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A Proofs

A.1 Proof of Proposition 1:

Define $\Delta f = f_{t+1} - f_t$. Note that

$$\Delta f = f_t(1 - f_t)\kappa(T_A(R_A) - T_P(R_P)) + \hat{\lambda}(q - f_t), \tag{A.1}$$

where $T_A(R_A) = r(R_A)s(R_A)$, and $T_P(R_P) = r(R_P)s(R_P)$. At date t, f_t is given.

Differentiating with respect to R_A twice, and using the earlier conditions that $r'(R_A), s'(R_A) > 0$, that $s''(R_A) = 0$ by (1), and that $r''(R_A) > 0$ by (2), gives

$$\frac{\partial T_A(R_A)}{\partial R_A} = r'(R_A)s(R_A) + r(R_A)s'(R_A) > 0 \tag{A.2}$$

$$\frac{\partial^2 T_A(R_A)}{\partial (R_A)^2} = r''(R_A)s(R_A) + 2r'(R_A)s'(R_A) > 0. \tag{A.3}$$

Since R_A affects T_A but not T_P, these formulas describe how active return affects both the expected net shift in the fraction of A's, and the expected unidirectional rate of conversion from P to A.

Furthermore, substituting for the sending function $s(R_A)$ from (1) and the receiving function $r(R_A)$ from (2) into (A.2) and (A.3) gives

$$\frac{\partial T_A(R_A)}{\partial R_A} = (2aR_A + b)(\beta R_A + \gamma) + \beta(aR_A^2 + bR_A + c) \tag{A.4}$$

$$\frac{\partial^2 T_A(R_A)}{\partial (R_A)^2} = 2a(\beta R_A + \gamma) + 2\beta(2aR_A + b). \tag{A.5}$$

The fact that sending and receiving functions and their first and second derivatives are all positive signs some of the terms in parentheses. So it follows immediately from (A.4) that the sensitivity of the transformation rate of investors to A as a function of past active return is increasing with the parameters of the sending and receiving functions, β, γ, a, b, and c. By (A.5), a similar point follows immediately for convexity as well, with the exception that c does not enter into convexity. Furthermore, since κ is multiplied by $T_A(R_A) - T_P(R_P)$ in equation (A.1), it also follows immediately that these effects are increasing with the intensity of social interactions, κ.

A.2 Proof of Proposition 2

From the discussion in the main text, it follows that we need to show that $\bar{T} = \bar{T}_A - \bar{T}_P > 0$ under the assumption of the proposition.

Using the definitions of the sending and receiving functions, direct calculation yields

$$\begin{aligned} \bar{T} &= \bar{T}^A - \bar{T}^P \\ &= a\beta[\gamma_A\sigma_A^3 - \gamma_P\sigma_P^3 + 3\mu_A(\sigma_A^2 - \sigma_P^2)] + B(\sigma_A^2 - \sigma_P^2) - a\beta D^3 \\ &\quad -3a\beta\mu_A D^2 - [3a\beta\mu_A^2 + 3(a\beta + B)\sigma_P^2 + C]D \end{aligned} \tag{A.6}$$

where

$$B = a\gamma + b\beta,$$

and

$$C = b\gamma + c\beta.$$

It is easy to verify that $\bar{T} > 0$ when $D = 0$ and $\frac{\partial \bar{T}}{\partial D} < 0$ when $D > 0$ (see the proof of Proposition 3, Part 1), so that $\bar{T}$ is decreasing with D when $D > 0$. On the other hand, when D is very large (relative to a fixed set of other model parameters), $\bar{T}$ would be negative. Therefore, there exists a positive $\bar{D}$ corresponding to which $\bar{T}$ is zero. Then for all $D < \bar{D}$, $\bar{T} > 0$, so the active strategy dominates.

∎

A.3 Proof of Proposition 3

To show Part 1, we differentiate (A.6) with respect to D to obtain that

$$\frac{\partial \bar{T}}{\partial D} = -3a\beta D^2 - 6a\beta\mu_A D - [3a\beta\mu_A^2 + 3(a\beta + B)\sigma_P^2 + C] < 0.$$

For Part 2, differentiating with respect to active volatility σ_A gives

$$\frac{\partial \bar{T}}{\partial \sigma_A} = 3a\beta\gamma_A\sigma_A^2 + 6a\beta\mu_A\sigma_A + 2B\sigma_A > 0 \tag{A.7}$$

Thus, the growth of A increases with active volatility σ_A. Greater return variance increases the effect of *SET* on the part of the sender. Although high salience to receivers of extreme returns ($a > 0$) is not required for the result, it reinforces this effect. Indeed, even if there were no *SET* ($\beta = 0$), since $a > 0$ implies that $B > 0$, the result would still hold. Intuitively, high volatility generates the extreme outcomes which receive high attention.

