Volatility and Expected Option Returns

Online Appendix

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I. Overview

In this online appendix, we report on various theoretical and empirical results that complement the analysis in the paper. Section II extends our benchmark theoretical results to any finite holding period. Section III extends our results to a stochastic volatility model. Section IV characterizes the relation between expected stock returns and expected option returns, and reports empirical results that control for the expected return on the underlying stock using double sorts. Section V discusses the relation between volatility and straddle returns. Section VI provides additional robustness results, Section VII reports on delta-hedged returns, and Section VIII presents time-series evidence using index option data.

II. Holding-Period Expected Option Returns

Propositions 1 and 2 also hold for expected option returns over any holding period in the Black-Scholes-Merton (BSM) model. We first derive expected holding-period option returns in the BSM model. To save space, we only focus on call options. The analysis of put options proceeds along the same lines. To facilitate the notation, we consider an European call option at time 0 that matures at time T. By definition, the expected return of holding the call option from time 0 to time h ($h < T$) is:

$$R^h_{call} = \frac{E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\}}{S_0 N(d_1) - e^{-rT} K N(d_2)}$$

where $S_h N(d_1') - e^{-r(T-h)} K N(d_2')$ is the future value of the option at time h, and

$$d_1' = \frac{\ln \frac{S_h}{K} + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}} \qquad d_2' = \frac{\ln \frac{S_h}{K} + (r - \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}$$

$$d_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln \frac{S_0}{K} + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

The expected future value of the option at time h can be split into two pieces:

$$\begin{aligned} E_0\{S_h N(d_1') - e^{-r(T-h)}KN(d_2')\} &= \int_{-\infty}^{\infty} [S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') - e^{-r(T-h)}KN(d_2')] \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &= \int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &\quad + \int_{-\infty}^{\infty} -e^{-r(T-h)}KN(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz. \end{aligned}$$

For the first integral, it can be shown that

$$\begin{aligned} &\int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &= S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-\sigma\sqrt{h})^2}{2}} N\left(\frac{\ln\frac{S_0}{K} + \mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h}z + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}\right) dz. \end{aligned} \tag{A.1}$$

Define a new variable $z^* = z - \sigma\sqrt{h}$. (A.1) becomes

$$S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N\left(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T-h}} + \sqrt{\frac{h}{T-h}} z^*\right) dz^*. \tag{A.2}$$

Using (see Rubinstein (1984))

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(A + Bz^*) = N\left(\frac{A}{\sqrt{1+B^2}}\right),$$

(A.2) can be further simplified as

$$S_0 e^{\mu h} N\left(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right). \tag{A.3}$$

Following the same steps, the second integral can be rewritten as

$$\int_{-\infty}^{\infty} -e^{-r(T-h)}KN(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = -e^{-r(T-h)}KN\left(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right). \tag{A.4}$$

Combining (A.3) and (A.4), we obtain

$$R_{call}^h = \frac{S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}) - e^{-r(T-h)} K N(\frac{\ln\frac{S_0}{K}+(\mu-r)h+(r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}})}{S_0 N(d_1) - e^{-rT} K N(d_2)}.$$

This can be further simplified to

$$\text{(A.5)} \qquad \begin{aligned} R_{call}^h &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \\ d_1^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu-r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\ d_2^* &= \frac{\ln\frac{S_0}{K} + [HP(\mu-r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \end{aligned}$$

where $HP = h/T$ is the ratio of the holding period to the life of the option contract.

Note that the expected holding-to-expiration option return derived in the paper is nested in (A.5), for $HP = 1$. We can use the structure of the proof of Proposition 1 to show $\frac{\partial R_{call}^h}{\partial \sigma} < 0$, by observing $r + (\mu - r)HP > r$. Thus, we conclude that expected call (put) option returns decrease (increase) with underlying volatility for any holding period in the BSM model.

III. Stochastic Volatility and Expected Option Returns

This section analyzes expected option returns using the Heston (1993) stochastic volatility model, which captures important stylized facts such as time-varying volatility and the leverage effect, while also allowing for quasi-closed form European option prices. As with option prices, we can express expected returns in the Heston model in quasi-closed form. The Heston (1993) model assumes that the asset return and its spot variance obey the following dynamics under the physical measure P

$$
\begin{aligned}
dS_t &= \mu S_t dt + S_t \sqrt{V_t} dZ_1^P \\
dV_t &= \kappa(\theta - V_t) dt + \sigma \sqrt{V_t} dZ_2^P
\end{aligned}
$$

where μ is the drift of the stock price, θ is the long run mean of the stock variance, κ is the rate of mean reversion, σ is the volatility of volatility, and Z_1 and Z_2 are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$. The dynamics under the risk-neutral measure Q are

$$
\begin{aligned}
dS_t &= r S_t dt + S_t \sqrt{V_t} dZ_1^Q \\
dV_t &= [\kappa(\theta - V_t) - \lambda V_t] dt + \sigma \sqrt{V_t} dZ_2^Q
\end{aligned}
$$

where r is the risk-free rate and λ is the market price of volatility risk. Again we consider the expected return of holding a call option to expiration:

$$
R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, V_t))} = \frac{E_t^P[\max(S_T - K, 0)]}{E_t^Q[e^{-r\tau} \max(S_T - K, 0)]}.
$$

