Internet Appendix to The Dividend Term Structure

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Abstract

In this Internet Appendix we discuss results for three other markets EuroStoxx 50: the Nikkei 225, the S&P 500 index, and the FTSE 100 index. We then compare the results for the S&P 500 index to implications of various asset pricing models. We also present several robustness checks.

IA.1 Other markets

IA.1.1 Data

We use data for exchange-traded dividend futures for the Nikkei 225 index. These data start in 2010. Maturities extend out to ten years with annual intervals. In Table 1 in the main text we show statistics on this market. For maturities up to two years notionals of over half a billion ($-equivalent) are observed and tens to hundreds of millions for longer maturities (Mixon and Onur, 2017). All maturities normally trade on a daily basis and in the same way as in the Eurostoxx 50 analysis we apply the estimation procedure to daily prices.

We obtain dividend swap price data from several investment banks for the S&P 500 index and FTSE 100 index.[1] Before 2008, dividend derivatives existed as dividend swaps traded over-the-counter (OTC) only. They date back to 2002, well before the onset of listed futures.

[1] Deutsche Bank, Goldman Sachs and Credit Suisse.

Mixon and Onur (2017) provide insight into the OTC market for dividend swaps. They investigated data from a Swap Data Repository to which participants in swap markets must report at transaction-level. It is shown that OTC swaps trade infrequently; even for the S&P 500, which is the largest OTC dividend market, they trade less than daily between dealers and only once every few weeks between a dealer and a non-dealer end-user.

Investment banks update their pricing sheets on a daily basis, but often prices remain stale and extended periods go by without a single trade taking place. The data set of OTC prices for dividend swaps, therefore, is impacted by the model investment banks use for pricing them. We find price differences for same maturity transactions among the pricing sheets of investment banks of on average 3% with a standard deviation of 3%.

Since the OTC market does not trade regularly, it seems likely that fitting the state space model to its price data is akin to mimicking the pricing models used by the investment banks. We nonetheless perform the same set of estimations and reconciliations as done for the exchange-trade futures, using the OTC price data of dividend swaps referring to the S&P 500 and the FTSE 100 indices. The results shown below are restricted to monthly frequencies, as the daily data are stale.

IA.1.2 Results

We first discuss the results for the Nikkei 225 index, for which we have exchange-traded contracts and daily prices. Table IA.1 presents the estimation results. The mean reversion of the first state variable is a bit slower than for the Eurostoxx 50 ($\varphi = 0.74$ for Nikkei 225), but still rather fast with a half life of about 1 year. Similar to the Eurostoxx 50, the second state variable reverts at business-cycle frequency, given the estimate for $\psi = \;\, 0.18$ which implies a half-life of almost 4 years. The minor differences between the Eurostoxx 50 and Nikkei 225 results are mostly driven by the difference in sample period. The global credit crisis in 2008/09 is included in the Eurostoxx 50 data period but not in the Nikkei 225 data period. Estimating the model for the Eurostoxx 50 data over a partial data period that coincides with the Nikkei 225 data period yields mean reversion parameters that are closer to those found for the Nikkei 225: $\varphi = 0.88$ and $\psi = \ 0.09$.

This two-factor model fits the Nikkei dividend futures data very well, even better than for the Eurostoxx 50 market. The shortest-maturity future has a mean absolute pricing error of 1.2% (versus 1.5% for the Eurostoxx 50), while all other futures have mean absolute pricing errors below 0.4% (versus about 0.5% for the Eurostoxx 50). A one-factor model has considerably larger pricing errors, for most futures more than twice as large.

Next we perform the same reconciliation regressions as for the Eurostoxx 50 (equation (27)), regressing observed stock index returns on the return of the first dividend future, the return of the model-implied price-dividend ratio, and the change in the risk-free rate.

Table IA.2 shows that this regression model explains 54% of the daily stock return variation, similar to the Eurostoxx 50 market. We also see that most of this explanatory power comes from the model-implied price-dividend ratio, and that the slopes on the first dividend future and the price-dividend ratio are not far below the expected values of 1. Only the effect of the risk-free rate is negligible. These results confirm our conclusion that a two-factor model, estimated on dividend futures data, can explain a substantial part of the actual stock market returns.

