Internet Appendix to accompany the paper

"Crowding and Tail Risk in Momentum Returns"

(Not for publication)

Pedro Barroso, Roger M. Edelen, and Paul Karehnke*

Abstract

This internet appendix accompanies the paper "Crowding and Tail Risk in Momentum Returns." It contains the literature review in Section IA.A, details on the model's derivation and solution in Sections IA.B to IA.D, and further simulation results in Section IA.E.

IA.A. Existing Literature

Our paper is related to the empirical and theoretical literature on momentum. Momentum was initially documented for US stock returns (Levy, 1967; Jegadeesh and Titman, 1993) and has since been documented for stock returns in most countries (Rouwenhorst, 1998) and across asset classes (Asness, Moskowitz, and Pedersen, 2013). Besides its very high average returns, momentum carries significant downside risk or negative skewness in the form of occasional large crashes (Daniel

*Barroso, pedro.barroso@ucp.pt, Universidade Católica Portuguesa Católica Lisbon School of Business and Economics; Edelen, edelenr@vt.edu, Virginia Tech Pamplin College of Business; and Karehnke, pkarehnke@escp.eu, ESCP Business School.

and Moskowitz, 2016). Existing research also shows that institutional investors are momentum traders, i.e., tilt their portfolios towards momentum stocks (Grinblatt et al., 1995; Lewellen, 2011; Edelen et al., 2016; Baltzer et al., 2019). Our paper contributes to this literature by directly examining whether uncertain institutional participation in the momentum strategy is the source of higher-moment return characteristics.

A recent empirical literature examining the time series properties of momentum finds results broadly consistent with an over-reaction explanation of the effect. The premium is stronger in periods of bull markets (Cooper et al., 2004), high liquidity (Avramov, Cheng, and Hameed, 2016), and high sentiment (Antoniou, Doukas, and Subrahmanyam, 2013). Hillert, Jacobs, and Müller (2014)'s finding that momentum is more pronounced in firms with more media coverage also supports an over-reaction interpretation, as does the evidence in Edelen et al. (2016) regarding institutional purchases in the portfolio-formation period.

On the other hand, the momentum premium is stronger in stocks experiencing frequent but small price changes that are less likely to attract attention (Da et al., 2014) or those characterized by small trades of investors under-reacting to past returns (Hvidkjaer, 2006). Also there is recent evidence that momentum is somehow explained by improvements in firm fundamentals (Novy-Marx, 2015; Sotes-Paladino, Wang, and Yao, 2016; DeMiguel, Martín-Utrera, Nogales, and Uppal, 2020). This evidence suggests momentum investors exploit under-reaction and as such (exogenous increases in) crowding should reduce its premium.

The related theoretical literature on momentum offers theories based on institutional investors and fund flows (Vayanos and Woolley, 2013) or behavioral biases such as over-reaction / self-attribution (see, e.g., Barberis, Shleifer, and Vishny, 1998; Daniel, Hirshleifer, and Subrahmanyam, 1998) or information externalities and gradual diffusion of information (see, e.g., Stein, 1987; Hong and Stein, 1999; Andrei and Cujean, 2017). Our work is most closely related to the latter branch of the literature.

Our model builds on the information externality that the actions of unanticipated momentum investors impose on their peers. Thus, it is closest in development to Stein (2009), but follows in a long line of research relating to arbitrageur information coordination and externalities. This literature dates to Stein (1987) who characterizes the externality, and Scharfstein and Stein (1990) and Froot, Scharfstein, and Stein (1992) who relate it to herding behavior. Hong and Stein (1999) relate the externality to persistence and reversal patterns in returns. A related branch of the literature identifies the positive feedback trading of momentum investors as a source of destabilizing noise in prices, e.g., De Long, Shleifer, Summers, and Waldmann (1990a,b).

More recently, Kondor and Zawadowski (2019) study whether the presence of more arbitrageurs improves welfare in a model of capital reallocation. Trades in the model can become crowded due to imperfect information, but arbitrageurs can also devote resources to learn about the number of earlier entrants. They find that if the number of arbitrageurs is high enough, more arbitrageurs do not change capital allocations, but decrease welfare due to costly learning.

