Online Appendix for "Financial vs. Strategic Buyers"

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Proof of Proposition 1. First we show that a debt-like contract between the firm and investors is always optimal. The proof of this claim follows Tirole (2005) closely.

Claim 1 *One optimal contract pays the manager only when the two projects are successful and nothing otherwise.*

Proof. *The manager is risk-neutral so she only cares about her expected payoff and thus an optimal contract provides her with incentives for a given expected payoff, which is,*

$$p_H^2 R_m^2 + 2p_H(1-p_H)R_m^1 + (1-p_H)^2 R_m^0,$$

where R_m^j is payoff when j projects are successful.

Assume there is an optimal contract $\{R_m^2, R_m^1, R_m^0\}$. By optimality it must satisfy two incentive compatibility constraints. The first, that she prefers to work on both projects rather than a single one:

$$p_H^2 R_m^2+2p_H(1-p_H)R_m^1+(1-p_H)^2 R_m^0 \geq p_H p_L R_m^2+(p_H+p_L-2p_H p_L)R_m^1+(1-p_H)(1-p_L)R_m^0+B,$$

the second that she prefers to work on both projects than none:

$$p_H^2 R_m^2 + 2p_H(1-p_H)R_m^1 + (1-p_H)^2 R_m^0 \geq p_L^2 R_m^2 + 2p_L(1-p_L)R_m^1 + (1-p_L)^2 R_m^0 + 2B.$$

Now, for such a given contract, take another one with $\left\{\widetilde{R}_{m2}, 0, 0\right\}$ such that the manager receives the same expected payoff, that is,

$$p_H^2 \widetilde{R}_{m2} = p_H^2 R_{m2} + 2p_H(1-p_H)R_{m1} + (1-p_H)^2 R_{m0}.$$

Then using the first IC

$$p_H^2 \widetilde{R}_{m2} \geq p_H p_L R_{m2} + (p_H + p_L - 2p_H p_L)R_{m1} + (1-p_H)(1-p_L)R_{m0} + B > p_H p_L \widetilde{R}_{m2} + B,$$

and using the second IC

$$p_H^2 \widetilde{R}_{m2} \geq p_L^2 R_{m2} + 2p_L(1-p_L)R_{m1} + (1-p_L)^2 R_{m0} + 2B > p_L^2 \widetilde{R}_{m2} + 2B;$$

where the first inequalities use the definition of $\widetilde{R}_{m2}$ and the second inequalities in both equations come from $p_H > p_L$. Hence $\left\{\widetilde{R}_{m2}, 0, 0\right\}$ is also optimal because the two IC are satisfied. ■

By definition, $\overline{V}^s$ is the largest number such that the manager's payoff satisfies the perceived IC constraint, $R_m^s \geq B/p'_H\Delta p'$, and the relevant IR, which is either $R_m^s \geq \gamma A_m/p_H^2$ or $R_m^s \geq (\gamma A_m - B)/p_H p_L$ depending on whether the manager's true IC constraint binds. Thus, the "if" statement in the proposition checks to see if the true IC constraint is met when the IR constraint binds. The manager's ability to extract value is constrained by the perceived IC and IR constraints.

First, if $\gamma A_m/p_H \geq B/\Delta p$, the true IC binds if the manager participates because $R_m^s \geq \gamma A_m/p_H^2 \Rightarrow R_m^s \geq B/p_H\Delta p$. Therefore the manager exerts effort and the relevant IR in this case is $R_m^s \geq \gamma A_m/p_H^2$. The highest willingness-to-pay is defined by the constraint that allows shareholders to extract the highest value. Since uninformed investors of the project expect to get $p_H'^2 R_u^s + 2p'_H(1-p'_H)R^s = 2\gamma(\overline{V}^s + I - A_m)$, and $R_u^s + 2R_m^s = 2R^s$, we can rewrite the perceived IC constraint as

$$R^s/p'_H - \gamma(V^s + I - A_m)/p_H'^2 \geq B/p'_H\Delta p'$$

and also the IR constraint as

$$R^s/p'_H - \gamma(V^s + I - A_m)/p_H'^2 \geq \gamma A_m/p_H^2.$$

It is easy to see that the maximum willingness-to-pay is constrained by the maximum value of the right hand side of the above constraints, which explains the max function in the proposition. If $B/p'_H\Delta p' \geq \gamma A_m/p_H^2$,

$$\overline{V}^s = \gamma^{-1}p'_H\left(R^s - B/\Delta p'\right) - I + A_m$$

and if $\gamma A_m/p_H^2 > B/p'_H\Delta p'$,

$$\overline{V}^s = \gamma^{-1}p'_H R^s - I + \left(1 - \frac{p_H'^2}{p_H^2}\right)A_m.$$

