bar{k}]$, investors post less collateral than in the benchmark. If collateral is expensive, investors forego this hedging value of collateral. The collateral requirement is then determined by the pledgeability constraint. Since payers' transfers are lower when the resource constraint binds, less collateral is needed.

IA.2. Additional Proofs

IA.2.1. Proof of Proposition B1

The first step of the proof is identical to that of Proposition 3; that is, we can rewrite the limited pledgeability constraint as equation (A19).

We first show two results about CCP capital e. First, CCP capital may be used only if $k < \kappa$. If this condition does not hold, we showed in Proposition 2 that collateral is preferred to CCP capital in the frictionless benchmark. This conclusion still applies under limited pledgeability because x (e) relaxes (tightens) constraint (A19). Second, if CCP capital is used, it must be that equation (A19) binds. Otherwise, it is optimal to increase e and decrease x while keeping $r_f(N) = 2x + e$ constant. With a small enough change, constraint (A19) still holds and the objective function increases because $k < \kappa$ must hold if CCP capital is used, as we just showed.

We now argue that we can consider two different cases for the analysis: Either $r_s = r_f = \hat{c}$ or constraint (A19) binds. This observation follows from Proposition 2, where we showed that $r_s = r_f = \hat{c}$ is optimal in the absence of constraint (A19). Additionally, the relative weight on these two variables is the same in the objective function in equation (A14) and in constraint (A19).

Suppose first that $r_s = r_f = \hat{c}$. We now derive conditions such that $r_f(N) = 2x + e = \hat{c}$.

Optimality of $r_f(N) = \hat{c}$

Case $k \leq (\nu - 1)(1-q)^N$

Increasing x until $r_f(N) = 2x + e = \hat{c}$ is then optimal because condition (A16) shows that investors' utility increases with x and because increasing x relaxes constraint (A19). If, in addition $k < \kappa$, CCP capital should not be used, as shown above. In this case, the contract is given by $r_s^{OM} = r_f^{OM} = \hat{c}$, $x^{OM} = \frac{\hat{c}}{2}$, and $e^{OM} = 0$. This case is thus identical to Case 1 of Proposition 3.

If instead $k > \kappa$, CCP capital should be used and, as shown above, constraint (A19) should bind. Hence, the contract is given by $r_s^{OM} = r_f^{OM} = \hat{c}$, with x^{OM} and e^{OM} such that $r_f^{OM}(N) = 2x^{OM} + e^{OM} = \hat{c}$ and equation (A19) binds. This corresponds to Case 2 of Proposition B1 when $k < \tilde{k}_N$.

Case $k > (\nu - 1)(1-q)^N$

Then, it is optimal to decrease x until constraint (A19) binds because $U'(x) < 0$. Equation (A15) shows that increasing e until $r_f(N) = 2x + e = \hat{c}$ can still be optimal if $\kappa \leq (\nu-1)(1-q)^N$. To determine the sufficient condition, we need to account for the effect of e on constraint (A19) when computing the total derivative of the objective function with respect to e. Maintaining r_s and r_f constant in equation (A19), we have

$$\frac{\partial x}{\partial e}_{|r_f=r_s=\hat{c},\text{ (A19) binds}} = \frac{\kappa + (1-q)^N}{2 - q\beta - 2(1-q)^N}. \tag{IA1}$$

We thus obtain

(IA2)

$$U'(e)_{|r_f=r_s=\hat{c},\text{ (A19) binds}} = \frac{\partial U}{\partial e} + \frac{\partial U}{\partial x}\frac{\partial x}{\partial e}_{|r_f=r_s=\hat{c},\text{ (A19) binds}}$$

$$= \frac{1}{2}\left[(\nu-1)(1-q)^N - \kappa\right] + \left[(\nu-1)(1-q)^N - k\right]\frac{\kappa + (1-q)^N}{2 - q\beta - 2(1-q)^N}. \tag{IA3}$$

This term is positive if and only if $k \leq \underline{\tilde{k}}_N$, with $\underline{\tilde{k}}_N$ defined in equation (B1). If this inequality holds, $r_f(N) = \hat{c}$ is optimal, and thus the OM contract is given by $r_s^{OM} = r_f^{OM} = \hat{c}$ and x^{OM} and

e^{OM} such that $r_f^{OM}(N) = 2x^{OM} + e^{OM} = \hat{c}$ and equation (A19) binds. Hence, we characterized all cases in which $r_f(N) = \hat{c}$ is optimal.

