

Online Appendix for Factor Structure in Commodity Futures Return and Volatility

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In this online appendix, we first present various descriptive statistics for commodity returns and realized volatility, analyze the relationship between volatility and time-to-delivery, and then provide some detail on the implementation of our realized volatility estimates. Subsequently, we construct subsample correlation estimates and look for seasonal effects in volatility. Finally, we present results on the relationship between commodity carry and volatility.

Descriptive Statistics

In Figure A.1 we plot daily log returns on the 15 commodity futures. Note that the scale differs across panels highlighting the difference in volatility levels across contracts. Volatility persistence patterns are evident for all contracts, as are the presence of occasional outliers.

In Figure A.2 we plot the empirical autocorrelations functions of the daily returns on the 15 contracts. Comparing this with the 99% confidence intervals marked by the horizontal grey lines, the evidence for significant daily return dynamics is weak at best.

Figure A.3 shows the empirical quantiles of daily returns scatter plotted against the corresponding quantiles from a normal distribution. The daily returns are generally below the 45 degree line to the left in the figures and above the 45 degree line to the right in the figures. This indicates that the daily return distribution is fat-tailed, as it is commonly found for speculative assets.

Figure A.4 plots the raw daily realized volatilities for the 15 contracts and Figure A.5 the corresponding logarithms. While the log transformation helps diminish the outliers, it is clear that the realized volatilities contain

some high-frequency estimation error, which we subsequently filter out with ARMA models. A strongly persistent lower frequency component is evident particularly from Figure A.5.

Figure A.6 shows the empirical quantiles of daily log realized volatility scatter plotted against the corresponding quantiles from a normal distribution. While certain contracts show some evidence of deviations from log-normality in the tails, our realized volatility estimates generally provide support for the log-normality assumption.

Figure A.7 plots the raw realized stock market betas for the 15 contracts. The realized betas are constructed as the ratio of two noisy estimates and so are themselves naturally quite noisy. We filter out some of this noise using ARMA models below.

Volatility and Time to Delivery

Kamara (1984), Anderson (1985) and Bessembinder et al. (1996) have found evidence that futures volatility is affected by the time to delivery. In Figure A.8 we therefore scatter plot $\log(RVol_t)$ for the most active futures contract against the number of days until the roll over to the next-maturity con-

tract.²³ The slope from regressing $\log(RVol_t)$ on the days-to-roll is always very close to zero but sometimes significant (positive or negative) due to the large number of observations. In addition, the R^2 is always $< 1\%$ except for corn (2.7%), cotton (5.1%), and soybeans (1.2%). When reporting the expected $\log(RVol_t)$, as we often do below, any maturity effect will be partly captured by the lagged log realized volatility in the ARMA model in equation 3, above.

Constructing Realized Volatility

We are interested in computing daily volatility corresponding to the standard 24-hour daily close-to-close returns used in the literature. To this end, we need to combine our realized volatility measures computed when markets are open with overnight squared returns from the overnight gaps. Table A.1 reports the trading intervals for each contract. The last two rows

²³In 2011, the March and May contracts for cotton were never traded more than the December contract, which is evident in Figure A.8. Similarly, the July contract for corn in 2004 was never the most traded. Also, recall that not all calendar months are delivery months and that the distance in months between the delivery months therefore varies. This sometimes leaves more observations with few days to roll than with many days to roll, which is evident in the figure for soybeans and sugar for instance.

of Table A.2 report the optimal loadings on the overnight squared return and the open-to-close realized volatility when computing 24-hour volatility.

We rely on ARMA(1,1) models to filter out some of the noise inherent in our model-free realized volatility, beta, and systematic risk ratio estimates.

Table A.3 reports the ARMA parameter estimates and various model diagnostics, including the first-order autocorrelation of the ARMA residuals, and the Ljung-Box test for 5 and 21 lags. While the residual dynamics are often significant, we prefer the simplicity of the ARMA(1,1) structure for the purpose of filtering the model-free estimates.

Correlation by Subsamples

Table 3 in the main paper reports the cross-commodity correlations for daily returns and log volatility. In Tables A.4-A.6 we report the same correlations for three subsamples corresponding to the pre-crisis period (2004-2008), the crisis period (2007-2011), and the post-crisis period (2010-2014). Note that the sub-periods overlap but are of equal length. The key findings are reported in Figure 7.a for returns and Figure 7.b for volatilities and are discussed in the main paper in Section III.C.

Seasonality in Volatility

It is well known that certain commodities, in particular agriculturals, have annual seasonal patterns in returns, see for example Kamara (1982) and Gorton et al. (2013). We therefore investigate if commodity realized variances display seasonal patterns as well. We regress the daily realized variance in levels for each commodity on monthly dummy variables as well as on twenty lags of the realized variance in order to pick up the strong persistence in volatility. Table A.7 shows that apart from natural gas, the seasonal dummy variables are rarely significant. We acknowledge that the lack of significance could be driven by our relatively simplistic model for seasonality as well as our relatively short sample period covering 11 years.

Commodity Carry and Volatility

In Table A.8 we compute a commodity carry return factor for each commodity and then average the carry return across the commodities within each of the commodity groups used above. We define carry using spot and

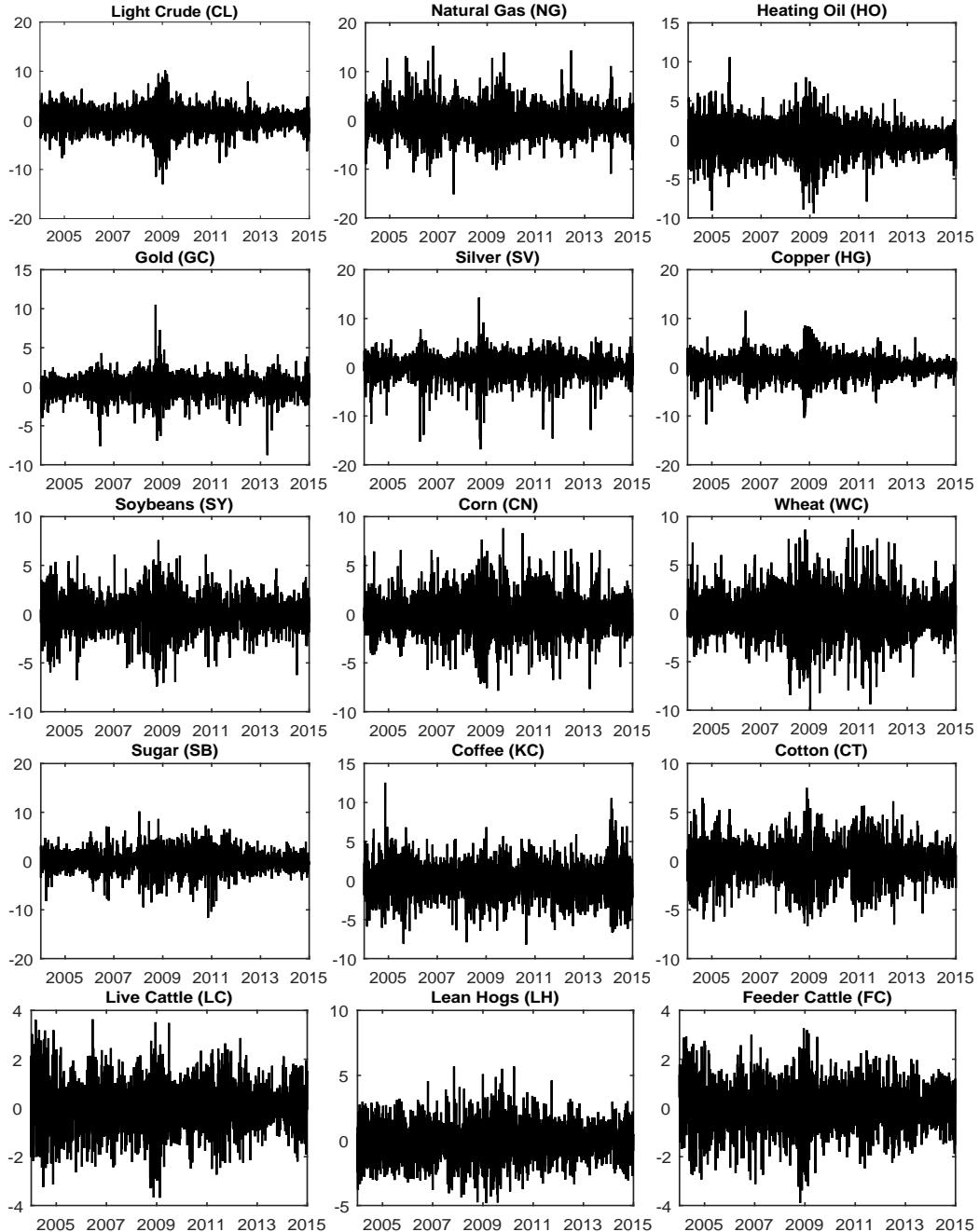
futures prices as follows,

$$(8) \quad Carry_t = \frac{S_t - F_t}{F_t} \approx \frac{F_{1m,t} - F_{2m,t}}{F_{2m,t}},$$

where we have used the one-month futures price as a proxy for the spot price. Carry has been studied in a pure commodity context by Gorton et al. (2013) among others. Kojien et al. (2018) find that assets with relatively large carry values (e.g. high dividend yields for stocks) tend to earn relatively high returns on average.

