Supplementary Material I: A Stylized Model of Operating Volatilities

With a stylized model of operating volatilities, we illustrate how capital market imperfections in underdeveloped countries can simultaneously lead to lower risk at the firm level and higher risk at the aggregate level. For simplicity, we assume that there are two firms, *A* and *B*. They are endowed with fixed supply of productive resources X_A and X_B. The economy lasts *T* periods and the resources do not depreciate. Firms have a linear production function: $F(e_{i,t}, K_{i,t}) = e_{i,t} K_{i,t}$ where $K_{i,t}$ is the amount of productive resources allocated to firm *i* at time *t*, $i \epsilon \{A,B\}$, and $t \epsilon \{0,\ldots,T\}$. The distribution of productivity e_i is binomial with two possible states: (1) $e_A=H$ and $e_B=L$ and (2) $e_A=L$ and $e_B=H$. The transition matrix between these two states is $\begin{bmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{bmatrix}$.[1] Also, $H\text{-}L = d$ and $d > 0$. The parameter *d* represents the true difference in firm-level productivities which does not vary across countries. From this set-up, the first best allocation of resources is $K_{A,t} = X_A + X_B$ and $K_{B,t} = 0$ if $e_{A,t} > e_{B,t}$ and $K_{A,t} = 0$ and $K_{B,t} = X_A + X_B$ if $e_{A,t} < e_{B,t}$. In other words, with a linear production function, the first best allocation is to transfer all the resources from the losing (low-productivity) firm to the winning (high-productivity) firm.

The key assumption here is that capital markets are less than perfect. Specifically, we assume that it is costly to reallocate resources across firms. For firm *B* to transfer δ units of resources to firm *A* for one period, the transaction cost $C(\delta)$ must be incurred. Examples of adjustment cost include external financing cost of the expanding firms or the cost associated with asymmetric information in the market for productive assets. These costs are expected to be high in developing countries. In the extreme case where the costs associated with market imperfections are infinitely large, firms always produce with their initial endowment. In a case where these costs are positive but not infinitely large, firms partially respond to productivity shocks.

With costly adjustment, the allocation of resources must solve the Bellman's equation below:

$$V(e_{A,t}, e_{B,t}) = \max_{\delta_t} e_{A,t} K_{A,t} + e_{B,t} K_{B,t} - C(\delta_t) + E[V(e_{A,t+1}, e_{B,t+1})]$$

where $K_{A,t} = X_A + \delta_t$, $K_{B,t} = X_B - \delta_t$, and $V(e_{A,T}, e_{B,T}) = 0$. Following Hayashi (1982), we use quadratic transaction cost: $C(\delta_t) = c/2\ (\delta_t)^2$. The transaction cost parameter *c* directly captures the degree of capital market imperfections. For simplicity, we further assume that transaction cost is large enough, $c > d/\min(X_A, X_B)$, so neither firms will be allocated all the resources in the economy, $X_A + X_B$ (i.e., corner solutions).

[1] The parameter ρ is the probability that the economy remains in the same state between *t* and *t+1*. ($\rho = 0.5$ represents the case where the distribution of productivity is identical and independent across time. $\rho > 0.5$ allows for persistency.) While the value of ρ will affect the level of firm-level and aggregate volatilities (V), it will not affect our analyses below which focus on relative volatilities between developed and developing countries ($\frac{\partial V}{\partial c}$). Conditional on a given realization of shock, firm-level changes in output and gains from reallocation are always greater in developed countries.

Lemma 1: The policy function is $\delta_t = (e_{A,t} - e_{B,t})/c$. This policy implies that:

$\boldsymbol{e_{A,t} = H}$ **and** $\boldsymbol{e_{B,t} = L}$	$\boldsymbol{e_{A,t} = L}$ **and** $\boldsymbol{e_{B,t} = H}$
Capital of Firm A = $X_A + d/c$	Capital of Firm A = $X_A - d/c$
Output of Firm A = $H(X_A + d/c)$	Output of Firm A = $L(X_A - d/c)$
Capital of Firm B = $X_B - d/c$	Capital of Firm B = $X_B + d/c$
Output of Firm B = $L(X_B - d/c)$	Output of Firm B = $H(X_B + d/c)$
Aggregate Output = $HX_A + LX_B + (H-L)d/c$	Aggregate Output = $LX_A + HX_B + (H-L)d/c$

Proof:

After substituting the resource constraints into the objective function, the maximization problem becomes

$max_{\delta_t}\ e_{A,t}(X_A + \delta_t) + e_{B,t}(X_B - \delta_t) - C(\delta_t) + E[V(e_{A,t+1}, e_{B,t+1})]$.