Related empirical research includes Hanson and Sunderam (2014) who construct a measure of the capital allocated to momentum and the valuation anomaly (book-to-market or B/M) using short-interest. They find some evidence that an increase in arbitrage capital has reduced the returns on B/M and momentum strategies. In addition, Lou and Polk (2013) proxy for momentum capital with the residual return correlations in the short and long leg of the momentum strategy and find that momentum profits are lower and crashes more likely in times of higher momentum capital. Baltzer et al. (2019) classify institutions as a whole as momentum traders and find that, in Germany, momentum trading peaked before the crash. While our analysis uses a different approach and insights in proxying for momentum capital, our results on unanticipated momentum capital and momentum returns are generally consistent with these findings, but we do not attribute momentum's crashes to crowding. Finally, Huang (2015) proposes a momentum gap variable, which is defined as the cross sectional dispersion of formation period returns. He shows that this measure predicts momentum returns and crashes, and argues that this is consistent with Stein (2009)'s crowded trade theory. Throughout our analysis, we control for momentum's past volatility, which has a correlation of 0.73 with the momentum gap measure. We also verify in Section IV.G that momentum gap's predictive power for crash risk is unrelated to various institutional measures of momentum crowding. This corroborates our finding that momentum's crashes are not explained by crowding.

We go beyond the usual focus on first moments to study the determinants of the risk of momentum. This relates our work to a recent strand of literature focusing on the predictability of the moments of momentum. Barroso and Santa-Clara (2015) show that the volatility of momentum is highly predictable and it is a useful variable to manage the risk of the strategy. Daniel and Moskowitz (2016) argues the crash risk of momentum is due to the optionality effect of the losers portfolio that resembles an out-of-the-money call option after extreme bear markets. Jacobs, Regele, and Weber (2015) examine the expected skewness of momentum as a potential explanation of its premium. They propose an enhanced momentum strategy but find that managing its risk results in a performance hard to reconcile with a premium for skewness. Grobys, Ruotsalainen, and Äijö (2018) find industry momentum has different risk properties from standard momentum but shows similar gains from risk management. Our results address the question of whether investors condition their exposure to momentum using this new-found predictability. Consistent with the economic case for managing the risk of momentum, we find less crowding in momentum after periods of high volatility.

IA.B. Derivation of Equation (3)

First notice that solving equation (2) is equivalent to solving each of the following (presuming $\gamma > 1$)

$$\max_{\varsigma} \quad \frac{K_{type,2}^{1-\gamma}}{1-\gamma} \cdot E\left[e^{(1-\gamma)\left(r_f + \log\left(1+\varsigma\left(e^{r_p - r_f} - 1\right)\right)\right)}\right] \quad \Leftrightarrow \quad \min_{\varsigma} \quad \log E\left[e^{(1-\gamma)\log\left(1+\varsigma\left(e^{r_p - r_f} - 1\right)\right)}\right],$$

where ς is the weight on the risky asset portfolio and r_p its log return, and r_f is the log risk-free rate. Second, to solve for the fraction of wealth invested in the risky portfolio, we follow Section 2.1.1 in Campbell and Viceira (2002, Internet Appendix) and approximate the function $g\left(r_p - r_f\right) = \log\left(1 + \varsigma\left(e^{r_p - r_f} - 1\right)\right)$ with a second-order Taylor expansion around 0:[1]

$$\begin{aligned} g\left(r_p - r_f\right) &\cong \log(1) + \frac{\varsigma e^0}{1+\varsigma\left(e^0 - 1\right)}\left(r_p - f\right) + \frac{1}{2}\frac{\varsigma\left[e^0\left(1+\varsigma\left(e^0-1\right)\right) - \varsigma e^{2\cdot 0}\right]}{\left(1+\varsigma\left(e^0-1\right)\right)^2}\left(r_p - f\right)^2 \\ &\cong \varsigma\left(r_p - f\right) + \frac{1}{2}\left(\varsigma - \varsigma^2\right)\sigma^2, \qquad \text{(IA.B.1)} \end{aligned}$$

where $\left(r_p - f\right)^2$ is replaced with its conditionally expectation σ^2 as in Campbell and Viceira (2002, Internet Appendix). Using equation (IA.B.1), we can rewrite the maximization problem to

