Cash Holdings, Capital Structure, and Financing Risk

— Online appendix, Not for publication —

A The Micro-Interpretation of the Enforcement Constraint

In this appendix, we provide a micro-interpretation of the enforcement constraint (4) used in Section III C of the paper:

$$p_t b_{t+1} \leq \max\left\{\xi_t k_{t+1}, (1-\delta_b) p_t b_t\right\} \tag{A.1}$$

We keep the variable ξ_t as a constant collateral rate and introduce a new variable η_t to capture the credit risk. We assume that firms need to search for lenders when they decide to tap the credit market. The probability of finding a lender depends on the financing condition. When the financing condition is η_t, the firm can find a lender with probability η_t, and with probability $1-\eta_t$ the firm cannot get financed. Here, the inverse of the financing condition $1/\eta_t$ can be interpreted as a measure of credit market tightness. Thus, the probability of finding a lender decreases with the credit market tightness. The variable η_t can also be interpreted as a measure of the lender's financial health.

In the case that the firm finds a lender, it can issue new debt. However, due to the firm's limited commitment on its debt obligations, the issuance of debt is subject to collateral constraints: when the lender provides loans to the firm in the current period, it wants to make sure that in the next period the liquidation value of the firm's assets is larger than the value of the firm's outstanding debt so that the firm does not default. To be specific, if the firm has capital assets k_{t+1} at the end of period t, its total credit limit during period t would be ξk_{t+1}, in which we assume that the assets in place $(1-\delta)k_t$ and the new investments i_t have the same collateral rate ξ. As a result, the firm's debt outstanding b_{t+1} at the end of period t should satisfy: $p_t b_{t+1} \leq \xi k_{t+1}$. The value of total debt should be less than or equal to the value of collateral assets.

In the case that the firm does not find a lender, it cannot issue new debt. However, according to the arrangement of long-term debt, the lender cannot force the firm to repay more than δ_b percent of its debt outstanding, without regard to the financing conditions. In this case, the borrowing constraint would be $p_t b_{t+1} \leq (1-\delta_b)p_t b_t$, where $(1-\delta_b)p_t b_t$ is the value of non-paid debt.

To sum up, during the period t the firm is subject to the following revised enforcement constraint, which is a stochastic version of the constraint (4):

$$p_t b_{t+1} \leq \omega_{t+1}, \tag{A.2}$$

where $p_t b_{t+1}$ is the value of debt outstanding and ω_{t+1} is the firm's total debt capacity as follow:

$$\underbrace{\omega_{t+1}}_{\text{debt capacity}} = \underbrace{\eta_t}_{\text{refinancing prob.}} \underbrace{\xi k_{t+1}}_{\text{collateral asset}} + (1-\eta_t)\underbrace{(1-\delta_b)p_t b_t}_{\text{non-paid debt}}. \tag{A.3}$$

The firm's debt capacity depends on the financing condition η_t, the value of collateral assets ξk_{t+1}, and the value of non-paid debt $(1-\delta_b)p_t b_t$.

Accordingly, the firm's unused lines of credit during period t would be defined as:

$$l_t = \eta_t \xi k_{t+1} + (1 - \eta_t)(1 - \delta_b) p_t b_t - p_t b_{t+1}. \tag{A.4}$$

The Feature of Enforcement Constraint There are three remarks on the enforcement constraint (A.2). First, as can be seen from equation (A.3), a better financing condition η_t eases the enforcement constraint, a lower repayment rate δ_b relaxes the enforcement constraint, and a larger the last period's debt outstanding b_t also relaxes the current period's enforcement constraint.

Second, if the repayment rate $\delta_b = 1$, the enforcement constraint (A.2) becomes $p_t b_{t+1} \leq \eta_t \xi k_{t+1}$. In this case, consider the constraint in period $t+1$: $p_{t+1} b_{t+2} \leq \eta_{t+1} \xi k_{t+2}$. Suppose there is a decline of the financing opportunity η_{t+1}. Then, as a result, the firm has to reduce its debt outstanding b_{t+2}, which in turn forces the firm to cut either investment or dividend. Thus, if it is costly for the firm to adjust capital or equity quickly within a period, concerns of the period $t+1$'s credit contractions would induce the firm to borrow less and save unused debt capacity in period t.

Third, if the repayment rate $\delta_b < 1$, the second term $(1 - \eta_t)(1 - \delta_b) p_t b_t$ on the right side of equation (A.3) comes up. In this case, the firm would have incentives to borrow more to hedge against future credit contractions. This is because an additional unit of borrowing Δb_t in period $t-1$ would relax the enforcement constraint in period t by $(1 - \eta_t)(1 - \delta_b) p_t \Delta b_t$ dollars. Further, if it is costly to adjust capital or equity quickly, the firm would temporally save the funds from the long-term borrowing in cash.

