Internet Appendix for
"Synthetic Options and Implied Volatility for the Corporate Bond Market"

Steven Shu-Hsiu Chen   Hitesh Doshi   Sang Byung Seo

A Black Formula for Credit Swaptions

A regular CDS contract is often called a spot CDS contract because protection immediately starts to apply as soon as a trade is made. In contrast, protection takes effect a certain period of time after the trade date in the case of a forward CDS contract. In a forward CDS contract, investors enter into a $(T-\tau)$-year CDS contract after a τ period from today either as a protection buyer (i.e., long protection forward) or as a protection seller (i.e., short protection forward) at a deal spread pre-established today. That is, while the deal spread is determined at time t, protection against credit events begins at future time $t+\tau$ and lasts until maturity $t+T$.

The deal spread should be a quantity that the protection seller and buyer can both agree upon. In other words, the deal spread is chosen so the value of the forward contract is zero at the beginning of the contract. This spread is observable in the market and is called the forward CDS spread. We denote the time-t forward CDS spread for firm i as $F_{i,t} = F_i(t, t+\tau, t+T)$. The forward CDS spread $F_{i,t}$ is the main object of interest in the Black model.

The Black formula is derived based on the payoff of a non-standard single-name credit swaption.[1] Recall that under the SNAC, when credit swaptions are exercised, the holder enters into a standard CDS contract with a fixed deal spread of 1% and receives/pays the strike upfront fee K_{U} at the beginning of the CDS contract. However, in the case of non-standard credit swaptions, the holder enters into a non-standard CDS contract at a given strike spread K_{S} without an upfront payment. Let $V^{\mathrm{Pay}}_{i,t+\tau}$ and $V^{\mathrm{Rcv}}_{i,t+\tau}$ denote the payoffs of non-standard payer and receiver swaptions at their maturity. It follows that

$$
\begin{aligned}
V^{\mathrm{Pay}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T) \max\left[S_i(t+\tau, t+T) - K_{\mathrm{S}}, 0\right], \\
V^{\mathrm{Rcv}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T) \max\left[K_{\mathrm{S}} - S_i(t+\tau, t+T), 0\right].
\end{aligned}
$$

A payer swaption is only exercised when the spot spread is higher than the strike spread. In this case, the holder obtains credit protection by paying a cheaper premium than the fair spot level. On the other hand, a receiver swaption is only exercised when the spot spread is lower than the strike spread. This is because the holder can sell credit protection for receiving a higher-than-the-fair spread. In either case, the gap between the spot spread and the strike spread is converted into the corresponding dollar value when it is multiplied by $\Pi_i(t+\tau, t+T)$, the present value of a risky annuity at time $t+\tau$.

Note that a spot CDS contract can be viewed as a special case of a forward contract that immediately becomes effective. Thus, we can re-express the spot CDS spread at time $t+\tau$ as

$$S_i(t+\tau, t+T) = F_i(t+\tau, t+\tau, t+T),$$

[1] Furthermore, the Black formula assumes knockout swaptions: when the firm defaults before maturity, it assumes that swaptions disappear without any payments. In the U.S., standard credit swaptions are non-knockout swaptions that do not cancel at default. In the case of a non-knockout payer swaption, the holder is still able to exercise the option at maturity and enters into a CDS contract as the protection buyer. Since the reference entity is already defaulted, this protection buy position immediately allows the holder to collect a protection payment from the counterparty or deliver a defaulted bond at par. In contrast, a non-knockout receiver swaption would never be exercised because it is not profitable to sell protection on an already defaulted entity.

which implies that

$$
\begin{aligned}
V^{\text{Pay}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T)\max\Big[F_i(t+\tau, t+\tau, t+T) - K_{\text{S}}, 0\Big], &\text{(A.1)}\\
V^{\text{Rcv}}_{i,t+\tau} &= \Pi_i(t+\tau, t+T)\max\Big[K_{\text{S}} - F_i(t+\tau, t+\tau, t+T), 0\Big]. &\text{(A.2)}
\end{aligned}
$$

