INTERNET APPENDIX

"Does Corporate Investment Respond to the Time-Varying Cost of Capital? Empirical Evidence"

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Appendix A. Identity Between Investment Return and Stock Return

In equation (3), the ex-dividend price of stock on date t is $\mathbb{E}_t\left[M_{t+1}V(A_{t+1}, K_{t+1})\right]$. The gross return from date t to $t+1$ is:

$$R_{E,t+1} = \frac{V_{t+1}}{\mathbb{E}_t\left[M_{t+1}V_{t+1}\right]}. \tag{A.1}$$

The numerator of equation (A.1) can be written as $K_{t+1}\partial V_{t+1}/\partial K_{t+1}$ due to the linear homogeneity of the firm value V_{t+1}. Similarly, the denominator becomes:

$$\mathbb{E}_t\left[M_{t+1}V_{t+1}\right] = \mathbb{E}_t\left[M_{t+1}K_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = K_{t+1}\phi'\left(\frac{I_t^*}{K_t}\right), \tag{A.2}$$

where the first-order condition of optimal investment is used in the last equality. The return on the stock can be written as:

$$\frac{V_{t+1}}{\mathbb{E}_t\left[M_{t+1}V_{t+1}\right]} = \frac{K_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}}{K_{t+1}\phi'\left(\frac{I_t^*}{K_t}\right)} = R_{I,t+1}, \tag{A.3}$$

confirming that the return on investment equals the stock return.

Appendix B. Decomposition of Marginal *q*

Given the total costs $\phi(I/K) = (I/K) + \gamma(I/K)^{\eta}$, equation (6) becomes:

$$1 + \gamma\eta\left(\frac{I_t}{K_t}\right)^{\eta-1} = \frac{\mathbb{E}_t\left[H(A_{t+1})\right]}{1+\mathrm{WACC}_t}. \tag{B.1}$$

Taking logarithms of the above equation leads to the following expression for the optimal investment:

$$\log\left(1 + \gamma\eta\left(\frac{I_t}{K_t}\right)^{\eta-1}\right) = \log\Big(\mathbb{E}_t\left[H(A_{t+1})\right]\Big) + \log\left(\frac{1}{1+\mathrm{WACC}_t}\right). \tag{B.2}$$

I log-linearize the left-hand side of equation (B.2), so that I can later relate the log of the investment-capital ratio to the components of the log of the marginal q. The left-hand side becomes:

$$
\begin{aligned}
\log\left(1+\gamma\eta\left(\frac{I_t}{K_t}\right)^{\eta-1}\right) &= \log\left(1+\gamma\eta e^{(\eta-1)\log\left(\frac{I_t}{K_t}\right)}\right) \\
&\approx 1+\gamma\eta e^{(\eta-1)\overline{\log\left(\frac{I}{K}\right)}} + \underbrace{\frac{\gamma\eta(\eta-1)e^{(\eta-1)\overline{\log\left(\frac{I}{K}\right)}}}{1+\gamma\eta e^{(\eta-1)\overline{\log\left(\frac{I}{K}\right)}}}}_{\equiv\kappa}\left(\log\left(\frac{I_t}{K_t}\right)-\overline{\log\left(\frac{I}{K}\right)}\right) \\
&= \underbrace{1+\gamma\eta e^{(\eta-1)\overline{\log\left(\frac{I}{K}\right)}} - \kappa\overline{\log\left(\frac{I}{K}\right)}}_{\equiv\xi} + \kappa\log\left(\frac{I_t}{K_t}\right),
\end{aligned}
\tag{B.3}
$$

where $\overline{\log(I/K)}$ is the unconditional mean of the log of the investment-capital ratio.

The first term on the right-hand side of equation (B.2) can be approximated through the Taylor series expansion around the unconditional mean of productivity $\overline{A}$. The first term then becomes:

$$
\begin{aligned}
\log\left(\mathbb{E}_t\left[H(A_{t+1})\right]\right) &\approx \log\left(\mathbb{E}_t\left[H(\overline{A})+\frac{\partial H}{\partial A}\left(A_{t+1}-\overline{A}\right)\right]\right) \\
&= \log\left(H(\overline{A})\mathbb{E}_t\left[1+\frac{1}{H(\overline{A})}\frac{\partial H}{\partial A}\left(A_{t+1}-\overline{A}\right)\right]\right) \\
&= \log\left(H(\overline{A})\right)+\log\left(1+\frac{1}{H(\overline{A})}\left(\mathbb{E}_t[A_{t+1}]-\overline{A}\right)\right),
\end{aligned}
\tag{B.4}
$$

where I use the fact that $\partial H/\partial A = 1$ as explained below. To determine the derivative $\partial H/\partial A$, I first apply the envelope theorem to the Bellman equation (3) and find $\partial V_t/\partial A_t = K_t$. Given that $V_t = H(A_t)K_t$, the derivative is:

$$
\frac{\partial H(A_t)}{\partial A_t} = \frac{1}{K_t}\frac{\partial V_t}{\partial A_t} = 1. \tag{B.5}
$$