Optimality of $r_s = r_s = \hat{c}$ **and** $r_f(N) < \hat{c}$

Suppose now that condition (B1) does not hold, while still assuming $r_s = r_f = \hat{c}$. Then the analysis above shows that setting $e = 0$ is optimal. Since $k > \tilde{\underline{k}}_N$ and thus $k > (\nu - 1)(1 - q)^N$, the collateral amount x is pinned down by saturating constraint (A19) with $e = 0$. The analysis is then similar to that of Case 2 of Proposition 3, and thus the same contract obtains. The threshold with the full-hedging region changes from $\underline{k}_N$ to $\tilde{\underline{k}}_N$, but threshold $\bar{k}$ over which $r_s = r_s = \hat{c}$ is no longer optimal remains the same.

Optimality of $r_s, r_f < \hat{c}$

For the same reasons, the analysis of the case $k \geq \bar{k}$ is similar to that of Case 3 of Proposition 3. This concludes the proof.

IA.2.2. Proof of Corollary 1 with optimal monitoring

We extend Corollary 1, taking into account the investor's optimal monitoring choice analyzed in Appendix A.A7. We show that the comparative statics with respect to N remains valid in this case.

The upper bound of the essential CCP region is again given by $\bar{k}$. For $k > \bar{k}$, monitoring is optimal, as shown in Appendix A.A7, and the optimal contract without monitoring can be implemented bilaterally. For k lower than but close to $\bar{k}$, monitoring and loss mutualization are optimal, which means that the upper bound is $\bar{k}$. This observation also implies that there exists a lower bound $\underline{k}_N^m < \bar{k}$ of the essential CCP region.

By Proposition 3 and A1, we have $\underline{k}_N^m \geq \underline{k}_N$ because the region with full hedging in which a CCP is not essential is larger without monitoring. Define $\hat{k}^m$ as the threshold such that investors are indifferent between the complete loss mutualization contract with monitoring and the full hedging contract without monitoring. This threshold solves

$$\text{(IA4)} \qquad 0 = U_{k=\hat{k}^m} - U^{\not{m}}_{|k=\hat{k}^m}$$

$$\text{(IA5)} \qquad = qR + \left[\nu - 1 - \hat{k}^m\right]\frac{\hat{c}}{2} - \left(\hat{k}^m - \underline{k}_N\right)\left(\frac{\hat{c}}{2} - x^{OM}\right) - \psi - \left\{qR + \left[\nu - 1 - \hat{k}^m\right]\frac{\hat{c}}{2}\right\}$$

$$\text{(IA6)} \qquad = \beta q\left(1 - \frac{\hat{c}}{2}\right)\frac{\hat{k}^m - \underline{k}_N}{2\left[1 - (1-q)^N\right] - \beta q} - \psi.$$

Two cases are then possible. Either $\hat{k}^m \leq \underline{k}_N^{\not{m}}$, which implies $\underline{k}_N^m = \hat{k}^m$, or $\hat{k}^m > \underline{k}_N^{\not{m}}$ and $\underline{k}_N^m = \underline{k}_N^{\not{m}}$. We thus have

$$\text{(IA7)} \qquad \underline{k}_N^m = \min\{\hat{k}^m, \underline{k}_N^{\not{m}}\},$$

with $\hat{k}^m$ defined implicitly by equation (IA6) and $\underline{k}_N^{\not{m}} = (\nu - 1)(1 - \alpha q)^N$.