Heston (1993) provides a closed-form solution to an European call option, up to a univariate numerical integral:

$$
C(t, T, S_t, V_t) = E_t^Q[e^{-r\tau} \max(S_T - K, 0)] = S_t P_1 - e^{-r\tau} K P_2 \tag{A.6}
$$

where P_1 and P_2 are given by[1]

$$
P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left(\frac{e^{-i\phi \ln K} f_j(x, V, \tau; \phi)}{i\phi}\right) d\phi
$$

[1]Note that $x = \ln S$.

$$f_j(x, V, \tau; \phi) = e^{C(\tau;\phi) + D(\tau;\phi)V + i\phi x}$$

$$C(\tau; \phi) = r\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1 - ge^{d\tau}}{1 - g}]\}$$

$$D(\tau; \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa + \lambda - \rho\sigma, b_2 = \kappa + \lambda.$$

By analogy, it can be shown that expected call option payoff at expiration is

$$E_t^P[\max(S_T - K), 0] = e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*] \tag{A.7}$$

where

$$P_j^* = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi\ln K} f_j^*(x, V, \tau; \phi)}{i\phi})d\phi$$

$$f_j^*(x, V, \tau; \phi) = e^{C(\tau;\phi) + D(\tau;\phi)V + i\phi x}$$

$$C(\tau; \phi) = \mu\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1 - ge^{d\tau}}{1 - g}]\}$$

$$D(\tau; \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa - \rho\sigma, b_2 = \kappa.$$

Combining (A.6) and (A.7), the expected holding-to-maturity call option return in the Heston model is given by

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*]}{S_t P_1 - e^{-r\tau} K P_2}. \tag{A.8}$$

Equation (A.8) computes the expected return conditional on current spot variance V_t. Because the unconditional spot variance has a gamma distribution, one can compute the unconditional expected return R_{Call}^{Heston} by taking a numerical integral over V_t. In this paper, we work with unconditional expected returns unless otherwise stated. The sign of the derivative $\frac{\partial R_{call}^{Heston}}{\partial \theta}$ can not be derived analytically. However, the expected option return in the Heston model can be easily calculated numerically given a set of parameter values.

IV. Controlling for Expected Stock Returns

In this section, we first characterize the theoretical relationship between expected stock returns and expected option returns. We then present empirical results that control for the expected return on the underlying security in two ways. First we present results for double sorts on volatility and average historical stock returns. Second, we specify a single-factor market model for the underlying security and control for the underlying stock's exposure to the market.

A. Expected Stock Returns and Expected Option Returns

We show that expected call (put) option returns increase (decrease) with expected stock returns: $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$. First, recall the expected return from holding a call option to maturity is:

$$R_{call} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

$$d_1^* = \frac{\ln \frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln \frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Taking the derivative with respect to μ

$$\frac{\partial R_{call}}{\partial \mu} = \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + e^{\mu\tau}[\tau e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

where ψ is the probability density function of standard normal distribution. Note that we apply the fact that the Rho of a call option is $\tau e^{-\mu\tau} K N(d_2^*)$ in deriving the above equation. $\frac{\partial R_{call}}{\partial \mu}$ can be further simplified:

$$\begin{aligned} \frac{\partial R_{call}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + \tau K N(d_2^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} \\ &= \frac{\tau e^{\mu\tau} S_t N(d_1^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} > 0. \end{aligned}$$

To see that the derivative is positive, notice that the denominator is just the price of call option which is always positive, and the numerator is obviously greater than zero.

Next we show that the expected put option return is a decreasing function of the expected stock return. Recall that the expected put option return is:

$$R_{put} = \frac{e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)}$$

where d_1^*, d_2^*, d_1, and d_2 are defined the same as the above. Taking the derivative with respect to μ yields:

$$\begin{aligned} \frac{\partial R_{put}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[e^{-\mu\tau} K N(-d_2^*) - S_t N(-d_1^*)] + e^{\mu\tau}[-\tau e^{-\mu\tau} K N(-d_2^*)]}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} \\ &= \frac{-\tau e^{\mu\tau} S_t N(-d_1^*)}{e^{-r\tau} K N(-d_2) - S_t N(-d_1)} < 0. \end{aligned}$$

Note the denominator is the price of a put option, which is always positive, and therefore the ratio itself is negative.