For Part 3, differentiating with respect to skewness γ_A of A gives

$$\frac{\partial \bar{T}}{\partial \gamma_A} = a\beta\sigma_A^3 > 0, \tag{A.8}$$

Thus, the advantage of A over P is increasing with return skewness of A. ∎

A.4 Proof of Proposition 4

For Part 1, we differentiate with respect to β, the strength of *SET*. This reflects how tight the link is between the sender's self-esteem and performance.

$$\begin{aligned} \frac{\partial \bar{T}}{\partial \beta} &= a[\gamma_A\sigma_A^3 - \gamma_P\sigma_P^3 + 3\mu_A(\sigma_A^2 - \sigma_P^2)] + b(\sigma_A^2 - \sigma_P^2) - aD^3 \\ &\quad -3a\mu_A D^2 - [3a\mu_A^2 + 3(a+b)\sigma_P^2 + c]D \\ &> 0 \end{aligned} \tag{A.9}$$

if $D \approx 0$ is sufficiently small. So greater *SET* increases the evolution toward A, because *SET* causes greater reporting of the high returns that make A enticing for receivers. A generates extreme returns for *SET* to operate upon through higher volatility, or more positive skewness.

For Part 2, differentiating with respect to conversability γ gives

$$\begin{aligned} \frac{\partial \bar{T}}{\partial \gamma} &= a(\sigma_A^2 - \sigma_P^2) - (b + 3a\sigma_P^2)D \\ &> 0 \end{aligned} \tag{A.10}$$

if D is sufficiently small. Greater conversability γ can help the active strategy spread because of the greater attention paid by receivers to extreme returns ($a > 0$), which are more often generated by the A strategy. (If $D < 0$, this effect is reinforced by the higher mean return of A. In this case an unconditional increase in the propensity to report returns tends to promote the spread of the sender's type more when the sender is A.) If A earns lower return than P on average, greater conversability incrementally produces more reporting of lower returns when the sender is A than P, which opposes the spread of A.

For Part 3, recall that the quadratic term of the receiving function a reflects greater attention on the part of the receiver to extreme profit outcomes communicated by the sender. Differentiating with respect to a gives

$$\begin{aligned} \frac{\partial \bar{T}}{\partial a} &= \beta[\gamma_A \sigma_A^3 - \gamma_P \sigma_P^3 + 3\mu_A(\sigma_A^2 - \sigma_P^2)] + \gamma(\sigma_A^2 - \sigma_P^2) - \beta D^3 \\ &\quad -3\beta\mu_A D^2 - [3\beta\mu_A^2 + 3(\beta+\gamma)\sigma_P^2]D \\ &> 0 \end{aligned} \tag{A.11}$$

if D is sufficiently small. So greater attention by receivers to extreme outcomes, a, promotes the spread of A over P because A generates more of the extreme returns which, when a is high, are especially noticed and more likely to persuade receivers. This effect is reinforced by *SET*, which causes greater reporting of extreme high returns.

For Part 4, differentiating with respect to how prone receivers are to extrapolating returns, b, gives

$$\begin{aligned} \frac{\partial \bar{T}}{\partial b} &= \beta(\sigma_A^2 - \sigma_P^2) - (3\beta\sigma_P^2 + \gamma)D \\ &> 0 \end{aligned} \tag{A.12}$$

if D is sufficiently small. Greater extrapolativeness of receivers helps A spread by magnifying the effect of *SET* (reflected in β), which spreads A because of the higher volatility of A returns.

