Online Appendix for "Moment Risk Premia and Stock Return Predictability"

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In this online appendix, we discuss the sources of high moments of stock return under a jump-diffusion model with stochastic volatility and stochastic jump probability. We also include two robustness checks that are omitted in the main text for brevity.

I. Sources of Higher Moments—An Example

In this section, we derive the moments of returns under a general discrete jump-diffusion model with stochastic volatility and stochastic jump probability. Note that we focus on moments of log returns rather than simple returns. Denote the stock price by S_t and its logarithm by s_t , the log stock forward price follows the following process:

(1)
$$\Delta s_{t+1} \equiv s_{t+1} - s_t = \phi_t + \sqrt{V_t Z_{t+1}} + x_{t+1} J_{t+1}$$

where Z_{t+1} is a Gaussian shock and V_t is the variance of the Gaussian shock. J_t is a jump component that takes the value 1 with probability λ_t and 0 with probability $(1 - \lambda_t)$. x_t is the jump amplitude, which is i.i.d. and independent of other risk factors. We denote the unconditional n^{th} moment of x_t by M(x, n). Here, we assume $s_{t+1} - s_t$ to be a martingale for simplicity.¹

The log return from t to T, denoted by r(t,T), is defined as $r(t,T) = \sum_{t=1}^{T} \Delta s_u$.

 $^{{}^{1}\}Delta s_{t+1}$ represents daily log returns in the paper, whose expected value is usually very small. The martingale assumption should not have much impact on the conclusion.

The second moment of r(t, T) can be calculated as,

(2)
$$E_t[r(t,T)^2] = E_t\left[\sum_{t+1}^T (\Delta s_u)^2\right] + 2E_t\left[\sum_{j=t+1}^T \sum_{i=t+1}^T \Delta s_i \Delta s_j\right].$$

The second term on the RHS is zero due to the martingale assumption of Δs_t . Therefore, we can express the second moment as

(3)
$$E_t[r(t,T)^2] = \sum_{j=t+1}^T E_t[V_{j-1}] + M(x,2) \sum_{j=t+1}^T E_t[\lambda_{j-1}].$$

The third moment of r(t, T) can be expressed as,

(4)
$$E_t[r(t,T)^3] = \underbrace{E_t\left[\sum_{t+1}^T (\Delta s_u)^3\right]}_{\mathrm{I}} + 3\underbrace{E_t\left[\sum_{j=i+1}^T \sum_{i=t+1}^T \Delta s_i(\Delta s_j)^2\right]}_{\mathrm{II}}$$

For a small time interval (t, t + 1), the first term, I, is the third moment of instantaneous returns. It is only a function of the jump component:

(5)
$$\mathbf{I} = \sum_{t+1}^{T} E_t[(x_u J_u)^3] = M(x,3) \sum_{t+1}^{T} E_t[\lambda_{u-1}].$$

Note that this term equals to 0 if M(x,3) = 0. That is, when the jump size distribution is symmetric. The second term, II, is a covariance term between return variance and returns, i.e., a term related to the leverage effect. It can be written as

(6)
$$II = \sum_{j=i+1}^{T} \sum_{i=t+1}^{T} E_t[\Delta s_i \Delta s_j^2] = \sum_{j=i+1}^{T} \sum_{i=t+1}^{T} E_t[E_i[\Delta s_i \Delta s_j^2]] = \sum_{j=i+1}^{T} \sum_{i=t+1}^{T} E_t[\Delta s_i \operatorname{Var}_i(\Delta s_j)]$$
$$= \sum_{j=i+1}^{T} \sum_{i=t+1}^{T} \operatorname{Cov}_t(\Delta s_i, E_i[V_{j-1}]) + M(x, 2) \sum_{j=i+1}^{T} \sum_{i=t+1}^{T} \operatorname{Cov}_t(\Delta s_i, E_i[\lambda_{j-1}]).$$

The first term of Equation (6) is the covariance of returns and instantaneous return variance, and the second term is the covariance of returns and jump intensities. As long as common factors drive the comovement of return, stochastic volatility, and jump intensity, the two covariance terms are non zero. Taken together, we can express the third moment as,

