Internet Appendix: The Term Structure of Expected Recovery Rates

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This document provides additional results that are omitted from the paper due to space considerations.

- **Section A** discusses estimates of the expected recovery and the default intensity for each firm in our sample.
- **Section B** presents a case study of Fannie Mae and examines its time-varying recovery rate dynamic.
- **Section C** presents results showing that the long-run equilibrium relationship between senior and subordinate CDS spreads does not breakdown during subprime crisis, indicating that our findings are not driven by structural change in the relative liquidity between senior and subordinate CDS.
- **Section D** examines the robustness of our model estimates to subperiod tests.
- **Section E** examines out-of-sample pricing performance of the CDS valuation model.
- **Section F** examines the robustness of our main conclusions to an alternative estimation procedure, i.e., a joint estimation approach.
- **Section G** presents evidence that the model can fit the simulated data reasonably well.
- **Section H** shows that the default intensity dynamic does not affect the relative CDS spreads between senior and subordinate contracts.
- **Section I** discusses the importance of using multiple-seniority CDS term structures for estimating time-varying recovery rates.

A. Expected Recovery: Individual Firms

Table IA1 reports the time-series averages of the expected recovery at three different maturities for each firm in our sample. The results are reported separately for senior and subordinate contracts. The table also reports the time-series averages of 1-year default probabilities implied by the model. The 1-year default probability at time t is calculated as $1 - Q_t[\tau > t + h]$, where $Q_t[\tau > t + h]$ is the survival probability between current period t up until time $t + h$, where $h = 12$ months. To see how the default probability is related to the expected recovery rates, the last two columns in Table IA1 report the time-series correlations between the 1-year expected recovery and the 1-year default probability for senior and subordinate contracts.

Table IA1 shows the expected recovery for senior contracts on Fannie Mae and Freddie Mac, the two government-backed entities, are among the highest in our sample. Their expected recovery for subordinate contracts are significantly lower. The correlations between the 1-year recovery and the 1-year default probability are mostly negative, suggesting that on average, the recovery rate at default decreases when the likelihood of default rises.

B. Time-varying Recovery: A Case Study

This section examines how important news arrivals impact the time-series dynamic of expected recovery rate at an individual firm level. We focus our study on a government sponsored entity Fannie Mae. We chose this firm because it was severely impacted by the subprime crisis as well as drawing widespread media attention.[23]

The top two panels in Figure IA1 plot the time series of 1-year expected recovery for Fannie Mae. The panels below plot its time series of market-observed and model-implied five-year CDS spreads, model-implied 1-year default probabilities, and 1-year trailing stock return. Looking at the second-row panels, the market-observed CDS spread (black line) and the model-implied CDS spread (grey line) have almost identical time-series dynamics. These results show that the CDS model performs well in fitting the spreads of senior and subordinate contracts.

Figure IA1 shows the expected recovery of Fannie Mae varies substantially over time. Its time-series average of expected recovery on a five-year senior contract is 70.15%. This value is relatively large compared to other firms in our sample, which reflects its sponsorship

[23]We observe similar results for Freddie Mac, another government-sponsored entity in our sample.

by the U.S. government. An important news item that significantly impacted Fannie Mae occurred on June 9, 2003.[24] Figure IA1 shows the expected recovery of Fannie Mae drops significantly in mid-2003, before climbing back to their conventional values.

Figure IA1 shows the expected recovery of Fannie Mae fell by as much as 50% in 2007 relative to its pre-crisis level. However, its recovery level started increasing in early 2008 and eventually exceeded its pre-crisis period values. Interestingly, while the expected recovery increased, CDS spreads of Fannie Mae widened, and its 1-year default probability rose sharply (bottom-left panel). In other words, the default risk of of Fannie Mae became positively correlated with their recovery rate during the crisis. The rising recovery rates of Fannie Mae during the subprime crisis can be linked to the government bail out when on July 24, 2008, the United States Congress passed the Housing and Economic Recovery Act of 2008 (HERA).[25] This bill was intended to restore confidence in Fannie Mae by strengthening regulations and injecting capital into their mortgage funding. As a result, Fannie Mae's CDS spread fell in the end of March 2008, while in the mean time, its 1-year expected recovery rose back to and eventually surpassing its pre-crisis level.