Finally, for κ, the result follows from the symmetric functional dependence of $\bar{f}$ on $\bar{T}$ and κ, $\bar{f} = \bar{f}(\kappa\bar{T})$, which implies that $\bar{f}$ is increasing in κ, just like it is increasing in $\bar{T}$. ∎

A.5 Proof of Proposition 5

In this and the next Proof, we assume the presence of a social network, as follows: Investors are connected in an undirected social network represented by the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of investors and $\mathcal{E}$ is the set of edges connecting them. The set of investors $\mathcal{N} = \{1, \ldots, N\}$, and $(m,n) \in \mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ if investors m and n are connected through a social tie. By convention, the network is undirected, i.e., $(m,n) \in \mathcal{E} \Leftrightarrow (n,m) \in \mathcal{E}$, and investors are not connected to themselves ($(n,n) \notin \mathcal{N}$).

In the model, social ties could represent friendship, professional collaboration, membership in the same country club, or involvement with the same online community. If $(m,n) \in \mathcal{E}$, there is a chance that investor m tells n his investment strategy and performance. The set of investors that n is socially linked to is $\mathcal{D}_n = \{m : (n,m) \in \mathcal{E}\} \subset \mathcal{N} \backslash \{n\}$, and n's *degree* (number of connections) is $d_n = |\mathcal{D}_n|$. An investor with a higher degree is said to be more connected. The total number of connections is $Q = \frac{1}{2}\sum_n d_n$.

Consider investor n, who has adopted a passive investment strategy. Given return realizations, R_A and R_P, the transition probability for a sender from A to P is $T_A(R_A)$. Denote the subset of neighbors of investor n that are type A (resp. P) by $\mathcal{D}_n^A$ (resp. $\mathcal{D}_n^P$).

We prove the result for a more general case than our base model in which, even within the same class of investment strategies (A or P), investors may have different returns. Specifically, the return of an A investor $m \in D_n^A$ is assumed to be R_{Am}. The main body considers the special case in which $R_{Am} \equiv R_A$ (is the same) for all active investors. In a period, the number $2\kappa N$ links are chosen randomly in $\mathcal{E}$, with equal probability, and such that (m,n) and (n,m) are not both chosen, and we assume that $2\kappa N << Q$. Here, n is the potential sender and m the potential receiver in the chosen link (n,m).

For a type P investor n to convert to A, he/she must (i) be selected for communication, which occurs with probability $2\kappa d_n/Q$, (ii) be selected to be receiver, which occurs with probability $1/2$, (iii) communicate with an A, $m \in \mathcal{D}_n^A$, and finally (iv) be converted, which occurs with probability $T_A(R_{Am})$. So the probability $\mathcal{C}$ that investor n switches from P to A is therefore

$$\mathcal{C} = \kappa \times \frac{|\mathcal{D}_n|}{Q} \times \frac{|\mathcal{D}_n^A|}{|\mathcal{D}_n|} \sum_{m \in \mathcal{D}_n^A} T_A(R_{Am}). \tag{A.13}$$

Clearly, this probability is increasing in the number of A connections, $|\mathcal{D}_n^A|$. It is also increasing and strictly convex in the performance of each of these connections, since T_A is and increasing strictly convex function of R_{Am}. Finally, an identical argument as in the proof of Proposition 3 applied to (A.13) implies that the probability is increasing in σ_{A_m}. ∎

A.6 Proof of Proposition 6

An identical argument as in the proof of Proposition 1 applied to (A.13) implies that the probability is increasing in a_n, b_n, β_m, and γ_m, $m \in \mathcal{D}_n$. ∎

B Endogenizing the Receiving and Sending Functions

We model here the determinants of the sending and receiving functions, and derive their functional forms.

B.1 The Sending Function

To derive a sending function that reflects the desire to self-enhance, we assume that the utility derived from sending is increasing with own-return. Conversation is an occasion for an investor to try to raise the topic of return performance if it is good, or to avoid the topic if it is bad. Suppressing i subscripts, let $\pi(R,x)$ be the utility to the sender of discussing his return R,

$$\pi(R,x) = R + \frac{x}{\beta'}, \tag{B.1}$$

where β' is a positive constant that measures the relative weight in the individual's utility on conversational context versus the desire to communicate higher returns. The more tightly the investor's self-esteem is tied to return performance, the higher is β'. The random variable x measures whether, in the particular social and conversational context, raising the topic of own-performance is appropriate or even obligatory.