$$\begin{aligned} &\min_{\varsigma} \quad \log E\left[\exp\left[\frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\gamma)\sigma_p^2\right] \cdot \exp\left[\varsigma(1-\gamma)\left(r_p - r_f\right)\right]\right] \\ \Leftrightarrow &\min_{\varsigma} \quad \frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\gamma)\sigma_p^2 + \varsigma(1-\gamma)\left(\mu_p - r_f\right) + \frac{1}{2}\varsigma^2(1-\gamma)^2\sigma_p^2 \\ \Leftrightarrow &\max_{\varsigma} \quad \varsigma\left(\mu_p - r_f + \frac{1}{2}\sigma_p^2\right) - \frac{1}{2}\varsigma^2\gamma\sigma_p^2, \end{aligned}$$

[1]See also, e.g., Peress (2004), for the use of this approximate solution to the CRRA portfolio choice problem in a noisy rational expectations setting.

which has the solution

$$\varsigma = \frac{\mu_p - r_f + \frac{1}{2}\sigma_p^2}{\gamma\sigma_p^2}.$$

To proceed, we assume that log returns and arithmetic returns are similar such that $\mu_p - r_f + \frac{1}{2}\sigma_p^2 \cong e^{\mu_p - r_f} \cong \mu_p - r_f$. We then determine μ_p and σ_p^2 for a portfolio that consists of the market investment plus a long-short momentum investment. Because the momentum portfolio is self-financing, feasible combinations of the market portfolio and the momentum portfolio are given by the weight vector $\boldsymbol{w}' = \begin{bmatrix} 1 & w_m \end{bmatrix}$, i.e., hold the market portfolio plus a proportionate long-short momentum overlay w_m. The optimal risky portfolio w_m solves the constrained optimization

$$\min_{\boldsymbol{w}} \quad \frac{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}{2}, \qquad \text{s.t.} \qquad \boldsymbol{\mu}'\boldsymbol{w} = r^* - r_f,$$

where

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ w_m \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} r - r_f \\ E_{type}\left[m + \epsilon\right] \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_\chi^2 & 0 \\ 0 & Var_{type}\left[m + \epsilon\right] \end{bmatrix},$$

and $r^* - r_f$ is a target return premium that traces out the efficient frontier, and r is the required return on the market portfolio. The solution is

$$w_m = \frac{E_{type}\left[m + \epsilon\right] / Var_{type}\left[m + \epsilon\right]}{\left(r - r_f\right) \big/ \sigma_\chi^2}. \tag{IA.B.2}$$

Using equation (IA.B.2), the parameters of the optimal risky portfolio are

$$\mu_p - r_f = \begin{bmatrix} r - r_f & E_{type}\left[m + \epsilon\right] \end{bmatrix} \begin{bmatrix} 1 \\ w_m \end{bmatrix} = r - r_f + w_m E_{type}\left[m + \epsilon\right], \text{ and} \tag{IA.B.3}$$

$$\sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} = \sigma_\chi^2 + w_m^2 Var_{type}\left[m + \epsilon\right] = \frac{\sigma_\chi^2}{r - r_f}\left(r - r_f + w_m E_{type}\left[m + \epsilon\right]\right). \tag{IA.B.4}$$

Thus,

$$\varsigma = \frac{r - r_f}{\gamma \sigma_{\chi}^2}. \tag{IA.B.5}$$

Combining Eqs. (IA.B.2) and (IA.B.5),

$$Demand = w_m \varsigma K_{type,2} = \frac{E_{type}\left[m + \epsilon\right]}{\gamma Var_{type}\left[m + \epsilon\right]} K_{type,2}.$$

IA.C. Negative Market-clearing Price for Momentum Portfolio

In the case of $k_M > \lambda$, the demand of momentum investors increases with a positive f faster than the supply can keep up with, implying an increasingly large buying imbalance as f rises (depicted in Figure IA.3, Plot C.1). This again suggests that momentum investors buy up to their capacity, leading to a subsequent momentum crash.