We then turn to the results for the two OTC markets: S&P 500 and FTSE 100 dividend swaps, where we use a monthly frequency given the stale daily prices. The results are shown in Table IA.3. The two-state model produces a high estimate for the long run growth constant $\overline{p}^*$ of S&P 500 dividends. Indeed, at – 1.3 percent for this constant, the S&P 500 present value as estimated by the model overestimates its observed values by a factor of more than 2. Both mean reversion parameters attain reasonable levels, but they attract fairly large standard errors. In the case of the FTSE 100, the two-state model estimate for long run growth equals – 5.3 percent, with a standard error even exceeding that level in absolute terms. At the same time, the second mean reversion parameter comes out low at 0.04, which translates into a half-value time running into decades. At such slow moving mean reversion, the role of the long run constant is essentially taken over by the medium-term factor. The single-state estimate for long run growth is more reasonable at − 3.3 percent.

We then turn to the reconciliation regressions (equation (27)). The variation in the modeled price-dividend ratio produced by the estimates does not depend on the long run constant and the dynamic reconciliation to the stock indices demonstrates that it has meaningful explanatory power. Table IA.4 shows that the model produces a coefficient of 0.1 to 0.2 for the price-dividend ratio, with reasonable significance for both the S&P 500 and the FTSE 100. Overall explanatory power is high for the S&P 500 with the adjusted R^2 reaching 0.58. However, most of it stems from the observed first dividend price F_t rather than the modeled price-dividend ratio.[2] The same applies to the FTSE 100, albeit with the adjusted R^2 at a lower level of explanatory power. In both markets variation in the price-dividend ratio accounts for 8%, against 28% and 43% for the Eurostoxx 50 and Nikkei 225.

IA.1.3 Comparison to structural Macro models

Our model is not a structural model that separates expected dividends from risk premiums. But although it does not provide means of dividend growth or expected dividend strip returns, we can still calculate the model-implied volatility of returns of dividends across maturities. In a similar way as Binsbergen and Koijen (2017), we compare the volatility term structure of dividend returns for the S&P 500 to those implied by theoretical asset pricing models. Binsbergen et al. (2012) report in their Figure 5 the volatility term structures for three important models: the habit formation model of Campbell and Cochrane (1999), the Bansal-Yaron (2004) long-run risk model, and the rare disasters model of Gabaix (2009). Figure IA.1 graphs the annualized volatility of log dividend price changes implied by our model estimates, for maturities of one to ten years[3]. The figure also plots the volatility curve for the habit-formation model of Campbell and Cochrane (1999) and long-run risk

[2] Similar regressions based on daily estimates and stock index data produce R^2 of less than 5 percent.

[3] We do not estimate the volatility of current dividend changes. We set this value equal to 11.2% in Figure 10, which is the value used by Campbell and Cochrane (1999).

model of Bansal and Yaron (2004), as reported in Figure 5 of Binsbergen et al. (2012).[4] It shows that the volatility curve calibrated using our model is increasing and concave, similar to the results for the Eurostoxx and Nikkei data. In sharp contrast, the volatility curve of the habit formation model is almost flat in this range and out of line with the data. The long run risk curve approximates the calibrated volatility curve better than the habit formation model, but is somewhat less steep than our calibrated volatility curve. The volatility of the long maturity futures implied by the long run risk model (17 to 18%) is also lower than the volatility fitted by our model, and lower than typical values for the stock market volatility. Notice that Bansal and Yaron (2004) also report that their long-run risk model produces a volatility of the price-dividend ratio that is lower than the value found in the data.[5]

IA.2 Robustness checks

IA.2.1 Serial correlation in residuals

The estimated growth rates fit the data well, but the residuals exhibit some serial correlation in the growth rate from the first to the second constant maturity futures and swaps. This estimated growth rate sometimes varies from the data by several percentage points, although mostly during periods of stronger than average negative growth. Growth rates of longer horizons do not share this pattern. In order to check for the importance of this phenomenon on the estimated parameters, we allow for serial correlation in the measurement equation of the first growth rate. Followingle Eraker (2004), we assume a first order autoregressive structure for the first measurement equation ($n = 2$):

$$\eta_{t+1,2} = \xi \eta_{t,2} + u_{t+1} \tag{1}$$

[4] We thank Ralph Koijen and Jules van Binsbergen for providing us with these data.