First Order Condition with respect to δ_t is $e_{A,t} - e_{B,t} - C'(\delta_t) + E[V'(.)] = e_{A,t} - e_{B,t} - c\,\delta_t = 0$.

The policy function is $\delta_t = (e_{A,t} - e_{B,t})/c$.

In other words, $\delta_t = \begin{cases} d/c, & e_{A,t} = H \text{ and } e_{B,t} = L \\ -d/c, & e_{A,t} = L \text{ and } e_{B,t} = H \end{cases}$ where $d = H - L$.

Q.E.D.

Lemma 1 implies that the amount of resources being reallocated from the losing firm to the winning firm is decreasing with the reallocation costs. Next, we link firm-level volatility (time-series standard deviation of output growth over a sample path) with the amount of resources being reallocated across firms and the reallocation costs.

Proposition 1: Output growth of firms in financially developed countries is more volatile. Firm-level volatility is decreasing in the market imperfection parameter, *c*.

Proof:

Without loss of generality, we focus on the output of firm A (denoted by Y_A). Let $\Delta = d/c$.

Y_{AH} is the output of firm A when $e_{A,t} = H$ and $e_{B,t} = L$. So, $Y_{AH} = H(X_A + \Delta)$.

Y_{AL} is the output of firm A when $e_{A,t} = L$ and $e_{B,t} = H$. So, $Y_{AL} = L(X_A - \Delta)$.

Output growth of firm A in percentage term is $g_t = (Y_{A,t} - Y_{A,t-1})/Y_{A,t-1}$. Given the binomial nature of the productivity process, output growth of firm A must take one of these three values: $g_t \in \{g_{up}, g_{down}, 0\}$ where $g_{up} = \frac{Y_{AH} - Y_{AL}}{Y_{AL}} = \frac{Y_{AH}}{Y_{AL}} - 1$ and $g_{down} = \frac{Y_{AL} - Y_{AH}}{Y_{AH}} = \frac{Y_{AL}}{Y_{AH}} - 1$.

Next, we show that positive output growth (g_{up}) is decreasing in c and negative output growth (g_{down}) is increasing in c. In other words, a large value of c will move the growth rate closer to zero.

$\frac{\partial g_{up}}{\partial \Delta} = \frac{\partial}{\partial \Delta}(Y_{AH}/Y_{AL}) = (1/{Y_{AL}}^2)(Y_{AL}\frac{\partial Y_{AH}}{\partial \Delta} - Y_{AH}\frac{\partial Y_{AL}}{\partial \Delta}) = (1/{Y_{AL}}^2)(Y_{AL}H + Y_{AH}L) \geq 0.$

Using the chain rule, $\frac{\partial g_{up}}{\partial c} = \frac{\partial}{\partial c}\left(\frac{d}{c}\right)\frac{\partial g_{up}}{\partial \Delta} = -\left(\frac{d}{c^2}\right)\frac{\partial g_{up}}{\partial \Delta} \leq 0.$

$\frac{\partial g_{down}}{\partial \Delta} = \frac{\partial}{\partial \Delta}(Y_{AL}/Y_{AH}) = (1/{Y_{AH}}^2)(Y_{AH}\frac{\partial Y_{AL}}{\partial \Delta} - Y_{AL}\frac{\partial Y_{AH}}{\partial \Delta}) = -(1/{Y_{AH}}^2)(Y_{AH}L + Y_{AL}H) \leq 0.$

Using the chain rule, $\frac{\partial g_{down}}{\partial c} = \frac{\partial}{\partial c}\left(\frac{d}{c}\right)\frac{\partial g_{down}}{\partial \Delta} = -\left(\frac{d}{c^2}\right)\frac{\partial g_{down}}{\partial \Delta} \geq 0$.