(7)
$$E_t[r(t,T)^3] = M(x,3) \sum_{t+1}^T E_t[\lambda_{u-1}] + 3M(x,2) \sum_{j=i+1}^T \sum_{i=t+1}^T \operatorname{Cov}_t(\Delta s_i, E_i[\lambda_{j-1}])$$

$$SJ: \text{ jump terms}$$

+ 3
$$\sum_{j=i+1}^{T} \sum_{i=t+1}^{T} \operatorname{Cov}_t(\Delta s_i, E_i[V_{j-1}]).$$

SV: stochastic volatility term

The fourth moment of log return, r(t,T), is,

(8)
$$E_t[r(t,T)^4] = \underbrace{E_t\left[\left(\sum_{t+1}^T \Delta s_u\right)^4\right]}_{\mathrm{I}} + \underbrace{4E_t\left[\sum_{j=i+1}^T \sum_{i=t+1}^T \Delta s_i \Delta s_j^3\right]}_{\mathrm{II}} + \underbrace{6E_t\left[\sum_{u=j+1}^T \sum_{i=t+1}^T \sum_{j=i}^T (\Delta s_i \Delta s_j) \Delta s_u^2\right]}_{\mathrm{III}}.$$

Equation (8) says that the fourth moment of monthly returns has three sources: the fourth moment of instantaneous returns (I), covariance between returns and the third moment (II), and covariance between return auto-covariance and variance (III). Following similar derivation, one can show that both jumps and stochastic volatility terms contribute to the fourth moment of monthly returns, similar to the case of the third moment.

II. Additional Robustness Checks

The moment risk premia are defined as the differences between risk neutral moments and their physical counterparts. In the baseline regressions in the paper, we use the lagged realized moments as proxies for the physical moments in the next month. The advantage of this specification is that both the risk-neutral moments and the lagged realized moments are available ex ante without specifying any forecasting model. However, by using the lagged realized moments, we implicitly assume that the realized moments are random walks. In this section, we discuss two additional robustness checks, in which we use predicted realized moments and intraday moments to construct moment risk premia. The moment risk premia are then used to predict aggregate stock returns.

A. Predicted Realized Moments

In this section, we specify forecasting models to obtain estimates of $\mathbb{E}_t[\text{QRV}_{t+1}]$, $\mathbb{E}_t[\text{RV}_{t+1}]$, $\mathbb{E}_t[\text{RM3}_{t+1}]$, and $\mathbb{E}_t[\text{RM4}_{t+1}]$ using the information up to time t. We estimate these physical moments using lagged realized moments and risk-neutral moments. The former accounts for persistence in realized moments and the latter is a naturally forward-looking estimate of return moments.

We use a moving window of 100 months to get the one-step-ahead forecasts of QRV, RV, RM3, and RM4. We use a superscript f (which stands for "forecast") to denote quantities that are predicted. We first run the predictive regressions and estimate the coefficients. Then we calculate the predicted realized moments as,

$$QRV_t^{f} = \mathbb{E}_t[QRV_{t+1}] = \hat{\alpha}_{1,t} + \hat{\beta}_{1,t}QRV_t + \hat{\beta}_{2,t}VIX_t^2,$$
$$RV_t^{f} = \mathbb{E}_t[RV_{t+1}] = \hat{\alpha}_{2,t} + \hat{\beta}_{3,t}RV_t + \hat{\beta}_{4,t}IV_t.$$
$$RM3_t^{f} = \mathbb{E}_t[RM3_{t+1}] = \hat{\alpha}_{3,t} + \hat{\beta}_{5,t}RM3_t + \hat{\beta}_{6,t}IM3_t.$$
$$RM4_t^{f} = \mathbb{E}_t[RM4_{t+1}] = \hat{\alpha}_{4,t} + \hat{\beta}_{7,t}RM4_t + \hat{\beta}_{8,t}IM4_t,$$

where $\hat{\alpha}_{i,t}$ (i = 1, 2, 3, 4) and $\hat{\beta}_{j,t}$ (j = 1, ..., 8) are estimated regression coefficients using information up to time t. QVRP^f, PVRP^f, M3RP^f, and M4RP^f are constructed as the difference between the option implied moments and the one-step-ahead forecasts of their realized counterparts.