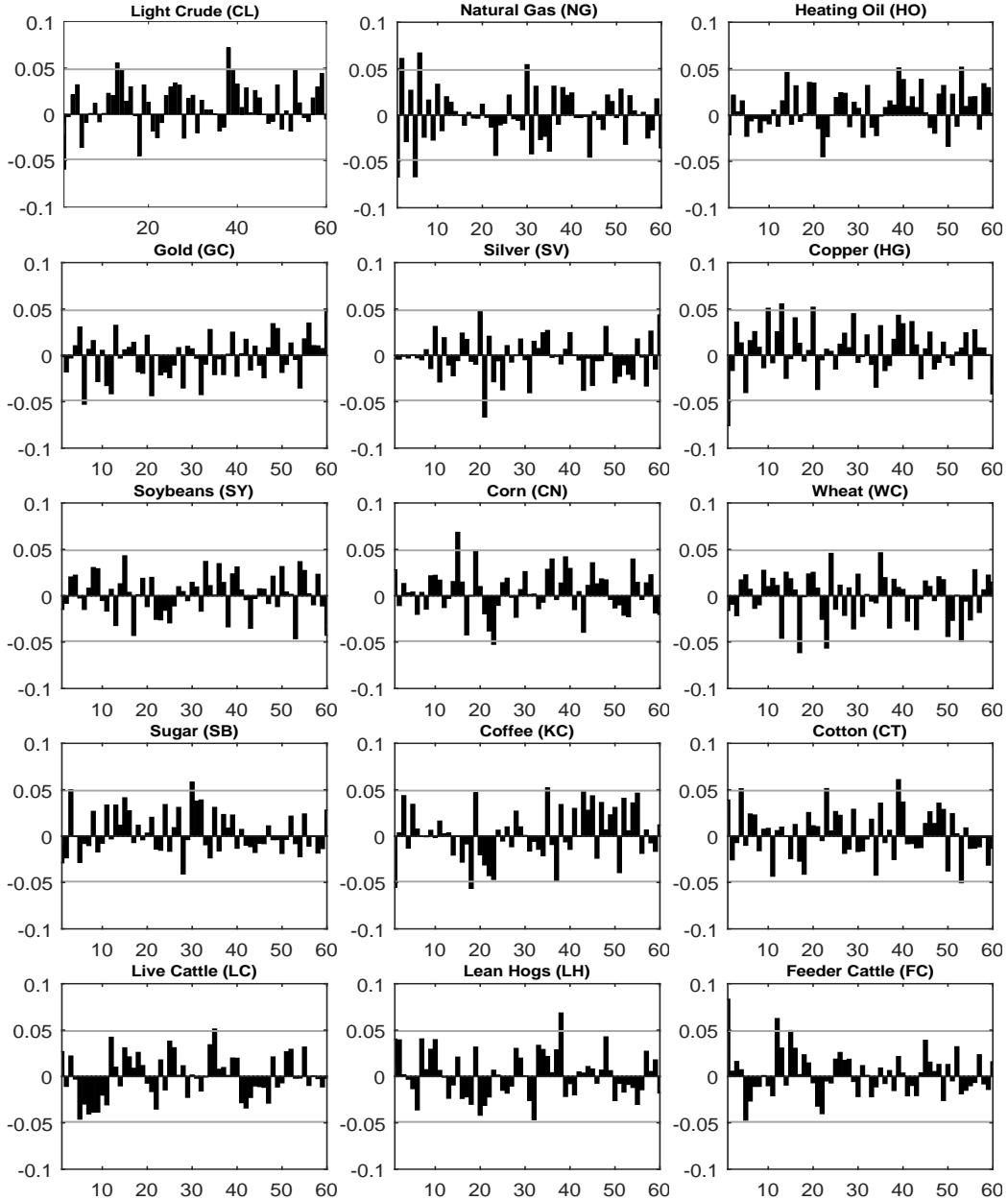
Panel a of Table A.8 reveals a negative (but not always significant) relationship between commodity carry returns and PC1 commodity volatility for each of our five commodity groups. For commodities, carry will be large when convenience yields are large or when interest rates and storage costs are low. Convenience yields are large when the physical commodity is scarce, for example corresponding to economic expansions when volatility is low as shown in Bloom (2014). Panel c in Table A.8 shows that PC3 also generally has a negative relationship with carry returns. The exception is for meats, which we have found to be unusual in other respects as well.

Figure A.1: Daily Log Returns of Commodity Futures.



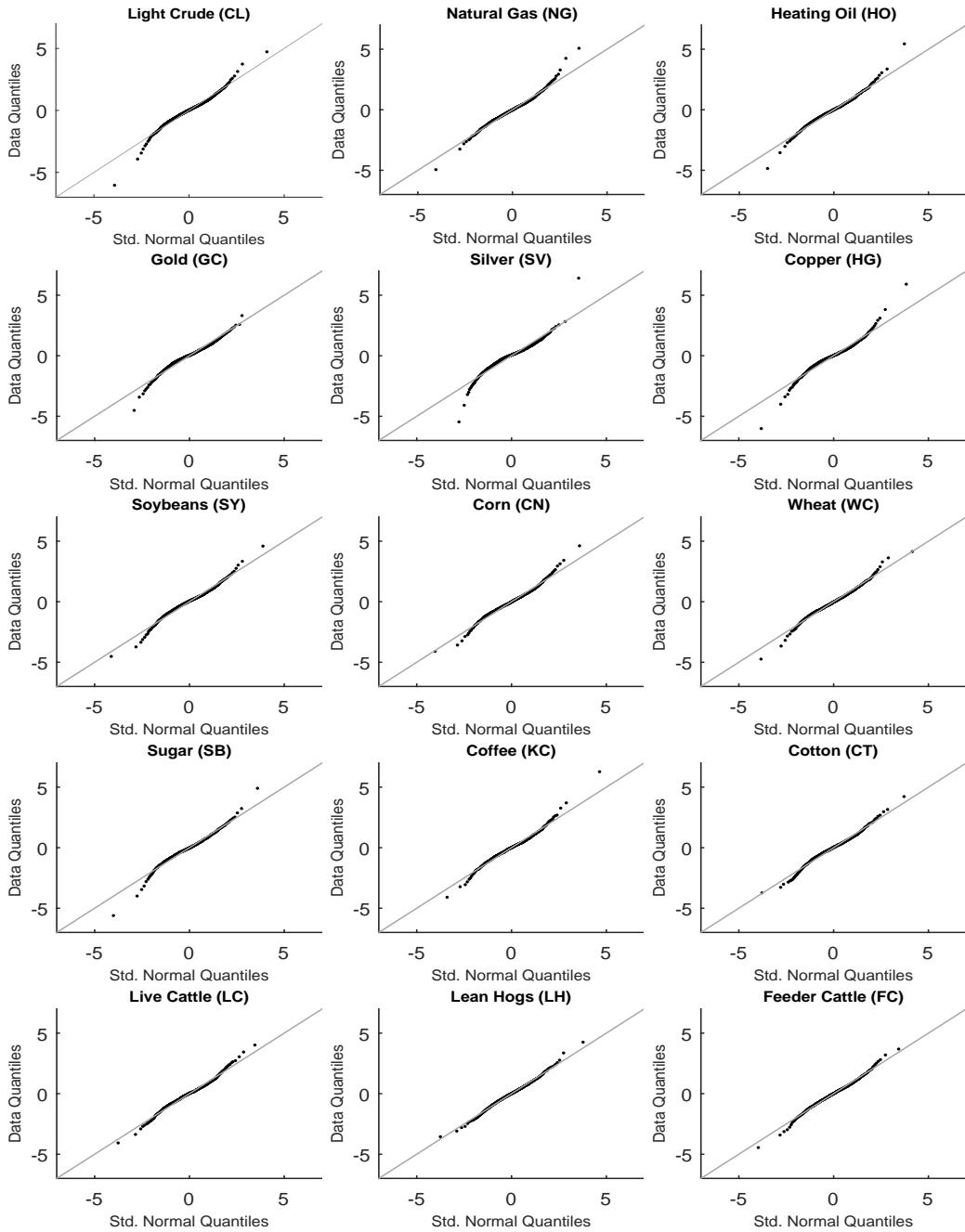
The figure shows the daily log returns of closing prices for 15 commodity futures during the 2004-2014 sample period. All returns are for the most active futures contract on a given day.

