Online Appendices for

Asset Variance Risk Premium and Capital Structure

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A. From Operating Income to Unlevered Value

Operating incomes (EBIT) from assets follow Geometric Brownian motion with stochastic variance under both measures:

$$(39)\qquad \mathbb{P}:\begin{cases}\frac{d\text{EBIT}}{\text{EBIT}}=(\mu-\delta)dt+\sqrt{V}dW_1^p\\ dV=\kappa(\theta-V)dt+\sigma\sqrt{V}dW_2^p\end{cases}\qquad \mathbb{Q}:\begin{cases}\frac{d\text{EBIT}}{\text{EBIT}}=(r-\delta)dt+\sqrt{V}dW_1\\ dV=\lambda(\theta^*-V)dt+\sigma\sqrt{V}dW_2\end{cases}$$

The EBIT and payout rate are strictly positive to assure unlevered asset value is always positive, where I borrow the assumption from Goldstein et al. (2001). The value of the unlevered assets, ν, that generate the income is the present value of all the future cash flows from operating income. Since $E^{\mathbb{Q}}(\text{EBIT}_s|\text{EBIT}_t)=\text{EBIT}_t e^{(r-\delta)\times(s-t)}$, the value is :

$$(40)\qquad \nu_t=E^{\mathbb{Q}}\Big(\int_t^{\infty}(1-\tau)e^{-r(s-t)}\text{EBIT}_s ds\Big)=\frac{(1-\text{TAX})}{\delta}\text{EBIT}_t$$

By applying Ito's lemma to Equation 40, the unlevered-asset return process also follows Geometric Brownian with stochastic variance under both measures as in Equation 1.

B. Economic Assumptions

In this economy, assets are traded and their premium can be determined. But, variance is not traded and in order to determine the instant variance premium, Heston (1993) assumes that the instant premium is proportional to variance itself following Cox et al. (1985).

Therefore, the stochastic discount factor, SDF, and the premiums of the returns follow:

$$\frac{d\text{SDF}}{\text{SDF}} = -rdt - \frac{\mu-r}{\sqrt{V}}dW_1^p - \frac{(\lambda-\kappa)\sqrt{V}}{\sigma}dW_2^p, \quad d\langle W_1^p, W_2^p\rangle = 0,$$
$$(41)\qquad \begin{cases} E^{\mathbb{P}}[\frac{d\nu}{\nu}] - E^{\mathbb{Q}}[\frac{d\nu}{\nu}] = -E^{\mathbb{P}}[\frac{d\text{SDF}}{\text{SDF}}\frac{d\nu}{\nu}] = (\mu-r)dt = \text{AP}.dt \\ E^{\mathbb{P}}[\frac{dV}{V}] - E^{\mathbb{Q}}[\frac{dV}{V}] = -E^{\mathbb{P}}[\frac{d\text{SDF}}{\text{SDF}}\frac{dV}{V}] = (\lambda-\kappa)dt = \text{VRP}.dt \end{cases}$$

where AP is the asset premium. When variance, V, is constant, the model collapses into classical Black-Scholes economy with classical stochastic discount factor:

$$(42)\qquad dW_1^p = \frac{\mu-r}{\sqrt{V}}dt - dW_1, \quad \text{AP} = \frac{\mu-r}{\sqrt{V}} \times \sqrt{V} = \mu - r, \quad \frac{d\text{SDF}}{\text{SDF}} = -rdt - \frac{\text{AP}}{\sqrt{V}}dW_1^p$$

In another representation, Barras and Malkhozov (2016) define the following as the premiums where the expected values are based on Equation 1:

$$E^{\mathbb{P}}[\frac{d\nu}{\nu}] - E^{\mathbb{Q}}[\frac{d\nu}{\nu}] = (\mu-r)dt = \text{AP}.dt,$$
$$(43)\qquad \begin{cases} E^{\mathbb{P}}[dV] - E^{\mathbb{Q}}[dV] = (\lambda-\kappa).Vdt = \text{VRP}.Vdt, \\ \frac{E^{\mathbb{P}}[dV]-E^{\mathbb{Q}}[dV]}{V} = \text{VRP}dt = (\lambda-\kappa)dt \end{cases}$$

In this setup which is used in options literature, $\lambda - \kappa$ is the relative difference between RN and historical instant variances. The results are similar to the assumptions in Heston (1993).

