Supplementary Material to:

Good Volatility, Bad Volatility, and the Cross-Section of Stock Returns

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A-1. The Distribution of Realized Moments

Fig. A.1: Realized Measures Sorted by REV.
Panels A-D display the 10-week moving average time series of the RSJ, RVOL, RSK and RKT realized measures within REV terciles.

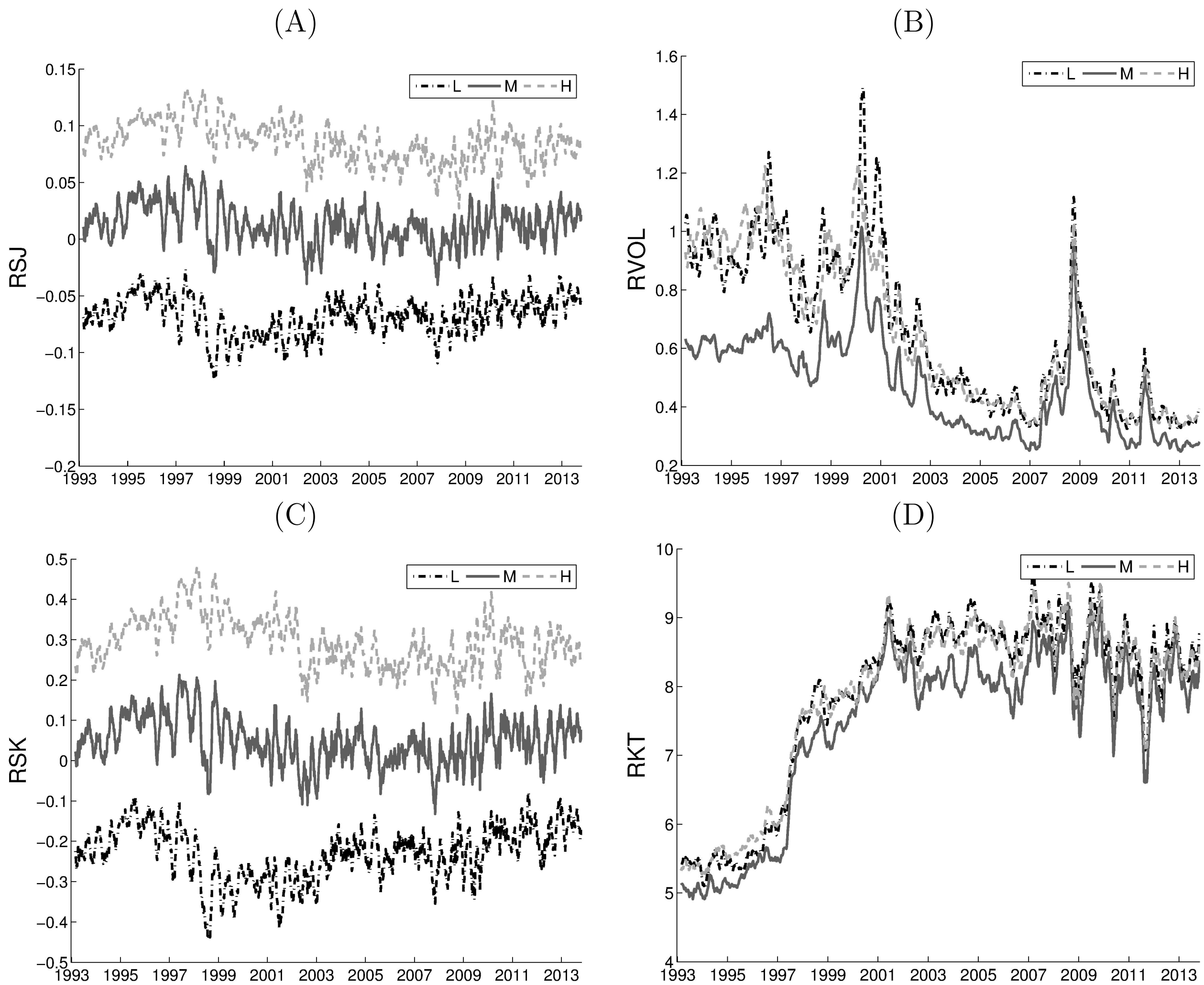


Fig. A.2: Realized Measures Sorted by RSJ.

Panels A-D display the 10-week moving average time series of the RSJ, RVOL, RSK and RKT realized measures within RSJ terciles.