B. Controlling for Expected Stock Returns Using Historical Averages

Expected call (put) option returns increase (decrease) with the expected return on the underlying asset. If the high volatility portfolios in Table 2 are primarily composed of stocks that have lower expected returns than those in the low volatility portfolios, the result that average call (put) options in the high volatility portfolios earn lower (higher) returns may not be due to volatility. We therefore start by documenting if the underlying stock returns affect our results by empirically controlling for expected stock returns. This is of course challenging because unlike volatility, expected stock returns are notoriously difficult to measure.

Our first approach follows Boyer and Vorkink (2014), who estimate expected stock returns as the simple average of daily returns over the past six months. Each month we first form five quintile portfolios based on estimated expected stock returns μ, and then within each μ quintile options are further sorted into five quintile portfolios according to underlying stock volatility. We once again measure underlying stock volatility by 30-day realized volatility.

Table A1 presents the results of this double sort. The columns correspond to different volatility levels, and the rows correspond to different average returns. Consistent with the single sort results, in each μ quintile call (put) option portfolio returns decrease (increase) with underlying volatility. In all μ quintiles, the average return differences between the two extreme call option portfolios are negative, ranging from -24% to -11% per month, and highly significant. For put options, the high minus low differences are all positive and statistically significant in four out of five μ quintiles. These findings suggest that our results are not driven by differences between the expected returns of the underlying stocks.

C. A Single-Factor Market Model

Estimates of expected returns from historical averages are notoriously imprecise. Our next approach controls for expected returns using the simple market or index model rather than the historical average. Panel A of Table A2 presents results for a double sort on market beta and volatility. Beta is estimated by the market model over the most recent 30 days preceding the portfolio formation date. The results are similar to those in Table A1, where we control for the expected return using lagged average returns, but the t-statistics are somewhat smaller. Average call option returns decrease with volatility for each beta quintile and the return spread between the two extreme portfolios is statistically significant across all beta quintiles. In contrast, average put option returns increase with volatility for each beta quintile and the return spread is significant for the top three beta quintiles.

Panel B of Table A2 uses the results from the market model in a slightly different way. We present results for sorts on idiosyncratic volatility based on the market model. Panel B indicates a negative relation between call option portfolio returns and idiosyncratic volatility, and a positive relation between put option portfolio returns and idiosyncratic volatility.[2] We obtain similar results when sorting on idiosyncratic volatility computed relative to the Fama-French three-factor model.

V. Volatility and Expected Straddle Returns

We study the relation between expected straddle returns and the volatility of the underlying. Straddle returns are not very sensitive to the expected returns on the underlying security. Therefore, several existing papers that investigate the cross-sectional relation between option

[2]Given the additional assumption of the market model, the results in Propositions 1 and 2 effectively establish a relation between option returns and idiosyncratic volatility. This interpretation is more in line with Johnson's (2004) analysis of the role of volatility in returns on levered equity.

returns and different aspects of volatility focus on straddle returns to separate the cross-sectional effect of volatility and volatility-related variables from that of the underlying stock returns. See for example Goyal and Saretto (2009) and Vasquez (2017).

A straddle consists of the simultaneous purchase of a call option and a put option on the same underlying asset. The call and put options have the same strike price and time to maturity. The expected gross return on a straddle is given by:

$$R_{straddle} = \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)}$$

where $C_t(\tau, S_t, \sigma, K, r)$ and $P_t(\tau, S_t, \sigma, K, r)$ are the call and put prices that an investor has to pay to build a long position in straddle. We investigate the impact of volatility on expected straddle returns by taking the derivative of $R_{straddle}$ with respect to σ. It follows that

$$\begin{aligned} \frac{\partial R_{straddle}}{\partial \sigma} &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1^*)A - 2e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1)B}{[S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \\ &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\{\psi(d_1^*)A - \psi(d_1)B\}}{[S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \end{aligned}$$

where $A = S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)$ and $B = S_tN(d_1^*) - e^{-\mu\tau}KN(d_2^*) + e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)$. It is clear that the sign of $\frac{\partial R_{straddle}}{\partial \sigma}$ is determined by $\psi(d_1^*)A - \psi(d_1)B$. This term can be positive or negative depending on underlying parameters. We now show that $d_2 > 0$ is a sufficient condition for a negative relation between straddle returns and underlying volatility.