The sender sends if and only if $\pi > 0$, so

$$\begin{aligned} s(R) &= Pr\left(x > -\beta' R | R\right) \\ &= 1 - F(-\beta' R), \end{aligned} \tag{B.2}$$

where F is the distribution function of x. If $x \sim U[\tau_1, \tau_2]$, where $\tau_1 < 0, \tau_2 > 0$, then

$$\begin{aligned} s(R) &= \frac{\tau_2 + \beta' R}{\tau_2 - \tau_1} \\ &= \frac{\tau_2}{\tau_2 - \tau_1} + \beta R, \end{aligned} \tag{B.3}$$

where $\beta \equiv \beta'/(\tau_2 - \tau_1)$, and where we restrict the domain of R to satisfy $-\tau_2/\beta' < R < -\tau_1/\beta'$ to ensure that the sending probability lies between 0 and 1. This will hold almost surely if $|\tau_1|, |\tau_2|$ are sufficiently large. Equation (B.3) is identical to the sending function (1) in Subsection 2.2 with

$$\gamma \equiv \frac{\tau_2}{\tau_2 - \tau_1}.$$

In the sender's utility $\pi(R,x)$ of discussing return R, the parameter β' captures the value placed on mentioning one's high return experience, versus the appropriateness of doing so. The more tightly bound is the sender's self-esteem or reputation to return performance, the larger is the parameter β', and hence the stronger is *SET*, as measured by β in the sending function (1) which is proportional to β'.

The constant γ in the sending function (1) reflects the *conversability* of the investment choice. When investment is a more attractive topic for conversation or when conversations are more extensive, as occurs when investors are more sociable, higher γ shifts the distribution of x to the right (i.e., an increase in τ_2, for given $\tau_2 - \tau_1$, implies higher γ).

B.2 The Receiving Function

We derive an increasing convex increasing shape for the receiving function as in equation (2) in Section 2.3 from the combination of two effects: greater receiver attention to extreme return outcomes, and, conditional upon paying attention, and, owing to the representativeness heuristic, greater persuasiveness of higher return.

The return on a sender or receiver strategy has unknown mean μ^i, $i = s, r$, where $R^i = \mu^i + \epsilon^i$, where for tractability the receiver perceives the distribution of the means as $\mu^i \sim N(\mu_0^i, \sigma_{\mu i}^2)$, $\epsilon^i \sim N(0, \sigma_{\epsilon i}^2)$. Assume all RHS random variables are independent.

The receiver is exposed to a realization of (R^s, R^r) and to the sender's type. A receiver can, at cost $\sim U(0, \bar{c}_1)$, pay attention, in which case, the receiver learns the direct cost of switching strategies, $c_2 \sim U(\underline{c}_2, \bar{c}_2)$, and optimizes over whether to switch. A non-attending receiver incurs no cost, and never switches. The costs of paying attention and of switching depends on situation-specific circumstances not observed by the econometrician.

We assume that $\underline{c}_2 < 0 < \bar{c}_2$. The possibility that the 'cost' of switching is negative reflects a possible favorable inference by the receiver about the sender's adoption of the sender's strategy. (It could alternatively reflect conformist preferences.)

The quasi-Bayesian update of μ^i, $i = s, r$ given observed returns

$$E[\mu^i | R^i] = \mu_0^i + \beta^i (R^i - \mu_0^i), \tag{B.4}$$

where

$$\beta^i = \frac{\sigma_{\mu i}^2}{\sigma_{\mu i}^2 + \sigma_{\epsilon i}^2}.$$

Here we capture representativeness/overextrapolation taking the form of β_i being an overestimate of the true relationship, i.e., the receiver regards past returns as being more indicative of future performance than they really are.[18]

We assume for simplicity that an attending receiver switches to the sender's strategy based on whether the difference in updated means $\mu^s - \mu^r$ exceeds the switch cost c_2.[19]

So conditional upon attending and the observed returns, the probability of switching strategies is

$$P(E[\mu^s | R^s] - E[\mu^r | R^r] - c_2 \geq 0) = \int_{c_2 = \underline{c}_2}^{\beta^s R^s - \beta^r R^r} \frac{dc_2}{\bar{c}_2 - \underline{c}_2} = \frac{\beta^s R^s - \beta^r R^r - \underline{c}_2}{\bar{c}_2 - \underline{c}_2} \tag{B.5}$$

when this quantity lies between 0 and 1, and is at the relevant probability boundary otherwise.