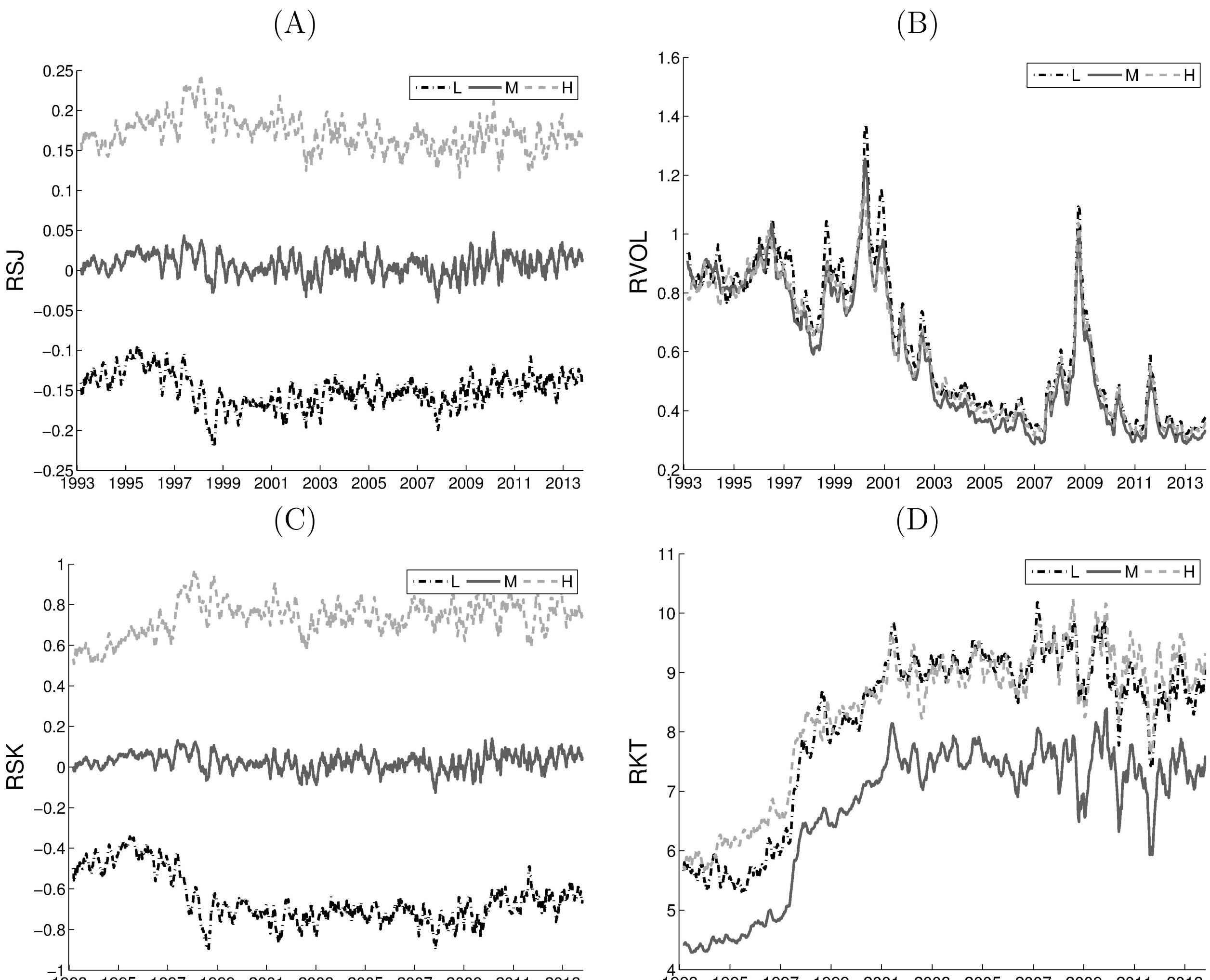
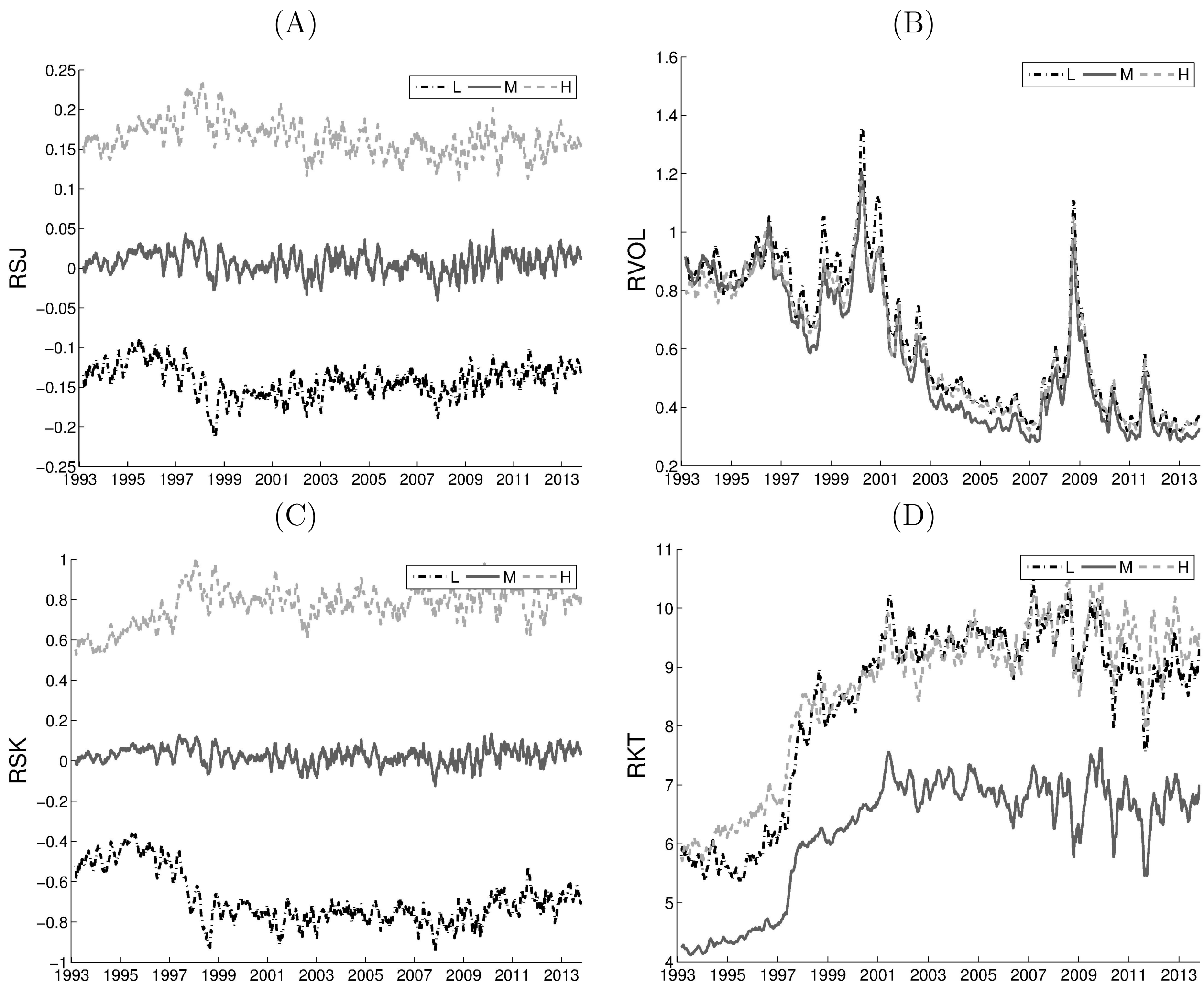