[5] Bansal and Yaron (2004, Table IV) report a price-dividend ratio volatility of 29% in the data and 18% in their benchmark model.

The fit as measured by the log likelihood contribution improves. Since the short term growth rate captures most of the short term mean reversion, there should be some effect from an extra degree of freedom on the speed of mean reversion of the factors. However, the mean reversion parameters do not change drastically. In the case of Eurostoxx 50 dividend futures, φ decreases from 1.51 to 1.43, and ψ decreases from 0.24 to 0.22. So, we conclude that our estimates are robust and the two factor model seems to be correctly specified.

IA.2.2 An alternative model

Our modeling approach focuses directly on discounted risk-adjusted dividend growth $\pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}$. We thus incorporate discounting at the risk-free rate when valuing future dividends. An obvious alternative to this approach would be to model $z_{t+1} = g_{t+1} - \theta_{t+1}$ using a term structure model to value dividend derivatives, and subsequently discount it at observed interest rates to calculate present values. This latter step requires the assumption that interest rates and $g_{t+1} - \theta_{t+1}$ are independent. In addition, we assume the expectations hypothesis holds for bonds, so that bond risk premiums equal zero and long-term interest rates equal expected future short rates. Given these assumptions we rewrite equation (3) in the main text as:

$$\begin{aligned} P_{t,n} &= D_t E_t \left[\exp\left(\sum_{i=1}^{n} g_{t+i} - y_t - \theta_{t+i} \right) \right] \\ &= D_t E_t \left[\exp\left(\sum_{i=1}^{n} -y_{t+i} \right) \right] E_t \left[\exp\left(\sum_{i=1}^{n} z_{t+i} \right) \right] \\ &= D_t \exp\left(-n y_{t,n}\right) E_t \left[\exp\left(\sum_{i=1}^{n} z_{t+i} \right) \right]. \end{aligned} \tag{2}$$

and the derivatives price equals:

$$F_{t,n} = D_t E_t \left[\exp\left(\sum_{i=1}^{n} z_{t+i} \right) \right]. \tag{3}$$

This shows that, to fit the futures price data, only a model for z_{t+1} is needed. To reconcile this model with the stock index level, the independence assumption and expectations hypothesis for bonds are necessary and equation (25) in the main text can be used to calculate present values of dividends and the stock index value. Using this pricing equation, one can again specify a two-state model, in this case for one period growth z_{t+1}, and estimate it using the Kalman filter in the same way as described for the base model.

As mentioned, this model assumes independence of interest rates and risk-adjusted growth rates. In the real world, however, correlation between the risk-free rate, dividend growth and the dividend risk premium is expected since often the same drivers apply: economic growth, the investment cycle, slack in the labor market and other economic variables will affect all of them. For estimating the term structure model, such correlation is not a problem if z_{t+1} is the subject of state space estimation instead of π_{t+1}, but it will cause misestimation of the implied stock market levels. It is easy to show that this separation of the two correlated variables would produce overestimation of the stock index in equation (25) if the actual correlation is positive.

Turning to the results for the Eurostoxx 50 index, the long-term estimate for risk-adjusted growth $\overline{z}$ is estimated rather high, at 0.3 percent, which translates to 1.9 percent once $\overline{z}$ is corrected for the convexity term (Table IA.5). Standard errors are larger than in the case of the base model. The mean reversion parameters obtained remain reasonable and significant. However, reconciling the dividend market to the stock market based on these estimates overstates the stock market by a large margin and reduces the fit of the dynamic return reconciliation (Table IA.6). The coefficient of the estimated price-dividend ratio is almost negligibly small for the Eurostoxx 50 market. In Europe there was a steep drop in interest rates in the period of 2008 to 2015, and likely both the independence assumption and the expectation hypothesis are not appropriate for this sample period. Therefore, these results demonstrate the correlation among the three elements of π_{t+1}, which confirms the advantage of estimating risk-adjusted dividend growth *after* discounting at the risk-free rate.

IA.2.3 Illiquidity

IA.2.3.1 Model estimates excluding days with little trading

Eurostoxx 50 dividend futures have maturities of 1 to 10 years. Not all of them trade on all trading days in the data set. In order to establish whether non-trading materially influences the results of the model estimation, we compare model runs with and without such trading days excluded. Specifically, when 2 or 3 (out of the 10 available) do not show volume on a given trading day, that trading day is excluded from the data set and the model is estimated without it. The full data set consists of 1,679 consecutive trading days. Excluding trading days during which at least 3 futures show no volume reduces the number of trading days to 1,234. Excluding trading days when 2 futures do not trade reduces it to 821 days. If only a single zero-volume future is allowed the data set would be reduced to only 447 trading days, which is too little to consider.