However, when $k_M > \lambda$ there is also a (finite) negative value for f that clears the market. While we discount this equilibrium as implausible, we note that even here the contrary pricing of winner and loser stocks implies a substantial negative momentum return because the formation-period 'winners' are actually the fundamental losers, and vice versa.

It is not clear how this $f < 0$ equilibrium could be found, because informed investors presumably seed formation-period returns with *buying* of the momentum portfolio (and an initially positive f). Nevertheless, it is a call auction and if they were to bizarrely trade contrary to their private information, seeding a negative value for f, then they might induce momentum investors into selling (buying) so much winner (loser) stock that their bizarre trade is preferred.

IA.D. Probability Density Function Conditional on f

Below we derive the expression for $p\,(\delta|f)$. By the definition of conditional probability, we have

$$p\,(\delta|f) = \frac{p_2(\delta, f)}{p_1(f)},$$

where numerical subscripts distinguish the functional form of each probability density function. To solve for these densities, we exchange the primitive random variable k_I with the observable random variable f. Formally, let

$$F : (\delta, k_I, k_M) \to f = \frac{1}{D}\left(\delta k_I + \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right),$$

which characterizes market clearing as in equation (4). We map the primitive random variables into

$$\begin{pmatrix} \delta \\ k_M \\ k_I \end{pmatrix} \to \begin{pmatrix} \delta \\ k_M \\ F\,(\delta, k_M, k_I) \end{pmatrix}.$$

Next, we need

$$|\ \mathrm{J}\ | = \det \begin{pmatrix} \frac{\partial \delta}{\partial \delta} & \frac{\partial \delta}{\partial k_M} & \frac{\partial \delta}{\partial k_I} \\ \frac{\partial k_M}{\partial \delta} & \frac{\partial k_M}{\partial k_M} & \frac{\partial k_M}{\partial k_I} \\ \frac{\partial F^{-1}}{\partial \delta} & \frac{\partial F^{-1}}{\partial k_M} & \frac{\partial F^{-1}}{\partial k_I} \end{pmatrix} = \frac{D}{\delta}.$$

Following a standard result (see, e.g., Theorem 2 in Section 4.4 of Rohatgi and Saleh, 2000), the density is then given by

$$\begin{aligned}
p_3\left(\delta, k_M, f\right) &= g\left(\delta\right) h\left(k_M, F^{-1}\right) \mid \mathrm{J} \mid \\
&= g\left(\delta\right) h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right)\right) \frac{D}{\delta},
\end{aligned} \tag{IA.D.1}$$

where D is as in equation (4). Integrating k_M and then δ out of equation (IA.D.1) gives

$$\begin{aligned}
p_2(\delta, f) &= \frac{g\left(\delta\right)}{\delta} \int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right)\right) D dk_M, \\
p_1(f) &= \int_0^\infty \frac{g\left(\delta\right)}{\delta} \int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right)\right) D dk_M d\delta.
\end{aligned}$$

We then obtain

$$p\left(\delta | f\right) = \frac{\frac{g(\delta)}{\delta} \int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right)\right) D dk_M}{\int_0^\infty \frac{g(\delta)}{\delta} \int_0^1 h\left(k_M, \frac{1}{\delta}\left(fD - \frac{\delta^E}{1+\frac{\delta^V}{\sigma_\epsilon^2}} k_M\right)\right) D dk_M d\delta}.$$

IA.E. Additional simulation results

This section of the internet appendix investigates the impact of changing the distributional assumptions for δ and higher concentration parameters in the simulation analysis of Section III. It also analyses the relation between expected momentum returns and unexpected momentum capital in the different simulations.

First, we ask whether our results are robust to using a higher concentration parameter for the Dirichlet distribution and a uniform distribution for δ instead of a log-normal distribution. In particular, we let δ follow a uniform distribution on $[0.06,\ 0.12]$, and let $\alpha_i = 12$. The results are reported in Figure IA.1 and Table IA.1. In summary, the results are very similar to those in Section

III. In the myopic beliefs case, momentum returns again have pronounced negative skewness, high volatility and large excess kurtosis, and they are well behaved with low volatility, slightly positive skewness, and no excess kurtosis in the rational beliefs case.