Fig. A.3: Realized Measures Sorted by RSK.

Panels A-D display the 10-week moving average time series of the RSJ, RVOL, RSK and RKT realized measures within RSK terciles.



A-2. Predictive Single Sorts for Alternative Estimators, Samples, and Returns

The main empirical results discussed in the paper are based on weekly realized variation measures spanning Tuesday-close to Tuesday-close constructed from five-minute transaction prices for all of the NYSE, NASDAQ and AMEX listed stocks. To assess the robustness of our results to these specific choices, this section reports the results in which we construct the realized measures based on mid-quotes, alternative subsampling procedures, restrict the sample to NYSE listed stocks only, rely on Monday-close to Monday-close or Wednesday-close to Wednesday-close realized measures and returns, as well as longer four-week prediction horizons. We begin by examining the sensitivity to the use of quote prices instead of transaction prices.

The bid-ask bounce effect in transaction prices tends to inflate the standard realized variation measure as well as the up and down semi-variance measures used in the calculation of the relative signed jump variation. Following common practice in the literature, our choice of a "coarse" 5-minute sampling frequency was explicitly intended to help mitigate these biases. To help further assess and reduce the impact of the bid-ask bounce effect, Panel A of Table A.1 reports the results for single-sorted portfolios in which the RSJ measures are calculated based on mid-quote returns instead of transaction price returns. The FFC4-adjusted High-Low spread for these alternative sorts equals -31.54 bps, with a t-statistic of -5.76, for the value-weighted portfolios, and -39.90 bps, with a t-statistic of -10.24, for the equal-weighted portfolios. Both the magnitude and the statistical significance of these abnormal returns are very similar to those reported in Panel A of Table 3 in the paper based on high-frequency transaction price returns.

A number of alternative realized variation estimators have also been proposed in the literature to help guard against the adverse effect of market microstructure “noise,” including the bid-ask bounce effect. The subsampling methodology originally proposed by Zhang, Mykland, and Ait-Sahalia (2005), in particular, is designed to improve on the efficiency of sparsely-sampled realized volatility measures by averaging the estimators obtained across different sampling schemes. Our implementation of this alternative estimator relies on six different grids of 30-minute returns (starting from 9:30 a.m., 9:35 a.m., 9:40 a.m., 9:45 a.m., 9:50 a.m. and 9:55 a.m., respectively) from which we calculate six different daily relative signed jump variation estimators. We then average these six different daily estimators to obtain a single subsampled daily RSJ, which we use to compute a weekly subsampled RSJ as in equation (10). Panel B of Table A.1 reports the results from the single sorts based on this weekly subsampled RSJ estimator. Interestingly, the predictability becomes even stronger. The FFC4-adjusted High-Low spreads now equals -40.66 bps, with a t-statistic of -7.18, for the value-weighted portfolios, and -47.04 bps, with a t-statistic of -10.90, for the equal-weighted portfolios, all of which exceed (in an absolute value sense) the corresponding values based on the simple RSJ estimator reported in Panel A of Table 3 in the paper.

Our previous results were based on all of the stocks listed on the NYSE, NASDAQ, and AMEX exchanges. The results reported in Panel C in Table A.1 show that our findings of a negative relation between the relative signed jumps and the future returns remain intact in a sample consisting solely of NYSE listed stocks. In particular, even though the difference in the average abnormal returns between the High and Low RSJ-sorted portfolios based exclusively on the NYSE listed stocks equal to -25.27 bps, with a t-statistic of -5.21, for the value-weighted portfolios, and -13.60 bps, with a t-statistic of -3.93, for the equal-weighted portfolios, are slightly weaker, the results are still very much in line with those obtained for the larger sample comprised of all the stocks. As a general rule, the NYSE listed firms tend be larger and more liquid, thus again underscoring that our results aren't merely driven by market frictions and issues having to do with lack of liquidity.

