Optimal portfolios under time-varying investment opportunities, parameter uncertainty and ambiguity aversion

Online Appendix

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Online Appendix

D. VAR Estimation

The first four moments of our five assets and two predictor variables are indicated in Table D.1.

Table D.1: Moments of tradable assets and predictor variables (based on monthly returns)

	bonds	stocks	cash	real estate	gold	spr	d-p
mean	0.0027	0.0057	0.0008	0.0045	0.0008	0.0181	-3.5834
standard deviation	0.0295	0.0431	0.0029	0.0057	0.0478	0.0147	0.3961
skewness	0.2735	-0.4059	0.2627	-0.4078	0.8323	-0.3132	-0.2645
kurtosis	5.6420	4.6219	5.9190	4.6647	10.1398	2.7883	2.3199

Table D.2 shows OLS parameter estimates of the VAR process. Apart from analyzing a different time interval and including the additional assets real estate and gold, our estimates are well in line with those of Campbell, Chan, and Viceira (2003).

The most noteworthy observation is the very high and significant persistence of the log dividend-price ratio and the term spread, and the relatively high persistence of returns of cash and of real estate, with the consequence that R^2 of the regression of these time series is high. Bond, stock and gold returns are harder to predict. For bonds the returns of stocks and cash as well as the term spread have significant predictive power. In line with previous literature, stock returns have the lowest R^2. As expected, real estate returns are negatively related to the short interest rate (return of cash). Table D.3 describes the correlation structure of the innovations in the VAR system, with annual standard deviations on the main diagonal. Consistent with previous results of Campbell et al. (2003), unexpected stock returns are highly negatively correlated with shocks to the

Table D.2: OLS based VAR estimation parameters (standard errors in parenthesis)

	Dependent variable:						
	bonds(t)	stocks(t)	cash(t)	real estate(t)	gold(t)	spr(t)	d(t)-p(t)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
bonds(t-1)	0.036	0.147^{**}	0.004	−0.001	0.052	0.043^{***}	-0.132^{**}
	(0.039)	(0.058)	(0.003)	(0.005)	(0.064)	(0.006)	(0.059)
stocks(t-1)	-0.105^{***}	0.028	-0.005^{**}	−0.001	−0.058	0.001	−0.042
	(0.027)	(0.039)	(0.002)	(0.004)	(0.043)	(0.004)	(0.040)
cash(t-1)	1.331^{***}	0.588	0.541^{***}	-0.255^{***}	1.219^{*}	−0.033	−0.504
	(0.426)	(0.630)	(0.035)	(0.058)	(0.698)	(0.061)	(0.636)
real estate(t-1)	0.179	0.131	-0.035^{**}	0.743^{***}	0.487	-0.061^{**}	−0.004
	(0.205)	(0.303)	(0.017)	(0.028)	(0.336)	(0.029)	(0.306)
gold(t-1)	−0.015	-0.059^{*}	−0.001	−0.003	0.095^{**}	−0.0004	0.064^{*}
	(0.024)	(0.035)	(0.002)	(0.003)	(0.039)	(0.003)	(0.035)
spr(t-1)	0.306^{***}	0.240^{**}	−0.001	0.015	0.164	0.954^{***}	-0.238^{**}
	(0.079)	(0.116)	(0.007)	(0.011)	(0.129)	(0.011)	(0.117)
d(t-1)-p(t-1)	−0.0001	0.008^{*}	0.0004	−0.00000	0.006	−0.00001	0.992^{***}
	(0.003)	(0.004)	(0.0002)	(0.0004)	(0.005)	(0.0004)	(0.004)
const	−0.005	0.027^{*}	0.002^{**}	0.001	0.018	0.001	−0.024
	(0.010)	(0.015)	(0.001)	(0.001)	(0.017)	(0.001)	(0.015)
Observations	671	671	671	671	671	671	671
R^2	0.056	0.027	0.301	0.533	0.029	0.922	0.988
Adjusted R^2	0.046	0.017	0.293	0.528	0.019	0.922	0.988
Residual Std. Error (df = 663)	0.029	0.043	0.002	0.004	0.047	0.004	0.043
F Statistic (df = 7; 663)	5.659^{***}	2.664^{**}	40.705^{***}	108.172^{***}	2.871^{***}	$1,125.882^{***}$	$8,015.214^{***}$

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

log dividend-price ratio. Unexpected bond returns are negatively correlated with shocks to the term spread.