The Trade-off Between Cash and Unused Lines of Credit Let's compare the efficiency of cash and unused lines of credit in providing future liquidity. Denote p_t^m by the price of cash at period t, then one additional dollar of cash in period $t-1$ leads to $\frac{1}{p_{t-1}^m}$ dollars of available funds in period t. Similarly, suppose the price of debt at period t is p_t, then one additional dollar of unused lines of credit in period $t-1$ leads to $\frac{\eta_t(1-\delta_b)p_t+\delta_b}{p_{t-1}}$ dollars of available funds in period t. The term $\frac{\eta_t(1-\delta_b)p_t+\delta_b}{p_{t-1}}$ depends on the credit market condition η_t, and it contains two parts. The first part $\frac{\eta_t(1-\delta_b)p_t}{p_{t-1}}$ represents the increase in debt issuance, and the second part $\frac{\delta_b}{p_{t-1}}$ is the reduction in debt repayment.

The firm makes a trade-off between low-return cash $\frac{1}{p_{t-1}^m}$ and contingent unused lines of credit $\frac{\eta_t(1-\delta_b)p_t+\delta_b}{p_{t-1}}$. And this trade-off depends on the maturity of debt ($\frac{1}{\delta_b}$), the opportunity cost of holding cash ($p_{t-1}^m - p_{t-1}$), and the future financing condition (η_t).

If $\delta_b = 1$, $\frac{\eta_t(1-\delta_b)p_t+\delta_b}{p_{t-1}} = \frac{1}{p_{t-1}} > \frac{1}{p_{t-1}^m}$, unused lines of credit are less costly than cash holdings in providing liquidity. However, if $\delta_b < 1$, unused lines of credit become contingent, and cash can be more efficient than credit lines in accumulating liquidity in some states, particularly when future credit market conditions become worse: it is more likely that $\frac{1}{p_{t-1}^m} > \frac{\eta_t(1-\delta_b)p_t+\delta_b}{p_{t-1}}$ when η_t becomes smaller.

The above results can also be explained by the features of long-term debt. With long-term debt, one unit of debt issuance in period $t-1$ not only brings in p_{t-1} dollars of proceeds in period $t-1$, but also relaxes the period t's enforcement constraint by $(1-\eta_t)(1-\delta_b)p_t$ dollars. The relaxation of enforcement constraint then increases the available credit the firm can use in period t. Meanwhile, one unit of debt retirement in period $t-1$ only leads to $\eta_t(1-\delta_b)p_t + \delta_b$ dollars of available funds in period t. This is because the financing opportunity is stochastic. If the firm does not borrow now, it may lose the chance to borrow in the future.

B Proofs

The firm's problem after detrending is (we remove the tilde on the detrended variables):

$$V(m,b;s) \quad = \quad \max_{g',m',b',d}\left\{d+\beta g'\mathbb{E}[(\frac{z_a'}{z_a})^{-\gamma}V(m',b';s')]\right\} \tag{B.1}$$

subject to:

$$p^m m' g' \geq z_i z_a \tag{B.2}$$

$$z_i z_a + m + pn = p^m m' g' + \delta_b b + i + \varphi(d) \tag{B.3}$$

$$g' = (1-\delta) + \chi\phi(i) \tag{B.4}$$

$$pb'g' = (1-\delta_b)pb + pn \tag{B.5}$$

$$pn \geq 0 \tag{B.6}$$

$$pn \leq \eta[\xi g' - (1-\delta_b)pb] \tag{B.7}$$

Let μ be the multiplier on the cash-in-advance constraint (B.2), λ_0 be the multiplier on the budget constraint (B.3), q be the multiplier on the investment equation (B.4), λ_1 be the multiplier on the debt dynamics equation (B.5), λ_2 be the multiplier on non-negative debt issuance constraint (B.6), and λ_3 be the multiplier on the enforcement constraint (B.7).

The Lagrangian equation is:

$$
\begin{aligned}
L &= d + \beta g' \mathbb{E}[(\frac{z_a'}{z_a})^{-\gamma} V(m', b'; s')] \\
&+\mu\big(p^m m' g' - z_i z_a\big) \\
&+\lambda_0\big(z_i z_a + m + pn - p^m m' g' - \delta_b b - i - \varphi(d)\big) \\
&+q\big(1 - \delta + \chi\phi(i) - g'\big) \\
&+\lambda_1 p\big(b' g' - (1-\delta_b) b - n\big) \\
&+\lambda_2 pn \\
&+\lambda_3\big(\eta[\xi g' - (1-\delta_b) pb] - pn\big)
\end{aligned}
$$

where $\lambda_2 \equiv 0$ and η is the probability of having a financing opportunity.