C. Asymmetric Equity Variance Effect

Even without negative correlation between asset returns and asset variance (variance asymmetry), this simplifying assumption does not reduce the power of the model in qualitatively replicating asymmetric variance observed at the equity level. It is a stylized fact that equity returns and return variance have negative correlation.[19]

In Figure B.1, model-implied equity volatility is asymmetric in this paper and has neg-

[19]See, e.g., Ait-Sahalia, Fan, and Li (2013), Bekaert and Wu (2000), Figlewski and Wang (2001), Wu (2001).

ative correlation with equity returns. Volatility has one-on-one relation with variance by square-root transformation. Equations 35 and 36 in Appendix E show the equity process which creates Figure B.1. Without debt, the firm is all equity and there is no correlation between the returns and volatility by the model assumption. As the fraction of debt in the capital structure increases, a negative shock to equity return raises equity volatility due to financial distress costs. Hence, there exists a negative correlation between equity return and its volatility. The higher is the leverage ratio of the firm, the higher is the asymmetric volatility.

Insert Figure B.1 about here.

D. Example of Different Exposures to Market Shocks and Market Variance Risk

Let's consider two firms: one firm has high exposure to market variance, α, with low beta, $\sqrt{b}$, and idiosyncratic variance, V_i. But, the other has low exposure to market variance and high beta and idiosyncratic variance. The first is relatively more exposed to variance shocks and the latter is relatively more exposed to return shocks. Low-VRP firm has high asset beta and total variance, but it has low VRP because it has relatively small proportion of systematic variance compared to the high-VRP firm. High-VRP firm, on the other hand, has low asset beta and total variance, but its asset VRP is high because of small idiosyncratic variance and relatively large systematic variance. While it has no effect on asset beta, idiosyncratic variance reduces the impact of market variance shocks and premium to be transferred to the firm.

Table B.1 shows the numerical example. Total volatility and asset VRP for high-VRP firm are within estimated ranges for IG firms and total volatility and asset VRP for low-VRP firm are within the range for B firms in Table 9. Market variance risk exposure for high-VRP firm is within the range for IG firms and the exposure for low-VRP firm is within the range for B firm in Table 7.

Place Table B.1 about here

E. Proof for Proposition1

All state variables are Markov. An optimal default boundary, L^*, is chosen based on Markov state variables, which also makes L^* a Markov variable.

Assumption 1. *Equity value at each point of time is monotonic and increasing in the firm's value, given all other parameters being constant ($\nu_1 < \nu_2 \rightarrow EQ(\nu_1) \leq EQ(\nu_2)$)*

The assumption is plausible because if $\mathrm{EQ}(\nu_1) > \mathrm{EQ}(\nu_2)$, then shareholders will destroy part of the firm's value to move it to ν_1. Therefore, the equity value at ν_2 cannot be smaller than ν_1, and the equity value at ν_2 is at least as large as the equity value at ν_1.

Assumption 2. *The equity function is a continuous function of the unlevered firm value for all parameters and other state variables.*

PROPOSITION 1. *The optimal default triggering boundary L^* is independent of the firm's current value, if the firm's value is above the boundary.*

The proposition is analogical to the optimal exercise policy of an American put option; the optimal exercise boundary of the put is independent from the underlying asset's value as long as the put is not exercised.

Proof: The rationale of the proof is about showing that the optimal boundary is the same for two different firm values. For the firm's value below the default boundary, the firm defaults, equity is valued at zero, and debt holders are in control of the firm. Hence, I only consider the values above the boundary. Based on the rationality of investors, equity value is positive for any asset value above the boundary, L^*. Shareholders maximize equity value, $\mathrm{EQ}(\{\nu, \Sigma\}; \Theta)$, where Θ={all model parameters} and {ν,Σ}={all state variables including current unlevered firm-value}. Based on the smooth pasting condition, the optimal control variable must satisfy $\partial \text{EQ}(\Theta; \{\nu, \Sigma\})/\partial \nu|_{(\nu = L^*)} = 0$. From Assumption 1 and Assumption 2, the equity function is monotonic and continuous. Therefore, the solution to the smooth pasting condition is unique. Ceteris paribus, this result implies that the optimal default policy is the same for two completely similar firms with only different unlevered assets' values ($L^*[\Theta; \{\nu_1, \Sigma\}] = L^*[\Theta; \{\nu_2, \Sigma\}]$).