To complete the decomposition in equation (B.2), I substitute equation (B.3) for the left-hand side and equation (B.4) for the productivity component. As a result:

$$
\log\left(\frac{I_t}{K_t}\right) \approx \underbrace{\frac{\log\left(H(\overline{A})\right)-\xi}{\kappa}}_{\equiv\nu} + \frac{1}{\kappa}\underbrace{\log\left(1+\frac{1}{H(\overline{A})}\left(\mathbb{E}_t(A_{t+1})-\overline{A}\right)\right)}_{\equiv L(A)_t} + \frac{1}{\kappa}\underbrace{\log\left(\frac{1}{1+\mathrm{WACC}_t}\right)}_{\equiv L(R)_t}. \tag{B.6}
$$

Appendix C. Estimating the State Prices

In the semiparametric approach to equation (9), the call option price is given by:

$$\text{Call}_{BSM}\left(S^F, X, \tau, R_{f,t}, \sigma(X/S^F, \tau)\right) \tag{C.1}$$

where S^F is the forward price of the stock, σ is the implied volatility, $\tau(= T - t)$ is the time to maturity, and Call_{BSM} is the Black-Scholes-Merton formula. The function of implied volatility is estimated for each month. To do so, I perform the kernel regression using option prices observed in that month as follows:

$$\widehat{\sigma}\left(X/S^F, \tau\right) = \frac{\sum_{i=1}^{n} k\left(\frac{X/S^F - X_i/S_i^F}{h_{X/S^F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right)\sigma_i}{\sum_{i=1}^{n} k\left(\frac{X/S^F - X_i/S_i^F}{h_{X/S^F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right)} \tag{C.2}$$

where i denotes each observation in the month, σ_i is the implied volatility of observation i, $k(z)$ is the Gaussian kernel function such that $k(z) = 1/\sqrt{2\pi}\exp\left(-z^2/2\right)$, and h_{X/S^F} and h_τ are bandwidth parameters. The bandwidth parameters are chosen to minimize the sum of squared errors of observations as suggested in Hardle (1994). Ait-Sahalia and Lo (2000) show that this semiparametric estimator captures the salient features in the option market, the volatility smile or smirk, which are likely to carry risk-relevant information. As a result, the expected return that will be recovered from the estimated state price is expected to reflect these option market features.

I next compute the state price as in equation (9) for each future stock price from the support $(S_1, S_2, \ldots, S_N)$. Specifically, the state price of S_j on date T when the current price is S_i is:

$$F_{i,j} = R_{f,t}\left.\frac{\partial^2 \text{Call}_{BSM}\left(S_i^F, X, \tau, R_{f,t}, \widehat{\sigma}\left(X/S_i^F, \tau\right)\right)}{\partial X^2}\right|_{X=S_j}\left(\frac{S_{j+1} - S_{j-1}}{2}\right). \tag{C.3}$$

Note that the increment of stock price, $(S_{j+1} - S_{j-1})/2$, is multiplied to obtain the state prices over the discrete states.

Appendix D. Connection between Dividend and Productivity Growth

Consider the all-equity-financed firm described in equation (3). According to the cash flow identity, the date-t dividend (cash flows to stock holders) is equal to date-t cash flow from assets, which is production output minus investment expenditure for the firm. The dividend growth is then:

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}K_{t+1} - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)K_{t+1}}{A_tK_t - \phi\left(\frac{I_t}{K_t}\right)K_t} = \frac{1 - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)/A_{t+1}}{1 - \phi\left(\frac{I_t}{K_t}\right)/A_t}\frac{K_{t+1}}{K_t}\frac{A_{t+1}}{A_t}. \tag{D.1}$$