Figure A.2: Empirical Autocorrelation of Daily Commodity Futures Returns.



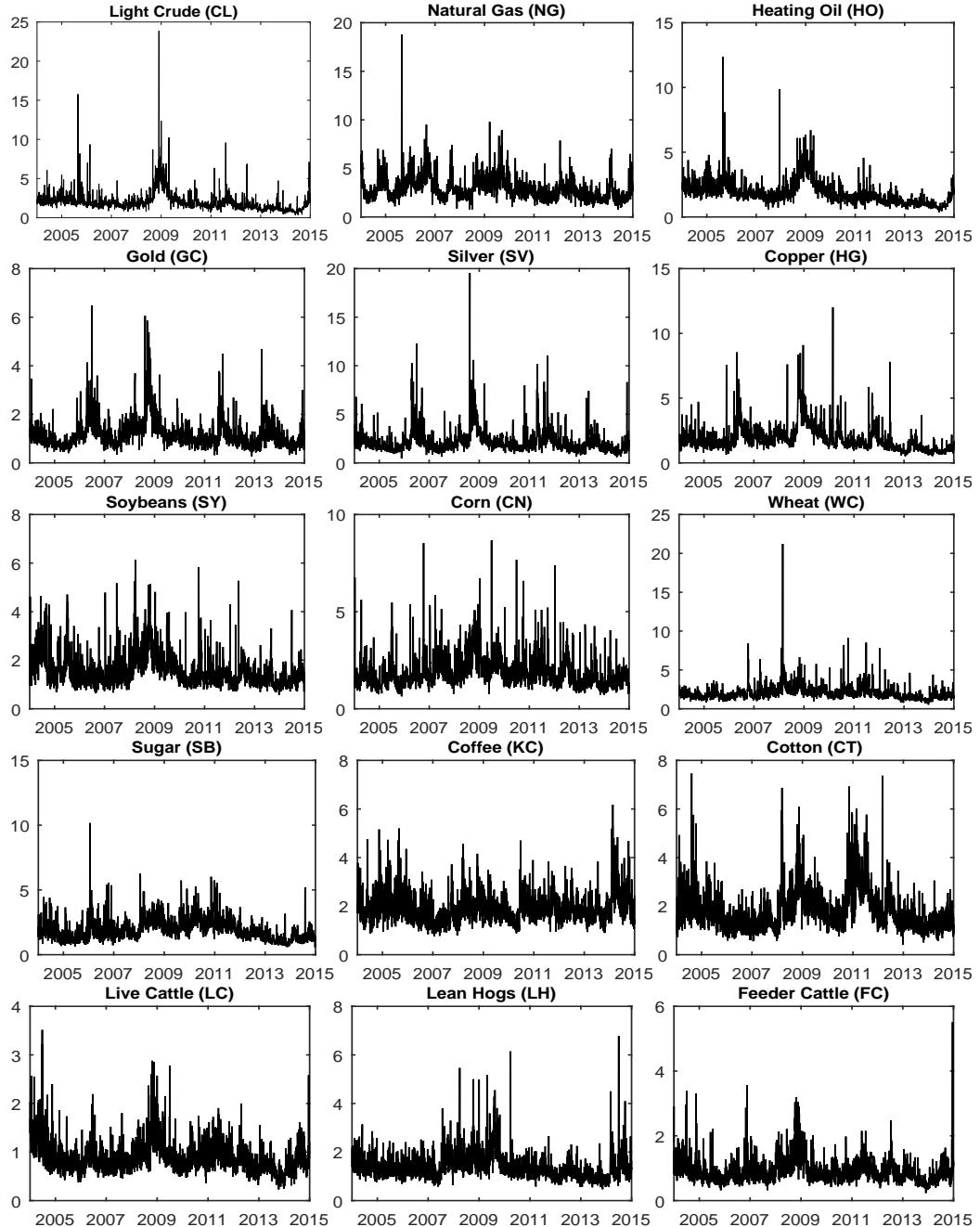
The figure shows the empirical autocorrelation of log returns for 15 commodity futures during the 2004-2014 sample period. All returns are for the most active futures contract on a given day. Grey lines indicate 99% confidence bounds assuming that the series are Gaussian white noise. The horizontal axis indicates the lag order in days.

Figure A.3: Quantile-Quantile of Daily Commodity Futures Returns.



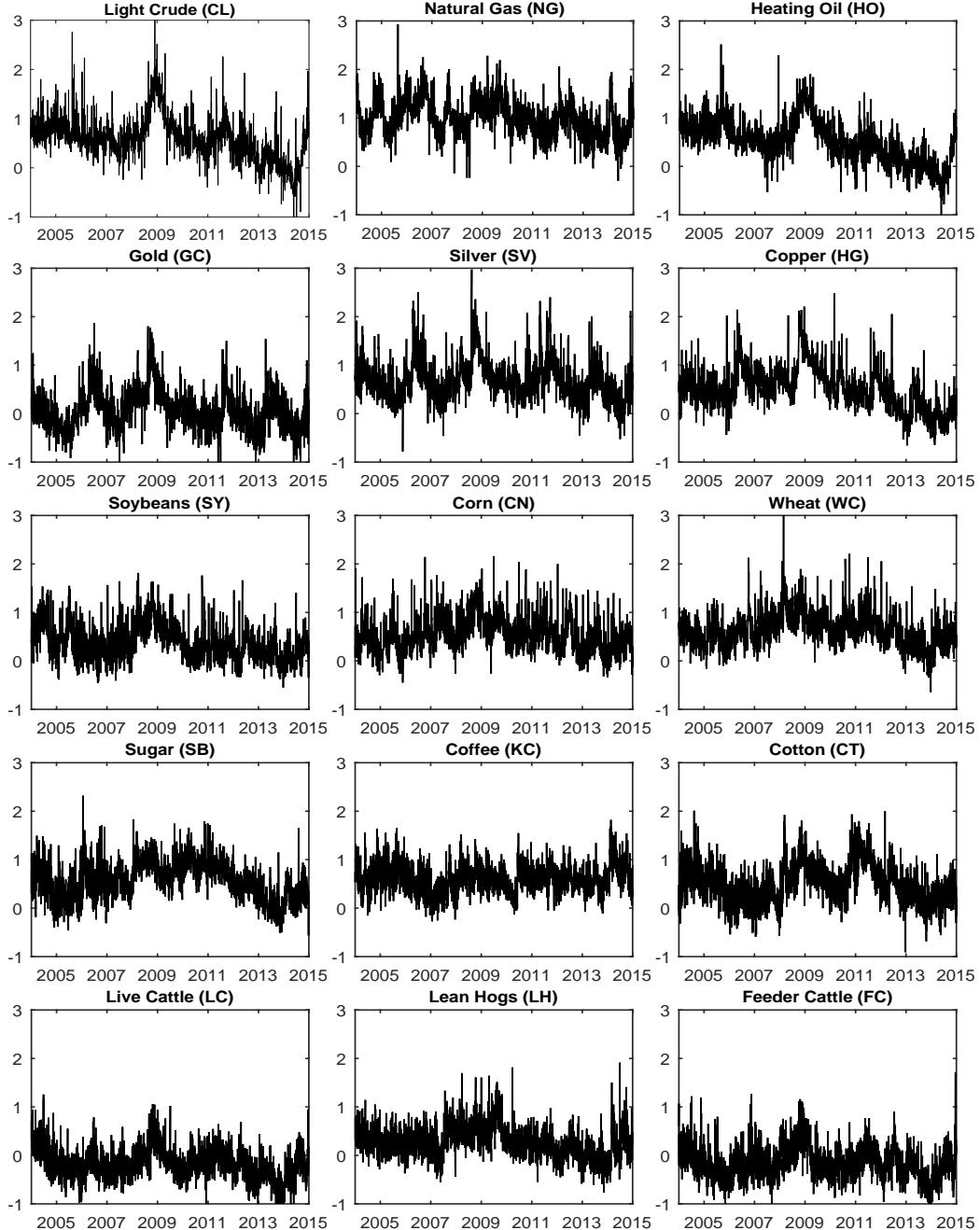
The figure shows the empirical quantile-quantile plots of daily log return for 15 commodity futures during the 2004-2014 sample period. All prices are for the most active futures contract on a given day.

Figure A.4: Daily Realized Commodity Volatility.