Our empirical analysis so far has been based on weekly realized variation measures and subsequent weekly returns calculated from Tuesday-close to Tuesday-close. Alternatively, we estimate the weekly RSJ measures using the intraday returns between Monday-close and Monday-close, and between Wednesday-close and Wednesday-close, and then forecast the subsequent one-week-ahead returns. Panels D and E of Table A.1 present the results based on these two alternative weekly portfolio formation and return holding periods. For the Monday-close to Monday-close sorts, the FFC4-adjusted High-Low spread equals -36.22 bps, with a t-statistic of -7.13, for the value-weighted portfolios, and -44.12 bps, with a t-statistic of -10.98 for the equal-weighted portfolios. Both the spread and the t-statistic thus demonstrate a slightly stronger predictability of RSJ than the previously reported Tuesday-close to Tuesday-close results in Panel A of Table 3 in the paper. On the other hand, for the Wednesday-close to Wednesday-close sorts reported in Panel E, the predictability of RSJ is slightly weaker, albeit still highly statistically and economically significant.

The final set of results reported in Panel F of Table A.1 expand the weekly return horizon to a longer four-week holding period. For each portfolio formation week, we form quintile portfolios based on that week's RSJ measures, and then compute the value- and equal-weighted cumulative returns over the subsequent four weeks.[1] The average risk adjusted return spread between the resulting High and Low quintile portfolios equals -23.85 bps, with a t-statistic of -2.26, for the value-weighted portfolios. For the equal-weighted portfolios, the spread is -49.90 bps with a t-statistic of -5.37. Thus, even though the weekly RSJ-based portfolio formation continues to generate abnormal returns over this longer four-week return horizon, the results are obviously not as strong as the results for predicting the one-week-ahead returns.[2] The fact that the return predictability diminishes with the return horizon directly illustrates that the information in the firm specific RSJ measures is relatively short-lived, and as such indirectly suggests that the information isn't necessarily related to any underlying systematic risk factors.

[1]By construction, this results in a three-week overlap between any two consecutive four-week returns, and return spreads. We rely on the Newey–West (1987) robust standard errors to adjust for this.

[2]The one-month-ahead return spreads based on the realized skewness measure RSK reported in ACJV are also not as significant as the one-week-ahead return spreads.

Table A.1: Predictive Single Sorts Based on Alternative Estimators, Samples, and Returns

This table reports the average returns for predictive single-sorted portfolios based on RSJ. At the end of each week, stocks are sorted into quintiles according to RSJ computed from the previous week's high-frequency returns. In Panel A, we use the high-frequency mid-quote prices to estimate weekly RSJ. In Panel B, we use RSJ estimated by subsampling methodology. In Panel C, we only consider stocks listed on NYSE. In Panel D/E, we compute RSJ estimates based on intraday data between Monday/Wednesday close and Monday/Wednesday close period and then forecast the subsequent weekly returns. In Panel F, we forecast the four-week ahead cumulative returns. The column labeled "Return" reports the average one-week or four-week ahead excess returns of each portfolio. The column labeled "FFC4" reports the corresponding Fama-French-Carhart four-factor alpha for each portfolio. The row labeled "High-Low" reports the difference in returns between portfolio 5 and portfolio 1, with Newey–West (1987) robust t-statistics in parentheses. In each panel, the first two columns report the value-weighted sorting results and the last two columns report the equal-weighted sorting results.

Panel A: RSJ Estimated by Mid-Quote Returns

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	34.91	18.24	57.67	38.07
2	25.92	10.06	34.31	14.98
3	14.49	-0.98	26.90	7.70
4	6.35	-9.21	21.56	2.58
5(High)	2.48	-13.30	17.00	-1.83
High-Low	-32.43	-31.54	-40.68	-39.90
	(-5.88)	(-5.76)	(-9.83)	(-10.24)

Panel B: RSJ Estimated by Subsampling

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	38.05	21.87	61.17	41.66
2	23.61	7.71	38.02	18.57
3	14.66	-1.08	25.60	6.12
4	9.91	-5.53	20.94	2.08
5(High)	-2.98	-18.79	12.88	-5.39
High-Low	-41.03	-40.66	-48.28	-47.04
	(-7.28)	(-7.18)	(-10.55)	(-10.90)

Panel C: NYSE Sample

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	28.91	12.73	34.85	14.67
2	22.74	6.31	29.05	9.21
3	13.44	-2.43	25.99	6.22
4	8.27	-7.72	20.96	1.42
5(High)	3.30	-12.54	20.17	1.07
High-Low	-25.61	-25.27	-14.69	-13.60
	(-5.33)	(-5.21)	(-4.22)	(-3.92)

Panel D: Monday-Close to Monday-Close

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	37.62	20.79	58.04	38.73
2	24.37	7.58	41.40	21.30
3	20.45	4.30	29.63	9.61
4	8.38	-7.78	21.24	1.36
5(High)	0.29	-15.42	13.50	-5.39
High-Low	-37.34	-36.22	-44.54	-44.12
	(-7.10)	(-7.13)	(-10.35)	(-10.98)

Panel E: Wednesday-Close to Wednesday-Close

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	32.60	16.32	50.55	31.57
2	22.64	6.89	37.21	18.02
3	19.46	4.13	30.76	11.58
4	9.75	-5.58	21.30	2.36
5(High)	4.83	-10.94	18.40	-0.44
High-Low	-27.76	-27.26	-32.15	-32.01
	(-5.71)	(-5.52)	(-8.59)	(-8.64)