F. Approximation Errors

The approximation errors are small and closed-form debt value with Taylor expansion to the second degree is close to the value from the simulations. The closed-form formula slightly overestimates the value of debt and leverage compared to the simulation. Hence, using the closed-form is more parsimonious because debt value is even slightly lower with VRP based on simulations, which implies stronger negative effect of VRP on leverage.

For the simulations, I draw 100,000 paths of both variance and unlevered asset value with weekly steps, $\Delta t = 1/50$, under RN measure:

$$\begin{aligned} \nu_t &= \nu_{t-\Delta t}\,\text{EXP}\Big((r - \delta - \tfrac{V_{t-\Delta t}}{2})\Delta t + \sqrt{\Delta t}\sqrt{V_{t-\Delta t}}z_1\Big) \\ V_t &= V_{t-\Delta t}\,\text{EXP}\Big(\tfrac{[\lambda(\frac{\kappa\theta}{\lambda} - V_{t-\Delta t}) - \frac{\sigma^2}{2}]\Delta t + \sqrt{\Delta t}\sigma\sqrt{V_{t-\Delta t}}z_2}{V_{t-\Delta t}}\Big) \end{aligned} \tag{44}$$

where z_1 and z_2 are respectively standard normal shocks to the asset return and variance under RN measure. Random numbers are all antithetic to increase convergence in the results. Although the path is discrete, the process is time continuous. At the end of each year, $T = 1$, I calculate and update the default boundary, L, as in Equation 12. For each path, if asset value hits the boundary, firm defaults and creditors control the firm. To value their debt, I discount the firm's value less the default costs plus all the coupons up to the default. Otherwise, I discount debt's market value, considering the bond sold after 50 years for the constant-variance price, plus all the coupons during the 50 years. This method allows me to calculate the debt value under the perpetual process. Although this violates the model assumption about stochastic volatility, it has a minor effect on the results because the discounting time is after 50 years, $e^{-50r} \approx 0.08$. If the path does not hit default, I calculate the market value of debt using Equation 21 on year 50 and discount it to time 0.

Simulating perpetual stochastic variance processes has a main obstacle, i.e. having an infinite time dimension. It is not yet possible to simulate this process with an infinite horizon. Therefore, I assume the variance to stay constant after 50 years at the mean level under the RN measure. At the end of 50 years with this assumption, there exists a closed-form formula and it is the closest value to estimate the terminal debt value.

Figures B.2 and B.3 show the results. Average and median of the debt value from all the paths provides the simulated value for the perpetual debt. VRP is 2 within the range of empirically estimated values. Instant and long-run variances are equal to 0.04, 0.2 squared. The rest of the parameters are also similar to calibrations. The results for the tax benefits and bankruptcy costs are similar because they follow a similar derivation.

Insert Figure B.2 about here.

Insert Figure B.3 about here.

G. Sample Statistics for Data between 2002 and 2015

Place Table B.2 about here

H. Robust Estimation with Bias Correction and Control for Small Sample Properties

The inferences are robust when I control for small sample properties with the dynamic-panel regressions and bootstrapped errors. Table B.3 shows the results where VRP is more significant for IG firms than SG firms. The estimated coefficients and the inferences are also close to the results in Table 11 in magnitude. Estimations are done using the method described in Bruno (2005), which uses bias-corrected least-squares dummy variable (LSDV) estimators (it is embedded in Stata's LSDVC). The procedure automatically includes lagged variable while corrects for biases. There are 200 iterations using estimates from the method recommended by Arellano and Bond (1991) as the initial points. Therefore, the dynamic regression does not require control for the industry effect and drops the control variables. This method not only resolves the issue with small sample properties, but also addresses possible concerns raised by the classical Nickell (1981)'s critique.

Place Table B.3 about here

I. 3-by-3 Calibration Statistics for Data (2002-15)

The results are robust to dropping leverage in the calibrations. In the model with VRP, dropping the leverage from the calibration reduces the equations into 3-by-3 match where I drop instant variance and assume it is equal to the long-run mean. This is a more restrictive assumption because it assumes that the variance term structure is flat and the asset VRP is more important. The 2-by-2 calibration for the model without VRP also misses the leverage and the equity value replaces leverage in the calibration. Table B.4 shows the results. The comparison between optimal leverage from each model shows that the model with VRP implies lower leverage closer to the observed leverage, especially for IG firm-years.