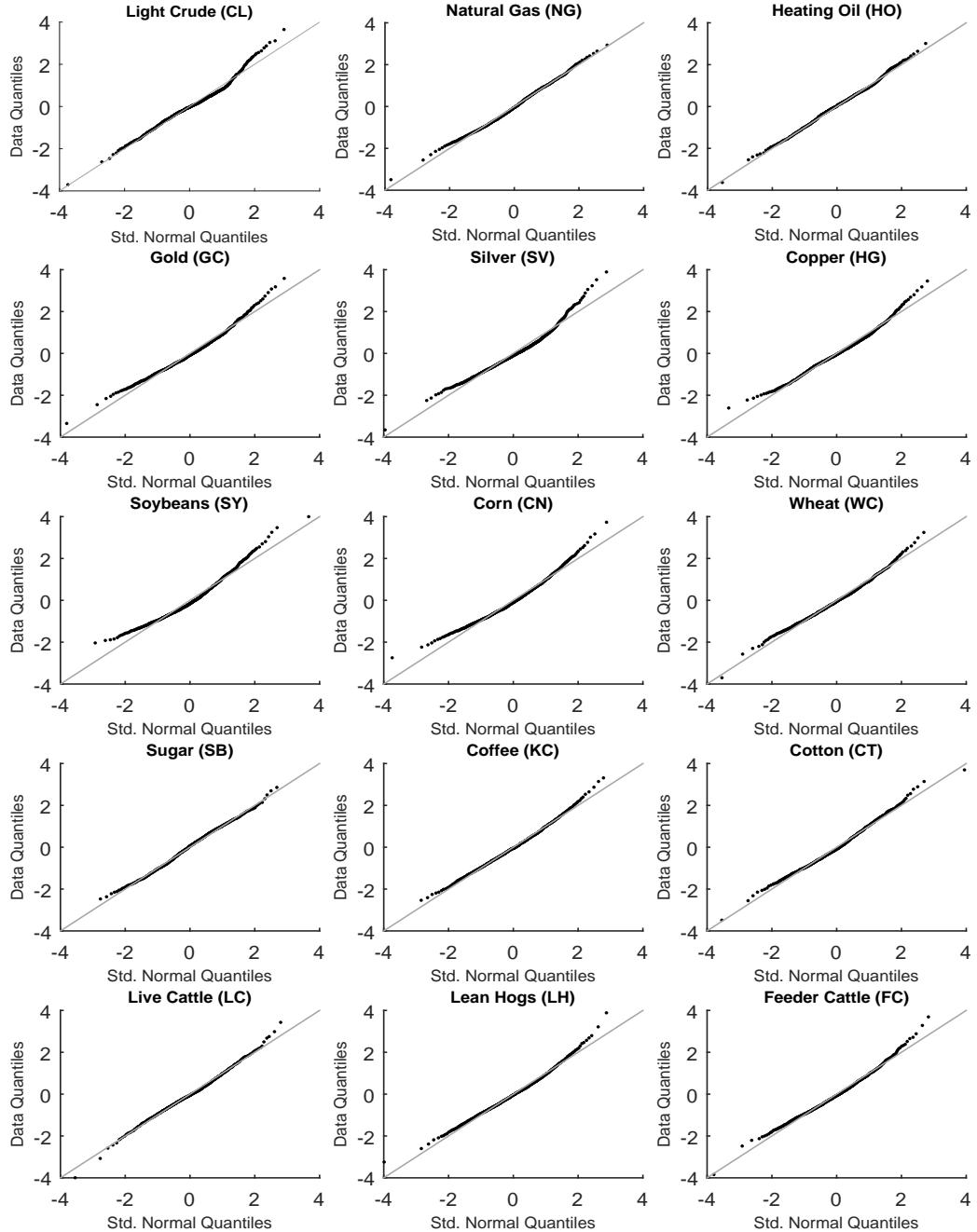
The figure shows the daily realized volatility for 15 commodity futures during the 2004-2014 sample period. All volatilities are for the most active futures contract on a given day.

Figure A.5: Daily Log Realized Commodity Volatility.



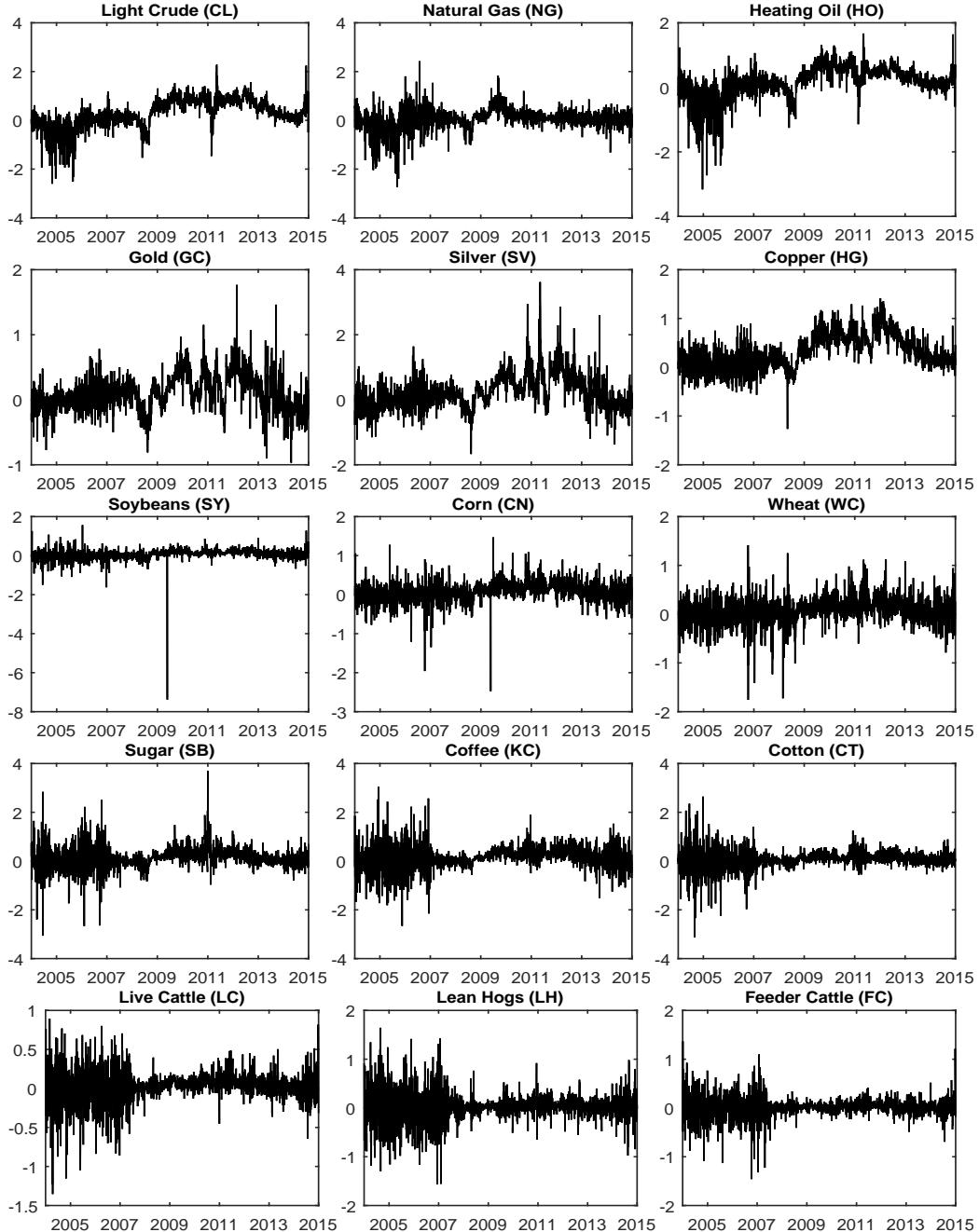
The figure shows the daily log realized volatility for 15 commodity futures during the 2004-2014 sample period. All volatilities are for the most active futures contract on a given day.

Figure A.6: Quantile-Quantile of Daily Log Realized Commodity Volatility.