Panel F: Four-Week Holding Period Return

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	77.63	15.05	139.9	71.57
2	74.47	13.67	118.16	48.98
3	57.87	-0.09	106.8	37.97
4	51.36	-6.83	93.21	23.86
5(High)	46.67	-8.8	90.62	21.67
High-Low	-30.96	-23.85	-49.28	-49.9
	(-2.98)	(-2.26)	(-5.19)	(-5.37)

A-3. Double-sorted portfolios with additional controls

To further investigate the robustness of our findings, we document the results from a series of additional double sorts based on the same sequential sorting approach employed in Section C, using other control variables. To conserve space, we do not report the average returns for all the 25 portfolios for each of the different sorts, focusing instead on the returns for the quintile portfolios averaged across the first sorting variable to produce portfolios with large variation in the second sorting variable, but small variation in the control.

Panel A of Table A.2 reports the results in which the final sorts are based on RSJ. The strong predictability of RSJ is preserved across all of the different control variables. The value-weighted average weekly return spread between the High and Low quintile portfolios ranges from -15.32 bps, with a t-statistic of -3.83, after controlling for REV, to -40.07 bps, with a t-statistic of -8.09, after controlling for RVOL. The spreads in the FFC4 alphas are almost always very close to those for the raw returns, both in terms of their absolute magnitudes and the t-statistics. The equal-weighted average weekly return spread ranges from -17.32 bps, with a t-statistic of -6.29, after controlling for REV, to -39.67 bps, with a t-statistic of -9.96, after controlling for IVOL. The Fama-French-Carhart four factors again provide little explanation for these return spreads between the equal-weighted High and Low RSJ-sorted portfolios. The past week's return and the realized skewness offer the highest explanatory power for the RSJ effect, as manifested by the smallest value-weighted FFC4 spreads of -21.39 bps in the RSK column and -14.35 bps in the REV column.

However, both of these spreads remain highly statistically significant and economically large, further corroborating the idea that RSJ contains unique predictive information.

Panel B shows the results for the corresponding sequential portfolio sorts, in which the final sorts are based on RSK. Consistent with the evidence in ACJV, the negative relation between RSK and future returns generally remains intact across the various control variables. At the same time, however, the abnormal returns and the associated t-statistics are generally smaller than their counterparts based on the RSJ double sorts reported in Panel A. Importantly, as discussed in the main text, the double sorts that control for RSJ completely changes the sign, and reveals a statistically significant *positive* relation between the realized skewness and the future returns. Also, after controlling for REV, the value-weighted return spread for the High-Low RSK-sorted portfolios and the corresponding FFC4 alpha are no longer significant. By contrast, RSJ-based sequential sorts always result in the same highly statistically significant negative return predictability across *all* of the controls.

Table A.2: Predictive Double-Sorted Portfolios with Additional Controls

The table reports the average returns for predictive double-sorted portfolios. The sample consists of all the NYSE, AMEX and NASDAQ listed common stocks with share codes 10 or 11 and prices between \$5 and \$1,000 over the 1993-2013 sample period. For each week, all stocks in the sample are first sorted into five quintiles on the basis of one control variable. Within each quintile, the stocks are then sorted into five quintiles according to their RSJ/RSK. These five RSJ/RSK portfolios are then averaged across the five control variable portfolios to produce RSJ/RSK portfolios with large cross-portfolio variation in their RSJ/RSK but little variation in the control variable. RSJ, RVOL, RSK, and RKT denote the relative signed jump, realized volatility, realized skewness, and realized kurtosis, respectively. BETA denotes the standard CAPM beta. ME denotes the logarithm of the market capitalization of the firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from day $t-252$ through day $t-21$. REV is the lagged one-week return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are the measures of coskewness and cokurtosis, respectively. MAX and MIN represent the maximum and minimum daily raw returns over the previous week. ILLIQ refers to the logarithm of the average daily ratio of the absolute stock return to the dollar trading volume over the previous week. The first five rows in both value-weighted and equal-weighted sorting results report time-series averages of weekly excess returns for the RSJ/RSK quintile portfolios. The row labeled "High-Low" reports the difference in the returns between portfolio 5 and portfolio 1. The row labeled "FFC4" reports the average Fama-French-Carhart four-factor alphas. The corresponding Newey–West (1987) robust t-statistics are reported in parentheses. Panels A and B display the results for the portfolios first sorted by the control variables listed in the columns and then by RSJ or RSK, respectively.