With these less liquid data removed from the data set we re-estimate the model. The parameter estimates are very similar to the full data set (Table IA.7).

Absolute pricing errors for the full sample and these two reduced samples are graphed in Figure IA.2. The errors are somewhat higher for the reduced samples. This increase could indicate that on days when there is little trading, the quotes put in the market are 'model-based' and hence smoother and easier to fit compared to days with substantial trading. However, even for the reduced samples, the economic size of the absolute pricing errors is quite small, and below 1% of the futures price for all futures (except the shortest-maturity future).

IA.2.3.2 The cause of small price changes of the shortest dated future

Eurostoxx 50 dividend future change less often in price in their last year of expiry. Prices of these close-to-expiry futures are stale about every other trading day, whereas longer-dated futures see daily price changes about 90% of the time. On such stale days, these futures nevertheless nearly always trade in significant volume. The cause of this phenomenon is that the reference value of dividend futures develops during the year of expiry. European companies pay dividends concentrated in a few months in spring (see Figure 7 in the main text). By June over 80% of annual dividends, to which Eurostoxx 50 dividend futures refer, has already been paid out. Only over the remaining portion some uncertainty remains as a result of which the future may change in price. Figure IA.3 shows that the average absolute daily price change falls as time to expiry reduces to zero.

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Table IA.1: **Base model of Discounted Risk-adjusted Dividend Growth**: $\pi_t = g_{t+1} - y_t - \theta_{t+1}$

Estimates using **listed Dividend Futures of the Nikkei 225 Index**.
Sample period: 17 June 2010 – 16 February 2015.

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. σ_η measures the standard deviations of the second until the eighth measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section 3. Standard errors in parentheses.

	Two state		Single state	
	$dp_t = \varphi(\tilde{p}_t - p_t)\,dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)\,dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi(\overline{p} - p_t)\,dt + \sigma_p dW_p$	
$\overline{p}$	-0.0320	-0.0371	-0.0719	-0.0487
	(1.3833)	(0.0264)	(3.8264)	(0.0304)
φ	0.7381	0.7381	0.2837	0.2837
	(0.2360)	(0.2345)	(0.0306)	(0.0306)
ψ	0.1784	0.1784		
	(0.0539)	(0.0537)		
β_p	-1.5513	*Set to 0*	-7.6306	*Set to 0*
	(92.822)		(211.137)	
$\beta_{\tilde{p}}$	-3.4234	-3.4229		
	(16.191)	(16.1862)		
σ_p	0.1531	0.1531	0.0630	0.0630
	(0.2193)	(0.2193)	(0.1189)	(0.1186)
$\sigma_{\tilde{p}}$	0.0251	0.0251		
	(0.0731)	(0.0730)		
σ_ε	0.0147	0.0147	0.0137	0.0137
	(0.0197)	(0.0197)	(0.0041)	(0.0041)
σ_η	0.0040	0.0040	0.0170	0.0170
	(0.0015)	(0.0015)	(0.0285)	(0.0285)
Log Likelihood per contribution	29.37	29.36	22.04	22.04

Table IA.2: Reconciliation of the Base Present Value Model (two state) constituent returns to stock market returns: **listed Dividend Futures.**

The modeled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (26) in the main text $\Delta s_t = \alpha + \beta_f \Delta f_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{pd}} \Delta \widehat{pd}_t + \varepsilon_t$, in which Δs_t is stock index log returns, Δf_t is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero-coupon swap rate and $\Delta \widehat{pd}_t$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two state space model. β is fixed at zero. Daily data for periods as in Tables 1 and 2. Standard errors in parentheses.