We endogenize the investor's attention heuristic by solving for the optimal decision of whether to pay attention, taking into account (R^s, R^r) and what this implies about (μ^s, μ^r). Owing to cognitive processing constraints, in general we expect this decision to be heuristic. However, a wide set of heuristics are possible, and the result we derive are not driven by bias in this decision. So as a benchmark case that is neutral with respect to bias in the attention decision, we model the attention decision as fully rational, i.e., making full use of R^s, R^r, and c_1, but not c_2 which is only

[18] Algebraically this could arise from overestimation of $\sigma_{\mu i}^2$ and/or underestimation of $\sigma_{\epsilon_i}^2$. The form of the receiving function that we derive here does not actually require this overextrapolation, but for realistic parameter values $\sigma_{\mu i}^2 / \sigma_{\epsilon_i}^2$ would be low, since most of the variance in strategy performance comes from chance rather than differences in means. This would lead to very weak updating, implying a very small slope of the receiving function.

[19] It would not be hard to allow for the effect of risk aversion via an adjustment for the difference in variances of the two strategies. Since prior variances are known, observation reduces posterior variances deterministically, i.e., by the same amount regardless of the signal.

observed after paying attention.[20] The approach of assuming rationality in attention allocation is also applied in the large literature on rational inattention (Sims 2003), and in other work on limited attention such as Peng and Xiong (2006).

The receiver's attention heuristic is tuned to pay attention if the expected improvement in portfolio expected returns, net of switch costs, and given the observed past returns, exceeds the cost of attention. Let $\mathbf{1}_{E[\mu^s|R^s]-E[\mu^r|R^r]-c_2\geq 0}$ be an indicator function for the receiver switching to the sender's strategy after attending and observing returns. The receiver attends iff the expected gain exceeds c_1,

$$E[(\mu^s - \mu^r - c_2)\mathbf{1}_{E[\mu^s|R^s]-E[\mu^r|R^r]-c_2\geq 0}|R^s, R^r] - c_1 \quad \geq \quad 0, \tag{B.6}$$

so substituting out expectations of μ's by (B.4), the condition becomes

$$\frac{(\beta^s R^s - \beta^r R^r)(\beta^s R^s - \beta^r R^r - \underline{c}_2)}{\bar{c}_2 - \underline{c}_2} - E[c_2 \mathbf{1}_{\mu^s-\mu^r-c_2\geq 0}|R^s, R^r] - c_1 \geq 0. \tag{B.7}$$

Now the expectation above is

$$E[c_2\mathbf{1}_{E[\mu^s|R^s]-E[\mu^r|R^r]-c_2\geq 0}|R^s, R^r] \quad = \quad \frac{(\beta^s R^s - \beta^r R^r)^2 - \underline{c}_2^2}{2(\bar{c}_2 - \underline{c}_2)}$$

So the receiver attends iff

$$\frac{(\beta^s R^s - \beta^r R^r - \underline{c}_2)^2}{2(\bar{c}_2 - \underline{c}_2)} - c_1 \geq 0. \tag{B.8}$$

Since c_1 is uniformly distributed,

$$P(\text{Attend}|R^s, R^r) = P\left(c_1 \leq \frac{(\beta^s R^s - \beta^r R^r - \underline{c}_2)^2}{2(\bar{c}_2 - \underline{c}_2)}\right) = \frac{(\beta^s R^s - \beta^r R^r - \underline{c}_2)^2}{2\bar{c}_1(\bar{c}_2 - \underline{c}_2)}, \tag{B.9}$$

which is quadratically increasing in the weighted return difference $\beta^s R^s - \beta^r R^r$.

The probability that the receiver switches conditional upon the returns is the product

$$P(\text{Attend}|R^s, R^r)P(\text{Switch}|\text{Attend}, R^s, R^r).$$

The first probability is given in (B.9), and the second in (B.5).