(9)
$$QVRP_t^f = VIX_t^2 - QRV_t^f, PVRP_t^f = IV_t - RV_t^f,$$
$$M3RP_t^f = IM3_t - RM3_t^f, M4RP_t^f = IM4_t - RM4_t^f.$$

Table A1 reports the predictive regression results on the moment risk premia constructed with predicted realized moments. The predictive power of the moment risk premia constructed with predicted moments is in general slightly weaker than the baseline results, consistent with the evidence in ?. We find that M3RP^f and M4RP^f significantly predict future market returns over 6- to 24-month horizons in univariate regressions with R^{2} 's ranging from 4% to 7%, while PVRP^f predicts market returns over 1- to 3-month horizons. Joint regressions of PVRP^f and higher moment risk premia outperform QVRP^f from 6- to 24-month horizons in terms of R^{2} 's.

[Insert Table A1 here.]

Table A2 reports the OOS R^2 's of moment risk premia constructed with predicted moments. At each time t, we use the historical observations of realized moments to predict the one-month ahead realized moments and calculate the moment risk premia as their differences with the risk-neutral moments. In a second step, we forecast returns over different horizons using these moment risk premia as predictors. We find that PVRP outperforms the prevailing historical mean over 1- and 3-month horizons, while higher moment risk premia generate outperformance over 6- to 12-month horizons. The OOS R^{2} 's are in general slightly weaker when using predicted realized moments than the baseline results in Panel A, consistent with the evidence in ?. A possible reason is that we have to first estimate realized moments for t + 1 and then forecast returns at t + h for every forecast at time t. This leads to errors-in-variable bias in the return forecasts.

[Insert Table A2 here.]

B. Intraday Realized Moments

In this section, we use 5-minute intraday returns to construct realized moments and test whether the moment risk premia constructed from high frequency data can produce similar prediction results, compared to those constructed from daily data in the baseline results. The intraday realized moments are calculated from the 5-min intraday returns. Moment risk premia based on intraday realized moments are calculated as the difference between risk-neutral moments and intraday realized moments, which are denoted with a superscript of "5-min".

The high-frequency second moment and high-frequency higher moments have different properties. Under reasonable assumptions, utilizing intraday return data provides a more consistent and efficient estimator for the return variance than using daily returns. But this is generally not the case for realized higher moments. As shown in ?, skewness estimates of long-horizon log returns can be very different from those of the high-frequency log returns due to the leverage effect. Note that we focus on the moments of log returns in this paper. For simple returns, skewness estimates of long-horizon returns shall be different from those of short-horizon returns due to compounding even in the absence of the leverage effect (see ?). In Section I of the online appendix, we derive the sources of higher moments of long-horizon log returns in an illustrative example.

To illustrate this point, we use 5-minute intraday returns to construct quasi realized variance (QRV^{5-min}), realized variance (RV^{5-min}), realized third (RM3^{5-min}) and fourth moments (RM4^{5-min}). The intraday S&P 500 price data is up to January 2017. Summary statistics are reported in Table A3. Consistent with ? and ?, we observe that the realized third and fourth moments have different means for daily and intraday data. Panel A of Table A3 shows that the mean of the intraday RM3^{5-min} is -1.2×10^{-3} , while the mean of the daily RM3 is -7.2×10^{-3} . The mean of the intraday RM4^{5-min} is 2.0×10^4 , while the mean of the daily RM4 is 4.4×10^3 . Panel B of Table A3 shows that the correlation between the daily and intraday second moment is as high as 0.96, while the correlation between the daily and intraday third moment is only -0.04. This evidence further confirms that the realized third moments constructed from daily and intraday data differ quite a lot.

[Insert Table A3 here.]

We report in-sample predictive regression results on the intraday moment risk premia in Table A4 and out-of-sample results in Table A5. Conclusions are qualitatively similar to the baseline regressions: variance-dominated risk premiums, such as $QVRP^{5-min}$ and $PVRP^{5-min}$, predict returns over short horizons, and higher moment risk premia perform better at longer horizons. We observe that $QVRP^{5-min}$ and $PVRP^{5-min}$ outperform their daily counterparts in univariate regressions with larger in-sample R^{2} 's and positive OOS R^{2} 's. This confirms the results in the literature that using intraday returns to estimate realized variance improves prediction performance of variance risk premia.