First Order Conditions for d, i, m', b', g', n are:

$$
\begin{aligned}
&\lambda_0 - \frac{1}{\varphi'(d)} = 0 \\
&q - \frac{\lambda_0}{\chi\phi'(i)} = 0 \\
&\beta g' \mathbb{E}[(\frac{z_a'}{z_a})^{-\gamma} V'_{m'}] - \lambda_0 p^m g' + \mu p^m g' = 0 \\
&\beta g' \mathbb{E}[(\frac{z_a'}{z_a})^{-\gamma} V'_{b'}] + \lambda_1 p g' = 0 \\
&\beta \mathbb{E}[(\frac{z_a'}{z_a})^{-\gamma} V'] + \mu p^m m' - \lambda_0 p^m m' - q + \lambda_1 p b' + \lambda_3 \xi \eta = 0 \\
&\lambda_0 p - \lambda_1 p + \lambda_2 p - \lambda_3 p = 0
\end{aligned}
$$

Envelope Conditions are:

$$V_m = \lambda_0$$

$$V_b = -\lambda_0\delta_b - \lambda_1(1-\delta_b)p - \lambda_3(1-\delta_b)p\eta$$

Proof of Proposition 3.1: If $\delta_b = 1$, then $\mu = \lambda_0 - \lambda_1 \frac{p}{p^m} > 0$, by using $\lambda_0 - \lambda_1 = \lambda_3 \geq 0$ and $p^m > p$. Thus, the cash-in-advance constraint is always binding.

Proof of Proposition 3.2: When $\delta_b < 1$,
$\mu = \lambda_0 - \frac{p}{p^m}\frac{\lambda_1}{\delta_b+(1-\delta_b)p\eta} + \frac{p}{p^m}\beta\mathbb{E}_s\left[(\frac{z_a'}{z_a})^{-\gamma}\frac{(1-\eta)(1-\delta_b)\lambda_1'}{\delta_b+(1-\delta_b)p\eta}\right]$. The size of lagrangian multiplier μ depends on the state s. Thus, it is possible that in some states $\mu = 0$ and therefore the cash-in-advance constraint can be occasionally non-binding.

C Numerical Methods

After writing down the first-order conditions and the envelope conditions, the firm's problem can be summarized by a system of non-linear equations associated with three expectation terms. Thus, by solving the non-linear equations, we get the solution of the firm's problem.

The numerical solution takes three steps. First, we approximate the three conditional expectation functions as follows:

$$\Phi_V(m, b; s) = \mathbb{E}_s\big[(z_a')^{-\gamma} V(m', b'; s')\big]$$

$$\Phi_m(m, b; s) = \mathbb{E}_s\big[(z_a')^{-\gamma} V_{m'}(m', b'; s')\big]$$

$$\Phi_b(m, b; s) = \mathbb{E}_s\big[(z_a')^{-\gamma} V_{b'}(m', b'; s')\big]$$

Second, given the parameterized expectations, we solve the system of non-linear equations on each grid. We discretize each shock on five grid points and each state variable on ten grid points. We also do robust check by increasing the number of grids. We interpolate linearly between grids when calculating the expectations. Finally, we iterate on the approximation functions until convergence.

A Main Programming Routine

The main numerical routine contains two loops:

The outside loop: given states (m, b, s), solve policies (m', b') and update the approximation functions (Φ_V, Φ_m, Φ_b).

The inside loop: solve a non-linear equation system with four unknowns.

The four unknowns are: m', b', i, V, and the four equations are:

$$EQ1 : \mu(p^m m' g' - z_i z_a) = 0$$

$$EQ2 : \lambda_2 pn = 0$$

$$EQ3 : \lambda_3 \big[\eta\big(\xi g' - (1-\delta_b)pb\big) - pn\big] = 0$$

$$EQ4 : d + \beta g'(z_a)^\gamma \Phi_V - V = 0$$

where,

$$g' = (1-\delta) + \chi\phi(i)$$

$$n = b'g' - (1-\delta_b)b$$

$$d = \varphi^{-1}(z_i z_a + m + pn - p^m m' g' - \delta_b b - i)$$

$$\lambda_0 = \frac{1}{\varphi'(d)}$$

$$q = \frac{\lambda_0}{\chi\phi'(i)}$$

$$\lambda_1 = \frac{-\beta(z_a)^\gamma \Phi_b}{p}$$

$$\mu = \lambda_0 - \frac{\beta(z_a)^\gamma \Phi_m}{p^m}$$

$$\lambda_3 = \frac{\lambda_0 p^m m' + q - \lambda_1 pb' - \mu p^m m' - \beta(z_a)^\gamma \Phi_V}{\xi\eta}$$