Panel A: Final Sorted by RSJ

Quintile	RVOL	RSK	RKT	BETA	ME	BM	MOM	REV	IVOL	CSK	CKT	MAX	MIN	ILLIQ
							Value-Weighted							
1(Low)	42.96	30.53	34.14	35.34	47.63	33.01	33.50	25.00	35.63	33.11	32.75	32.71	32.61	42.15
2	27.88	20.37	21.67	23.19	38.50	22.49	25.00	20.00	25.00	23.70	21.55	22.61	23.97	28.95
3	21.96	16.81	14.83	15.92	28.24	16.68	15.98	16.28	15.36	13.56	15.16	14.66	14.67	21.17
4	9.32	12.33	12.32	10.72	19.72	11.94	9.49	15.40	8.55	9.13	9.67	12.42	13.92	15.17
5(High)	2.89	8.15	7.06	2.68	14.41	6.10	3.36	9.68	-1.28	3.10	4.09	2.66	7.78	9.34
High-Low	-40.07	-22.38	-27.08	-32.66	-33.22	-26.91	-30.14	-15.32	-36.91	-30.01	-28.66	-30.04	-24.83	-32.81
	(-8.09)	(-5.43)	(-6.07)	(-8.05)	(-8.63)	(-6.04)	(-6.62)	(-3.83)	(-7.47)	(-6.30)	(-6.16)	(-5.76)	(-6.17)	(-8.09)
FFC4	-40.03	-21.39	-26.38	-32.58	-32.58	-26.11	-29.66	-14.35	-36.74	-29.25	-27.77	-28.64	-25.04	-32.21
	(-8.19)	(-5.16)	(-5.98)	(-8.14)	(-8.77)	(-5.89)	(-6.73)	(-3.60)	(-7.57)	(-6.28)	(-5.98)	(-6.18)	(-6.06)	(-8.16)
							Equal-Weighted							
1(Low)	53.90	45.74	54.47	53.73	50.25	51.71	54.00	40.41	47.07	46.74	46.38	52.47	47.72	56.34
2	39.59	38.27	38.71	36.53	39.87	37.41	37.82	35.31	32.86	31.61	31.61	37.58	38.05	38.33
3	31.48	31.27	31.00	30.95	30.98	30.02	30.87	30.76	25.72	24.08	24.29	29.78	31.07	30.40
4	22.85	28.47	23.75	21.21	22.55	20.86	20.59	27.40	17.11	17.72	18.14	22.31	26.59	22.28
5(High)	16.67	20.46	16.43	15.59	16.88	16.62	16.38	23.10	7.40	9.84	9.57	18.63	17.05	16.87
High-Low	-37.23	-25.28	-38.03	-38.14	-33.36	-35.10	-37.62	-17.32	-39.67	-36.90	-36.81	-33.84	-30.67	-39.47
	(-10.89)	(-7.40)	(-8.79)	(-11.03)	(-8.60)	(-9.39)	(-10.59)	(-6.29)	(-9.96)	(-7.85)	(-8.02)	(-7.61)	(-10.26)	(-8.67)
FFC4	-36.68	-24.22	-37.25	-37.60	-32.74	-34.22	-36.95	-17.02	-38.92	-36.13	-36.01	-32.14	-31.08	-39.00
	(-11.07)	(-7.38)	(-9.09)	(-11.19)	(-8.78)	(-9.39)	(-11.12)	(-6.28)	(-10.02)	(-8.04)	(-8.16)	(-8.71)	(-10.31)	(-8.88)

Panel B: Final Sorted by RSK

Quintile	RSJ	RVOL	RKT	BETA	ME	BM	MOM	REV	IVOL	CSK	CKT	MAX	MIN	ILLIQ
							Value-Weighted							
1(Low)	13.07	35.08	28.62	28.83	43.20	27.11	26.82	21.15	29.71	28.68	27.61	26.93	28.45	37.26
2	15.05	31.77	20.86	21.19	37.60	22.45	24.76	18.22	25.85	21.79	23.79	21.34	21.80	29.62
3	14.57	18.18	16.73	17.07	28.11	17.38	16.62	18.14	14.45	16.45	13.42	16.50	17.25	20.41
4	20.90	13.73	15.42	11.89	22.71	12.73	11.48	14.74	11.43	10.23	11.03	11.12	15.49	16.97
5(High)	24.91	7.67	9.00	6.18	16.75	10.12	6.94	16.49	4.04	5.53	7.91	8.98	11.42	12.10
High-Low	11.84	-27.41	-19.62	-22.65	-26.46	-16.99	-19.88	-4.66	-25.67	-23.15	-19.69	-17.95	-17.03	-25.16
	(3.50)	(-6.94)	(-5.19)	(-6.64)	(-8.77)	(-4.45)	(-5.50)	(-1.32)	(-6.41)	(-5.86)	(-5.20)	(-4.35)	(-4.86)	(-7.72)
FFC4	11.73	-26.67	-18.45	-22.66	-26.01	-16.05	-19.20	-3.70	-24.80	-22.50	-18.67	-16.39	-16.89	-24.43
	(3.45)	(-6.74)	(-4.91)	(-6.65)	(-8.89)	(-4.17)	(-5.45)	(-1.04)	(-6.23)	(-5.69)	(-4.89)	(-4.25)	(-4.72)	(-7.68)
							Equal-Weighted							
1(Low)	30.08	48.96	49.99	49.34	46.23	47.69	49.31	38.61	42.48	43.02	42.13	47.44	44.61	51.71
2	32.06	39.50	38.84	36.31	39.12	37.26	37.02	34.16	32.29	30.65	31.72	36.90	37.44	38.60
3	31.58	30.93	30.14	29.51	30.76	28.92	29.56	31.16	23.44	22.95	22.28	30.18	30.61	29.24
4	33.77	25.52	27.10	24.11	25.37	23.55	24.07	26.42	20.48	20.55	19.52	24.29	27.00	25.35
5(High)	37.15	19.60	18.41	18.70	19.13	19.34	19.65	26.58	11.81	13.40	14.31	21.78	20.82	19.36
High-Low	7.07	-29.36	-31.57	-30.63	-27.10	-28.35	-29.66	-12.03	-30.66	-29.62	-27.82	-25.66	-23.79	-32.35
	(2.93)	(-10.95)	(-8.72)	(-11.07)	(-8.96)	(-9.53)	(-10.27)	(-5.16)	(-9.80)	(-7.87)	(-7.72)	(-7.40)	(-9.58)	(-8.90)
FFC4	6.27	-29.11	-31.11	-30.38	-26.87	-28.08	-29.46	-11.81	-30.34	-29.35	-27.52	-24.55	-24.25	-32.15
	(2.63)	(-11.10)	(-9.02)	(-11.17)	(-9.20)	(-9.62)	(-10.74)	(-5.09)	(-9.88)	(-8.07)	(-7.89)	(-8.30)	(-9.76)	(-9.16)