	Eurostoxx 50				**Nikkei 225**			
Constant	0.0005	0.0002	0.0006	-0.0001	-0.0002	0.0002	0.0005	0.0002
	(0.0003)	(0.0003)	(0.0004)	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0003)
f_t	0.8978	1.0009			0.8488	0.6582		
	(0.0337)	(0.0426)			(0.0508)	(0.0719)		
Δy_t	0.1446		0.2022		0.0751		-0.0081	
	(0.0127)		(0.0178)		(0.0619)		(0.0912)	
$\widehat{pd}_t$	0.6587			0.6893	0.8619			0.8156
	(0.0216)			(0.027)	(0.0251)			(0.0278)
Adj. R^2	0.540	0.248	0.071	0.280	0.540	0.068	0.000	0.429

Table IA.3: **Base model** of Discounted Risk-Adjusted Dividend Growth: $\pi_t = g_{t+1} - y_t - \theta_{t+1}$

Estimates using **OTC Dividend Swaps** (monthly).

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years. σ_η measures the standard deviations of the second until the eighth measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section 3. Standard errors in parentheses.

	Two state		Single state	
	$dp_t = \varphi\left(\tilde{p}_t - p_t\right)dt + \sigma_p dW_p$ $d\tilde{p}_t = \psi\left(\overline{p} - \tilde{p}_t\right)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}$		$dp_t = \varphi\left(\overline{p} - p_t\right)dt + \sigma_p dW_p$	
	S&P 500	**FTSE 100**	**S&P 500**	**FTSE 100**
Sample period	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 – June 2014	Dec 2005 June 2014
$\overline{p}$	-0.0188	-0.0841	-0.0186	-0.0430
	(0.0231)	(0.1513)	(0.0108)	(0.0093)
φ	1.0651	1.6347	0.3537	1.7702
	(0.7296)	(0.5865)	(0.0583)	(0.5828)
ψ	0.1809	0.0371		
	(0.1431)	(0.1422)		
β_p	*Set to 0*	*Set to 0*	*Set to 0*	*Set to 0*
$\beta_{\tilde{p}}$	-2.5935	-2.1624		
	(10.4524)	(8.5798)		
σ_p	0.1756	0.5865	0.0584	0.6770
	(0.2975)	(0.9707)	(0.0674)	(1.1518)
$\sigma_{\tilde{p}}$	0.0293			
	(0.0642)			
σ_ε	0.0167	0.0199	0.0138	0.0141
	(0.0072)	(0.0326)	(0.0044)	(0.0056)
σ_η	0.0078	0.0054	0.0261	0.0298
	(0.0021)	(0.0026)	(0.0167)	(0.0687)
Log Likelihood per contribution	22.62	23.79	19.79	19.29

Table IA.4: Reconciliation of the Base Present Value Model (two state) constituent returns to stock market returns: **OTC Dividend Swaps** (monthly data).

The modeled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (26) in the main text $\Delta s_t = \alpha + \beta_f \Delta f_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{pd}} \Delta \widehat{pd}_t + \varepsilon_t$, in which Δs_t is stock index log returns, Δf_t is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero swap rate and $\Delta \widehat{pd}_t$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two state space model. β is fixed at zero. Monthly data for periods as in Table IA.6. Standard errors in parentheses.

	S&P 500				**FTSE 100**			
Constant	-0.0019	-0.0016	0.0065	0.0038	0.0005	0.0005	0.0032	0.0016
	(0.0029)	(0.0028)	(0.0042)	(0.0040)	(0.0034)	(0.0035)	(0.0040)	(0.0039)
f_t	1.1677	1.1901			0.5812	0.6273		
	(0.1118)	(0.1073)			(0.1111)	(0.1033)		
Δy_t	-0.0041		0.0514		0.0047		0.0320	
	(0.0158)		(0.0221)		(0.0163)		(0.0173)	
$\widehat{pd}_t$	0.1869			0.2561	0.1190			0.1428
	(0.0600)			(0.0881)	(0.0410)			(0.0459)
Adj. R^2	0.582	0.552	0.051	0.078	0.307	0.269	0.033	0.088

Table IA.5: **Alternative Model** of Undiscounted Risk-adjusted Dividend Growth: $z_t = g_{t+1} - \theta_{t+1}$

Estimates using **listed Dividend Futures of the Eurostoxx 50 index.**

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years, except for its begin until 13^{th} May 2009 in which the number is five due to a lack of data. σ_η measures the standard deviations of the second until the eighth measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section 3. Standard errors in parentheses.