Secondly, if $\gamma A_m/p_H < B/\Delta p$ then $(\gamma A_m - B)/p_H p_L < \gamma A_m/p_H^2$. Therefore, if $B/p'_H\Delta p' < (\gamma A_m - B)/p_H p_L$ then the manager chooses the lower probability project and the relevant IR is $R_m^{pe} \geq \left(\gamma A_m - B\right)/p_H p_L$. If $B/p'_H\Delta p' \geq \left(\gamma A_m - B\right)/p_H p_L$,

$$\overline{V}^s = \gamma^{-1}p'_H\left(R^s - B/\Delta p'\right) - I + A_m$$

and if $(\gamma A_m - B)/p_H p_L > B/p'_H \Delta p'$,

$$\overline{V}^s = \gamma^{-1} p'_H (R^s + B/p_H p_L) - I + \left(1 - \frac{p'^2_H}{p_H p_L}\right) A_m.$$

Q.E.D.

Proof of Proposition 2.

By definition, $\overline{V}^{pe}$ is the largest number such that the manager's payoff satisfies the perceived IC constraint, $R^{pe}_m \geq b/\Delta p'$, and the relevant IR, which is either $R^{pe}_m \geq \gamma A_m/p_H$ or $R^{pe}_m \geq (\gamma A_m - B)/p_L$ depending on whether the manager's true IC constraint binds. Thus, the "if" statement in the proposition checks to see if the true IC constraint is met when the IR constraint binds. The manager's ability to extract value is constrained by the perceived IC and IR constraints.

First, if $\gamma A_m/p_H \geq b/\Delta p$, the true IC binds if the manager participates because $R^{pe}_m \geq \gamma A_m/p_H \Rightarrow R^{pe}_m \geq b/\Delta p$. Therefore the manager exerts effort and the relevant IR in this case is $R^{pe}_m \geq \gamma A_m/p_H$. The highest willingness-to-pay is defined by the constraint that allows shareholders to extract the highest value. Since uninformed investors of the project expect to get $p'_H R^{pe}_u = \gamma(\overline{V} + I - A_m)$, and $R^{pe} - R^{pe}_u = R^{pe}_m + R^{pe}_{pe}$, we can rewrite the perceived IC constraint as

$$R^{pe} - \gamma(V^{pe} + I - A_m)/p'_H \geq (b + c)/\Delta p'$$

and also the IR constraint as

$$R^{pe} - \gamma(V^{pe} + I - A_m)/p'_H \geq \gamma A_m/p_H + c/\Delta p.$$

It is easy to see that the maximum willingness-to-pay is constrained by the maximum value of the right hand side of the above constraints, which explains the max function in the proposition. If $(b + c)/\Delta p' \geq \gamma A_m/p_H + c/\Delta p$,

$$\overline{V}^{pe} = \gamma^{-1} p'_H (R^{pe} - (b + c)/\Delta p') - I + A_m,$$

and if $\gamma A_m/p_H + c/\Delta p > (b + c)/\Delta p'$,

$$\overline{V}^{pe} = \gamma^{-1} p'_H R^{pe} - I + (1 - p'_H/p_H) A_m - p'_H c/\Delta p.$$

Secondly, if $\gamma A_m/p_H < b/\Delta p$ then $(\gamma A_m - B)/p_H < (\gamma A_m - b)/p_H < \gamma A_m/p_L$. Therefore, if $(b + c)/\Delta p' < (\gamma A_m - B)/p_L$ then the manager chooses the lower probability project and the relevant IR is $R_m^{pe} \geq (\gamma A_m - B)/p_L$. If $(b+c)/\Delta p' \geq (\gamma A_m - B)/p_L$,

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left(R^{pe} - (b+c)/\Delta p'\right) - I + A_m$$

and if $(\gamma A_m - B)/p_L > (b+c)/\Delta p'$,

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left(R^{pe} + B/p_L\right) - I + \left(1 - p'_H/p_L\right) A_m.$$

Q.E.D.