As shown in Figure IA1, we argue that the increase in expected recovery of Fannie Mae in 2008 is due to the government bail out. The model-implied default probability of Fannie Mae increase by roughly 400% from mid-2007 to Sep. 2008. However, its CDS spreads only increase by about 100%, suggesting the recovery rates of its debt must increase in order to balance the rapid rise in the default probability. Our results in Figure IA1 provide an economic insight linking the effect of government bailout to the borrowing costs in the debts market.

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[24] On that day, Freddie Mac was re-audited for three prior years because its previous auditor, Arthur Andersen, mis-applied accounting rules. As a result of the news, Freddie Mac's stock plunged 16% on that day. This accounting malpractice received wide media attention and its effect spread to Fannie Mae because lawmakers were pushing for more oversights among these government-sponsored entities.

[25] The bill authorized the Federal Housing Administration to guarantee up to $300 billion in new fixed rate mortgages for subprime borrowers, if lenders would write-down principal loan balances to 90% of current appraisal value.

C. Robustness Check: Price Discovery during the Financial Crisis

We follow Blanco, Brennan and Marsh (2005), and Norden and Weber (2009, 2012), among others, and estimate a two-stage vector error correction model (VECM) on daily changes in senior and subordinate CDS spreads. In the first stage, we estimate the long-run relationship between senior and subordinate CDS spreads according to

$$CDS^{SUB}_{i,\tau,t} = \alpha_\tau + \rho_1 CDS^{SEN}_{i,\tau,t} + \rho_2 CRISIS_t + E_{i,\tau,t}, \tag{C-9}$$

where $CDS^{SEN}_{i,\tau,t}$ and $CDS^{SUB}_{i,\tau,t}$ are senior and subordinate spreads of firm i with maturity τ on day t. $CRISIS_t$ is a time-series dummy equal to 1 from Dec. 2007 to June 2009, which corresponds to the subprime recession period defined on NBER's website. We estimate the above model (C-9) for each firm across all maturities. We include maturity fixed effects in the regression.

The residual term, $E_{i,\tau,t}$, in equation (C-9) can be interpreted as the error correction term. In the second stage, we apply lags of residual $E_{i,\tau,t}$ and estimate the short-run relation between daily changes in senior and subordinate CDS spreads. The second-stage regression model is given by

$$\Delta CDS^{SEN}_{i,\tau,t} = \delta_{1\tau} + \lambda_1 E_{i,\tau,t-1} + \beta_1 \Delta CDS^{SEN}_{i,\tau,t-1} + \gamma_1 \Delta CDS^{SUB}_{i,\tau,t-1} + \varepsilon_{i,\tau,t} \tag{C-10}$$

$$\Delta CDS^{SUB}_{i,\tau,t} = \delta_{2\tau} + \lambda_2 E_{i,\tau,t-1} + \beta_2 \Delta CDS^{SEN}_{i,\tau,t-1} + \gamma_2 \Delta CDS^{SUB}_{i,\tau,t-1} + \varepsilon_{i,\tau,t}, \tag{C-11}$$

where $\Delta CDS^{SEN}_{i,\tau,t}$ and $\Delta CDS^{SUB}_{i,\tau,t}$ denote the change in senior and subordinate CDS spreads, respectively. Equation (C-9) suggests that when the residual term $E_{i,\tau,t-1}$ is positive, the subordinate CDS spread level is too high relative to that of the senior contract. Consequently, the subordinate CDS spread will decrease while the senior CDS spread will increase. Therefore, if the equilibrium relation between senior and subordinate CDS spreads holds, we expect the sign on λ_1 in equation (C-10) to be positive, and the sign on λ_2 in equation (C-11) to be negative.