The second argument of the min in equation (IA7) strictly decreases with N by definition. We are thus left to show that $\hat{k}^m$ strictly decreases with N as well. For this, define $g : (y, k) \mapsto \frac{k + y(\nu-1)}{2 + 2y - \beta q}$ and apply the Implicit Function Theorem to equation (IA6). We obtain

$$\text{(IA8)} \qquad \frac{\partial k}{\partial N} = -\frac{\frac{\partial g}{\partial y}\frac{\partial \bar{y}}{\partial N}}{\frac{\partial g}{\partial k}},$$

with $y = -(1-q)^N$. As $\frac{\partial g}{\partial k} > 0$ and $\frac{\partial \bar{y}}{\partial N} > 0$, the derivative is negative if and only if

$$\text{(IA9)} \qquad 0 < \frac{\partial g}{\partial y} \quad \Leftrightarrow \quad 0 < \frac{(\nu-1)(2-\beta q) - 2k}{[2+2y-\beta q]^2} = \frac{2(\bar{k}-k)}{[2+2y-\beta q]^2}.$$

The last inequality holds because by Proposition 4, $\hat{k}^m$ lies below $\bar{k}$. This concludes the proof.

IA.2.3. Proof of Proposition IA1

As explained above, the resource constraint in state $d = 2$ may only bind in Case 2 of Proposition 3. Hence, the optimal contract is the same as in Proposition 3 for $k \notin [\underline{k}_3, \bar{k}]$.

For the case $k \in (\underline{k}_3, \bar{k})$, we need to determine the collateral amount x_{IC} such that constraint (LP) binds. By construction, under condition (IA2), this level satisfies $x_{IC} < \hat{x}_3(2)$. Building on the argument in Proposition 3, it is optimal to set the receiver transfer to its maximum value when the pledgeability constraint (LP) is slack. Hence, we can determine x_{IC} by saturating equation (LP) and setting $r(0) = r(1) = \hat{c}$, $r(2) = \bar{r}_3(2, x)$, and $r(3) = 2x$. Using budget constraint (6), we obtain

$$\text{(IA10)} \qquad \mathbb{E}[r_o(d)] = \left[q^3 + 3(1-q)q^2\right]\hat{c} + 3q(1-q)^2\bar{r}_3(2,x) + (1-q)^3 2x = x(2-q\beta) + q\beta.$$

Solving for x in equation (IA10), we find x_{IC} as given by equation (IA4). The inequality $x_{IC} < x^{OM}$ obtains because the proof of Proposition 3 shows that x_{IC} solves the same equation as x^{OM}, substituting $\bar{r}_3(2, x)$ for $\hat{c} > \bar{r}_3(2, x)$.

The optimal amount of collateral $\tilde{x}^{OM}$ when $k \in [\underline{k}_3, \bar{k}]$ is given either by x_{IC} or $\hat{x}_3(2)$ because the marginal value of collateral is piecewise constant, and it jumps only at these points. Totally differentiating equation (3) with respect to x, we obtain

$$\text{(IA11)} \qquad U'(x) = \begin{cases} (\nu-1)\Big[(1-q)^3 + q(1-q)^2(3-R)\Big] - k & \text{if} \quad x \in [x_{IC}, \hat{x}_2(3)] \quad \text{and} \\ \underline{k}_2 - k & \text{if} \quad x \in [\hat{x}_2(3), \frac{\hat{c}}{2}]. \end{cases}$$

To obtain the derivative $\frac{\partial \mathbb{E}[r_o(d)]}{\partial x}$ for the first expression, we use the middle term of equation (IA10). By definition of $\underline{k}_3(2)$, this first term is equal to $\underline{k}_3(2) - k$. Hence, as stated in the result, $\tilde{x}^{OM} = \hat{x}_2(3)$ is optimal when $k \in [\underline{k}_3, \underline{k}_3(2)]$, while $\tilde{x}^{OM} = x_{IC}$ is optimal when $k \in [\underline{k}_3(2), \bar{k}]$