So the probability of switching, i.e., the receiving function, is

$$r(R^s, R^r) = \frac{(\beta^s R^s - \beta^r R^r - \underline{c}_2)^3}{2(\bar{c}_2 - \underline{c}_2)^2\bar{c}_1}$$

[20]Modelling the attention choice as fully rational may seem paradoxical, since it can take more calculations to allocate attention optimally than to simply solve the decision problem at hand. However, again, we view full rationality of the attention decision as merely the most convenient benchmark case. Furthermore, it is not necessary to view our benchmark case as involving full conscious rationality in the attention allocation decision. The calculations needed to allocate attention correctly do not necessarily use cognitive resources at the time of each attentional decision. Attention heuristics can be viewed as having been designed in human evolutionary prehistory to balance the cost of paying attention against the benefits achieving better decision outcomes. Alternatively, the attention mechanism can be viewed as a rule-of-thumb heuristic that the investor has learned through previous experience over the investor's lifetime.

when this quantity lies between 0 and 1. This is a cubic function of $\beta^s R^s - \beta^r R^r$ with all nonnegative coefficients since $\underline{c}_2 \leq 0$.

A special case of this development is when $\beta^r << \beta^s$, in which case the expression approximately simplifies to

$$r(R^s) = \frac{1}{2(\bar{c}_2 - \underline{c}_2)^2 \bar{c}_1}[(\beta^s R^s)^3 - 3\underline{c}_2(\beta^s R^s)^2 + 3(\underline{c}_2)^2 \beta^s R^s - (\underline{c}_2)^3]$$

when this quantity lies between 0 and 1.

A quadratic Taylor approximation leads to a quadratic expression for $r(R^s, R^r)$ or, when β^r small, for $r(R^s)$, as in equation (2) in Section 2.3, where we assume that most of the probability mass of R is in the range where the coefficients of this quadratic approximation are positive, consistent with a convex increasing shape for the receiving function. Specifically, performing this Taylor expansion around $R^s = 0$ yields the quadratic receiving function coefficients $a = -3\underline{c}_2(\beta^s)^2/[2(\bar{c}_2 - \underline{c}_2)^2\bar{c}_1]$, $b = 3(\underline{c}_2)^2\beta^s/[2(\bar{c}_2 - \underline{c}_2)^2\bar{c}_1]$, and $c = -(\underline{c}_2)^3/[2(\bar{c}_2 - \underline{c}_2)^2\bar{c}_1]$. By varying the free parameters, any positive vector of values of (a, b, c) is feasible.

C Homophily

Consider a variation of the network model in which there is homophily. Specifically, senders and receivers (or, equivalently links) are no longer randomly selected, but rather the probabilities are tweaked such that the probability that communication (a link) is selected between two agents with the same strategy is relatively higher than the probability that agents with different strategies are selected, by a factor χ.

Specifically, when there is no homophily, the fraction of selected sender-receivers with an active potential sender and a passive potential receiver in a period is $\kappa f_t(1 - f_t)$. When there is homophily, it is instead $\frac{\kappa}{\chi} f_t(1 - f_t)$, where $\chi > 1$ denotes the degree of homophily. This is also the fraction of passive potential senders and active potential receivers selected.

The transformation rate will then be

$$E[\Delta f] = \frac{1}{\chi} f(1 - f)\kappa\bar{T} + \hat{\lambda}(q - f).$$

It is easy to show that compared with the transformation rate in the base model (3), the transformation rate with homophily is lower, and this also carries over to a lower steady state fraction of active investors.

D Equilibrium Model

So far, we have modeled the economy in a partial equilibrium setting with exogenous return distributions for A and P, along with informal arguments that when there are more A's in the investor population, demand for this strategy increases, decreasing future returns. In practice, after extensive inflow of investors into active strategies, we expect the equilibrium price of acquiring strategy positions to rise, reducing expected future returns. So evolution toward A is self-limiting. We now extend the model in a stylized way to capture such equilibrium effects. Without loss of generality, we assume that one agent is chosen in each time period, i.e., that $\kappa = 1/2N$.

The Investment Technology

We model the supply-side of the economy as a set of short-term investment opportunities with diminishing returns to scale, which implies imperfectly elastic supply. We assume that the output elasticity is lower for investments associated with active than for passive strategies, reflecting the idea that active strategies may be less scalable. For simplicity, we assume that investments associated with P's are perfectly elastic, whereas investments associated with A's are not. For example, if A is buying IPO stocks, and if the supply of excellent new business opportunities is limited, then there will be diminishing returns to aggregate investment in A. As a special case, the passive investment could, for example, represent a low-risk storage technology.