Second, we ask whether the beliefs specifications for unknown capital become more similar to the known capital case when $var(k)$ is very small. To achieve this, we set the concentration parameters $\alpha_i = 60$ in the Dirichlet distribution, and leave the setting otherwise identical to the one in the paper. The results in Figure IA.2 and Table IA.2 verify that crashes disappear in the myopic beliefs case once capital uncertainty is negligible. Momentum returns in all four specifications are now well behaved and have similar return characteristics.

Finally, we analyze the relation between unexpected momentum capital $k_M - Ek_M$ and expected momentum returns m. To do this, we rank the simulation trials for the different specifications in Figure 1, Figure IA.1, and Figure IA.2 into 100 bins according to $k_M - Ek_M$, and report the averages within each bin to approximate a conditional expectation. All nine subplots of Figure IA.3 corresponding to different distributional and beliefs assumptions show that the residual information m not incorporated into prices decreases with crowd size. Thus, the model supports the negative relation between momentum capital and expected momentum returns we document empirically.

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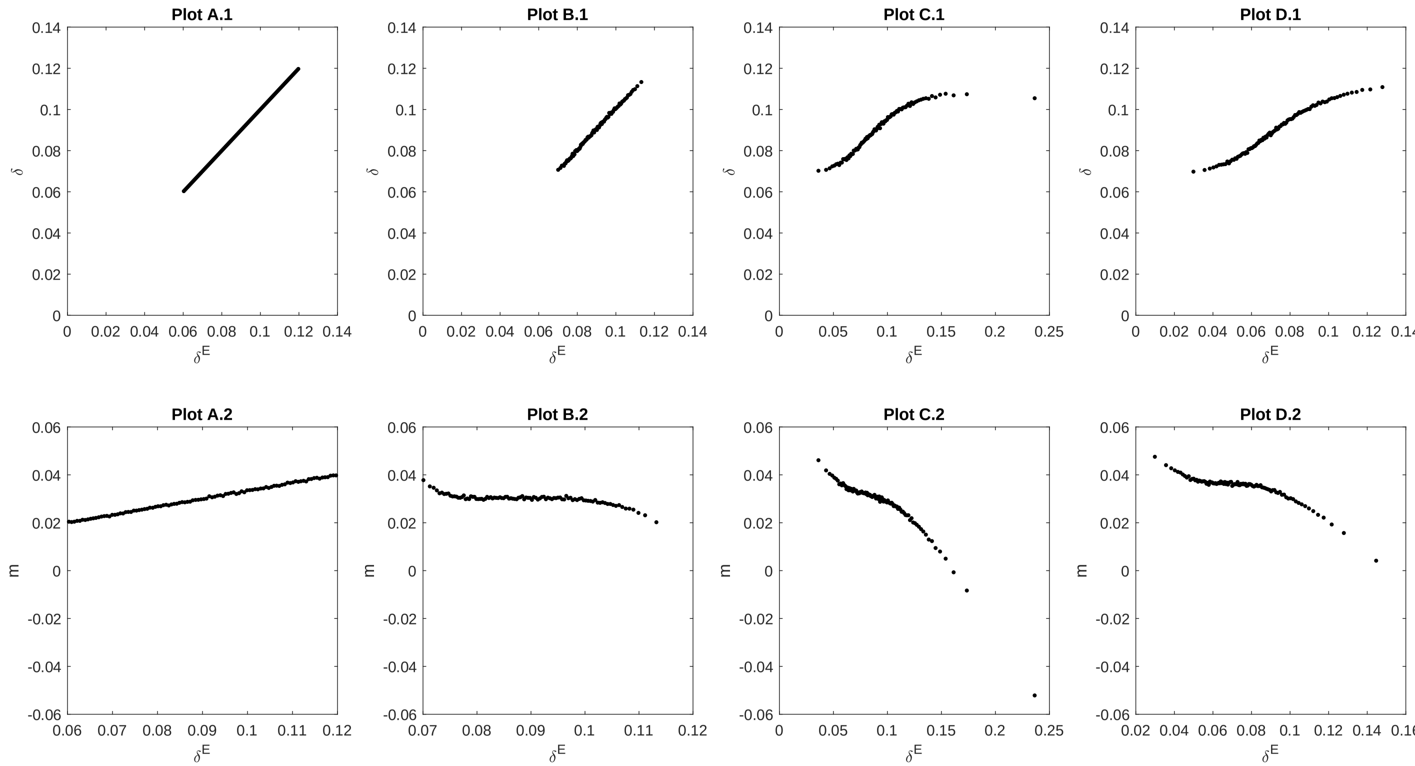