[Insert Table A4 here.]

[Insert Table A5 here.]

The predictive performance of higher moment risk premia is weaker at short horizons when using intraday data, compared with the baseline results. Medium-term-predictability of the higher moment risk premia is enhanced when constructed with intraday returns. From 9- to 24-month horizons, the higher moment risk premia constructed with intraday returns have more significant coefficients and give larger in-sample and OOS R^{2} 's both in the univariate as well as in the joint predictive regressions, compared with those in the baseline regressions.

Table A1: Market Return Predictive Regressions: Using Predicted Moments

The table reports estimated regression coefficients and R^2 of the predictability regressions for one to 24-month excess return of the S&P 500 index, using moment risk premia constructed with predicted moments. Heteroskedasticity- and autocorrelation-robust t-statistics are reported in the parenthesis. For each horizon, we report predictive regression results of the univariate regressions for QVRP^f, PVRP^f, M3RP^{$\perp f$}, M4RP^{$\perp f$}, bivariate regression for PVRP^f and M3RP^{$\perp f$} jointly, and bivariate regression for PVRP^f and M4RP^{$\perp f$} jointly (see Equation (9) for definitions of these predictors). Returns are observed monthly with the sample period ranging from January 1990 to July 2019.

			1-mo	onth		3-month						
$\rm QVRP^{f}$	0.23 (2.24)						$\begin{vmatrix} 0.70 \\ (4.34) \end{vmatrix}$					
$\mathrm{PVRP}^{\mathrm{f}}$	~ /	0.27 (2.36)			0.27 (3.54)	0.27 (3.70)		0.82 (4.58)			0.82 (5.27)	0.82 (5.21)
$\rm M3RP^{\perp f}$		(2.00)	-0.41 (-1.18)		-0.41 (-1.97)	(0.10)		(1.00)	-1.08 (-1.38)		(-2.87)	(0.21)
$\rm M4RP^{\perp f}$			(-1.10)	0.60 (1.21)	(-1.97)	0.60 (2.04)			(-1.58)	1.54 (1.55)	(-2.87)	1.54 (3.07)
Adj. R^2	1.80	2.37	0.83	1.00	3.21	3.38	6.24	7.68	2.35	(1.55) 2.60	10.07	(3.01) 10.32
	12-month						24-month					
$\rm QVRP^{f}$	0.50 (1.90)						0.18 (0.52)					
$\mathrm{PVRP}^{\mathrm{f}}$	~ /	0.69 (2.16)			0.69 (2.18)	0.69 (2.22)		$0.38 \\ (0.90)$			$\begin{array}{c} 0.39 \\ (0.92) \end{array}$	0.39 (0.94)
$\rm M3RP^{\perp f}$		(-)	-2.41 (-2.74)		-2.41 (-3.19)	()		()	-3.18 (-3.29)		-3.18 (-3.23)	()
$\rm M4RP^{\perp f}$			()	3.17 (2.74)	(0120)	3.17 (3.14)			(0.20)	4.15 (3.21)	(0.20)	4.15 (3.16)
Adj. R^2	1.19	2.20	5.82	5.37	8.06	7.61	-0.28	0.10	6.39	5.75	6.53	5.88
QVRP ^f	-0.04		12-m	onth			-0.24		24-m	onth		
-	(-0.11)						(-0.41)					
$\mathrm{PVRP}^{\mathrm{f}}$		$0.16 \\ (0.35)$			0.17 (0.31)	0.17 (0.32)		$0.10 \\ (0.13)$			0.12 (0.12)	0.12 (0.13)
$\rm M3RP^{\perp f}$. /	-3.56 (-3.17)		-3.56 (-3.09)	、 <i>/</i>		、 ,	-6.22 (-3.23)		-6.23 (-3.17)	、 ,
$\rm M4RP^{\perp f}$			(•)	4.52 (2.98)	(4.52 (2.90)			(7.64 (2.84)	()	7.65 (2.79)
Adj. R^2	-0.41	-0.35	5.71	4.84	5.39	4.52	-0.37	-0.42	6.98	5.53	6.59	5.13