Place Table B.4 about here

J. Calibration to Median Values

The results remain robust when I calibrate the models to the median of data for each rating and make sure the results are robust to potential convexity effects.[20]

The median for yield spread is the time-series median while the rest are panel medians. The middle columns in Table B.5 show larger underleverage when median leverage represents top ratings' leverage. Median leverage for the other ratings remains close the mean. There

[20] The convexity is addressed here because yield is for an average bond maturity in the representative-firm calibrations, unlike Huang and Huang (2012) and Chen, Collin-Dufresne, and Goldstein (2008) who control for bond maturity.

is also one-on-one relation between the credit-spread and underleverage puzzles in the model without VRP. Table B.6 reports the calibrations with VRP. IG firms have the largest VRP and underleverage improves the most for them when the model accounts for asset VRP.

Place Table B.5 about here

Place Table B.6 about here

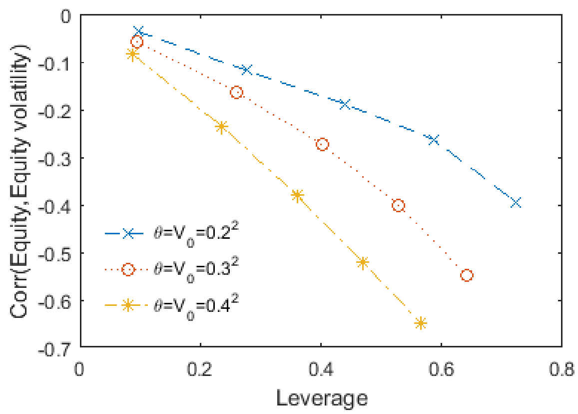
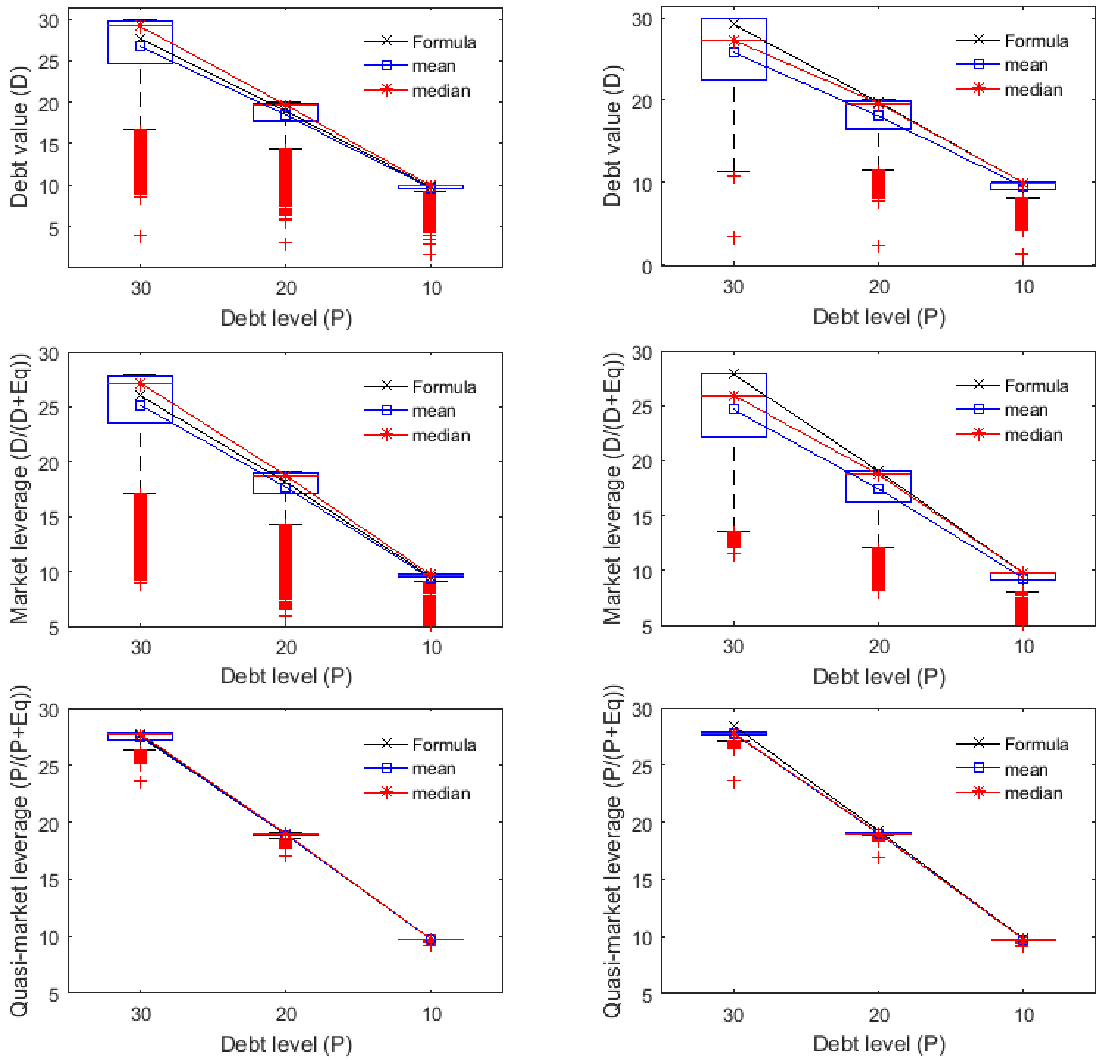
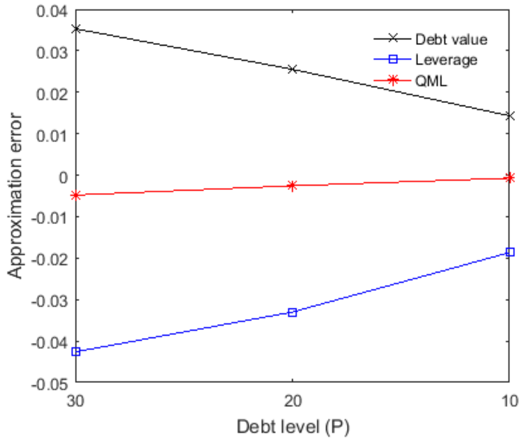