To see this, first recall from previous analysis $d_1^* > d_1 > d_2$. We then have

(A.9) $$d_2 > 0 \Rightarrow 0 < \psi(d_1^*) < \psi(d_1).$$

Moreover, note that

$$\frac{\partial A}{\partial r} = \tau e^{-r\tau}K[N(d_2) - N(-d_2)]$$

and therefore,

$$d_2 > 0 \Rightarrow \frac{\partial A}{\partial r} > 0$$

which further implies

$$0 < A < B \tag{A.10}$$

by noting that B is obtained by replacing r with μ in A. Putting together (A.9) and (A.10),

$$d_2 > 0 \Rightarrow \psi(d_1^*)A - \psi(d_1)B < 0 \Rightarrow \frac{\partial R_{straddle}}{\partial \sigma} < 0.$$

Recall that $d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$. The condition $d_2 > 0$ is thus likely to hold for straddles with strike prices below the current stock price, and we investigate if average straddle returns decrease with underlying volatility for such straddles. Table A.3 confirms that this relation indeed holds in the data. However, effectively the return on the straddle is dominated by the call option when $d_2 > 0$, which means that the negative sign in theory and in the data effectively merely confirms the results for call options.

VI. Robustness

In this section, we first examine if the negative (positive) relation between call (put) option portfolio returns and underlying volatility persists if different weighting methods are used for computing option portfolio returns. We calculate option volume weighted, option open interest weighted and option value weighted average portfolio returns. Option value is defined as the product of the option's open interest and its price.

Table A.4 contains return spreads for option portfolios sorted on 30-day realized volatility, using these alternative weighting methods. Regardless of the weighting method, the return spreads are negative (positive) for call (put) option portfolios, and they are statistically significant in most cases. These results suggest that our empirical findings are not due to the equal-weighting method used in the main analysis.

We also repeat the analysis in Table 2 using one-month option returns instead of holding-to-maturity returns. Table A5 presents the results for ATM, ITM, and OTM call and put options. The results are again statistically significant and consistent with Propositions 1 and 2. However, the magnitudes of the long-short returns are smaller, especially for calls.

Table A6 evaluates the statistical significance of our results by bootstrapping. Following Bakshi, Madan and Panayotov (2010), we draw 25000 bootstrap samples of returns on the two extreme portfolios and we compute the return difference for each bootstrap sample. The p-values are calculated as the proportion of positive (negative) differences in the 25000 bootstrap samples of call (put) portfolio returns. Table A6 also reports the p-values corresponding to the computation of standard errors using a Newey-West correction, as in Table 2. Table A6 indicates that the bootstrapped p-values are similar to the significance levels computed using Newey-West.

Finally, Table A7 repeats the analysis in Table 2 using only options on common stocks. In doing so, we exclude options written on mutual and investment trust funds, ADRs and ETFs. These options represent about 15% of our sample. The results in Table A7 are very similar to those in Table 2.

VII. Delta-Hedged Returns

Table A8 shows that the negative relationship between volatility and delta-hedged call and put returns documented in Table 10 continues to hold when including other control variables in a Fama-MacBeth regression. Table A8 also shows that the results of Goyal and Saretto (2009) are confirmed for our sample: delta-hedged returns on puts and calls are positively related to the variance risk premium.

VIII. Volatility and the Time Series of Index Option Returns

In this section, we explore the time-series implications of Propositions 1 and 2 by studying the relation between monthly S&P 500 index option (SPX) returns and S&P 500 index volatility. Consistent with Proposition 1 and 2, we find that SPX call (put) options tend to have lower (higher) returns in the month following a high volatility month. On the first trading day after each month's option expiration date, we collect index options with moneyness $0.9 \leq K/S \leq 1.1$ that mature in the next month. Table A9 provides summary statistics for SPX option data by moneyness. Index put options (especially out-of-the-money puts) generate large negative returns, consistent with the existing literature (see for example Bondarenko (2003)). For example, for the moneyness interval $0.94 < K/S \leq 0.98$, the average return is -40.6% per month. Table A9 also shows that in our sample, out-of-the-money SPX calls have large negative returns. This is consistent with the results in Bakshi et al. (2010).

Comparing Tables A9 and 1 highlights several important differences between index options and individual stock options. First, the volatility skew, the slope of implied volatility against moneyness, is much less pronounced for individual stock options. Second, the average realized volatility for index options is approximately 17%, and therefore the volatility risk premium for index options exceeds the volatility risk premium for stock options. This is consistent with existing findings, but note that the index variance risk premium in our paper is smaller than many existing findings due to our sample period.