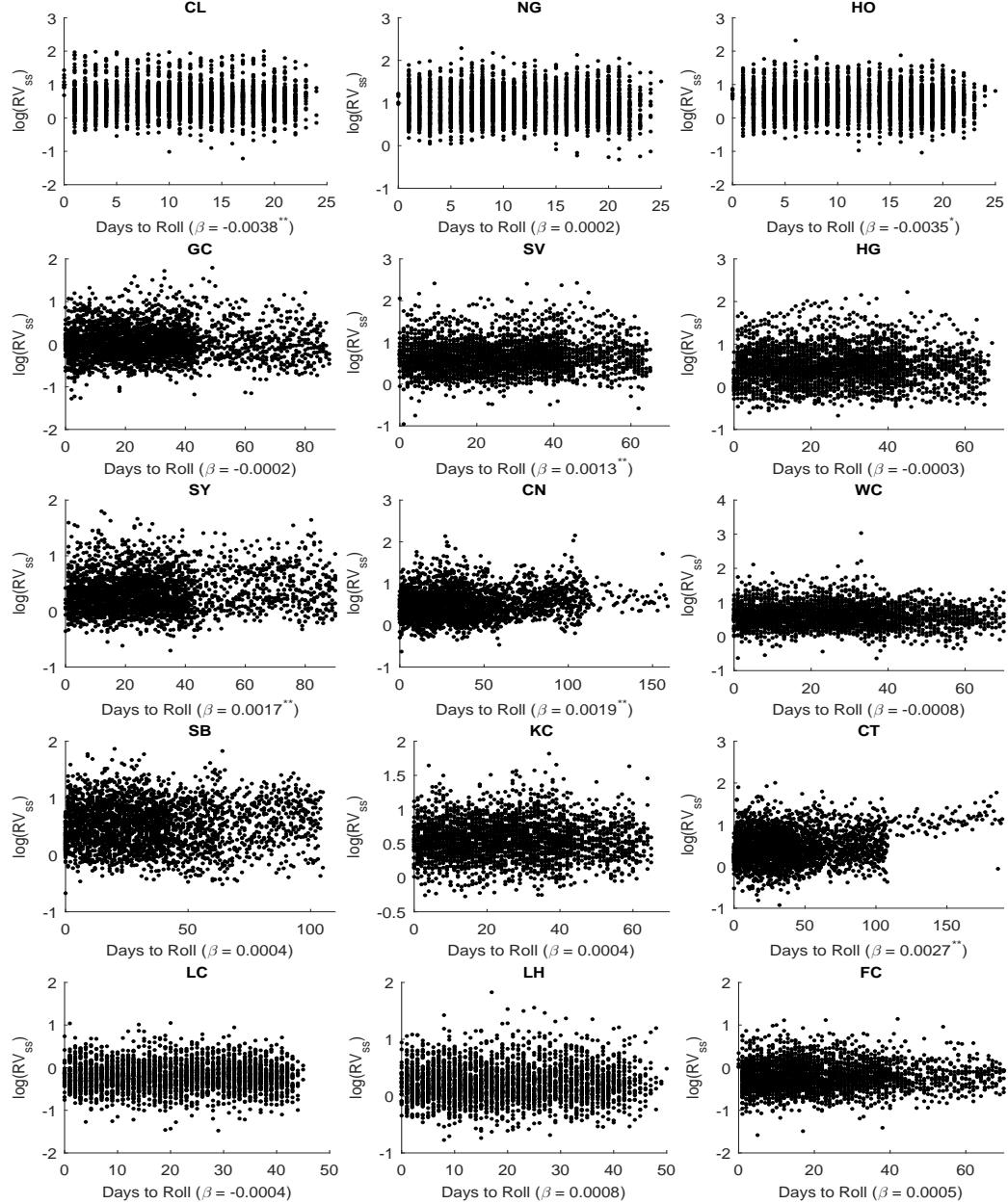
The figure shows the empirical quantile-quantile plots of daily log realized volatility for 15 commodity futures during the 2004-2014 sample period. All volatilities are for the most active futures contract on a given day.

Figure A.7: Realized Stock Market Beta for 15 Commodities.



The figure shows the daily realized stock market beta for 15 commodity futures during the 2004-2014 sample period. The calculation follows equation 6. All co-volatilities used are for the most active futures contract on a given day.

Figure A.8: Daily Log Realized Volatility versus Days to Roll.



The figure shows log realized volatility (y-axis) against days to next roll (x-axis) for 15 commodity futures during the 2004-2014 sample period. All volatilities are for the most active contract on a given day (see Section C). One asterisk indicates significance at the 95% level and two asterisks at the 99%.

Table A.1: Intervals of Trading for 15 Commodities.

Commodity	Period	Trading Interval(s)
Crude Oil, Natural Gas, and Heating Oil	5/1-2004 to 9/6-2006	00.00 - 14.30 15.15 - 12.00
	12/6-2006 to 31/12-2014	00.00 - 17.15 18.00 - 12.00
Gold, Silver	5/1-2004 to 28/5-2004	00.00 - 13.30 15.15 - 12.00
	1/6/2004 to 1/12-2006	00.00 - 13.30 14.00 - 12.00
Copper	4/12-2006 to 31/12-2014	00.00 - 17.15 18.00 - 12.00
	5/1-2004 to 4/6-2004	00.00 - 13.00 15.15 - 12.00
Live Cattle, Lean Hogs, and Feeder Cattle	7/6-2004 to 1/12-2006	00.00 - 13.00 14.00 - 12.00
	4/12-2006 to 31/12-2014	00.00 - 17.15 18.00 - 12.00
Corn, Soybeans, and Wheat	5/1-2004 to 1/6-2007	10.05 - 14.00
	4/6-2007 to 31/12-2014	00.00 - 17.00 18.00 - 12.00
Sugar	5/1-2004 to 7/10-2005	00.00 - 7.00 10.30 - 14.15
	10/10-2005 to 11/1-2008	20.30 - 12.00 00.00 - 7.00 10.30 - 14.15 19.30 - 12.00
Coffee	11/1-2008 to 30/6-2009	00.00 - 7.00 10.30 - 14.15 19.00 - 12.00
	1/7-2009 to 18/5-2012	00.00 - 8.15 10.30 - 14.15 19.00 - 12.00
Cotton	21/5-2012 to 5/4-2013	00.00 - 15.00 18.00 - 12.00
	8/4-2013 to 31/12-2014	00.00 - 8.45 9.30 - 14.15 20.00 - 12.00

All times are Eastern Standard Times. The list indicates for each commodity the intraday trading intervals available in our dataset in the sample period. * Minor changes in trading interval occurred during this period.

Table A.2: Optimal Weights on Open-to-Close Realized Volatility and Overnight Returns

	Light Crude	Natural Gas	Heating Oil	Gold	Silver	Copper	Soy- beans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Hogs	S&P 500 E-Mini
$\hat{E} \left[\left(r_{i,t}^{co} \right)^2 + RV_{i,t} \right]$	4.340	8.958	3.779	1.472	5.079	3.819	2.601	3.631	4.249	4.097	3.764	3.637	0.841	2.004	0.882
$\hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right]$	0.024	0.264	0.036	0.009	0.026	0.031	0.100	0.152	0.116	0.259	0.202	0.228	0.052	0.129	0.062
$\hat{E} [RV_{i,t}]$	4.316	8.694	3.744	1.463	5.053	3.787	2.501	3.480	4.133	3.838	3.562	3.409	0.790	1.874	0.820
$\frac{\hat{E}[RV_{i,t}]}{\hat{E}\left[\left(r_{i,t}^{co}\right)^2\right]}$	179.302	32.923	105.022	168.295	196.034	120.503	25.009	22.929	35.690	14.832	17.630	14.953	15.331	14.517	13.153
$\hat{E} \left[\left(\left(r_{i,t}^{co} \right)^2 - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \right)^2 \right]$	0.005	0.008	0.001	0.004	0.006	0.081	0.191	0.086	0.261	0.185	0.293	0.018	0.076	0.019	
$\hat{E} \left[\left(RV_{i,t} - \hat{E} [RV_{i,t}] \right)^2 \right]$	23.313	44.820	12.139	2.091	28.068	18.293	5.105	8.944	9.729	9.896	4.966	9.725	0.296	1.793	0.383
$\frac{\hat{E} \left[\left(RV_{i,t} - \hat{E} [RV_{i,t}] \right)^2 \right]}{\hat{E} \left[\left(r_{i,t}^{co} - \hat{E} [r_{i,t}^{co}] \right)^2 \right]}$	4813.945	60.717	1542.648	3357.871	6468.419	3090.076	62.908	46.894	113.169	37.973	26.775	33.240	16.560	23.677	19.874
$\hat{E} \left[\left(r_{i,t}^{co} - f \left[\left(r_{i,t}^{co} \right)^2 \right] \right)^2 \right]$	0.095	0.057	0.198	0.120	0.151	0.279	0.187	0.208	0.172	0.083	0.124	0.185	0.094	0.014	0.091
$\frac{\hat{E} [RV_{i,t}] \left(\left(r_{i,t}^{co} \right)^2 - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \right)}{\hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \cdot \hat{E} [RV_{i,t} - \hat{E} [RV_{i,t}]]}$	0.895	0.959	0.934	0.925	0.898	0.912	0.958	0.972	0.962	0.875	0.951	0.923	0.955	0.902	0.920
$\hat{\varphi}$	18.917	1.404	7.020	12.620	20.063	10.683	1.097	0.672	1.399	1.981	0.913	1.227	0.728	1.513	1.128
$\hat{\omega}_1^*$	0.900	0.988	0.943	0.931	0.903	0.920	0.996	1.014	0.989	0.934	1.005	0.985	1.018	0.965	0.990
$\hat{\omega}_2^*$															