Our economy lasts T periods. Let V be the volatility of a sample path:$\{g_1, g_2, \dots, g_t, \dots, g_T\}$. Volatility is defined as the time series standard deviation of the growth rates along the sample path.

$V^2 = \frac{1}{T-1}\sum_{t=1}^{T}(g_t - \bar{g})^2 = \frac{1}{T-1}\sum_{t=1}^{T}(g_t)^2 - 2\, g_t\, \bar{g} + (\bar{g})^2$ where $\bar{g} = \frac{1}{T}\sum_{t=1}^{T} g_t$.

Now, we show that V^2 is decreasing in c by taking the first partial derivative with respect to c.

$\frac{\partial V^2}{\partial c} = \frac{1}{T-1}\sum_{t=1}^{T}(2g_t \frac{\partial g_t}{\partial c} - 2\bar{g}\frac{\partial g_t}{\partial c} - 2g_t\frac{\partial \bar{g}}{\partial c} + 2\bar{g}\frac{\partial \bar{g}}{\partial c}) = \frac{2}{T-1}\sum_{t=1}^{T}(g_t - \bar{g})\frac{\partial g_t}{\partial c} - (g_t - \bar{g})\frac{\partial \bar{g}}{\partial c}$.

According to the definition of $\bar{g}$, the last term in the expression above is equal to zero.

$\sum_{t=1}^{T}(g_t - \bar{g})\frac{\partial \bar{g}}{\partial c} = \frac{\partial \bar{g}}{\partial c}\sum_{t=1}^{T}(g_t - \bar{g}) = 0$.

Therefore, $\frac{\partial V^2}{\partial c} = \frac{2}{T-1}\sum_{t=1}^{T}(g_t - \bar{g})\frac{\partial g_t}{\partial c}$.

Since $g_t \in \{g_{up}, g_{down}, 0\}$, the average growth rate is bounded by g_{up} and g_{down} ($g_{down} \leq \bar{g} \leq g_{up}$).

If $g_t = g_{up}$, then $(g_t - \bar{g})\frac{\partial g_t}{\partial c} \leq 0$ because $(g_{up} - \bar{g}) \geq 0$ and $\frac{\partial g_{up}}{\partial c} \leq 0$.

If $g_t = g_{down}$, then $(g_t - \bar{g})\frac{\partial g_t}{\partial c} \leq 0$ because $(g_{down} - \bar{g}) \leq 0$ and $\frac{\partial g_{down}}{\partial c} \geq 0$.

If $g_t = 0$, then $(g_t - \bar{g})\frac{\partial g_t}{\partial c} = 0$ because $\frac{\partial 0}{\partial c} = 0$.

So, $(g_t - \bar{g})\frac{\partial g_t}{\partial c}$ is always (weakly) negative.

We can conclude that $\frac{\partial V^2}{\partial c} = \frac{2}{T-1}\sum_{t=1}^{T}(g_t - \bar{g})\frac{\partial g_t}{\partial c} \leq 0$: V^2 is decreasing in c. Given that volatility V cannot be negative, $\frac{\partial V}{\partial c} \leq 0$: V is decreasing in c as well.

Q.E.D.

Proposition 2: Aggregate output growth is less volatile in financially developed countries. Aggregate volatility is increasing in the market imperfection parameter, c.

The intuition of our proof is as follows. We decompose aggregate output into two components: output generated by the initial endowment (as if reallocation is not feasible) and gain from reallocation. The volatility in aggregate output comes from fluctuation in the first component (because the second component is state invariant in our stylized model).

Without reallocation, aggregate output is high when the initial endowment is "right" (the firm with larger initial endowment receives higher productivity shock) and aggregate output is low when the initial endowment is "wrong" (the firm with the smaller initial endowment receive the higher productivity shock). With reallocation, the gain from reallocation is increasing with the amount of resources being reallocated across firms. As the reallocation costs decrease, the amount of resources being reallocated and gain from reallocation increase. As the gain from the reallocation component grows, output generated by initial endowment as a fraction of aggregate output shrinks. Therefore, the aggregate volatility (which is driven by fluctuation in output generated by initial endowment) is increasing with the reallocation costs. In other words, we expect that developing economies where financial market imperfections impede resource reallocation are subject to higher aggregate volatility.