Figure B.1: Replicated volatility asymmetry at the equity level implied by the model: The x-axis shows the market leverage ratio of the firm. The y-axis shows negative correlation between equity returns and equity volatility, known as stock volatility asymmetry. Volatility is the standard deviation of equity returns. Coupon for outstanding debt is the risk-free rate times the face-value of debt, $C = rP$. Initial and mean variances are the same, $\theta = V_0$. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, VRP, $|\lambda - \kappa|$, is 2, risk-free rate, r, is 5%, asset payout rate, δ, is 3%, PBC rate, ρ, is 45%, tax rate is 25%, and debt rollover rate, m, is 10%.

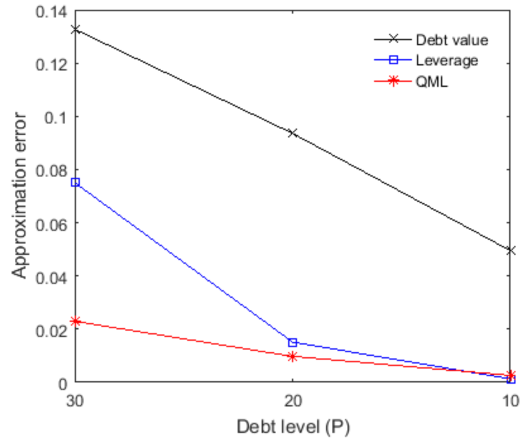


(a) Without debt roll over   (b) With debt roll over

Figure B.2: Comparing the approximate closed-form values with the simulations: The x-axis shows face value of debt, P. The y-axis shows the value of the variables of interest: the first row is for debt value, the second row is the market leverage ratio, $D/(D+\mathbf{EQ})$, and the third row is the quasi-market leverage (QML), $P/(P+\mathbf{EQ})$. Coupon for outstanding debt is the risk-free rate times the face-value of debt, $C = rP$. Initial and mean variances are the same, $\theta = V_0$, and set to 0.04, 0.2 squared. Initial asset value is \$100 and it is scalable. Historical variance mean-reversion speed, κ, is 4, VRP, $|\lambda - \kappa|$, is 2, risk-free rate, r, is 5%, asset payout rate, δ, is 3%, PBC rate, ρ, is 45% and tax rate is 25%. Debt rollover rate, m, is 10%. There are 100,000 simulation paths. For each path, I discount the value of debt to time zero, either from default or after 50 years at estimated perpetual value, and also measure leverage and QML. The box is between 25 and 75 percentiles of the simulated values.



(a) **Without debt roll over**



(b) **With debt roll over**

Figure B.3: Median approximation errors: The x-axis shows face value of debt P. The y-axis shows the approximation error for each value. All the parameters are the same as Figure B.2.