Propositions 1 and 2 characterize a general property of expected option returns: call (put) option returns decrease (increase) with underlying volatility. This property should hold in the time series of option returns as well as in the cross-section. We investigate the time-series implications of Propositions 1 and 2 by using index option returns to estimate the following time-series regression:

$$R^i_{t+1} = constant + \beta_1 VOL_t + \beta_2 Moneyness^i_t + \beta_3 R^I_t + \epsilon \tag{A.11}$$

where R^i_{t+1} is the return on holding index option i from month t to month $t+1$, R^I_t is the return on the S&P 500 in month t and VOL_t is the index volatility. Moneyness (K/S) is also included in the regression because previous studies (e.g., Coval and Shumway (2001)) have shown that moneyness is an important determinant of option returns. Here we consider four proxies for S&P 500 index volatility: 14-day realized volatility, 30-day realized volatility, 60-day realized volatility, and implied volatility. These volatilities are defined as in the cross-sectional analysis and are known in month t.

The estimate of β_1 is the main object of interest. According to Propositions 1 and 2, we expect β_1 to be negative for SPX call options and positive for SPX put options.

Table A10 presents the coefficient estimates, t-statistics, and adjusted R-squares for the regressions in equation (A.11). Consistent with Propositions 1 and 2, the slope coefficient on index volatility is always negative (positive) for SPX call (put) options, regardless of the index volatility proxy. For example, column 2 of Panel A of Table A10 shows that when using 30-day realized volatility as the volatility proxy, the slope coefficient on index volatility is -0.92 for SPX calls and is highly significant with a t-statistic of -3.78. For a 1% increase in S&P 500 volatility, the return to holding an SPX call option over the next month is expected to decrease by 0.92%. In contrast, in column 2 of Panel B of Table A10, the slope coefficient on index volatility for SPX puts is 1.39 and it is also highly statistically significant.

These results are based on the full sample that also contains in-the-money SPX options. However, Table A9 indicates that in-the-money SPX options are much less traded than their at-the-money and out-of-the-money counterparts. To ensure our results are not driven by illiquid in-the-money options, we repeat the regressions in (A.11) using only liquid options. Specifically, we only consider SPX calls with $0.98 \leq K/S \leq 1.10$ and SPX puts with

$0.90 \leq K/S \leq 1.02$.

The regression results using only liquid options are presented in columns 5 through 8 in Table A10. Consistent with the results using the full sample, we find that the slope coefficient estimate on index volatility is always negative (positive) and statistically significant for SPX calls (puts) regardless of the volatility proxy. For example, when using 60-day realized volatility as a proxy, we find a slope coefficient of -1.62 for SPX calls and 1.58 for SPX puts, and both are highly significant with t-statistics of -3.77 and 2.98 respectively. These results confirm that our findings are not due to illiquid index options.

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Table A1: Option Portfolio Returns Double-Sorted on Expected Stock Return and Underlying Volatility

We report average equal-weighted monthly returns on option portfolios sorted on expected stock return (μ) and 30-day realized volatility. Panel A reports on call options and Panel B on put options. Every month, all available one-month at-the-money options are first ranked into five quintile portfolios according to the underlying stocks' expected returns. Then, within each μ quintile, options are further sorted into five portfolios based on 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatility. Following Boyer and Vorkink (2014), the expected stock return is estimated as the simple average of daily returns over the past six month preceding the portfolio formation date. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5% and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Options		Low	3	3	4	High	H-L
	1	0.246	0.151	0.075	0.033	0.001	-0.245***
							(-5.889)
	2	0.190	0.148	0.117	0.085	-0.006	-0.195***
μ Quintiles							(-3.628)
	3	0.146	0.170	0.125	0.082	0.021	-0.125***
							(-2.769)
	4	0.131	0.122	0.136	0.094	0.018	-0.112***
							(-2.854)
	5	0.154	0.106	0.101	0.066	0.038	-0.116***
							(-2.823)
Panel B: Put Options		Low	2	3	4	High	H-L
	1	-0.113	-0.079	-0.067	-0.028	-0.044	0.069*
							(1.799)
	2	-0.162	-0.136	-0.117	-0.102	-0.056	0.107**
							(2.092)
μ Quintiles	3	-0.153	-0.187	-0.162	-0.095	-0.078	0.074*
							(1.762)
	4	-0.154	-0.158	-0.136	-0.116	-0.133	0.021
							(0.450)
	5	-0.182	-0.095	-0.132	-0.102	-0.079	0.103***
							(3.165)

Table A2: Controlling for Expected Stock Returns Using the CAPM

Panel A reports average equal-weighted monthly returns on option portfolios sorted on market beta and 30-day realized volatility. Every month one-month at-the-money options are first ranked into five portfolios according to the underlying stocks' betas. Then, within each beta quintile, options are further sorted into five portfolios based on realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatility. The beta is estimated using daily returns over the past 30 days preceding the portfolio formation date. Panel B reports average equal-weighted returns on option portfolios sorted on stock idiosyncratic volatility. Idiosyncratic volatility is estimated from the market model (CAPM) using daily returns over the past 30 days. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5% and 1% level is denoted by *, **, and *** respectively.