Table A.2 is similar to Table 2 in Hansen and Lunde (2005) and contains weighting parameters as per Hansen and Lunde (2005) that are used to compute the optimal measure of daily volatility for 15 commodities and the S&P E-Mini futures contract during the 2004–2014 sample period. The 1% largest squared overnight returns and the 0.5% largest realized covariances were omitted from the estimation. The estimated weights in equation 1 are given by

$$\begin{aligned} \hat{\varphi} &\equiv \frac{\hat{E} [RV_{i,t}]^2 \hat{E} \left[\left(\left(r_{i,t}^{co} \right)^2 - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \right)^2 \right] - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right]^2}{\hat{E} [RV_{i,t}]^2 \hat{E} \left[\left(\left(r_{i,t}^{co} \right)^2 - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \right)^2 \right] + \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right]^2 \hat{E} \left[\left(RV_{i,t} - \hat{E} [RV_{i,t}] \right)^2 \right] - 2 \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \hat{E} [RV_{i,t}] \hat{E} \left[RV_{i,t} \left(\left(r_{i,t}^{co} \right)^2 - \hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right] \right) \right]} \\ \hat{\omega}_1^* &\equiv (1 - \hat{\varphi}) \frac{\hat{E} \left[\left(r_{i,t}^{co} \right)^2 + RV_{i,t} \right]}{\hat{E} \left[\left(r_{i,t}^{co} \right)^2 \right]}, \\ \hat{\omega}_2^* &\equiv \hat{\varphi} \frac{\hat{E} \left[\left(r_{i,t}^{co} \right)^2 + RV_{i,t} \right]}{\hat{E} [RV_{i,t}]} \end{aligned}$$

Table A.3: ARMA(1,1) Estimates for Log Realized Volatility, Realized Beta, and Realized Systematic Risk Ratio.

Table A.3.a: ARMA(1,1) on Log Realized Volatility.

	Light Crude Gas	Natural Heating Oil	Gold	Silver	Copper	Soy-beans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle
ϕ_0	0.007	0.017	0.005	0.001	0.026	0.008	0.020	0.018	0.012	0.025	0.008	-0.003	0.005	-0.003
ϕ_1	0.988	0.983	0.991	0.975	0.961	0.984	0.977	0.963	0.971	0.979	0.985	0.986	0.980	0.979
θ_1	-0.752	-0.753	-0.753	-0.695	-0.641	-0.675	-0.748	-0.704	-0.722	-0.734	-0.726	-0.837	-0.804	-0.802
R^2	0.699	0.619	0.760	0.608	0.574	0.750	0.533	0.478	0.526	0.608	0.384	0.560	0.483	0.442
σ_e	0.242	0.227	0.211	0.253	0.261	0.222	0.249	0.258	0.237	0.252	0.230	0.270	0.234	0.238
$ACF_e(1)$	0.046	-0.008	0.050*	0.087*	0.081*	0.060*	0.066*	0.090*	0.065*	0.083*	0.033	0.086*	0.070*	0.095*
$Q_e(5)$	23.576*	63.908*	19.957*	44.738*	55.528*	39.032*	26.410*	46.752*	19.911*	28.903*	7.510	31.974*	21.025*	36.415*
$Q_e(21)$	60.936*	306.840*	70.399*	77.970*	75.821*	66.974*	48.710*	67.759*	44.328*	84.175*	39.614*	52.883*	68.405*	36.936*

Table A.3.b: ARMA(1,1) on Realized Beta.

	Light Crude Gas	Natural Heating Oil	Gold	Silver	Copper	Soy-beans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle
ϕ_0	0.002	0.000	0.001	0.002	0.004	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.030	0.012
ϕ_1	0.993	0.991	0.994	0.984	0.984	0.996	0.991	0.993	0.994	0.989	0.998	0.997	0.963	-0.312
θ_1	-0.783	-0.891	-0.844	-0.759	-0.739	-0.845	-0.940	-0.940	-0.953	-0.927	-0.975	-0.979	-0.905	-0.341
R^2	0.773	0.366	0.667	0.624	0.664	0.726	0.120	0.179	0.137	0.148	0.122	0.050	0.027	0.006
σ_e	0.273	0.321	0.268	0.163	0.292	0.180	0.241	0.214	0.229	0.375	0.408	0.340	0.176	0.184
$ACF_e(1)$	0.029	0.013	-0.001	0.014	0.021	0.043	0.014	0.014	0.002	0.011	0.026	0.019	0.006	-0.000
$Q_e(5)$	18.720*	6.084	6.731	1.264	2.329	9.251	0.890	5.293	10.907	12.209	8.000	10.887	5.728	10.241
$Q_e(21)$	109.795*	41.201*	59.946*	42.179*	40.464*	43.229*	8.384	16.096	35.237	29.491	36.771	78.505*	37.451	38.391

Table A.3.c: ARMA(1,1) on Realized Systematic Risk Ratio.

	Light Crude Gas	Natural Heating Oil	Gold	Silver	Copper	Soy-beans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle
ϕ_0	0.001	0.000	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000
ϕ_1	0.991	0.985	0.987	0.975	0.982	0.993	0.974	0.982	0.981	0.979	0.977	0.993	0.959	0.994
θ_1	-0.704	-0.861	-0.739	-0.693	-0.703	-0.760	-0.843	-0.883	-0.880	-0.861	-0.856	-0.959	-0.816	-0.965
R^2	0.817	0.332	0.713	0.617	0.681	0.789	0.253	0.214	0.221	0.245	0.242	0.071	0.202	0.070
σ_e	0.057	0.015	0.047	0.047	0.044	0.046	0.021	0.019	0.013	0.022	0.029	0.016	0.019	0.016
$ACF_e(1)$	0.042	0.009	0.024	0.076*	0.090*	0.110*	0.027	0.043	-0.004	0.065*	0.053*	0.018	0.025	-0.021
$Q_e(5)$	13.110	20.348*	22.631*	38.648*	49.604*	78.039*	5.198	9.402	3.096	18.695*	8.542	3.508	9.728	1.958
$Q_e(21)$	95.991*	69.401*	136.054*	85.403*	175.769*	221.140*	35.648	22.981	20.712	74.763*	35.961	17.709	30.951	21.781

The sample period covers 5 January 2004 to 31 December 2014.

Table A.4: Correlations for Daily Returns (upper diagonal) and Log Volatility (lower). “Pre-crisis” sample.

	Light Crude	Natural Gas	Heating Oil	Gold	Silver	Copper	Soybeans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle
Light Crude	42.7	88.7	33.5	29.5	24.4	23.3	19.9	14.7	18.3	14.2	17.0	4.1	7.9	-3.6	
Natural Gas	0.4	45.0	14.1	11.5	9.4	12.8	8.1	1.8	8.0	5.6	5.7	1.9	0.3	2.5	
Heating Oil	82.6	11.7	28.8	25.0	20.3	21.2	18.6	13.3	16.2	14.2	15.9	0.5	5.6	-6.0	
Gold	-9.0	25.3	-13.7	80.3	50.3	22.0	23.5	18.3	18.7	18.5	20.6	4.6	2.5	-1.9	
Silver	5.4	19.8	-0.3	83.0	49.2	24.3	23.3	17.9	18.9	18.4	20.2	6.7	2.6	0.1	
Copper	-15.9	5.2	-26.4	63.1	61.5	16.3	13.3	13.2	19.4	14.1	15.4	6.7	5.8	0.5	
Soybeans	28.4	-35.6	21.4	0.0	13.0	-5.6	60.0	41.7	19.4	16.3	32.3	1.2	4.3	-14.3	
Corn	-20.1	-19.4	-35.8	16.2	7.7	23.7	45.4	55.6	19.3	15.3	28.7	1.0	-0.6	-25.7	
Wheat	-4.1	-18.0	-25.2	39.9	27.7	27.2	43.0	68.5	18.7	15.6	20.2	3.1	4.0	-13.6	
Sugar	-11.9	-7.5	-20.1	46.8	31.8	15.0	25.2	42.1	52.3	18.2	19.7	1.8	-1.5	-5.8	
Coffee	41.0	6.1	48.1	-11.3	1.6	-35.2	28.1	-25.3	-1.6	-10.7	17.2	5.9	4.5	1.2	
Cotton	36.3	-32.8	28.7	-17.6	4.2	-9.6	62.2	13.8	27.0	9.7	41.4	3.8	-0.2	-5.1	
Live Cattle	21.3	-16.8	20.9	26.9	44.4	16.5	44.5	7.1	19.3	23.4	9.2	38.3	31.3	74.0	
Lean Hogs	17.9	1.4	2.6	28.1	29.8	7.0	35.2	6.5	37.1	14.9	19.1	29.4	49.3	25.8	
Feeder Cattle	27.2	-23.0	11.1	27.5	29.1	16.5	38.3	25.9	36.3	36.4	-3.1	15.1	67.4	34.1	
<i>Average Correlation</i>															
For Returns	29.0	18.0	27.1	28.9	28.5	23.9	25.4	24.0	21.6	19.3	18.6	20.8	16.4	12.8	8.5
For E(log RVol)	20.0	1.1	13.7	27.0	30.6	16.2	29.6	17.1	28.6	23.2	13.8	23.1	31.4	27.5	29.3

Average Correlation

	With SP 500 Returns	With SP 500 E(log RVol)
With SP 500 Returns	0.1	-0.1
With SP 500 E(log RVol)	5.6	-24.3

The table shows unconditional correlations for daily returns (upper diagonal) and log realized volatility (lower diagonal) for 15 commodity futures during the 2004–2008 (“pre-crisis”) sample period. All observations are for the most active futures contract on a given day. Average correlations with the S&P 500 E-Mini futures contract are shown in the lower panel for each commodity.