Table B.1: -Examples of two firms with different exposure to market risk and market variance risk: This table shows that a firm with low asset beta can have high asset VRP and vice versa. Asset beta measures the exposure of the assets to market return premium and return shocks. Asset VRP measures the exposure of the assets to market VRP and variance shocks. Proportional variance from market exposure is also correlation of total asset variance with market variance. Market variance is set to 4% and market VRP is set to 4 reported by Ait-Sahalia and Kimmel (2007). The correlations are in the range of the empirical values in Table 7. Total volatility and asset VRP are within ranges in Table 9. All the numbers are calculated based on Equation 3.

	High-VRP firm		Low-VRP firm
Positive constant, b_1	1	b_2	1.2
Idiosyncratic variance, V_{i1}	$(24\%)^2$	V_{i2}	$(45\%)^2$
Proportional variance from market exposure, α_1	50%	α_2	20%
Total volatility, $\sqrt{V_1}$	32%	$\sqrt{V_2}$	50%
Asset beta, β_1	1.00	β_2	1.10
Total asset variance, V_1	10%	V_2	25%
VRP, $\lvert\lambda_1 - \kappa_1\rvert$	2.00	$\lvert\lambda_2 - \kappa_2\rvert$	0.8

Table B.2: - More descriptive statistics for the sample of the rated firm-years used in the regressions (2002-15): The sample is the same as in Table 9 from merged Compustat and Optionmetrics. Table A.2 in Appendix A has the details of the variable calculations. Tobin's Q is market to book value of assets. Profitability is the operating income ratio. Cash is the ratio of the cash holdings. Tangibility is the ratio of the tangible assets and the proxy for bankruptcy costs. Log(Sales) is the natural log of the revenues. Asset payout is the total payout to debt and equity holders relative to size. Leverage is the total liabilities to the market cap of equity plus the liabilities. Unlevered volatility is the historical volatility of stock returns unlevered with the leverage to proxy for the assets' historical volatility. VRP proxy represents the asset VRP and is the ratio of equity option-implied to historical volatilities.

Rating	statistic	Tobin's Q	Profitability	Cash	Tangibility	Log(sales)	Unlevered volatility	VRP proxy	Asset payout
AAA	Mean	2.34	18.9%	23.0%	12.9%	11.17	15.5%	1.12	4.7%
32	Std	0.80	7.2%	16.2%	4.2%	0.44	8.2%	0.23	1.6%
AA	Mean	2.23	16.8%	10.1%	28.5%	10.85	14.3%	1.14	4.0%
83	Std	0.84	5.7%	6.4%	15.9%	1.21	6.2%	0.18	1.8%
A	Mean	2.11	17.4%	8.5%	32.2%	9.75	17.1%	1.07	4.0%
365	Std	0.87	6.4%	7.9%	20.8%	0.96	7.2%	0.17	2.0%
BBB	Mean	1.48	13.2%	8.8%	33.3%	9.35	16.7%	1.03	4.3%
331	Std	0.50	4.9%	7.8%	24.6%	0.94	7.1%	0.16	4.8%
IG firms	Mean	1.87	15.7%	9.3%	31.5%	9.75	16.6%	1.06	4.1%
811	Std	0.80	6.2%	8.7%	22.0%	1.10	7.1%	0.17	3.4%
BB	Mean	1.27	10.5%	10.6%	28.7%	8.63	17.7%	1.03	4.0%
90	Std	0.35	5.4%	8.6%	15.8%	1.13	8.9%	0.18	3.9%
B	Mean	1.35	8.1%	11.9%	31.9%	9.06	16.2%	1.02	3.5%
51	Std	0.35	7.0%	8.6%	16.4%	1.25	9.0%	0.22	1.8%
CCC	Mean	1.24	4.0%	13.2%	57.8%	8.95	13.7%	1.00	4.3%
8	Std	0.25	4.8%	3.5%	16.1%	1.31	6.3%	0.22	0.6%
All	Mean	1.78	14.7%	9.6%	31.5%	9.61	16.6%	1.06	4.1%
960	Std	0.78	6.6%	8.6%	21.3%	1.16	7.4%	0.18	3.4%