Panel A: Double Sorts on Beta and Volatility

	Beta\Vol	Low	2	3	4	High	H-L
	1	0.156	0.126	0.094	0.067	-0.01	-0.165***
							(-3.384)
	2	0.162	0.165	0.15	0.135	0.061	-0.101**
							(-2.082)
Call	3	0.149	0.194	0.147	0.112	0.05	-0.099*
							(-1.969)
	4	0.113	0.133	0.107	0.106	0.031	-0.082**
							(-2.024)
	5	0.09	0.076	0.104	0.022	0.005	-0.085**
							(-2.115)
	Beta\Vol	Low	2	3	4	High	H-L
	1	-0.15	-0.095	-0.132	-0.126	-0.121	0.029
							(0.618)
	2	-0.149	-0.165	-0.156	-0.101	-0.107	0.041
							(0.896)
Put	3	-0.147	-0.198	-0.112	-0.069	-0.065	0.082**
							(1.997)
	4	-0.14	-0.122	-0.14	-0.071	-0.047	0.093**
							(2.087)
	5	-0.133	-0.106	-0.085	-0.067	-0.065	0.068*
							(1.957)

Panel B: Sorts on Idiosyncratic Volatility

	Low	2	3	4	High	H-L
Call	0.156	0.13	0.119	0.083	0.021	-0.133***
						(-3.245)
Put	-0.157	-0.151	-0.119	-0.069	-0.077	0.080**
						(2.271)

Table A3: Straddle Portfolio Returns Sorted on Volatility

We report average monthly returns for five straddle portfolios sorted on the volatility of the underlying stock. We use three samples of straddles based on moneyness: $0.95 \leq K/S \leq 1$, $0.875 \leq K/S < 0.95$, and $0.80 \leq K/S < 0.875$. Every month, we select call and put options on the same stock with the same strike price and maturity to form straddles. These straddles are then sorted into five quintile portfolio based on the realized volatility over the preceding month. Portfolio Low (High) contains straddles with the lowest (highest) underlying volatility. We report equal-weighted and volume-weighed portfolio returns. Straddle volume is computed as the average volume for the call and put options that form the straddle. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

Panel A: $0.95 \leq K/S \leq 1$						
	Low	2	3	4	High	H-L
Equal Weighted	0.028	0.010	0.022	0.014	-0.026	-0.054***
						(-2.904)
Volume Weighted	0.026	-0.006	0.005	0.014	-0.037	-0.063**
						(-2.139)
Panel B: $0.875 \leq K/S < 0.95$						
Equal Weighted	0.022	0.034	0.025	0.015	-0.033	-0.055***
						(-3.249)
Volume Weighted	0.013	0.044	-0.010	0.003	-0.044	-0.057**
						(-2.522)
Panel C: $0.80 \leq K/S < 0.875$						
Equal Weighted	0.020	0.013	0.014	0.000	-0.065	-0.085***
						(-4.972)
Volume Weighted	0.021	0.001	-0.005	-0.021	-0.046	-0.067**
						(-2.328)

Table A4: Option Portfolio Returns Using Different Weighting Methods

We report long-short monthly returns for portfolios sorted on 30-day realized volatility, using different option samples. Alternative weighting methods are used: volume weighted, open interest weighted, and option value weighted. Option value is defined as the product of option price and option open interest. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	Volume Weighted	Open Interest Weighted	Option Value Weighted
Panel A: Call Option Portfolios			
One-month ATM	-0.182***	-0.133***	-0.107**
	(-3.557)	(-3.094)	(-2.352)
Two-month ATM	-0.204**	-0.235***	-0.216***
	(-2.401)	(-3.640)	(-2.893)
One-month ITM	-0.113***	-0.060**	-0.066***
	(-3.978)	(-2.512)	(-2.652)
Two-month ITM	-0.210***	-0.188***	-0.191***
	(-3.925)	(-4.334)	(-4.456)
One-month OTM	-0.171**	-0.059	-0.137*
	(-2.145)	(-0.897)	(-1.747)
Two-month OTM	-0.242**	-0.292**	-0.438***
	(-1.992)	(-2.422)	(-2.985)
Panel B: Put Option Portfolios			
One-month ATM	0.073	0.089*	0.052
	(1.433)	(1.869)	(1.061)
Two-month ATM	0.081	0.187***	0.170**
	(0.971)	(2.969)	(2.497)
One-month ITM	0.035	0.099***	0.090***
	(1.094)	(3.679)	(2.934)
Two-month ITM	0.154***	0.134***	0.134***
	(3.451)	(2.857)	(2.896)
One-month OTM	0.268***	0.278***	0.274***
	(3.448)	(4.139)	(3.481)
Two-month OTM	0.310***	0.307***	0.349***
	(3.047)	(2.932)	(3.706)