The one-period returns in this case depend on total active investments, X, as

$$R_A(N_A) = (\beta_A r + \epsilon_A + \upsilon) \times (\rho X)^{-1/2} - \upsilon, \tag{D.1}$$
$$R_P = \beta_P r + \epsilon_P, \tag{D.2}$$

where the N_A is the total number of active investors, $\upsilon > 0$, $\rho > 0$ are parameters, and X in equilibrium will depend on N_A.[21] Also, r denotes a common component of returns shared by A and P (e.g., the market portfolio), $E[r] = 0$, β_i is the sensitivity of strategy return to the common return component, ϵ_i is a strategy-specific component, $E[\epsilon_i] = 0$, $i = A, P$. We assume that r, ϵ_A and ϵ_P are independent, with skewness γ_A and γ_P, respectively, and that the skewness of r is zero.

The Investor Objective

The objective of investors is to maximize the mean-variance expected utility function

$$U = E[R] - \left(\frac{\zeta}{2}\right) Var(R), \tag{D.3}$$

where for simplicity we set the risk aversion coefficient $\zeta = 1$. The riskfree asset has return r_f. Here, since we have normalized such that $E[r] = E[\epsilon_A] = E[\epsilon_P] = 0$, we assume that $r_f < 0$. The negative riskfree rate could, for example, represent a storage technology with some depreciation. This assumption could easily be modified, at the cost of greater algebraic complexity, by allowing for additional intercept components of returns in (D.1) and (D.2).

By assumption, the P's maximize expected utility of investing in a portfolio consisting of a risky investment alternative that is available to P investors, and the riskfree asset. Similarly, A's optimize a portfolio of a risky investment alternative that is available to A investors, and the riskfree asset. Investors optimize rationally, but do not consider including both passive and active assets in their portfolios at the same time.[22] In equilibrium, active investors' total demand is X, where they optimize expected utility given the return distribution in (D.1).

[21] The return specification in (D.1) corresponds to a concave production function where input X leads to stochastic production $(\beta_A r + \epsilon_A + \upsilon) \times \left(\frac{X}{\rho}\right)^{1/2} - \upsilon X$. The parameters are such that a higher ρ is associated with a lower expected output, and a higher υ corresponds to a more concave production function.

[22] Our assumption of an increasing receiving function was based on the idea that investors overextrapolate reports about past return performance. This suggests that investors will have overoptimistic expectations about the strategies they have been persuaded to adopt. It would be easy to incorporate such overestimation

Joint Determination of Strategy Popularity and Asset Returns

In this specification, the return penalty, D_{N_A}, depends on N_A, the number of A's. We choose a specific value for the ρ parameter,

$$\rho = \frac{2(\beta_A^2 \sigma_r^2 + \sigma_A^2)}{N|r_f|},$$

which in equilibrium implies an initial return penalty of zero, $D_{N/2} = 0$. Here, we have assumed that $f_0 = \frac{1}{2}$, i.e., that half of the population initially invests in each strategy. Moreover, we assume that $q = \frac{1}{2}$, so that new investors also invests equally in the two strategies.

The case of a zero return penalty to active investing is a simple benchmark case that is useful for identifying what influences the spread of competing investment strategies when the obvious effect of expected return differences is neutralized. It follows from the dependence here of equilibrium return on the number of A's that the transformation probability also depends on N_A,

$$\overline{T}_{N_A}^{A} = E[T_A(R_A(N_A))]. \tag{D.4}$$

The following proposition provides conditions under which the results from Sections 3.2-3.3 generalize to the equilibrium setting.

Proposition 7 *Under the parameter restrictions that $|r_f|$ is small, $\kappa \geq |r_f|$, $\gamma_P = 0$, $\gamma_A \geq 0$, and*

$$\beta_A > 2\beta_P \tag{D.5}$$

$$\sigma_A > 2\sigma_P, \tag{D.6}$$

Proposition 2, Proposition 3:2-3:3 , and Proposition 4 continue to hold in equilibrium. Moreover, the returns an investor is expected to receive from active investments is nonpositive and strictly decreasing over time.

In equilibrium, active investments thus dominate, and the return penalty is positive, in line with the core results of the partial equilibrium model. Intuitively, transmission bias causes A to spread, putting a downward pressure on the returns to the A strategy, and thereby inducing a return penalty to active investing. In other words, owing to transmission bias, A investing persists despite needing to climb uphill against a return penalty.