Table D.3: Cross-correlation of residuals. Standard deviation (p.a.) on the main diagonal.

	bonds	stocks	cash	real estate	gold	spr	d-p
bonds	0.100	0.137	0.233	0.052	0.048	-0.365	-0.119
stocks	–	0.148	0.110	0.111	0.104	0.001	-0.989
cash	–	–	0.008	0.470	0.091	-0.147	-0.065
real estate	–	–	–	0.014	0.022	-0.023	-0.087
gold	–	–	–	–	0.164	-0.032	-0.091
spr	–	–	–	–	–	0.014	-0.011
d-p	–	–	–	–	–	–	0.149

E. Pope Correction

We apply the Pope (1990) correction, to adjust for the small-sample bias of OLS estimates of the persistency of autoregressive processes (e.g., the OLS parameter for term spread of 0.9544 is corrected to 0.9618, that of the dividend-price ratio of 0.9919 is corrected to 0.9921). That this seemingly "minor" correction is relevant also in terms of risk is shown by Table E.1, in which we compare elements on the main diagonal of the decomposed covariance matrices from the OLS estimated VAR model with those of the Pope-corrected VAR model, see Equation (9) of the main text. It can be seen that the Pope correction considers the higher estimation risk imposed by very persistent processes and that the difference to OLS increases with the investment horizon.

Table E.2 presents the ratio of the overall asset-return variance (i.e. $\bar{\Sigma} + \bar{\Omega} + \Lambda$) for Pope-corrected estimates and OLS estimates. It shows that bonds are affected most by

Table E.1: Elements on the main diagonal of OLS versus Pope estimated variance.

	OLS			Pope		
	$\bar{\Sigma}_{120}$	$\bar{\Omega}_{120}$	Λ_{120}	$\bar{\Sigma}_{120}$	$\bar{\Omega}_{120}$	Λ_{120}
bonds	0.213	0.038	0.0002	0.255	0.046	0.0002
stocks	0.161	0.028	0.0001	0.178	0.031	0.0002
cash	0.005	0.001	0.00000	0.005	0.001	0.00000
real estate	0.031	0.006	0.00003	0.036	0.007	0.00003
gold	0.445	0.082	0.0004	0.472	0.092	0.0005
	$\bar{\Sigma}_{600}$	$\bar{\Omega}_{600}$	Λ_{600}	$\bar{\Sigma}_{600}$	$\bar{\Omega}_{600}$	Λ_{600}
bonds	1.809	1.616	0.008	2.490	2.345	0.012
stocks	0.332	0.248	0.001	0.407	0.304	0.002
cash	0.051	0.044	0.0002	0.065	0.057	0.0003
real estate	0.264	0.245	0.001	0.355	0.319	0.002
gold	3.939	3.551	0.019	4.547	4.255	0.023

the small-sample adjustment. However, the overall variance of all assets significantly increases when addressing the small-sample bias.

F. Term Structure of Correlations

The corresponding term structure of correlations for the different asset pairs is illustrated in Figure F.1. In general, the 95% confidence intervals of the correlation between all asset classes indicated with dashed-dotted lines are wide, with values between -0.8 and $+0.8$. For the pairwise correlation between bond returns, stock returns and real estate returns we observe a small positive short-term and a significantly higher long-term average dependence. Furthermore, the long-term average pairwise correlation between each of these assets and cash is negative, i.e., bonds, stocks and real estate benefit in the long

Table E.2: Ratio of the overall variance between Pope corrected and OLS variance.

	bonds	stocks	cash	real estate	gold
T=120	1.199	1.104	1.131	1.153	1.069
T=600	1.412	1.225	1.282	1.324	1.175

run from a decline in the short-term interest rate, and vice versa.[1] While the long-term average correlation between gold and stocks/real estate is negative, the pairwise correlation between gold and bonds is positive. Our tentative explanation is that during times of financial turmoil stock and real estate prices decline, while safe-haven instruments as governmental bonds and gold are in high demand. All confidence bounds are wide, so ignoring estimation errors and giving asset-allocation advice based on the expected term structure of return correlations is not recommended.

[Figure F.1 about here]

G. Alternatives to Determine Expected Returns

Let us mention two alternative approaches to specifying expected returns: First, expected returns can be determined endogenously in a general equilibrium setup. If assets are in limited supply and different types of investors form their optimized portfolios, aggregate demand drives prices and consequently expected returns. We have worked out the optimization analysis in general equilibrium for two types of investors, an investor that shows only risk aversion but no ambiguity aversion and an investor with both risk and ambiguity aversion. Such a model leads to interesting cross-dependencies since changes in

[1]The cash returns are given by the monthly T-bill rate.

the parametrization of one investor type influence optimal portfolios of the other through the demand channel. While these effects are undoubtedly very interesting, we think that the estimation and the comprehensive interpretation of such a model is beyond the scope of this paper. Thus, we will discuss optimal asset allocation and the treatment of ambiguity aversion in a partial equilibrium with market-implied expected returns as described above.

Second, implicit return expectations are given directly by the VAR model. In Section VI of the main text we discuss portfolio properties when applying the VAR estimate of risk premia for a variety of different investment horizons and different levels of ambiguity aversion. It can be seen that under reasonable levels of ambiguity aversion optimized long-term portfolios based on expected premia directly from the VAR estimates are very similar to optimal portfolios based on premia implied by SCF data.

References

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Figure F.1: Term structure of return correlations of pairs of asset classes based on the full predictive covariance $\bar{\Sigma}_T + \bar{\Omega}_T + \Lambda_T$ (95% confidence bounds as dashed-dotted lines).