Table A.5: Correlations for Daily Returns (upper diagonal) and Log Volatility (lower). “During-crisis” sample.

	Light Crude	Natural Gas	Heating Oil	Gold	Silver	Copper	Soy- beans	Corn	Wheat	Sugar	Cotton	Livc	Lean	Feeder Cattle
Light Crude	28.1	88.5	31.2	42.8	59.7	43.3	38.3	34.2	32.6	34.0	29.0	32.3	16.4	24.4
Natural Gas	39.1	29.6	9.5	14.8	18.6	19.3	17.7	15.0	18.9	18.6	13.5	13.3	0.1	10.2
Heating Oil	95.7	49.5	30.8	41.1	56.1	44.9	36.9	31.1	30.8	32.7	30.1	28.5	14.4	21.3
Gold	74.1	21.6	68.4	78.0	32.7	27.2	27.2	16.2	21.5	17.1	6.5	6.3	-2.4	
Silver	65.2	11.2	57.3	82.1	45.9	38.1	35.8	31.7	24.6	31.3	25.6	18.8	12.3	9.0
Copper	80.7	45.6	79.2	72.3	63.6	44.4	36.7	32.9	31.9	37.3	33.2	30.8	15.5	22.2
Soybeans	59.0	41.9	66.8	61.6	54.8	59.8	67.2	54.7	35.0	38.7	41.1	26.6	14.9	8.1
Corn	54.3	38.7	57.6	43.3	48.9	47.3	76.2	64.9	35.1	34.5	37.4	28.2	12.7	-1.1
Wheat	33.0	3.7	34.3	39.2	45.6	28.4	63.0	67.1	31.3	34.7	35.6	25.8	16.3	4.7
Sugar	8.7	-10.9	6.7	15.0	23.3	-0.7	17.7	26.6	22.4	35.9	30.2	23.4	9.9	14.0
Coffee	23.7	-5.8	19.7	27.9	36.9	20.6	38.2	39.3	50.4	21.0	31.9	23.1	11.8	15.8
Cotton	34.2	-6.0	29.0	22.5	47.7	15.8	37.1	45.6	42.8	35.7	47.2	21.8	9.2	11.5
Live Cattle	68.4	5.0	61.9	47.3	55.2	53.2	48.9	54.3	42.7	19.1	37.3	59.3	35.4	76.3
Lean Hogs	31.4	55.0	40.4	26.9	18.7	38.2	56.7	47.0	29.3	-5.1	9.4	5.9	9.4	30.0
Feeder Cattle	67.9	-0.5	62.7	63.3	63.2	53.6	56.4	53.6	49.7	23.6	34.0	51.1	87.1	17.4

Average Correlation

For Returns	42.3	21.8	41.1	28.3	36.7	39.9	40.2	38.1	35.7	31.3	33.4	31.1	32.7	20.4	22.9
For E(log RVol)	55.7	25.9	55.3	51.0	51.6	50.5	55.9	53.3	43.4	20.2	33.3	37.9	49.9	32.0	52.2

Average Correlation

With SP500 Returns	46.9	11.3	43.4	4.7	24.2	54.2	23.0	21.7	20.3	21.6	27.4	25.3	32.6	9.9	31.3
With SP500 E(log RVol)	81.8	28.5	76.2	76.6	65.5	80.2	47.7	46.7	33.5	0.2	21.7	25.1	60.8	34.5	68.5

The table shows unconditional correlations for daily returns (upper diagonal) and log realized volatility (lower diagonal) for 15 commodity futures during the 2007-2011 (“during crisis”) sample period. All observations are for the most active futures contract on a given day. Average correlations with the S&P 500 E-Mini futures contract are shown in the lower panel for each commodity.

Table A.6: Correlations for Daily Returns (upper diagonal) and Log Volatility (lower). “Post-crisis” sample.

	Light Crude	Natural Gas	Heating Oil	Gold	Silver	Copper	Soybeans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle
Light Crude	12.2	81.9	28.6	40.8	51.5	24.7	18.9	20.2	25.4	18.4	19.9	15.0	11.9	14.7	
Natural Gas	28.2	11.8	3.5	7.4	5.6	6.4	13.1	12.8	6.9	8.6	1.1	5.2	4.3	2.9	
Heating Oil	94.5	31.8	47.4	28.0	39.3	45.1	26.1	19.1	20.8	22.9	17.4	22.5	9.8	10.0	9.3
Gold	52.7	3.2	47.4	80.7	34.9	18.2	14.6	14.1	11.2	14.6	12.6	8.3	3.6	5.5	
Silver	67.6	11.6	64.8	82.5	48.8	24.0	20.0	19.3	16.4	19.4	18.3	10.2	7.2	6.7	
Copper	71.0	18.5	68.3	64.8	75.1	29.4	18.8	20.6	22.9	21.6	22.6	11.2	7.4	10.0	
Soybeans	47.6	24.9	44.3	16.6	29.9	37.3	60.5	49.5	26.1	15.7	23.4	11.0	7.2	-6.8	
Corn	56.2	17.7	56.2	27.8	43.1	54.0	73.0	68.3	27.9	16.2	23.9	15.9	9.8	-13.7	
Wheat	61.9	26.0	63.9	12.7	37.2	57.1	56.7	79.0	25.3	18.5	24.1	13.7	7.8	-9.6	
Sugar	58.0	33.6	62.6	12.0	37.7	55.9	39.0	53.9	77.2	26.5	21.7	16.2	12.0	8.4	
Coffee	-3.1	15.2	-5.1	-11.2	-7.9	4.1	16.4	19.0	28.5	24.9	11.1	8.8	8.5	5.2	
Cotton	58.4	23.2	61.7	21.0	47.7	54.3	41.3	62.0	71.5	79.0	3.9	8.1	3.4	2.3	
Live Cattle	49.4	12.2	48.6	5.4	25.9	45.9	33.5	54.8	73.2	65.8	23.9	67.9	34.7	74.4	
Lean Hogs	29.6	1.2	26.2	-12.4	-0.9	15.5	37.4	43.3	58.8	52.0	43.6	40.6	56.6	25.1	
Feeder Cattle	47.8	5.9	43.9	3.1	22.2	34.8	41.1	52.9	66.0	51.4	7.9	59.7	83.2	50.1	
<i>Average Correlation</i>															
For Returns	32.3	13.5	30.9	25.2	30.6	30.0	27.7	27.6	27.0	24.7	20.7	21.0	22.8	16.9	15.6
For E(log RVol)	54.6	23.5	53.9	28.4	42.4	50.4	42.6	52.9	58.0	53.5	17.3	52.8	49.8	36.1	44.7

	With SP 500 Returns	With SP 500 E(log RVol)	With SP 500 E(log RVol)	With SP 500 E(log RVol)
With SP 500 Returns	49.0	0.5	41.8	6.3
With SP 500 E(log RVol)	72.6	19.4	67.9	53.6

The table shows unconditional correlations for daily returns (upper diagonal) and log realized volatility (lower diagonal) for 15 commodity futures during the 2010-2014 (“post-crisis”) sample period. All observations are for the most active futures contract on a given day. Average correlations with the S&P 500 E-Mini futures contract are shown in the lower panel for each commodity.

Table A.7: Seasonal Regression Dummies for Realized Variance.