The sufficient condition on σ_A is stricter in the equilibrium setting, as seen by the extra factor 2 in (D.6). This factor arises because the restriction $\overline{T}_{N_A}^{A}$ depends on the number of active investors, N_A, and $\overline{T}_{N_A}^{A} > \overline{T}_{N_A}^{P}$ needs to be satisfied for all $1 \leq N_A \leq N$. Of course, this is just a sufficient condition.

The only result from Section 3.2 that does not extend to the equilibrium setting is Proposition 3:1, the comparative static with respect to the return penalty. Such comparative statics are not well defined in the equilibrium model since the return penalty is endogenous.

into the investor's mean-variance portfolio optimization here, but doing so would not affect the general nature of our conclusions. These are driven by the fact that in our specification, an increase in the fraction of A's drives down the equilibrium expected return of this strategy. So for simplicity we assume rational mean-variance optimization.

E Trading Volume

If we interpret A as active trading in the market for individual stocks, with a preponderance of long positions, then a high market return implies high average returns to A's. Proposition 1 therefore suggests that when the stock market rises, volume of trade in individual stocks increases. This implication is consistent with episodes such as the rise of day trading, investment clubs, and stock market chat rooms during the millennial internet boom, and with evidence from 46 countries including the U.S. that investors trade more when the stock market has performed well (Statman, Thorley, and Vorkink 2006; Griffin, Nardari, and Stulz 2007). We next study trading volume in the equilibrium model, to verify that evolution toward A is associated with high trading volume.

The total demand of N_A active investors, given a risky investment opportunity with expected return $E[R_A]$ and return variance $Var(R_A)$ is $X = N_A \frac{E[R_A]-r_f}{Var(R_A)}$, and market clearance, by (D.1), leads to

$$X = \frac{\rho\kappa^2 N_A^2}{(\rho\upsilon N_A - \rho N_A|r_f| + (\beta_A^2\sigma_r^2 + \sigma_A^2))^2}. \tag{E.1}$$

When an investor switches from P to A, he liquidates his passive portfolio position of

$$\frac{|r_f|}{\sigma_P^2},$$

the number of active investors increases from N_A to $N_A + 1$, and he invests

$$\frac{1}{N_A+1} X_{N_A+1}$$

in the active investment. Here, in equilibrium,

$$X_{N_A} = \frac{2\upsilon^2 N N_A^2 |r_f|}{(2N_A(\upsilon - |r_f|) + N|r_f|)^2(\beta_A^2\sigma_r^2 + \sigma_A^2)}. \tag{E.2}$$

Moreover, the N_A investors that are already active rebalance from a total position of X_{N_A} to $\frac{N_A}{N_A+1} X_{N_A+1}$. The total trading volume is thus: $\frac{|r_f|}{\sigma_P^2} + Z_{N_A}$, where

$$Z_{N_A} \stackrel{\text{def}}{=} \frac{1}{N_A+1} X_{N_A+1} + N_A \left| \frac{X_{N_A}}{N_A} - \frac{X_{N_A+1}}{N_A+1} \right|.$$

It is easy to verify that when $\upsilon + r_f \approx 0$, i.e., when $|r_f|$ is of similar size as υ, then $\frac{X_n}{n}$ is increasing in n, and therefore

$$Z_{N_A} = X_{N_A+1} - X_{N_A}.$$

Moreover, when $\upsilon = -r_f$,

$$Z_{N_A} = \frac{2\upsilon}{N(\beta_A^2\sigma_r^2 + \sigma_A^2)}(1 + 2N_A), \tag{E.3}$$

which is strictly increasing in N_A. Therefore, by continuity, for $\upsilon + r_f \approx 0$, total trading volume, is also strictly increasing in N_A.

An identical argument applies to the situation when an investor switches from A to P. Specifically, if there are initially $N_A + 1$ investors, and an investor switches from A to P, that investor invests $\frac{|r_f|}{\sigma_P^2}$ in the passive strategy, sells $\frac{1}{N_{A+1}} X_{N_{A+1}}$ in the active investment, whereas the other N_A investors in total rebalance from $N_A \frac{X_{N_{A+1}}}{N_{A+1}}$ to X_{N_A}. Again, the total trading volume is described by $\frac{|r_f|}{\sigma_P^2} + Z_{N_A}$.