Proof of Corollary 1. The result follows directly from Proposition 2 by simply setting $R^{pe} = R$, $c = 0$ and $b = B$. *Q.E.D.*

Proof of Corollary 2. As explained in the main text, when both projects are perfectly positively correlated there is no co-insurance of cash flows hence the difference $\overline{V}^{pe} - \overline{V}^{s}$ equals 0 (recall that we are assuming that $R^{pe} = R^{s}$). When the projects are independent (Proposition 3) that difference is instead proportional to $\left(p'_H/p_H\right)^2 - p'_H/p_H$. Therefore, for projects that are positively correlated the difference will be anywhere between the two values, or in other words, strategic acquirers will be more able to outbid financial buyers compared to the case of independent projects. *Q.E.D.*

Proof of Proposition 4.

From Corollary 1 we know that if $B/\Delta p'$ is larger than or equal to either $\gamma A_m/p_H$ or $(\gamma A_m - B)/p_L$ then

$$\overline{V} = \gamma^{-1} p'_H \left(R - B/\Delta p'\right) - I + A_m.$$

Note that from the perceived IC constraint, the moral hazard cost is equal to $B/\Delta p' = B/(1-\theta)\, p'_H$. Therefore, the perceived moral hazard cost declines with overvaluation:

$$\frac{\partial B/\Delta p'}{\partial \mu} = -\frac{B}{(1-\theta)\, p'^2_H} \frac{\partial p'_H}{\partial \mu},$$

whose sign is the opposite of the sign of $\partial p'_H/\partial \mu$, since $\theta < 1$.

Note that overvaluation loosens the perceived IC constraint, and moreover it is possible for the perceived IC constraint to hold at the same time as the true IC constraint does not. To show this, assume that $\gamma A_m < B/\Delta p$ (which implies that $(\gamma A_m - B)/p_L < \gamma A_m$). In this case, it is possible that $B/\Delta p' > \gamma A_m$, implying that investors believe that the manager is exerting effort and it is perceived to be individually rational to do so, but since $B/\Delta p' > B/\Delta p$, the true IC is not met. *Q.E.D.*

Proof of Proposition 5.

First, note that $b/\Delta p > \gamma A_m/p_H$ implies $B/\Delta p > \gamma A_m/p_H$. Therefore, the equation determining maximum willingness-to-pay by a strategic acquirer is given by (4b). And the equation that determines the highest willingness-to-pay of a PE buyer is given by (8b).

Second, overvaluation, $\mu > 0$, implies $\Delta p' > \Delta p$, $\forall \mu$. Therefore, $B/\Delta p' < B/\Delta p$, $\forall \mu$. Absent misvaluation, $B/\Delta p > (\gamma A_m - B)/p_L$ because $B/\Delta p > \gamma A_m/p_H$ implies $\gamma A_m/p_H > (\gamma A_m - B)/p_L$. However, with overvaluation, $B/\Delta p'$ can fall either above or below $(p'_H/p_H)(\gamma A_m - B)/p_L$. Therefore we need to evaluate two cases.

Case 1. If $B/\Delta p' \geq (p'_H/p_H)(\gamma A_m - B)/p_L$, then $(b+c)/\Delta p' > (p'_H/p_H)(\gamma A_m - B)/p_L > (\gamma A_m - B)/p_L$, and altogether, we must consider

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left[R^{pe} - (b+c)/\Delta p'\right] - I + A_m$$

and

$$\overline{V}^{s} = \gamma^{-1} p'_H (R^{s} - B/\Delta p') - I + A_m.$$

The difference is

$$\overline{V}^{pe} - \overline{V}^{s} = \gamma^{-1} p'_H \left[R^{pe} - R^{s} - (b+c-B)/\Delta p'\right],$$

and the set of parameter values such that $\overline{V}^{pe} - \overline{V}^{s} > 0$ include any set $\{R^{pe}, R^{s}, b, c, B, \Delta p'\}$ such that

$$R^{pe} - R^{s} > \frac{b+c-B}{\Delta p'} \equiv \Delta R^{*}(\mu). \tag{13}$$

Furthermore, for a given $\{b, c, B, \Delta p'\}$ there is a set $\{R^{pe}, R^{s}\}$ such that $\overline{V}^{pe} - \overline{V}^{s} > 0$, i.e., $R^{pe} - R^{s} > \Delta R^{*}(\mu)$.