	Light Crude	Natural Gas	Heating Oil	Gold	Silver	Copper	Soy- beans	Corn	Wheat	Sugar	Coffee	Cotton	Live Cattle	Lean Hogs	Feeder Cattle	
C	1.202	2.645**	0.082	0.179	0.893**	0.587*	0.388	0.979*	1.576*	1.068*	0.564**	0.326	0.138**	0.407**	0.232**	
AR(1)	0.049	0.177**	0.100**	0.296**	0.282	0.200**	0.237**	0.230**	0.280	0.363**	0.240**	0.170**	0.160**	0.158**	0.145**	
AR(2)	0.020	0.124*	0.089**	0.051	0.044	0.134*	0.099**	0.052*	-0.025	0.039	0.174**	0.103**	0.152*	0.096**	0.091*	0.075**
AR(3)	0.068	0.094*	0.073*	0.039	0.096*	0.082*	0.048**	0.024	0.038	0.024	0.040	0.026	0.096**	0.075**	0.052*	
AR(4)	0.044	0.051	0.092**	0.026	0.046	0.094*	0.044	0.062**	0.030*	0.069	0.024	0.091**	0.043	0.076**	0.076**	
AR(5)	0.089	0.097	0.053*	0.049	0.061	0.066	0.070*	0.066*	0.013	0.078*	0.023	0.032	0.047	0.026	0.096*	
AR(6)	0.015	-0.003	-0.002	0.017	-0.000	0.043	0.075*	0.037	0.030**	-0.007	0.053	0.105*	0.020	0.040*	0.044	
AR(7)	0.048**	0.025	0.051*	0.030	0.029	0.038	0.067**	0.074**	0.047*	0.006	0.047*	0.072**	0.021	0.037	0.032	
AR(8)	0.016	0.014	0.042	0.063	0.024	0.024	0.044	0.024	0.041	0.048**	0.042	0.026	-0.002	0.030	-0.001	
AR(9)	0.058**	-0.008	0.017	0.026	0.016	0.049	0.041	0.031*	-0.012	-0.034*	0.046	-0.005	0.035	0.014		
AR(10)	0.042**	0.042	0.031	0.058	0.060**	0.070	0.041	0.006	0.064	0.066**	0.089**	0.022	0.052	0.045	0.021	
AR(11)	0.050**	-0.001	0.023	-0.010	-0.019	-0.011	-0.022	-0.008	-0.103	-0.000	-0.015	0.045	0.029	0.008	0.046	
AR(12)	0.060**	0.040**	0.045	0.037	0.035	0.030	0.021	0.016	-0.035	0.023	0.040	-0.022	0.048	0.023	0.090	
AR(13)	0.029	-0.004	0.027	0.046	0.004	-0.013	-0.001	-0.007	0.069	0.032	-0.019	0.060	0.029	0.000	0.008	
AR(14)	0.014	0.059*	0.037*	0.018	-0.005	-0.018	0.072*	0.038*	-0.020	-0.026	-0.049	-0.024	0.027	0.024	0.023	
AR(15)	0.042	0.084**	0.038*	0.055	0.114**	0.030	0.017	0.019	0.017	0.031**	0.089**	0.007	-0.012	0.048	-0.003	
AR(16)	0.014	-0.028	-0.019	-0.019	-0.003	-0.009	-0.012	0.025	0.029*	0.003	0.001	-0.008	0.023	-0.018		
AR(17)	0.024	0.009	0.026	0.018	0.008	-0.004	-0.018	0.002	0.001	-0.012	-0.020	0.072	0.019	0.052*	0.041	
AR(18)	0.022	0.038	-0.007	-0.015	0.038**	0.007	0.014	0.019	0.019*	0.077	0.038	0.062	-0.012	0.009	0.020	
AR(19)	0.120**	0.010	0.195*	0.024	-0.013	-0.021	0.054*	0.041*	0.012	0.015	0.019	-0.014	0.062	0.022	0.032	
AR(20)	-0.007	0.015	0.016	0.072	0.037	0.036	0.004	0.013	0.040*	0.024	0.035	0.052	0.028	0.020		
Feb	-0.399	-1.877**	0.066	-0.099	-0.124	-0.035	-0.263	-0.035	-0.643	1.701	-0.459	0.316	-0.115*	-0.083	-0.104	
Mar	-0.842	-2.104**	0.130	-0.080	0.052	-0.456	0.372	0.278	-0.313	-0.284	0.085	0.251	0.020	0.282	-0.037	
Apr	-0.492	-1.735**	-0.163	0.030	1.199	0.010	-0.548*	-0.676	-0.618	-0.744	0.199	-0.217	-0.021	-0.178	-0.110	
May	-0.610	-1.356*	0.268	-0.110	0.275	0.047	-0.006	-0.187	-0.111	-0.502	-0.337	-0.013	-0.092	-0.293*	-0.106	
Jun	-0.381	-1.286*	0.170	-0.060	-0.248	-0.227	0.274	0.804	0.371	-0.423	0.240	0.412	0.115	0.381	0.051	
Jul	-0.747	-0.936	0.102	-0.101	-0.169	-0.724*	-0.131	-0.778	-0.586	-0.726	-0.049	-0.219	-0.193**	-0.139	-0.110	
Aug	0.703	0.992	0.660	0.035	1.148	-0.180	-0.156	-0.286	0.018	-0.362	0.135	0.040	-0.072	0.171	-0.163**	
Sep	-0.602	-0.754	0.308	0.308	1.147	-0.102	-0.058	0.019	0.105	-0.123	0.298	-0.057	0.007	0.168	-0.074	
Oct	-0.122	-1.768**	0.104	-0.085	0.595	0.613	0.126	0.094	0.306	-0.579	0.066	0.217	0.020	0.007	0.011	
Nov	-0.198	-1.031	0.294	-0.152	-0.195	-0.472	-0.217	-0.553	-0.425	-0.183	0.077	-0.109	-0.257	-0.046		
Dec	1.002	-1.067	0.698	-0.169	-0.106	-0.646	-0.266	-0.650	-0.731	-0.805	-0.072	-0.404	-0.077	-0.143	-0.091	

The table shows parameter estimates for an AR(20) model for (1-minute sub-sampled) 5-minute realized variance for 15 commodities with monthly dummies during the 2004-2014 sample period. One asterisk indicates significance at the 95% level and two asterisks at the 99%.

Table A.8: Contemporary PC Regressions on Commodity Carry**Table A.8.a:** $PC_{1,t} = \alpha + \beta_1 PC_{1,t-1} + \beta_2 X_t + \epsilon_t$

	<i>Energy</i>	<i>Meats</i>	<i>Metals</i>	<i>Softs</i>	<i>Grains</i>
α	-0.027*	-0.013	-0.000	-0.018	-0.013
β_1	0.819**	0.830**	0.835**	0.829**	0.829**
β_2	-2.844**	-2.115**	-0.075	-3.303**	-2.411**
R^2	0.700	0.698	0.697	0.698	0.698

Table A.8.b: $PC_{2,t} = \alpha + \beta_1 PC_{2,t-1} + \beta_2 X_t + \epsilon_t$

	<i>Energy</i>	<i>Meats</i>	<i>Metals</i>	<i>Softs</i>	<i>Grains</i>
α	-0.010	-0.005	-0.001	-0.018	-0.008
β_1	0.671**	0.673**	0.675**	0.660**	0.669**
β_2	-1.012*	-0.891	-0.487	-3.463**	-1.596*
R^2	0.458	0.456	0.456	0.461	0.458

Table A.8.c: $PC_{3,t} = \alpha + \beta_1 PC_{3,t-1} + \beta_2 X_t + \epsilon_t$

	<i>Energy</i>	<i>Meats</i>	<i>Metals</i>	<i>Softs</i>	<i>Grains</i>
α	-0.010	0.008	-0.001	-0.014*	-0.007
β_1	0.727**	0.727**	0.734**	0.722**	0.727**
β_2	-1.067**	1.404**	-0.810*	-2.580**	-1.459**
R^2	0.542	0.542	0.540	0.544	0.542

Table A.8.d: $PC_{4,t} = \alpha + \beta_1 PC_{4,t-1} + \beta_2 X_t + \epsilon_t$

	<i>Energy</i>	<i>Meats</i>	<i>Metals</i>	<i>Softs</i>	<i>Grains</i>
α	0.025**	0.002	-0.001	0.000	0.014
β_1	0.534**	0.575**	0.575**	0.575**	0.552**
β_2	2.620**	0.383	-0.625	0.034	2.695**
R^2	0.348	0.331	0.331	0.331	0.341

The table shows output from regression of the first four principal components of commodity log volatility on its lag and commodity carry defined in equation 8. The commodity category carry for day t is the simple arithmetic average of the day t carry for the individual commodities in the respective categories following Table 1. The principal components are constructed as the matrix of (demeaned) log volatility for all 15 commodities multiplied by the eigenvectors of the covariance matrix. The sample period covers 2004-2014. Two asterisks indicates rejection of the null of zero coefficients at the 1% level and one asterisk at the 5% level.