	Two state	Single state
	$dz_t = \varphi\left(\tilde{z}_t - z_t\right)dt + \sigma_z dW_p$ $d\tilde{z}_t = \psi\left(\overline{z} - \tilde{z}_t\right)dt + \sigma_{\tilde{z}} dW_{\tilde{z}}$	$dz_t = \varphi\left(\overline{z} - z_t\right)dt + \sigma_z dW_z$
$\overline{z}$	0.0027	-0.0147
	(0.0241)	(0.0212)
φ	1.4108	1.4142
	(0.2552)	(0.4588)
ψ	0.1860	
	(0.0779)	
β_z	*Set to 0*	*Set to 0*
$\beta_{\tilde{z}}$	-3.0428	
	(6.8937)	
σ_z	0.5125	0.4858
	(0.6433)	(1.0167)
$\sigma_{\tilde{z}}$	0.0362	
	(0.0687)	
σ_ε	0.0233	0.0259
	(0.0311)	(0.0136)
σ_η	0.0060	0.0569
	(0.0023)	(0.1331)
Log Likelihood per contribution	24.78	15.75

Table IA.6: Reconciliation of the **Alternative Model** of Undiscounted Risk-Adjusted Dividend Growth (two state) constituent returns to stock market returns.

The modeled present values of dividends are tested for their explanatory power of the dynamics of the stock market. The OLS regression estimates equation (26) in the main text $\Delta s_t = \alpha + \beta_f \Delta f_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{pd}} \Delta \widehat{pd}_t + \varepsilon_t$, in which Δs_t is stock index log returns, Δf_t is the log return of the first constant maturity dividend derivative, Δy_t is the change in the one year zero swap rate and $\Delta \widehat{pd}_t$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the two state space model. β is fixed at zero. Daily data for periods as in Tables 1 and 2. Standard errors in parentheses.

Constant	0.0006	0.0002	0.0006	0.0000
	(0.0003)	(0.0003)	(0.0004)	(0.0004)
f_t	0.9555	1.0009		
	(0.0419)	(0.0426)		
Δy_t	0.1693		0.2022	
	(0.0163)		(0.0178)	
$\widehat{pd}_t$	0.0725			-0.0113
	(0.0157)			(0.0179)
Adj. R^2	0.293	0.248	0.071	0.000

Table IA.7: **Base model of Discounted Risk-adjusted Dividend Growth.**

Estimates of the base model with all daily data, data with days excluded when 3 or more futures do not trade on that day and data with days excluded when 2 or more futures do not trade on that day.

Maximum Likelihood estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting one year following the observation date. The estimates include eight measurement equations: from one to eight years, except for its begin until 13^{th} May 2009 in which the number is five due to a lack of data. σ_η measures the standard deviations of the second until the eighth measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between first and second derivatives to expire. See the Data section 3. Standard errors in parentheses. Parameter $\overline{p}$ is estimated in the Kalman filter, $\overline{p}*$ equals $\overline{p}$ plus the convexity term.

Estimates using listed Dividend Futures of the Eurostoxx 50 Index.
Sample period: 4 August 2008 – 16 February 2015.

	No exclusions	3 futures excluded	2 futures excluded
$\overline{p}$	-0.0404	-0.0415	-0.0434
	(0.0197)	(0.0246)	(0.0355)
φ	1.5132	1.5032	1.5158
	(0.3158)	(0.3404)	(0.4126)
ψ	0.2434	0.2223	0.1980
	(0.1088)	(0.1221)	(0.1480)
β_p	*Set to 0*	*Set to 0*	*Set to 0*
$\beta_{\tilde{p}}$	-2.6693	-2.7719	-2.9184
	(6.2523)	(6.6359)	(7.8932)
σ_p	0.5701	0.6065	0.6847
	(0.787)	(0.9237)	(1.2622)
$\sigma_{\tilde{p}}$	0.0437	0.0418	0.0397
	(0.0946)	(0.0968)	(0.1059)
σ_ε	0.0219	0.0229	0.0238
	(0.0295)	(0.0372)	(0.0497)
σ_η	0.0063	0.0062	0.0062
	(0.0025)	(0.0028)	(0.0036)
$\overline{p}^*$	-0.0258	-0.0242	-0.0213
Log Likelihood per contribution	24.57	18.21	11.97
Number of observations	1,679	1,234	821

Figure IA.1: **Model implied dividend return volatility**: $\sigma_t\left(\ln P_{t+1,n} - \ln P_{t,n}\right)$. Our S&P 500 data is contrasted with the data for Long Run Risk and Habit Formation as compiled in Binsbergen et al. (2012).