The Black model calculates the time-t values of payer and receiver swaptions by assuming that between times t and $t+\tau$, the forward spread $F_{i,t}$ follows a geometric Brownian motion under the τ-forward survival measure.[2] Coupled with the payoff structures in equations (A.1) and (A.2), this assumption results in the Black formula for payer and receiver credit swaptions:

$$
\begin{aligned}
V^{\text{Pay}}_{i,t} &= \Pi_i(t, t+\tau, t+T)\Big[F_i(t, t+\tau, t+T)\Phi(d_1) - K_{\text{S}}\Phi(d_2)\Big],\\
V^{\text{Rcv}}_{i,t} &= \Pi_i(t, t+\tau, t+T)\Big[K_{\text{S}}\Phi(-d_2) - F_i(t, t+\tau, t+T)\Phi(-d_1)\Big],
\end{aligned}
$$

where

$$
d_1 = \frac{\log(F_i(t, t+\tau, t+T)/K_{\text{S}}) + \sigma^2\tau/2}{\sigma\sqrt{\tau}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{\tau}.
$$

The function $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal distribution, and $\Pi_i(t, t+\tau, t+T)$ the time-t present value of a risky annuity between times $t+\tau$ and $t+T$.[3]

B Description of Variables in the Predictive Regressions

From Panel A of Table 1, we observe that the variance risk premium in the corporate bond market has a sample mean of 0.03 and a sample standard deviation of 0.12. This variable is positively skewed with its median lesser than its mean, and is fat-tailed with a large kurtosis. In addition, the corporate bond variance risk premium has a monthly AR(1) coefficient of

[2] For more details about the forward survival measure and its relation with other probability measures, see, for instance, O'Kane (2011).

[3] The present value of a risky annuity Π_i is estimated from market CDS spreads. Specifically, we estimate it by exactly fitting the term structures of spot and forward CDS spreads every day.

0.24, which suggests that this variable quickly mean-reverts with relatively low persistence. From this table, we can also see the characteristics of the variance risk premium in the equity market. The equity variance risk premium has a sample mean of 9.59 and a sample standard deviation of 6.36. This variable has a skewness near zero, albeit slightly negative, and a kurtosis of 4.73, which indicates that the equity variance risk premium also has a fat-tailed sample distribution.

Looking at the summary statistics for implied variances and realized variances, we can see that in both markets, on average, the implied variance has a larger magnitude than the realized variance, while the standard deviations are about the same. However, in terms of kurtosis and skewness, the patterns are different in the two markets. In the equity market, the realized variance has a larger skewness and kurtosis compared to those of the implied variance. In the case of the corporate bond market, this pattern is reversed such that the implied variance has a larger skewness and kurtosis.

The bottom two rows in Panel A summarize synthetic bond returns and equity returns in our sample. Bond returns are, on average, 0.12% per month (1.43% annually), with a standard deviation of 0.36% (1.25% annually). The skewness value is close to zero but the kurtosis value is large at 6.15. The average equity return in our sample period is approximately 1.12% per month (13.39% annually), which is higher than the post-war sample, but the standard deviation is 2.83% per month (9.80% annually), which is lower than the post-war sample. This is because our sample coincides with the post Great Recession period when the stock market has steadily recovered from the financial crisis.

Panel B of Table 1 lists the monthly correlations among our eight variables of interest. The positive correlations among the six variance-related variables imply that they tend to comove in the same direction (with the exception of the equity variance risk premium and the equity realized variance). For example, the corporate bond variance risk premium and the equity variance risk premium are moderately correlated at 0.27.

We can also see from this panel that the returns on the bond and the returns on the equity are positively correlated at 0.77. As anticipated, the first six variance-related variables exhibit negative contemporaneous correlations with the two return time series: during bad times when markets suffer, variance risk as well as the compensation for variance risk typically go up. For instance, the corporate bond variance risk premium has a correlation of -0.66 with the bond return and a correlation of -0.60 with the equity return.

References

O'Kane, Dominic, 2011, *Modelling Single-Name and Multi-Name Credit Derivatives*. (John Wiley & Sons).