Differentiating $\Delta R^{*}(\mu)$ with respect to the misvaluation parameter, μ, gives

$$\frac{\partial \Delta R^{*}(\mu)}{\partial \mu} = -\frac{b+c-B}{(1-\theta)\, p'^{2}_H} \frac{\partial p'_H}{\partial \mu} < 0,$$

since $b+c-B > 0$ and $\frac{\partial p'_H}{\partial \mu} > 0$. Therefore, overvaluation increases the set of $\{R^{pe}, R^{s}\}$ such that $R^{pe} - R^{s} > \Delta R^{*}(\mu)$. This proves the first part of the proposition. Moreover,

$$\frac{\partial \left(\overline{V}^{pe} - \overline{V}^{s}\right)}{\partial \mu} = \gamma^{-1} \frac{\partial p'_H}{\partial \mu} \left[R^{pe} - R^{s} - \Delta R^{*}(\mu)\right] - \gamma^{-1} p'_H \frac{\partial \Delta R^{*}(\mu)}{\partial \mu},$$

which is positive when $R^{pe} - R^{s} > \Delta R^{*}(\mu)$. This proves the second part of the proposition.

Case 2. If $B/\Delta p' < (p'_H/p_H)(\gamma A_m - B)/p_L$, then $(b+c)/\Delta p'$ can be either larger or smaller than $(\gamma A_m - B)/p_L$, and one must consider two subcases.

Subcase 2.1: $(b+c)/\Delta p' \geq (\gamma A_m - B)/p_L$. In

$$\overline{V}^{pe} - \overline{V}^{s} = \gamma^{-1} p'_H \left[R^{pe} - R^{s} - (b+c)/\Delta p' + p'_H (\gamma A_m - B)/p_H p_L \right],$$

and the set of parameters such that $\overline{V}^{pe} - \overline{V}^{s} > 0$ include any set $\{R^{pe}, R^{s}, b, c, B, p_H, p_L, p'_H, \theta, \gamma, A_m\}$ such that

$$R^{pe} - R^{s} > \frac{b+c}{\Delta p'} - \frac{p'_H(\gamma A_m - B)}{p_H p_L} \equiv \Delta R^*(\mu). \tag{14}$$

Furthermore, for a given $\{b, c, B, p_H, p_L, p'_H, \theta, \gamma, A_m,\}$ there is a set $\{R^{pe}, R^{s}\}$ such that $\overline{V}^{pe} - \overline{V}^{s} > 0$, i.e., $R^{pe} - R^{s} > \Delta R^*(\mu)$.

Proceeding as before, we differentiate $\Delta R^*(\mu)$ with respect to the misvaluation parameter, μ. The derivative is given by the expression

$$\frac{\partial \Delta R^*(\mu)}{\partial \mu} = -\frac{b+c}{(1-\theta)\, p'^2_H} \frac{\partial p'_H}{\partial \mu} - \frac{\gamma A_m - B}{p_H p_L} \frac{\partial p'_H}{\partial \mu} < 0,$$

since $\partial p'_H/\partial \mu > 0$ and $\gamma A_m - B > 0$. Therefore, overvaluation increases the set of $\{R^{pe}, R^{s}\}$ such that $R^{pe} - R^{s} > \Delta R^*(\mu)$. This proves the first part of the proposition. Moreover,

$$\frac{\partial \left(\overline{V}^{pe} - \overline{V}^{s} \right)}{\partial \mu} = \gamma^{-1} \frac{\partial p'_H}{\partial \mu} \left[R^{pe} - R^{s} - \Delta R^*(\mu) \right] - \gamma^{-1} p'_H \frac{\partial \Delta R^*(\mu)}{\partial \mu},$$

which is positive when $R^{pe} - R^{s} > \Delta R^*(\mu)$. This proves the second part of the proposition.

Subcase 2.2: $(b+c)/\Delta p' < (\gamma A_m - B)/p_L$. In this case, it is easy to verify that the difference in willingness to pay is given by

$$\overline{V}^{pe} - \overline{V}^{s} = \gamma^{-1} p'_H \left[R^{pe} - R^{s} - (\gamma A_m - B)(1 - p'_H/p_H)/p_L \right],$$

and the set of parameters such that $\overline{V}^{pe} - \overline{V}^{s} > 0$ include any set $\{R^{pe}, R^{s}, b, c, B, p_H, p_L, p'_H, \gamma, A_m\}$ such that

$$R^{pe} - R^{s} > \frac{\gamma A_m - B}{p_L} \left(1 - \frac{p'_H}{p_H} \right) \equiv \Delta R^*(\mu). \tag{15}$$

Furthermore, for a given $\{b, c, B, p_H, p_L, p'_H, \gamma, A_m,\}$ there is a set $\{R^{pe}, R^{s}\}$ such that $\overline{V}^{pe} - \overline{V}^{s} > 0$, i.e., $R^{pe} - R^{s} > \Delta R^*(\mu)$.