Table A2: Out-of-sample R^2 of the Moment Risk Premia (Predicted Moments)

The table reports out-of-sample R^2 of the predictability regressions using moment risk premia constructed from predicted moments. For each horizon, we report OOS R^2 's for the univariate regressions for QVRP^f, PVRP^f, M3RP^{$\perp f$}, M4RP^{$\perp f$}, forecast combination for PVRP^f and M3RP^{$\perp f$}, and forecast combination for PVRP^f and M4RP^{$\perp f$} (see Equation (9) for definitions of these predictors). The sample period is from January 1990 to July 2019.

	$\rm QVRP^{f}$	PVRP ^f	$M3RP^{\perp f}$	$\rm M4RP^{\perp f}$	$\begin{array}{c} \rm PVRP^{f} \\ \& \ \rm M3RP^{\perp f} \end{array}$	$\begin{array}{c} \rm PVRP^{f} \\ \& \ \rm M4RP^{\perp f} \end{array}$
1-month	1.87	2.83	0.73	-1.41	2.31	1.37
3-month	-9.25	0.81	-4.74	-3.90	7.69	7.96
6-month	-14.71	-4.42	12.39	11.88	10.71	10.52
9-month	-8.35	-3.08	7.65	7.31	4.93	4.86
12-month	-4.58	-2.03	2.91	2.67	1.73	1.64
24-month	-10.61	-9.60	-8.35	-9.54	-6.58	-7.11

Table A3: Intraday vs. Daily Realized Moments

Panel A compares the summary statistics of daily and intraday realized moments: realized variance (RV), realized third moment (RM3), and realized fourth moment (RM4). Variables with superscript 5-min are intraday realized moments, calculated from 5-min returns of the S&P 500 index. Panel B reports the correlation matrix of the daily and intraday realized moments. All variables are annualized and denoted in percent. The sample period extends from January 1990 to January 2017.

	RV	RM3	RM4	$\mathrm{RV}^{5\text{-min}}$	$ m RM3^{5-min}$	$\mathrm{RM4^{5-min}}$
Mean	3.03	-7.22×10^{-3}	4.39×10^{-3}	2.39	-1.18×10^{-3}	2.17×10^{-4}
Std	5.25	9.55×10^{-2}	2.76×10^{-2}	4.32	1.22×10^{-2}	1.45×10^{-3}
Max	60.18	0.56	0.44	60.26	2.67×10^{-2}	2.35×10^{-2}
Min	0.19	-0.96	3.82×10^{-6}	0.22	-0.14	2.70×10^{-7}

RV	RM3	RM4
0.96	-0.38	0.71
-0.01	-0.04	0.18
0.91	-0.50	0.87
	0.96 -0.01	0.96 -0.38 -0.01 -0.04

Panel B: Correlations

Table A4: Market Return Predictive Regressions: Using Intraday Realized Moments

The table reports estimated regression coefficients and R^2 of the predictability regressions for one to 24-month excess return on the S&P 500 index, using moment risk premia whose realized moments are calculated using 5-minute intraday returns, denoted by a superscript 5-min. Heteroskedasticity- and autocorrelation-robust t-statistics are reported in the parenthesis. For each horizon, we report predictive regressions results of the univariate regressions on the quasi variance risk premium (QVRP^{5-min}), the pure variance risk premium (PVRP^{5-min}), the residual of the third moment risk premium after regressing on PVRP^{5-min} (M3RP^{\pm 5-min}), the residual of the fourth moment risk premium after regressing on PVRP^{5-min} (M4RP^{\pm 5-min}), bivariate regression on PVRP^{5-min} and M3RP^{\pm 5-min} jointly, and bivariate regression for PVRP^{5-min} and M4RP^{\pm 5-min} jointly. Returns are observed monthly with the sample period ranging from January 1990 to January 2017.