Table IA3 reports results for the regression model in equations (C-10)-(C-11). We report estimates for two samples. The first is the full sample for which we have data available from Jan. 2001 to May 2012. The second period corresponds to the subprime crisis spanning from Dec. 2007 to June 2009. Table IA3 shows the coefficients on the error correction terms $E_{i,\tau,t-1}$ are correctly signed and statistically significant in the full sample as well as during the crisis period. The results suggest that there is a long-run equilibrium relation between senior and subordinate CDS spreads, and importantly, the relationship holds during the financial crisis. Table IA3 shows the negative coefficients on lagged CDS spread changes, supporting the mean-reverting dynamic of CDS spreads. Importantly, we find that the mean-reverting property of CDS spreads changes is statistically significant for senior and subordinate contracts, and that this finding holds during the financial crisis.

D. Robustness Check: Subperiod Analysis

This section reports results from the subperiod analysis of the CDS valuation model. We provide evidence showing that our estimation results are fairly stable when we estimate the model using different sample periods.

For each of 46 firms in our sample, we reestimate the model on 2 subperiods. The first is from Jan. 1, 2001 to June 30, 2007, corresponding to the "pre-crisis" period. The second subperiod is from July 1, 2007 to May 31, 2012, which includes the subprime crisis period and the post-crisis period; we refer to it as the "crisis/post-crisis" period. This analysis yields $2 \times 46 = 92$ sets of new parameter estimates. We compare the model parameters and the recovery rate dynamics estimated from these two subperiods against those estimated using the full-sample period, which is from Jan. 1, 2001 to May 31, 2012. We examine the stability of our model estimates by looking at their parameter deviations in Table IA4, and visually by plotting their recovery dynamics in Figure IA2. We discuss our findings below.

Table IA4 reports cross-firm average deviations (in %) of model parameters estimated from the two subperiod samples against those obtained from the full sample. The percentage deviation of each model parameter is first calculated firm by firm. For instance, if θ_s and θ_f are the two parameters that we estimate from the subsample and full-sample periods, respectively, we calculate its percentage deviation as $\left(\theta_s - \theta_f\right)/\theta_f \times 100$. We then report the average deviation of each parameter (in %) across 46 firms in Table IA4. Panel A reports average deviations of the loading coefficients, while Panel B reports average deviations of state-variable dynamics. In each panel, results from the two subperiods are reported: the pre-crisis period, and the crisis/post-crisis period.

Panel A of Table IA4 shows that the loading coefficients are very stable across sample periods. Their percentage deviations range between -6.15% and 3%. On average, we find the loading coefficients estimated from the two subperiods tend to be smaller than those obtained from the full-sample period (i.e., negative percentage deviations). Nevertheless, these deviations are fairly small suggesting the stability in loading-coefficient estimates of the default intensity dynamic, and the loss-given-default (LGD) dynamics of senior and subordinate contracts. At this point, we note that it is difficult to interpret, by looking at these parameter deviations alone, how the levels of model-implied default intensity and recovery rate levels are directionally affected. This is because the dynamics of default intensity and LGDs are quadratic in the state variables; see equations (5) and (6) in the main text. We will later discuss deviations in the model-implied default intensity and recovery rate dynamics using a visual representation in Figure IA2.

Panel B of Table IA4 reports parameters that govern the dynamic of latent state factors X_3, X_4, and X_5. Overall, we find that deviations in the factor-dynamic parameters are larger than the loading-coefficient parameters shown in Panel A. The deviations in their parameters are between -19.24% and 18.33%. We expect this finding because X_3, X_4, and X_5 are latent-state variables. Therefore, the length and when the sample starts can substantially affect the results.

We find the dynamic of latent factor X_5, which drives the loss-given-default (LGD) dynamic is more stable than the dynamic of latent factors X_3 and X_4, which drive the default intensity dynamic. This finding is intuitive and expected because the default intensity dynamic is explained by two latent factors (X_3 and X_4), while only one factor drives the LGD dynamic (X_5). As a result, there is more degree of freedom in the modeling of the intensity dynamic.