Table A5: Holding Period Option Returns Sorted on Underlying Volatility

We report one-month holding period returns of options sorted on underlying volatility. On the first trading day of each month, we collect options that expire in the following month and compute the returns of holding these options to the month end. ATM options are defined by moneyness $0.95 \leq K/S \leq 1.05$, ITM options are defined by moneyness $0.80 \leq K/S < 0.95$ for calls and $1.05 < K/S \leq 1.20$ for puts, and OTM options are defined by moneyness $1.05 < K/S \leq 1.20$ for calls and $0.80 \leq K/S < 0.95$ for puts. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Option Portfolios						
	Low	2	3	4	High	H-L
1-month ATM	0.010	0.024	0.008	0.005	-0.047	-0.056**
						(-2.046)
1-month ITM	0.002	0.003	0.003	-0.007	-0.051	-0.053***
						(-2.931)
1-month OTM	0.056	0.032	0.030	0.008	-0.039	-0.095**
						(-2.482)
Panel B: Put Option Portfolios						
	Low	2	3	4	High	H-L
1-month ATM	-0.082	-0.069	-0.047	-0.037	-0.027	0.055**
						(2.016)
1-month ITM	-0.044	-0.021	-0.013	-0.018	0.007	0.051**
						(2.537)
1-month OTM	-0.159	-0.099	-0.051	-0.057	-0.048	0.111***
						(2.915)

Table A6: Bootstrapped p-Values

We report average equal-weighted monthly returns for option portfolios sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B on put options. Every month, all available one-month at-the-money options are sorted into five quintile portfolios according to their 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. We report the Newey-West t-statistics from Table 2, and the corresponding p-values. In the last column, we report p-values computed from a bootstrap with 25000 resamples following Bakshi, Madan and Panayotov (2010). The sample period is from January 1996 to July 2013. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Option Portfolios								
Low	2	3	4	High	H-L	t-stat	p-value	bootstrapped p-value
0.147	0.128	0.111	0.084	0.009	-0.138***	(-3.422)	(0.000)	(0.000)
Panel B: Put Option Portfolios								
Low	2	3	4	High	H-L	t-stat	p-value	bootstrapped p-value
-0.146	-0.153	-0.109	-0.077	-0.075	0.071**	(2.004)	(0.023)	(0.035)

Table A7: Option Portfolio Returns Sorted on Underlying Volatility

We report average equal-weighted monthly returns for option portfolios sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B on put options. The sample contains only options on common stocks. Every month, all available one-month at-the-money options are sorted into five quintile portfolios according to their 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. The sample period is from January 1996 to July 2013. Newey-West t-statistics using four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

Panel A: Call Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	0.155	0.136	0.108	0.083	0.019	-0.136***
						(-3.124)
Panel B: Put Option Portfolios						
	Low	2	3	4	High	H-L
$0.95 \leq K/S \leq 1.05$	-0.152	-0.158	-0.115	-0.085	-0.080	0.072**
						(1.984)

Table A8: Fama-MacBeth Regressions Using Delta-Hedged Option Returns

We report results for the Fama-MacBeth regressions $R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon$, where R^i_{t+1} is the delta hedged option return as in Goyal and Saretto (2009), VOL^i_t is the underlying stock volatility, and Z^i_t is a vector of control variables that includes volatility risk premium (vrp), the stock's beta (beta), firm size (size), book-to-market (btm), momentum (mom), stock return reversal (reversal), and option characteristics such as moneyness, Delta, Vega, Gamma, and option beta. The sample consists of one-month at-the-money options. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

	calls			Puts		
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-0.001	0.132***	0.129***	0.003*	0.015	0.017
	(-0.351)	(3.604)	(3.524)	(1.906)	(0.480)	(0.555)
Vol	**-0.009*** **	**-0.018*** **		**-0.010*** **	**-0.018*** **	
	(-3.115)	(-6.204)		(-4.097)	(-6.124)	
Ivol			-0.021***			-0.020***
			(-5.689)			(-5.494)
Vrp		0.039***	0.038***		0.033***	0.032***
		(11.049)	(11.036)		(10.680)	(10.641)
Beta		0.000	-0.001**		0.000	-0.001
		(-0.054)	(-1.981)		(0.725)	(-1.201)
Size		0.000	0.000		0.000	0.000
		(-1.300)	(-1.389)		(-1.231)	(-1.292)
Btm		0.000	0.000		0.000	0.000
		(-0.088)	(-0.048)		(0.321)	(0.334)
Mom		0.001	0.001		0.000	0.000
		(0.489)	(0.507)		(0.353)	(0.326)
Reversal		-0.017***	-0.014***		-0.015***	-0.013***
		(-3.370)	(-2.888)		(-3.458)	(-2.986)
Moneyness		-0.112***	-0.111***		-0.011	-0.016
		(-3.600)	(-3.549)		(-0.333)	(-0.484)
Delta		-0.028***	-0.027***		-0.005	-0.008
		(-2.789)	(-2.707)		(-0.617)	(-0.928)
Vega		0.000	0.000		0.000	0.000
		(0.416)	(0.405)		(0.080)	(0.042)
Gamma		0.008	0.008		0.007	0.006
		(1.049)	(1.095)		(0.722)	(0.692)
Option beta		-0.000*	-0.000*		0.000	0.000
		(-1.872)	(-1.696)		(0.900)	(0.548)