A-4. Relative Signed Jumps and Other Firm Characteristics

This section reports the results from additional cross-sectional regressions designed to investigate the relationship between the relative signed jump variation measure RSJ and other firm characteristics. Motivated by previous empirical findings related to the cross-sectional variation in standard measures of return skewness (e.g., Hong, Wang, and Yu (2008) and Engle and Mistry (2014)), we focus on size (ME), book-to-market ratio (BM), leverage (LEVERAGE), and credit rating (CREDIT). ME and BM are computed as discussed in the main text. LEVERAGE refers to the ratio of book value of debt over market value of equity.[3] CREDIT refers to the Standard and Poor's credit rating available monthly from Compustat.[4]

Turning to the results, the coefficient associated with market size ME in Regression I is small and insignificant. In Regression II the coefficient of BM equals 0.0044, with a t-statistic of 5.57. The average cross-sectional standard deviations of BM and RSJ in our sample are 0.63 and 0.16, respectively. A two-standard deviation increase in BM thus leads to an increase in RSJ of $2 \times 0.63 \times 0.0044$, or about 3.5% of the standard deviation of RSJ. In Regression III the coefficient for LEVERAGE equals -0.0002, with a t-statistic of -2.04, again suggesting that a firm's leverage is negatively related to it's relative signed jump

[3]The book value of debt is the total asset, minus the book value of common equity, minus deferred taxes and investment tax credit.

[4]Following Amaya, Christoffersen, Jacobs, and Vasquez (2016), we assign numerical grades to the ratings: 1-AAA, 2-AA+, 3-AA, 4-AA-,5-A+, until 22-D.

variation. However, the economic effect is small. A two-standard deviation increase in LEVERAGE is associated with a decrease in the firm RSJ of only $2 \times 3.72 \times 0.0002$, or around 1% of the standard deviation of RSJ. Lastly, the t-statistic of -1.13 in Regression IV indicates that CREDIT is not significantly related to RSJ. All-in-all, no strong empirical relationship between RSJ and the other explanatory variables emerges from these regressions.

Table A.3: Fama-MacBeth Cross-Sectional Regressions for RSJ

The table displays the average estimated regression coefficients and robust t-statistics (in parentheses) from Fama-MacBeth cross-sectional regressions for the weekly RSJs. The sample consists of all the NYSE, AMEX and NASDAQ listed common stocks with share codes 10 or 11 and prices between $5 and $1,000 over the 1993-2013 sample period. ME refers to the logarithm of the market capitalization of the firms. BM denotes the ratio of the book value of common equity to the market value of equity. LEVERAGE denotes book debt divided by market equity. CREDIT is assigned a numerical value as follows: AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21 and D=22.

Regression	ME	BM	LEVERAGE	CREDIT
I	-0.0002			
	(-0.70)			
II		0.0044		
		(5.57)		
III			-0.0002	
			(-2.04)	
IV				-0.0001
				(-1.13)
V	-0.0005	0.0061	-0.0005	-0.0004
	(-1.43)	(7.43)	(-4.27)	(-2.67)

A-5. Frequency Analysis of RSJ and RSK

The actual sample values of the different realized measures invariably depend on the sampling frequency of the returns used in their estimation. Along these lines, RSJ generally affords more robust measurements than RSK. The table below, in particular, presents the correlations between weekly RSJs and RSKs computed with returns sampled at different frequencies. As the table shows, the correlations between the differently estimated weekly RSJ measures are in general higher than the correlations among the weekly RSK measures.

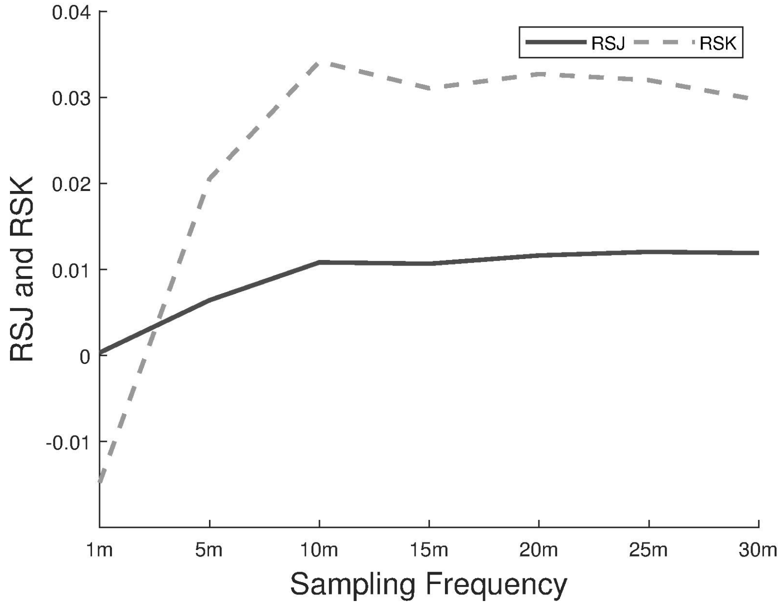
Panel A: Weekly RSJ Correlation							
	1m	5m	10m	15m	20m	25m	30m
1m	1	0.88**	0.82**	0.79**	0.78**	0.76**	0.75**
5m		1	0.93**	0.90**	0.88**	0.86**	0.85**
10m			1	0.93**	0.93**	0.90**	0.89**
15m				1	0.93**	0.93**	0.93**
20m					1	0.93**	0.93**
25m						1	0.93**
30m							1