			1-m	onth					3-m	onth		
$\rm QVRP^{5-min}$	0.37 (4.05)						0.96 (7.29)					
$PVRP^{5-min}$	(1.00)	$0.39 \\ (3.78)$			$0.39 \\ (3.56)$	$0.39 \\ (3.46)$	(1.20)	1.02 (7.20)			1.02 (6.89)	1.02 (6.75)
$M3RP^{\perp 5-min}$		(0.10)	-0.14 (-0.57)		-0.14 (-1.23)	(0110)		(-0.31 (-0.43)		-0.31 (-1.14)	(0110)
$M4RP^{\perp 5-min}$			(0.01)	0.23 (0.69)	(1120)	0.23 (1.55)			(0.10)	0.42 (0.49)	()	0.43 (1.58)
Adj. R^2	3.99	4.32	-0.14	-0.05	4.20	4.29	8.92	9.56	-0.06	-0.02	9.54	9.58
	6-month						9-month					
$\rm QVRP^{5-min}$	1.23 (5.03)						$ 1.06 \\ (3.20)$					
$PVRP^{5-min}$	()	$1.39 \\ (5.37)$			$1.39 \\ (4.47)$	1.39 (4.28)		$1.26 \\ (3.66)$			1.26 (3.01)	1.26 (2.85)
$\mathrm{M3RP}^{\perp5\text{-min}}$		()	-1.51 (-1.73)		-1.52 (-3.34)	(-)		()	-2.18 (-2.73)		-2.18 (-3.44)	()
$\rm M4RP^{\perp 5\text{-}min}$			(1.10)	1.92 (1.77)	(0.01)	1.92 (3.82)			(2.10)	2.80 (2.84)	(0.11)	2.81 (3.99)
Adj. R^2	6.55	8.01	2.42	2.43	10.48	(0.02) 10.49	2.87	3.99	3.25	(2.04) 3.37	7.28	(0.55) 7.40
			12-m	nonth					24-m	nonth		
$\rm QVRP^{5-min}$	1.05 (2.72)						$ 1.24 \\ (1.74)$					
$PVRP^{5-min}$	()	1.28 (3.10)			1.28 (2.46)	1.28 (2.34)		1.56 (1.90)			1.57 (1.61)	1.57 (1.57)
$M3RP^{\perp 5-min}$		、	-2.54 (-2.99)		-2.55 (-3.18)	、)		~ /	-3.72 (-2.30)		-3.73 (-2.26)	
$\rm M4RP^{\perp 5\text{-}min}$			(3.27 (3.15)	()	3.28 (3.68)			()	4.75 (2.52)	()	4.78 (2.58)
Adj. R^2	1.88	2.79	3.09	3.21	5.93	6.04	0.75	1.32	2.29	2.33	3.64	3.70

Table A5: Out-of-sample \mathbb{R}^2 of the Intraday Moment Risk Premia

The table reports out-of-sample R^2 of the predictability regressions using moment risk premia constructed from intraday realized moments. For each horizon, we report the OOS R^2 's of the univariate regressions for the quasi variance risk premium (QVRP^{5-min}), the pure variance risk premium (PVRP^{5-min}), the residual of the third moment risk premium after regressing on PVRP^{5-min} (M3RP^{\perp 5-min}), the residual of the fourth moment risk premium after regressing on PVRP^{5-min} (M4RP^{\perp 5-min}), forecast combination for PVRP^{5-min} and M3RP^{\perp 5-min}, and forecast combination for PVRP^{5-min} and M4RP^{\perp 5-min}. The sample period is from January 1990 to January 2017.

	$QVRP^{5-min}$	PVRP ^{5-min}	$M3RP^{\perp 5-min}$	$M4RP^{\perp 5-min}$	$\mathrm{PVRP}^{5\text{-min}}$ & M3RP $^{\perp5\text{-min}}$	$\mathrm{PVRP}^{5\text{-min}}$ & M4RP $^{\perp 5\text{-min}}$
1-month	7.77	8.08	-5.66	-6.40	2.27	2.00
3-month	11.57	13.04	-31.21	-33.24	3.15	2.96
6-month	7.29	10.17	5.37	4.22	13.11	13.59
9-month	2.32	5.18	10.42	10.88	10.93	12.23
12-month	2.53	4.66	9.45	10.82	9.22	10.58
24-month	2.28	2.34	-0.44	2.44	1.43	2.73