Proof:

Let O be aggregate output if there is no reallocation in the economy.

When $e_A = H$ and $e_B = L, O = O_H = HX_A + LX_B$. When $e_A = L$ and $e_B = H, O = O_L = LX_A + HX_B$.

Without loss of generality, assume that initial endowment of firm A is greater than the initial endowment of firm B ($X_A \geq X_B$) so that, without resource reallocation, aggregate output is higher when firm A receives higher productivity than firm B ($O_H \geq O_L$).

Define $\Delta = \frac{d}{c}$ and $d = H - L$.

Let $\tilde{O}$ be aggregate output if there is reallocation of resources in the economy.

When $e_A = H$ and $e_B = L, \tilde{O} = \widetilde{O_H} = O_H + d\Delta$. When $e_A = L$ and $e_B = H, \tilde{O} = \widetilde{O_L} = O_L + d\Delta$. That is, $d\Delta$ is the gain from reallocation.

Aggregate output growth in percentage term is $G_t = (\tilde{O}_t - \tilde{O}_{t-1})/\tilde{O}_{t-1}$. Given the binomial nature of the productivity process, aggregate output growth must take one of these three values: $G_t \in \{G_{up}, G_{down}, 0\}$ where $G_{up} = \frac{\widetilde{O_H} - \widetilde{O_L}}{\widetilde{O_L}} = \frac{O_H - O_L}{O_L + d\Delta}$ and $G_{down} = \frac{\widetilde{O_L} - \widetilde{O_H}}{\widetilde{O_H}} = \frac{O_L - O_H}{O_H + d\Delta}$.

Next, we show that positive output growth (G_{up}) is increasing in c and negative output growth (G_{down}) is decreasing in c. In other words, a small value of c will move the growth rate closer to zero.

$\frac{\partial G_{up}}{\partial \Delta} = \frac{\partial}{\partial \Delta}\left(\frac{O_H - O_L}{O_L + d\Delta}\right) = -\frac{O_H - O_L}{(O_L + d\Delta)^2}(d) \leq 0.$

Using the chain rule, $\frac{\partial G_{up}}{\partial c} = \frac{\partial}{\partial c}\left(\frac{d}{c}\right)\frac{\partial G_{up}}{\partial \Delta} = -\left(\frac{d}{c^2}\right)\frac{\partial G_{up}}{\partial \Delta} \geq 0.$

$\frac{\partial G_{down}}{\partial \Delta} = \frac{\partial}{\partial \Delta}\left(\frac{O_L - O_H}{O_H + d\Delta}\right) = -\frac{O_L - O_H}{(O_H + d\Delta)^2}(d) = \frac{O_H - O_L}{(O_H + d\Delta)^2}(d) \geq 0.$

Using the chain rule, $\frac{\partial G_{down}}{\partial c} = \frac{\partial}{\partial c}\left(\frac{d}{c}\right)\frac{\partial G_{down}}{\partial \Delta} = -\left(\frac{d}{c^2}\right)\frac{\partial G_{down}}{\partial \Delta} \leq 0.$

Let V be the volatility of a sample growth path $\{G_1, G_2, \ldots, G_t, \ldots, G_T\}$. Volatility is defined as the time series standard deviation of the growth rates along the sample path.

$V^2 = \frac{1}{T-1}\sum_{t=1}^{T}(G_t - \bar{G})^2 = \frac{1}{T-1}\sum_{t=1}^{T}(G_t)^2 - 2\, G_t\, \bar{G} + (\bar{G})^2$ where $\bar{G} = \frac{1}{T}\sum_{t=1}^{T} G_t$.

Now, we show that V^2 is increasing in c by taking the first partial derivative with respect to c.