Table B.3: - Regression results with small-sample controls: the results are similar to Table 11. Stata's LSDVC procedure estimates bias-corrected least-squares dummy variable (LSDV) model. The procedure automatically includes lagged variable and corrects its inclusion bias. The model is $\mathbf{LEV}_{i,t} = (1-\Psi)\mathbf{LEV}_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$ where X has standardized independent variables and control dummies. Table A.2 in Appendix A has the details of the variable calculations. Independent variables are described Independent-variable statistics are described in Online Appendix G's Table B.2. IG-firms sample only has all the firm-quarters rated as investment grade by S&P. SG-firms sample only has all the firm-quarters rated as none investment-grade or speculative grade by S&P. Dummies for years and firms control for time and firm fixed effects. Standard errors are corrected for small sample properties with bootstrapping 200 iterations and they are reported below the estimates. The p-values test the null hypothesis that the coefficient is zero:$***p < 0.01, **p < 0.05, *p < 0.1$. Coefficients for the standardized variables show the relative importance of each variable in determining the target leverage.

Parameter	Estimated Coefficients for each statistical regression		
Model	(1)	(2)	(3)
Sample	All	IG firms	SG firms
Dependent variable	Leverage	Leverage	Leverage
Lag Leverage	0.415***	0.317***	0.504***
	(0.026)	(0.033)	(0.092)
Asset volatility	-0.0617***	-0.0504***	-0.0608***
	(0.004)	(0.004)	(0.011)
VRP proxy	-0.0316***	-0.0304***	-0.0162
	(0.003)	(0.003)	(0.010)
Tobin Q	-0.0420***	-0.0544***	-0.0605
	(0.005)	(0.005)	(0.040)
Profitability	-0.0269***	-0.0186***	-0.0259
	(0.004)	(0.005)	(0.017)
Cash	-0.0029	-0.00364	-0.0193
	(0.004)	(0.004)	(0.019)
Tangiblity	0.0659***	0.0383***	0.085
	(0.011)	(0.013)	(0.068)
Log sales	0.0328***	0.00124	0.0134
	(0.010)	(0.012)	(0.046)
Time and firm fixed effect	yes	yes	yes
Obs	814	683	131

Table B.4: -Results for calibrations without leverage on the rated firm-years (2002-15): This table is comparable to Table 9. Table A.2 in Appendix A has the details of the variable calculations. The last four columns report the statistics for the parameters in a 3-by-3 calibration with VRP to the firm-years as in Equation 37 and the error without having leverage. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. MAPE is the mean average percentage error between model-implied and observed values in the calibration. The first two columns report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. These columns are comparable with actual leverage. The optimal leverage for the model without VRP is implied by the parameters in a 2-by-2 calibration as in Equation 37 where equity value replaces leverage.

		Optimal leverage		Calibrated values in the model with VRP			
Rating	statistic	With VRP	Without VRP	Asset VRP	Size	Mean volatility	MAPE
AAA	Mean	44.2%	54.5%	3.45	340,754	17.2%	20%
32	Std	9.9%	11.2%	0.92	230,505	8.2%	15%
AA	Mean	42.9%	54.9%	3.66	188,772	16.3%	22%
83	Std	7.0%	9.1%	0.87	172,226	6.4%	16%
A	Mean	40.1%	50.5%	3.47	49,312	19.3%	16%
365	Std	5.2%	7.8%	1.04	44,886	7.4%	15%
BBB	Mean	40.6%	51.0%	3.48	29,539	19.6%	13%
331	Std	6.9%	9.1%	0.99	42,168	7.9%	13%
IG firms	Mean	40.7%	51.3%	3.49	67,014	19.0%	16%
811	Std	6.5%	8.8%	1.00	108,907	7.6%	15%
BB	Mean	39.7%	51.1%	3.56	14,649	21.6%	13%
90	Std	7.4%	11.2%	0.85	32,627	10.5%	13%
B	Mean	41.3%	54.0%	3.09	23,087	22.3%	15%
51	Std	9.2%	14.6%	1.36	58,314	10.9%	23%
CCC	Mean	40.2%	53.4%	3.65	34,160	21.4%	21%
8	Std	10.2%	7.8%	0.64	68,392	9.2%	28%
All	Mean	40.7%	51.4%	3.48	59,283	19.5%	15%
960	Std	6.8%	9.4%	1.01	102,979	8.2%	15%