$$\lambda_2 = \lambda_3 - \lambda_0 + \lambda_1$$

B Occasionally Binding Constraints

We first solve the equation system by assuming that the two constraints (B.2) and (B.7) are both binding, and then check the Lagrangian multipliers μ and λ_3. According to the sign of μ and λ_3, we specify four cases and resolve the system case by case.

Case A, both binding, neither precautionary cash nor unused lines of credit:

$$EQ1 : \eta\big(\xi g' - (1-\delta_b)pb\big) - pn = 0$$

$$EQ3 : p^m m' g' - z_i z_a = 0$$

Case B, one non-binding, only has precautionary cash:

$$EQ1 : \eta\big(\xi g' - (1-\delta_b)pb\big) - pn = 0$$

$$EQ3 : p^m m' g' - z_i z_a > 0$$

Case C, one non-binding, only has unused lines of credit:

$$EQ1 : \eta\big(\xi g' - (1-\delta_b)pb\big) - pn > 0$$

$$EQ3 : p^m m' g' - z_i z_a = 0$$

Case D, both non-binding, precautionary cash *coexists* with unused lines of credit:

$$EQ1 : \eta\big(\xi g' - (1-\delta_b)pb\big) - pn > 0$$

$$EQ3 : p^m m' g' - z_i z_a > 0$$

D Simulated Method of Moments

The choice of model parameters is done by the simulated method of moments (SMM). The basic idea of SMM is to choose the model parameters such that the moments generated by the model are as close as possible to the corresponding real data moments.

The real data is a panel of heterogeneous firms, but the simulated data is generated by a representative firm. To keep consistency between the actual data and the simulated data, we estimate the parameters of an *average* firm in the data. More specifically, given the panel structure of the data, we first calculate moments for each firm, and then compute the average of moments across firms and use it as the target moment. We use the bootstrap method to calculate the variance-covariance matrix associated with the target moments.

The estimation procedure is as follows.[11] First, for each firm i, we choose moments $h_i(x_{it})$, where x_{it} is a vector representing variables in the actual data, and subscript i and t indicates firm and year respectively. Second, for each firm i, we calculate the within-firm sample mean of moments as $f_i(x_i) = \frac{1}{T}\sum_{t=1}^{T} h_i(x_{it})$, where T is the number of fiscal years in the data. Third, we compute the average of the within-firm sample mean as $f(x) = \frac{1}{N}\sum_{i=1}^{N} f_i(x_i)$, where N is the number of firms in the data.

Correspondingly, we use the model to simulate a panel data of N number of firms and S periods. We set $S = 100T$ to make sure that the representative firm would visit all the states in the model. We calculate the average sample mean of moments in the model as

[11] We also use the estimation procedure described in DeAngelo, DeAngelo and Whited (2011), and the estimation results are robust.

$f(y, \theta) = \frac{1}{NS}\sum_{i=1}^{N}\sum_{s=1}^{S} h(y_{is}, \theta)$, where y_{is} is the simulated data from the model, and θ represents the parameters to be estimated.

The estimator $\widehat{\theta}$ is the solution to

$$\min_{\theta} : [f(x) - f(y, \theta)]^T \Omega [f(x) - f(y, \theta)].$$

The weighting matrix Ω is defined as $\widehat{\Sigma}^{-1}$, where $\widehat{\Sigma}$ is the variance-covariance matrix associated with the average of sample mean $f(x)$ in the data. We use the bootstrap method to calculate the variance-covariance matrix $\widehat{\Sigma}$. First, given the population of N number of firms from the real data, we draw J random samples with size $\frac{N}{2}$. Second, for each draw j, we compute the statistics of the drawn sample, denote by $f(x)^j$. Third, we approximate the variance-covariance matrix by the variance of $f(x)^j$, i.e., $\widehat{\Sigma} \approx \frac{1}{J}\sum_{j=1}^{J}\left(f(x)^j - \frac{1}{J}\sum_{j=1}^{J} f(x)^j\right)^T\left(f(x)^j - \frac{1}{J}\sum_{j=1}^{J} f(x)^j\right)$. Finally, we set J=50,000 to have enough accuracy of the bootstrap method.