Proceeding as before, we differentiate $\Delta R^*(\mu)$ with respect to the misvaluation parameter, μ. The derivative is given by the expression

$$\frac{\partial \Delta R^*(\mu)}{\partial \mu} = -\frac{\gamma A_m - B}{p_L p_H}\frac{\partial p'_H}{\partial \mu} < 0,$$

since $\frac{\partial p'_H}{\partial \mu} > 0$ and $(\gamma A_m - B) > 0$. Therefore, overvaluation increases the set of $\{R^{pe}, R^s\}$ such that $R^{pe} - R^s > \Delta R^*(\mu)$. This proves the first part of the proposition. Moreover,

$$\frac{\partial\left(\overline{V}^{pe} - \overline{V}^{s}\right)}{\partial \mu} = \gamma^{-1}\frac{\partial p'_H}{\partial \mu}\left[R^{pe} - R^s - \Delta R^*(\mu)\right] - \gamma^{-1}p'_H\frac{\partial \Delta R^*(\mu)}{\partial \mu},$$

which is positive when $R^{pe} - R^s > \Delta R^*(\mu)$. This proves the second part of the proposition. *Q.E.D.*

Proof of Proposition 6. The first part of the proposition can be shown by taking the derivative of $\overline{A}_m^{pe}$ with respect to μ. The case of the stand-alone and strategic are parallel and we will not reproduce them here. Using (10) and deriving gives

$$\frac{\partial \overline{A}_m^{pe}}{\partial \mu} = -\gamma^{-1}R^{pe}\frac{\partial p'_H}{\partial \mu} < 0, \tag{16}$$

where we have used the simplification $\Delta p' = (1-\theta)\,p'_H$. For the second half of the proposition, first use (10) and (12) in order to express $\overline{A}_m^{pe} - \overline{A}_m^{s}$ as

$$\overline{A}_m^{pe} - \overline{A}_m^{s} = -\gamma^{-1}p'_H\left[R^{pe} - R^s - (b+c-B)/\Delta p'\right];$$

and the set of parameter values such that $\overline{A}_m^{pe} - \overline{A}_m^{s} < 0$ include any $\{R^{pe}, R^s, b, c, B, \Delta p'\}$ such that

$$R^{pe} > R^s + (b+c-B)\,\Delta p' \equiv \widetilde{R}(\mu).$$

Furthermore, for a given $\{b, c, B, \Delta p', R^s\}$ there is a set of values for R^{pe} such that $\overline{A}_m^{pe} - \overline{A}_m^{s} < 0$, i.e., $R^{pe} > \widetilde{R}(\mu)$. Taking derivatives, we find

$$\frac{\partial\left(\overline{A}_m^{pe} - \overline{A}_m^{s}\right)}{\partial \mu} = \gamma^{-1}\left(R^{pe} - R^s\right)\frac{\partial p'_H}{\partial \mu}.$$

Note that $R^{pe} > \widetilde{R}(\mu)$ implies $(R^{pe} - R^s) > 0$ because $b + c - B > 0$.

Proof of Corollary 3.

Taking derivatives of $\overline{A}_m - \overline{A}_m^{pe}$ with respect to μ results in the following expression:

$$\frac{\partial \left(\overline{A}_m - \overline{A}_m^{pe}\right)}{\partial \mu} = \gamma^{-1} \left(R^{pe} - R\right) \frac{\partial p'_H}{\partial \mu}.$$

If $R^{pe} - R > 0$ then $\partial \left(\overline{A}_m - \overline{A}_m^{pe}\right) / \partial \mu > 0$ when $\partial p'_H / \partial \mu > 0$.

Taking derivatives of $\overline{A}_m - \overline{A}_m^{s}$ with respect to μ results in the following expression:

$$\frac{\partial \left(\overline{A}_m - \overline{A}_m^{s}\right)}{\partial \mu} = \gamma^{-1} \left(R^{s} - R\right) \frac{\partial p'_H}{\partial \mu},$$

which is positive as long as $R^s > R$ when $\partial p'_H / \partial \mu > 0$. *Q.E.D.*

Proof of Proposition 7.

A first step requires rewriting Proposition 2 to take into account the financial buyer's decision to contribute her own funds in the deal. The lemma below is the equivalent of Proposition 2 with (endogenously determined) PE capital, once we take into account that the PE only invests her own capital A_{pe} if her IC constraint holds (which can only occur when the manager's IC holds). In which case the PE capital, A_{pe}, is determined in equilibrium by the IR constraint, provided that this increases the willingness-to-pay (if adding more capital decreases the offer price then the PE firm adds no capital).