Panel B: Weekly RSK Correlation							
	1m	5m	10m	15m	20m	25m	30m
1m	1	0.78**	0.68**	0.63**	0.61**	0.58**	0.56**
5m		1	0.88**	0.81**	0.78**	0.75**	0.72**
10m			1	0.87**	0.88**	0.82**	0.80**
15m				1	0.87**	0.87**	0.86**
20m					1	0.86**	0.86**
25m						1	0.86**
30m							1

The signature plots below, depicting the time-series averages of the cross-sectional means of RSJ and RSK for sampling frequencies ranging from 1 minute to 30 minutes, further underscore this same robustness of RSJ vis-a-vis RSK.

Fig. A.4: Signature Plots of RSJ and RSK.

This figure shows the signature plots of weekly RSJ and RSK computed from different return frequencies ranging from 1 minute to 30 minutes.



As discussed in the main text, the RSJ measure also provides a much "cleaner" and easier to interpret asymptotic limit (for increasingly higher sampling frequencies). By contrast, the expected value of RSK depends on the sampling frequency used in the estimation. Koike and Liu (2018), "Asymptotic properties of the realized skewness and related statistics," *Annals of the Institute of Statistical Mathematics*, provides a recent attempt at establishing a more complicated limit theory (for increasingly higher sampling frequencies) for the RSK measure.

A-6. Relative Signed Return

It is possible that the superiority of RSJ compared to RSK stem from the fact that RSJ only uses second powers of high-frequency returns, while RSK relies on third powers. If so, the use of absolute returns instead of squared returns in the definition of positive and negative semi-variances in equation (4) could possibly result in even stronger predictability results.[5]

To investigate this conjecture, we replace the squared returns in the calculation of the RSJ measure with the corresponding absolute returns, referring to this alternative measure as relative signed return (RSR). Not surprisingly, RSR does indeed predict future returns, as is evident by the single-sorts in the table below.

Single Sort by RSR

	Value-Weighted		Equal-Weighted	
Quintile	Return	FFC4	Return	FFC4
1(Low)	37.15	20.83	58.22	38.82
2	23.55	7.81	39.54	20.21
3	15.26	-0.59	26.27	6.88
4	9.22	-6.39	20.99	2.03
5(High)	-2.11	-17.97	13.60	-4.90
High-Low	-39.26	-38.80	-44.62	-43.72
	(-6.73)	(-6.69)	(-9.61)	(-9.97)

These predictability results for RSR also closely mirror the RSJ-based single-sorts reported in Table 3 in the paper. Furthermore, in Fama-MacBeth regressions that simultaneously control for RSJ, RSK and RSR, both RSJ and RSR significantly (and negatively) predict the one-week-ahead returns, while the effect of RSK is again reversed.

[5]We thank the referee for this suggestion.

Meanwhile, the asymptotic limit (for increasingly higher sampling frequencies) of RSJ is much "cleaner" and easier to understand than the limit for RSR. In particular, as discussed in the main text, in the absence of jumps, RSJ should be identically equal to zero (again, under the usual in-fill asymptotic arguments). By contrast, the unscaled sum of the absolute high-frequency returns formally diverges for increasingly finer sampled returns (the sum needs to be scaled by $\Delta^{1/2}$, where $\Delta = 1/M$ denotes the length of the high-frequency return interval). However, if we scale the sum of the absolute returns by $\Delta^{1/2}$ in order to obtain a well-defined limit, that same scaling then "kills" the jumps. Formally, for the jump-diffusion process,

$$dp(t) = \mu(t) + \sigma(t)dW(t) + \kappa(t)dq(t),$$

it follows that for $M \rightarrow \infty$ (see, e.g., Barndorff-Nielsen and Shephard (2004), "Power and bipower variation with stochastic volatility and jumps," *Journal of Financial Econometrics*),

$$\mathrm{RV}_t \xrightarrow{M\rightarrow\infty} \mathrm{IV}_t + \Sigma_{s\in[0,t]:dq(s)=1}\kappa^2(s),$$

$$\mathrm{RAV}_t \equiv \mu_1^{-1}M^{-1/2}\Sigma_{i=1}^{M}|r_{t,i}| \xrightarrow{M\rightarrow\infty} \int_0^t \sigma(s)ds,$$

with the obvious extensions to the signed semi-versions of the measures. Hence, in the limit RSJ and RSR formally measure different things. In practice, of course, with a fixed Δ (or fixed M) that is somewhat mute. This also explains why the two measures provide very complimentary empirical evidence. However, in lieu of the "cleaner" asymptotic theory and interpretation associated with RSJ we prefer that measure.