Table A9: Summary Statistics for S&P 500 Index Options

We report averages of monthly S&P 500 index option returns (return), implied volatility (implied vol), option volume (volume), and option Greeks by moneyness. Panel A reports on call options and Panel B reports on put options. We compute the monthly option return using the midpoint of the bid and ask quotes. The sample consists of S&P 500 index options (SPX) with moneyness $0.90 \leq K/S \leq 1.10$ and one-month maturity. The sample period is from January 1996 to July 2013.

Moneyness K/S	[0.90−0.94]	(0.94−0.98]	(0.98−1.02]	(1.02−1.06]	(1.06−1.10]
Panel A: SPX Call Options					
Return	0.027	0.057	0.060	-0.112	-0.617
Implied vol	27.30%	22.75%	19.68%	17.42%	17.28%
Volume	251	306	2029	2867	2156
Open interest	9679	11770	15236	15388	14807
Delta	0.88	0.76	0.51	0.20	0.06
Gamma	0.002	0.005	0.007	0.005	0.002
Vega	60.32	93.12	119.86	80.66	32.99
Panel B: SPX Put Options					
Return	-0.540	-0.406	-0.224	-0.133	-0.171
Implied vol	26.56%	22.87%	19.66%	18.20%	22.68%
Volume	3699	2662	2619	391	338
Open interest	19604	18649	14674	8992	12322
Delta	-0.11	-0.23	-0.48	-0.75	-0.88
Gamma	0.002	0.005	0.007	0.006	0.003
Vega	55.13	90.56	119.80	93.61	53.04

Table A10: Regressions of Index Option Returns on Index Volatility

Using a pooled sample of S&P 500 index options (SPX) with $0.9 \leq K/S \leq 1.1$ and one-month maturity, we report results for the regression of monthly SPX option returns on lagged index volatility:

$$R_{t+1}^{i} = constant + \beta_1 VOL_t^i + \beta_2 Moneyness_t^i + \beta_3 R_t^I + \epsilon$$

where R_{t+1}^{i} is the option return from month t to month $t+1$, R_t^I is the S&P 500 index return in month t and VOL_t is the index volatility. Columns (1)-(4) consider four index volatility measures: realized volatility over the previous 14 days, realized volatility over the preceding month, realized volatility over the previous 60 days, and option-implied volatility. Columns (5)-(8) consider the same regressions using only liquid SPX options, consisting of calls with $0.98 \leq K/S \leq 1.1$ and puts with $0.90 \leq K/S \leq 1.02$. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses.

Panel A: SPX Calls	Full Sample				Only Liquid Options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	4.091	4.178	4.285	7.113	9.273	9.211	9.128	10.849
	(5.988)	(6.360)	(6.676)	(14.025)	(9.440)	(9.311)	(9.154)	(11.931)
14 day realized vol	-0.460				-0.194			
	(-1.913)				(-0.548)			
30 day realized vol		-0.921				-0.858		
		(-3.781)				(-2.516)		
60 day realized vol			-1.456				-1.617	
			(-4.778)				(-3.767)	
implied vol				-3.697				-4.444
				(-5.571)				(-4.473)
Adjusted R-square	1.13%	1.22%	1.37%	2.01%	1.67%	1.73%	1.90%	2.54%

Panel B: SPX Puts	Full Sample				Only Liquid Options			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	-4.243	-4.219	-4.216	-4.499	-6.324	-6.139	-6.069	-6.771
	(-8.418)	(-8.230)	(-8.155)	(-9.067)	(-8.971)	(-8.622)	(-8.444)	(-8.704)
14 day realized vol	2.106				2.664			
	(3.918)				(3.881)			
30 day realized vol		1.393				1.887		
		(2.951)				(3.140)		
60-day realized vol			1.070				1.582	
			(2.619)				(2.978)	
implied vol				0.263				0.920
				(0.652)				(1.732)
Adjusted R-square	2.70%	1.73%	1.46%	1.13%	3.18%	2.06%	1.76%	1.27%