Figure IA.1: Beliefs and expected momentum returns in simulations—uniform distribution

This figure is constructed in the same way as Figure 1 in the paper. In particular, the simulations use 100,000 independent random draws of $\{k_I, k_M, \delta\}$, where k_I and k_M are informed and momentum capital, respectively, and δ is the signal of differential fundamental value for winners minus losers. k_I and k_M (and k_C) follow a Dirichlet distribution with concentration parameters $\alpha_i = 12$, and $Ek_I = Ek_M = 1/3$. δ follows a uniform distribution on [0.06 0.12]. The market clearing formation period return f is solved for each $\{k_I, k_M, \delta\}$ pair by iteration using different specifications for momentum traders' beliefs δ^E: known crowding in Plots A.1-2, rational beliefs in Plots B.1-2, myopic beliefs in Plots C.1-2, and optimal linear beliefs in Plots D.1-2. The expected momentum return is then $m = \delta - f$. The values of $\{\delta, m\}$ are ranked into 100 equally populated bins according to the horizontal-axis variable, and the plots represent the averages for the indicated variables within these bins.

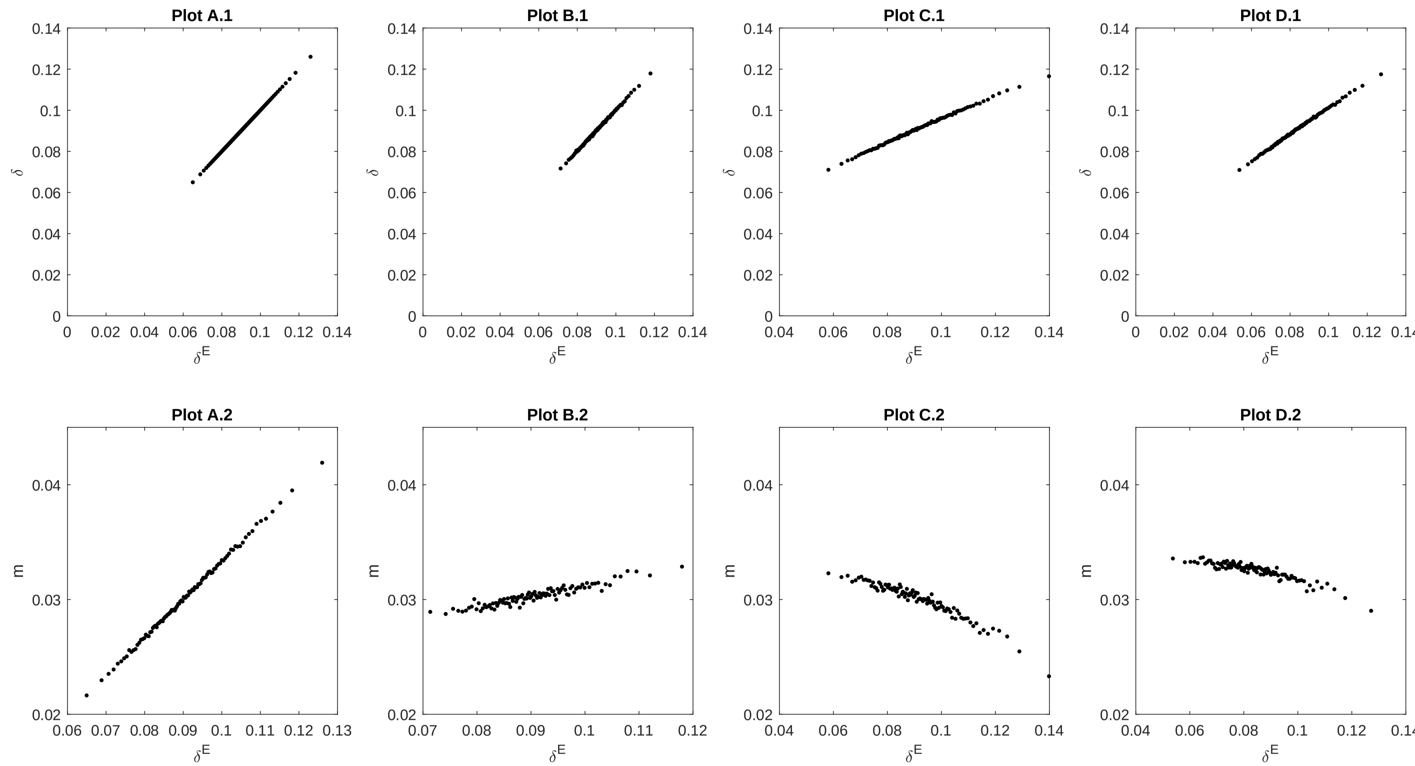

Figure IA.2: Beliefs and expected momentum returns in simulations—very low $var(k)$

This figure is constructed in the same way as Figure 1 in the paper, and the simulations are identical to those in Figure 1 except that k_I and k_M (and k_C) follow a Dirichlet distribution with concentration parameters $\alpha_i = 60$, which still implies $Ek_I = Ek_M = 1/3$. In addition, as opposed to Figure IA.1 δ follows a log-normal distribution with $\mu = -2.405$ and $\sigma = 0.125$ implying an average δ of 9.1% with standard deviation of 1.14% as in the paper.