Taking logarithms of both sides of the above equation, I express the log of dividend growth:

$$\underbrace{\log D_{t+1} - \log D_t}_{\equiv \Delta d_{t+1}} = \log\left(1 - \frac{\phi_{t+1}}{A_{t+1}}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + \log\left(1 - \delta + e^{\xi_t}\right) + \underbrace{\log A_{t+1} - \log A_t}_{\equiv a_{t+1}}, \tag{D.2}$$

where ϕ_t denotes $\phi\left(I_t/K_t\right)$, and ξ_t is the log of investment-capital ratio $\log(I_t/K_t)$. I next approximate dividend growth using Taylor expansion with respect to ξ_t. Hence:

$$\Delta d_{t+1} \approx \log\left(1 - \frac{\phi_{t+1}}{A_{t+1}}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + \underbrace{\log\left(1 - \delta + e^{\overline{\xi}}\right)}_{\equiv \kappa_1} + \underbrace{\frac{e^{\overline{\xi}}}{1 - \delta + e^{\overline{\xi}}}}_{\equiv \rho}(\xi_t - \overline{\xi}) + \Delta a_{t+1}. \tag{D.3}$$

where $\overline{\xi}$ is the unconditional mean of the investment-capital ratio, and κ_1 is log of the unconditional mean of the capital growth rate $1 - \delta + \overline{I/K}$.

Adding the dividend growth from $t+1$ to T leads to:

$$\sum_{s=t+1}^{T} \Delta d_s \approx \log\left(1 - \frac{\phi_T}{A_T}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + (T-t)\kappa_1 + \rho\sum_{s=t}^{T-1}(\xi_s - \overline{\xi}) + \sum_{s=t+1}^{T} \Delta a_{t+1}. \tag{D.4}$$

As a result, the long-run average of dividend growth can be written as:

$$\begin{aligned}
\lim_{T\to\infty}\frac{1}{T-t}\sum_{s=t+1}^{T}\Delta d_s \approx{}& \lim_{T\to\infty}\frac{1}{T-t}\log\left(1 - \frac{\phi_T}{A_T}\right) - \lim_{T\to\infty}\frac{1}{T-t}\log\left(1 - \frac{\phi_t}{A_t}\right) + \kappa_1 \\
&+ \lim_{T\to\infty}\frac{1}{T-t}\rho\sum_{s=t}^{T-1}(\xi_s - \overline{\xi}) + \lim_{T\to\infty}\frac{1}{T-t}\sum_{s=t+1}^{T}\Delta a_{t+1} \\
\approx{}& \kappa_1 + \lim_{T\to\infty}\frac{1}{T-t}\sum_{s=t+1}^{T}\Delta a_{t+1},
\end{aligned} \tag{D.5}$$

where I use the fact that $\lim_{T\to\infty}\sum_{s=t}^{T-1}\left(\xi_s - \overline{\xi}\right)$ is zero due to the definition of $\overline{\xi}$.

Appendix E. Data on Individual Equity Options

To minimize bias from possible illiquidity in options markets, I exclude price observations with the trading volume of five contracts or fewer. The option-based estimation requires a cross-section of call options with different strike prices, but in-the-money calls tend to be illiquid. Thus, for these calls, I instead use the price of the put option with the same strike price and maturity as described below. With the application of the above filters, the bid-ask spread is, on average, 7.3% of the mid-point price. For each of these filtered observations of individual equity options, which are mostly American, I compute the price of its European equivalent, following Carr and Wu (2009) and Martin and Wagner (2019). Specifically, I use the volatility surface reported by OptionMetrics and enter the reported volatility into the Black-Scholes-Merton formula to obtain the European option price. Next, for put options that are included as a substitute for illiquid calls, I apply the put-call parity to the European put price and obtain the price of the call with the same strike price and maturity. Lastly, to reliably estimate the expected return for each month, I require a firm to have at least 30 observations of option prices in a month.