$\frac{\partial V^2}{\partial c} = \frac{1}{T-1}\sum_{t=1}^{T}(2G_t \frac{\partial G_t}{\partial c} - 2\bar{G}\frac{\partial G_t}{\partial c} - 2G_t\frac{\partial \bar{G}}{\partial c} + 2\bar{G}\frac{\partial \bar{G}}{\partial c}) = \frac{2}{T-1}\sum_{t=1}^{T}(G_t - \bar{G})\frac{\partial G_t}{\partial c} - (G_t - \bar{G})\frac{\partial \bar{G}}{\partial c}.$

According to the definition of $\bar{G}$, the last term in the expression above is equal to zero.

$\sum_{t=1}^{T}(G_t - \bar{G})\frac{\partial \bar{G}}{\partial c} = \frac{\partial \bar{G}}{\partial c}\sum_{t=1}^{T}(G_t - \bar{G}) = 0.$

Therefore, $\frac{\partial V^2}{\partial c} = \frac{2}{T-1}\sum_{t=1}^{T}(G_t - \bar{G})\frac{\partial G_t}{\partial c}$.

Since $G_t \in \{G_{up}, G_{down}, 0\}$, the average growth rate is bounded by G_{up} and G_{down} ($G_{down} \leq \bar{G} \leq G_{up}$).

If $G_t = G_{up}$, then $(G_t - \bar{G})\frac{\partial G_t}{\partial c} \geq 0$ because $(G_{up} - \bar{G}) \geq 0$ and $\frac{\partial G_{up}}{\partial c} \geq 0$.

If $G_t = G_{down}$, then $(G_t - \bar{G})\frac{\partial G_t}{\partial c} \geq 0$ because $(G_{down} - \bar{G}) \leq 0$ and $\frac{\partial G_{down}}{\partial c} \leq 0$.

If $G_t = 0$, then $(G_t - \bar{G})\frac{\partial G_t}{\partial c} = 0$ because $\frac{\partial 0}{\partial c} = 0$.

So, $(\mathrm{G_t} - \overline{\mathrm{G}})\frac{\partial \mathrm{G_t}}{\partial \mathrm{c}}$ is always (weakly) positive.

We can conclude that $\frac{\partial V^2}{\partial c} = \frac{2}{T-1}\sum_{t=1}^{T}(G_t - \overline{G})\frac{\partial G_t}{\partial c} \geq 0$: V^2 is increasing in c. Given that volatility V cannot be negative, $\frac{\partial V}{\partial c} \geq 0$: V is increasing in c as well.

Q.E.D.

Supplementary Material II: Private Firms and Competition

The primary data source of this paper, WorldScope, only contains publicly-listed companies. In this supplementary material, we provide additional evidence from Bureau van Dijk's Orbis database, which includes unlisted companies from around the world.

Even though Orbis contains a large cross section of public and private companies, its time-series coverage is very limited, making it inappropriate for a study of firm-level risk which requires some within firm time-series information. To get around its lack of time-series coverage, we use Orbis to compute HHI to shed some light on private companies and competitive environment in different countries.[2]

Below, we present examples of HHI in five major developed countries (France, Germany, Japan, United Kingdom, and United States) and five major emerging markets (Brazil, China, India, Thailand, and Turkey) in year 2007. We choose these countries because they represent different geographical regions and the coverages are relatively extensive. We choose the year 2007 because it has a sizeable number of observations but avoids the criticism that greater competition in developed countries are consequences of the 2008 financial crisis. Similar to Section VI, we exclude all financial firms and regulated utilities and exclude all observations with zero or negative total assets. *HHI* is computed from sales in each country, two-digit SIC industry, and year. That is, $\mathrm{HHI} = \sum j(\mathrm{SHARE}_j^2)$, where SHARE_j is the market share of firm *j* and the index ranges from 0 to 1.

The table below reports HHI in each country (averaged across two-digit SIC industries). The results confirm that even with the sample that covers both public and private firms, developed economies are more competitive.

Developed Countries	HHI	Developing Countries	HHI
France	0.018	Brazil	0.258
Germany	0.044	China	0.129
Japan	0.090	India	0.231
United Kingdom	0.032	Thailand	0.139
United States	0.062	Turkey	0.324

[2] To the extent that volatility of a listed company is driven by its competition with both public and private competitors, our volatility results already reflect the presence of private companies in different countries. Therefore, the direct inclusion of unlisted companies in the sample is less critical for tests involving volatilities but more critical for the calculation of HHI.