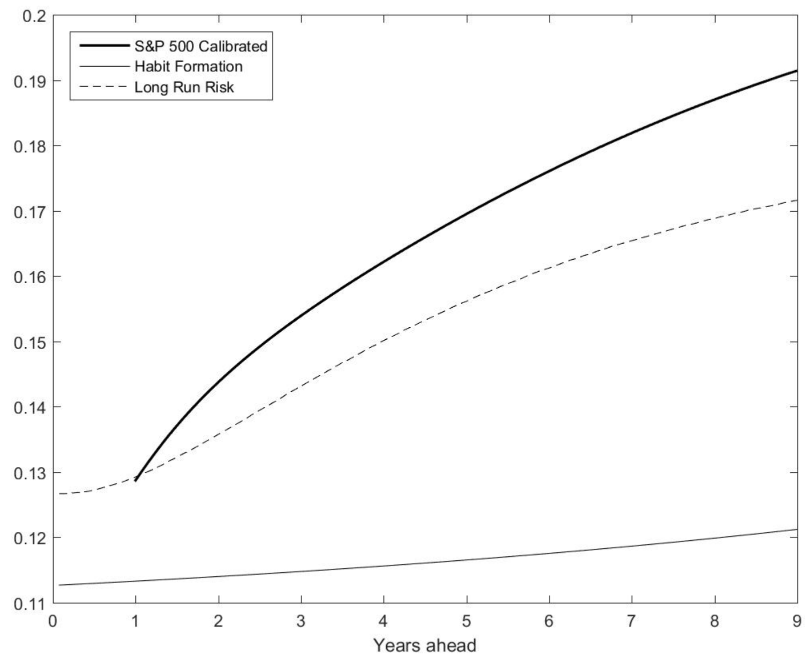

Figure IA.2: **Mean absolute estimation errors**.

The average of the absolute estimation error of the two-state base model for the Eurostoxx 50 index for various compositions of the data set. The measurement variables are discounted dividend risk-adjusted growth rates of 1 to 8 years. When 2 or 3 futures do not show trading volume on a given trading day, the measurement equation is skipped for that day.

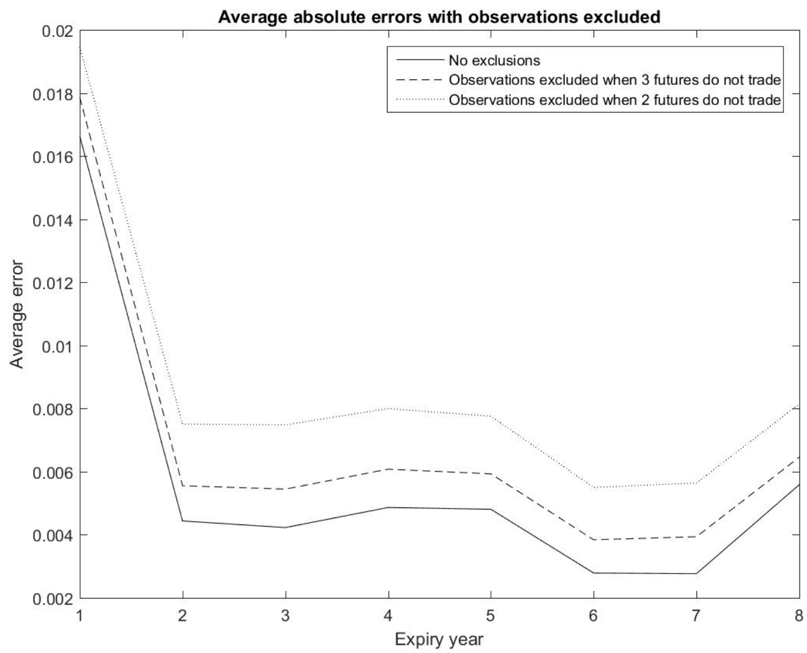

Figure IA.3: **Average absolute daily price changes of the shortest dividend future**.

Daily absolute price changes of the Eurostoxx 50 dividend future in the year before expiry. Price changes are expressed in € as a function of the number of days until expiry.