Appendix F. Additional Regression Results

Table F.1: Alternative Estimation of Factor-Based Cost of Equity and Capital Investment

This table presents panel regressions of capital investment on the alternative estimates of the factor-based cost of equity. The dependent variable is $\text{INVEST}_{i,t}$, firm i's investment-capital ratio in quarter t. The regression specification is:

$$\text{INVEST}_{i,t} = \alpha_i + \beta \times \text{ER}_{i,t-1} + \gamma \times X_{i,t-1} + \epsilon_{i,T}$$

where $\text{ER}_{i,t-1}$ is the factor-based cost of equity, $\text{ER}^{\text{CAPM}}_{i,t-1}$ or $\text{ER}^{\text{FF}}_{i,t-1}$, and $X_{i,t-1}$ denotes the control variables. In determining the cost of equity, the factor risk premiums are measured in multiple ways: historical mean of the factor returns in the past six months, one year, three years, or five years, or the mean in the expanding window. Each stock's risk exposures are derived from the regression of the stock's daily returns in the past year. Combining the risk exposures with each of the risk premium estimates results in alternative estimates of the five-factor-based cost of equity: $\text{ER}^{\text{FF, 6-month roll}}_{i,t}$, $\text{ER}^{\text{FF, 1-year roll}}_{i,t}$, $\text{ER}^{\text{FF, 3-year roll}}_{i,t}$, $\text{ER}^{\text{FF, 5-year roll}}_{i,t}$, and $\text{ER}^{\text{FF, expand}}_{i,t}$. The CAPM-based cost of equity is measured similarly. The controls include Tobin's q ($\text{Q}_{i,t-1}$), the log of the book value of total assets ($\text{SIZE}_{i,t-1}$), the leverage ratio ($\text{LEV}_{i,t-1}$), the value-weighted yields on corporate bonds ($\text{YIELD}_{i,t-1}$), 10-year treasury constant maturity (r^f_{t-1}), and cash flow-to-asset ratio ($\text{CF}_{i,t-1}$). The standard errors are clustered by firm. The t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Dependent variable:	INVEST$_{i,t}$									
Specification:	1	2	3	4	5	6	7	8	9	10
$\text{ER}^{\text{CAPM, 6-month roll}}_{i,t-1}$	-0.000733									
	(-0.26)									
$\text{ER}^{\text{CAPM, 1-year roll}}_{i,t-1}$		0.00540								
		(1.11)								
$\text{ER}^{\text{CAPM, 3-year roll}}_{i,t-1}$			-0.0429***							
			(-3.42)							
$\text{ER}^{\text{CAPM, 5-year roll}}_{i,t-1}$				-0.0181						
				(-1.28)						
$\text{ER}^{\text{CAPM, expand}}_{i,t-1}$					0.145*					
					(1.91)					
$\text{ER}^{\text{FF, 6-month roll}}_{i,t-1}$						0.00142				
						(0.51)				
$\text{ER}^{\text{FF, 1-year roll}}_{i,t-1}$							0.00895*			
							(1.72)			
$\text{ER}^{\text{FF, 3-year roll}}_{i,t-1}$								-0.0210*		
								(-1.79)		
$\text{ER}^{\text{FF, 5-year roll}}_{i,t-1}$									-0.00645	
									(-0.47)	
$\text{ER}^{\text{FF, expand}}_{i,t-1}$										-0.0273
										(-1.24)
N	7,136	7,136	7,136	7,136	7,136	7,136	7,136	7,136	7,136	7,136
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
adj. R^2	0.550	0.550	0.552	0.550	0.551	0.550	0.551	0.551	0.550	0.551

Table F.2: The Expected Future Productivity from Forward Prices and Realized Productivity

This table presents panel regressions of realized productivity on the expected future productivity. The dependent variable is the log of realized productivity in quarter t, $a_{i,t+1}$. In specification 1, the explanatory variable is the log of productivity lagged by one quarter, $a_{i,t-1}$, and the expected growth in productivity $\mathbb{E}_{t-1}^{F}\left[\Delta a_{i,t}\right]$, which is derived from forward prices as in equation (19). The regression specification is:

$$a_{i,t} = \alpha_i + \beta_1 \times a_{i,t-1} + \beta_2 \times \mathbb{E}_{t-1}^{F}\left[\Delta a_{i,t}\right] + \epsilon_{i,t}.$$

In specification 2, the explanatory variable is the estimate of future productivity $\mathbb{E}_{t-1}^{XS}\left[\Delta a_{i,t}\right]$, which is derived from the cross-sectional earnings model. The standard errors are clustered by firm. The t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Dependent variable:	$a_{i,t}$	
Specification:	1	2
$a_{i,t-1}$	0.5642***	
	(21.02)	
$\mathbb{E}_{t-1}^{F}\left[\Delta a_{i,t}\right]$	0.0540***	
	(8.27)	
$\mathbb{E}_{t-1}^{XS}\left[a_{i,t}\right]$		0.9670***
		(13.80)
N	22,408	27,638
adj. R^2	0.900	0.804

References

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