Lemma 1 *Including A_{pe} will modify the amount borrowed from uninformed investors and also the IR for the PE manager, therefore, (8a) becomes*

$$R^{pe} - \gamma \left(V^{pe} + I - A_m - A_{pe}\right) / p'_H = \max\left[(b+c)/\Delta p', \gamma A_m / p_H + \gamma_{pe} A_{pe} / p_H\right] \qquad if \;\; \gamma A_m / p_H \geq b / \Delta p$$

and (8b) can be rewritten as

$$R^{pe} - \gamma \left(V^{pe} + I - A_m - A_{pe}\right) / p'_H = \max\left[(b+c)/\Delta p', \left(\gamma A_m - B\right) / p_L\right] \qquad if \;\; \gamma A_m / p_H < b / \Delta p.$$

If $\gamma A_m / p_H \geq b / \Delta p$, there are two cases that we must assess. If $\max[(b+c)/\Delta p', \gamma A_m / p_H + \gamma_{pe} A_{pe} / p_H] = (b+c)/\Delta p'$, then A_{pe} increases the bid and equals $p_H c / \gamma_{pe} \Delta p$, based on the PE IR constraint, which is given by

$$p_H R_{pe}^{pe} = \gamma_{pe} A_{pe}$$

The true IC constraint is

$$R_{pe}^{pe} \geq c/\Delta p.$$

Therefore the PE's willingness to pay is given by

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left[R^{pe} - (b+c) / \Delta p' \right] - I + A_m + p_H c / \gamma_{pe}^{-1} \Delta p.$$

On the other hand if $\max[(b+c)/\Delta p', \gamma A_m/p_H + \gamma_{pe} A_{pe}/p_H] = \gamma A_m/p_H + \gamma_{pe} A_{pe}/p_H$ then

$$\overline{V}^{pe} = p'_H R^{pe}/\gamma - I + A_m + A_{pe} - p'_H A_m/p_H - p'_H \gamma_{pe} A_{pe}/\gamma p_H.$$

Since $p'_H > p_H$ and $\gamma_{pe} > \gamma$ then A_{pe} negatively affects the maximum bidding price, and in equilibrium $A_{pe} = 0$, and

$$\overline{V}^{pe} = p'_H R^{pe}/\gamma - I + A_m - p'_H A_m/p_H$$

If $\max[(b+c)/\Delta p', (\gamma A_m - B)/p_L] = (\gamma A_m - B)/p_L$, then when $\gamma A_m/p_H < b/\Delta p$ the manager's true IC does not hold, and therefore the PE manager does not monitor and $A_{pe} = 0$. Therefore,

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left(R^{pe} + B/p_L \right) - I + \left(1 - p'_H/p_L \right) A_m.$$

If $\max[(b+c)/\Delta p', (\gamma A_m - B)/p_L] = (b+c)/\Delta p'$, then

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left[R^{pe} - (b+c)/\Delta p' \right] - I + A_m + A_{pe}.$$

Furthermore, if $(b+c)/\Delta p' > \gamma A_m/p_H + \gamma_{pe} A_{pe}/p_H$ (with $A_{pe} = p_H c/\gamma_{pe} \Delta p$) then the IC holds and

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left[R^{pe} - (b+c)/\Delta p' \right] - I + A_m + p_H c/\gamma_{pe} \Delta p.$$

otherwise the IC does not hold and $A_{pe} = 0$ so

$$\overline{V}^{pe} = \gamma^{-1} p'_H \left[R^{pe} - (b+c)/\Delta p' \right] - I + A_m.$$

As we can see from comparing the new price expressions with Proposition 2, $\overline{V}^{pe}$ weakly increases in the amount $A_{pe} = p_H c/\gamma_{pe} \Delta p$. In other words, $\overline{V}^{pe}$ increases by $p_H c/\gamma_{pe} \Delta p$ when $A_{pe} > 0$. Otherwise it is equal to the case with no PE capital. This proves the first part of the proposition. Furthermore, since

$$\overline{V}^{pe} = \overline{V}^{pe}(\text{No PE capital}) + p_H c/\gamma_{pe}\Delta p,$$

it is immediate to realize that

$$\frac{\partial \overline{V}^{pe}}{\partial \mu} = \frac{\partial \overline{V}^{pe}(\text{No PE capital})}{\partial \mu},$$

which concludes the proof. $Q.E.D.$