Table B.5: -Sample statistics and calibration results for representative firms across the ratings in the model without VRP (1997-2015): This table is comparable to Table 2 where the sample means are replaced with medians for the calibrations. Table A.2 in Appendix A has the details of the variable calculations. The first 2 columns on the left report the medians used as target values in calibration for the representative firm in each rating. Volatility is square-root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. The last 2 columns on right report the outcomes of 2-by-2 calibration in the model without VRP to the medians in the first 2 columns (details are in Appendix E). Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of mean asset variance, $\sqrt{\theta}$. The two columns in the middle report yield spread and optimal leverage implied by the calibrated parameters in the last 2 columns. These middle columns are comparable with empirical leverage and yield spread in 2 columns on their left. Empirical yield spreads are the Bank-of-America Merrill-Lynch (BOA-ML) US Corporate Option-Adjusted spreads for an average rated firm. Empirical leverage is book liabilities divided by book liabilities plus equity market cap.

			1997-2015 Data statistics				Model-implied		Calibrated values in model without VRP	
Rating	Obs	statistic	Equity value	Equity volatility	Yield spread	Leverage	Optimal leverage	Yield spread	Size	Mean volatility
AAA	113	Median	165,680	25.3%	0.7%	19.9%	44.8%	0.1%	194,582	21.5%
AA	369	Median	54,269	27.7%	0.9%	21.9%	43.5%	0.1%	65,173	22.9%
A	1,975	Median	12,780	28.6%	1.2%	28.7%	44.5%	0.2%	16,552	21.8%
BBB	3,681	Median	4,637	31.7%	2.0%	38.5%	45.2%	0.4%	6,834	21.1%
IG firms	6,138	Median	22,464	33.1%	1.5%	35.3%	43.3%	0.4%	31,725	23.1%
BB	3,741	Median	1,647	40.4%	3.5%	45.2%	42.7%	0.9%	2,734	23.8%
B	2,321	Median	743	54.0%	5.3%	56.4%	41.1%	2.0%	1,551	25.8%
CCC or below	142	Median	305	77.5%	10.3%	79.2%	49.9%	3.3%	1,344	17.3%

Table B.6: -Sample statistics and calibration results for representative firms across the ratings in the model with VRP (1997-2015): This table is comparable to Table 4 where the sample means are replaced with medians for the calibrations. Table A.2 in Appendix A has the details of the variable calculations. The first 3 columns on the left report the sample medians used as target values in calibration for the representative firm in each rating. Volatility is square-root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. Empirical yield spreads are the Bank-of-America Merrill-Lynch (BOA-ML) US Corporate Option-Adjusted spreads for an average rated firm. The last 3 columns on right report the outcomes of 3-by-3 calibration in the model with VRP to the medians in the first 3 columns. Instant volatility is set equal to mean variance (details are in Appendix E). Similar to Table 4, yield spread in the model with VRP matches its empirical value as part of the calibration process. Size, ν_0, is the unlevered value of the firm's assets. Mean volatility is the square-root of the mean asset variance, $\sqrt{\theta}$. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. The two columns in the middle report the statistics for optimal leverage with and without VRP as implied by the calibrated parameters. Optimal leverage without VRP is borrowed from Table B.5. These columns are comparable with actual leverage on their left. Empirical leverage is book liabilities divided by book liabilities plus equity market cap.

			1997-2015 Data statistics				Optimal leverage		Calibrated values in the model with VRP		
Rating	Obs	statistic	Equity value	Equity volatility	Yield spread	Leverage	With VRP	Without VRP	Size	Mean volatility	Asset VRP
AAA	113	Median	165,680	25.3%	0.69%	19.9%	36.0%	44.8%	195,418	21.5%	3.08
AA	369	Median	54,269	27.7%	0.87%	21.9%	35.7%	43.5%	65,444	22.9%	2.91
A	1975	Median	12,780	28.6%	1.18%	28.7%	36.0%	44.5%	16,641	21.9%	2.97
BBB	3681	Median	4,637	31.7%	1.95%	38.5%	35.9%	45.2%	6,849	21.7%	3.08
IG firms	6138	Median	22,464	33.1%	1.48%	35.3%	36.5%	43.3%	31,806	23.5%	2.53
BB	3741	Median	1,647	40.4%	3.51%	45.2%	34.5%	42.7%	2,690	25.5%	2.98
B	2321	Median	743	54.0%	5.25%	56.4%	34.4%	41.1%	1,491	28.8%	2.52
CCC or below	142	Median	305	77.5%	10.26%	79.2%	35.3%	49.9%	1,208	24.4%	2.87