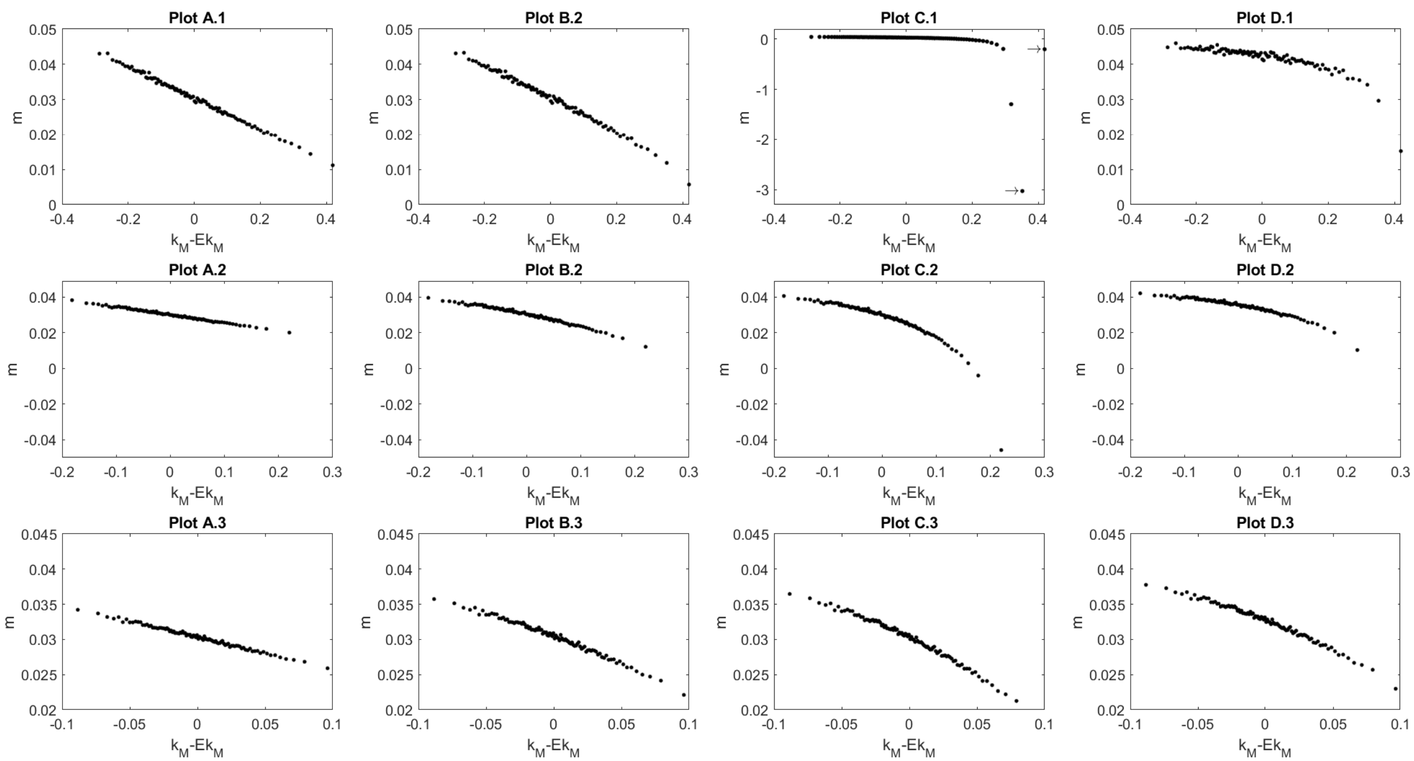

Figure IA.3: Expected momentum returns and unexpected momentum capital

Rows one, two, and three correspond to the simulations of Figure 1, Figure IA.1, and Figure IA.2, respectively. For each simulation $\{m, k_M\}$ are ranked into 100 equally populated bins according to the horizontal-axis variable, and the plots represent the averages for the indicated variables within these bins.

Table IA.1: Momentum returns in simulations—uniform distribution

The table reports unconditional return statistics for the simulations described in the caption of Figure IA.1. Panel A contains the descriptive statistics of expected momentum returns across all simulations. Mean, stdev, skew, kurt, min and max refer to average, standard deviation, skewness, kurtosis, minimum, and maximum, respectively. The simulations are computed under the indicated belief specification, and the Dirichlet distribution has the concentration parameters $\alpha_i = 12$, and δ follows a uniform distribution on $[0.06,\ 0.12]$. Optimal linear beliefs are chosen to maximize the utility of a CRRA investor with $\gamma = 2$, and they are reported in the row λ^{-1}. Profits are likewise the expected portfolio returns of a $\gamma = 2$ investor, and certainty equivalents 'cer(γ)' are calculated for $\gamma = 2, 4, 10$, with portfolio weights calculated as in (3). Realized momentum returns in Panel B are given by $m + \epsilon$ where ϵ is randomly drawn from a zero-mean normal distribution with standard deviation 0.125. Cer(γ) is an arithmetic return, and all other statistics are based on log returns.

Belief spec.	known	rational	myopic	optimal linear
λ^{-1}			1.50	1.34
Panel A. Expected momentum returns m				
mean	3.0%	3.0%	2.8%	3.5%
stdev	0.9%	1.4%	2.7%	1.4%
skew	0.5	0.3	-92.9	-0.1
kurt	3.1	2.9	17386.3	9.6
min	0.61%	-2.04%	-534.41%	-35.57%
max	7.51%	8.56%	9.22%	9.38%
Panel B. Realized momentum returns $m + \epsilon$				
profit	3.11%	2.78%	1.75%	1.99%
cer(2)	2.30%	2.16%	-100.00%	1.85%
cer(5)	1.14%	1.07%	-100.00%	0.92%
cer(10)	0.45%	0.43%	-100.00%	0.37%

Table IA.2: Momentum returns in simulations—very low $var(k)$

The table reports unconditional return statistics for the simulations described in the caption of Figure IA.2 and is constructed in the same fashion as Table IA.1.

Belief spec.	known	rational	myopic	optimal linear
λ^{-1}			1.50	1.44
Panel A. Expected momentum returns m				
mean	3.0%	3.0%	3.0%	3.3%
stdev	0.5%	0.7%	0.8%	0.7%
skew	0.4	0.3	0.2	0.3
kurt	3.4	3.3	3.3	3.3
min	1.39%	0.18%	-0.39%	0.41%
max	6.13%	7.35%	7.12%	7.35%
Panel B. Realized momentum returns $m + \epsilon$				
profit	3.02%	2.91%	2.90%	2.64%
cer(2)	2.29%	2.24%	2.15%	2.24%
cer(5)	1.14%	1.11%	1.07%	1.11%
cer(10)	0.45%	0.44%	0.43%	0.44%