Overall, results in Panel A of Table IA4 indicate that the loading coefficients are stable when estimated using a subperiod. The latent-factor dynamic, on the other hand, shows more deviations but we believe these magnitude are within reasonable range. Looking at parameter deviations in IA4 does not give a clear picture of how the default intensity and LGD dynamics are affected when the model is estimated using a subperiod sample. As mentioned previously, this is because we assume a quadratic specification in the default intensity and LGD.[26] For a better representation of deviations in the default intensity and recovery rate dynamics, we present our findings visually in Figure IA2.

Figure IA2 plot results comparing the recovery rate and the default intensity implied by subperiod estimates against those implied by full-sample estimates. We plot daily cross-firm averages of the 1-year expected recovery, the slope of expected recovery, and the 1-year default probability. The left-column panels compare pre-crisis results against full-sample results. The right-column panels compare crisis/post-crisis results against full-sample results. In all panels, the dark solid line represents subperiod estimates, while the dotted line represents full-sample estimates. We find very little deviation in the time series of 1-year expected recovery (top panels) and 1-year default probability (bottom panels). This

[26]We use the quadratic specification because it guarantees that our the default intensity dynamic and the LGD dynamics are always positive. See for examples, Longstaff (1989), Constantinides (1992), Ahn, Dittmar and Gallant (2002), Leippold and Wu (2002), Li and Zhao (2006), Ang, Boivin, Dong and Loo-Kung (2011), and Doshi, Ericsson, Jacobs, and Turnbull (2013) for applications of quadratic term structure models.

indicates that our estimates of the recovery-rate level and the default-intensity level are remarkably stable and robust to a subperiod test.

We find some deviations in the slope of expected recovery levels between the full-sample and subperiod results. This is shown in the middle panels of Figure IA2. Recall that the slope of expected recovery is the relative difference in expected recovery levels at the 10-year and 1-year horizons. Thus, while we find that the level of expected recovery at the 1-year horizon can be precisely pinned down as shown in the top panels of Figure IA2, its estimate is less stable at a longer horizon (i.e., 10 years). Nevertheless, the two time series of expected-recovery shown in the middle panels share close resemblance; they tend the move in the same direction. We also emphasize that these deviations are economically small.

E. Out-of-sample Tests

We examine out-of-sample pricing implications for our model. For continuity with the subperiod analysis shown in Section D, we split our sample into two periods: the "pre-crisis" period, which is from Jan. 2001 to June 2007; and the "crisis/post-crisis" period, which is from July 2007 to May 2012. We use parameter estimates from the pre-crisis period to price CDS out-of-sample in the crisis/post-crisis period. Table IA5 reports the results.

The procedure for calculating out-of-sample CDS pricing errors are as follows. We first re-estimate the model firm-by-firm using CDS data from the pre-crisis period. This yields 46 sets of new parameters, one for each firm. Before we can apply these parameters to price CDS contracts in the out-of-sample period, we need to we filter out daily latent factors $X_{3,t}, X_{4,t}$ and $X_{5,t}$.[27] For each firm, we apply its estimates obtained from the pre-crisis period to filter daily state variables $X_{3,t}, X_{4,t}$ and $X_{5,t}$ from July 2007 to May 2012.[28] We then apply these parameter estimates together with daily filtered state variables to price CDS contracts.

Table IA5 reports average CDS pricing errors for the crisis/post-crisis period, i.e., July 2007 to May 2012. We report two measures of pricing errors: the relative root-mean-squared-error (RMSE) in Panel A, and the absolute percentage error (APE) in Panel B. In each panel, we report both in-sample pricing errors and out-of-sample pricing errors.

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[27]We do not need to filter latent factors $X_{1,t}$ and $X_{2,t}$ because they are identified from the term structure of default-free rates.

[28]This procedure implies that while our parameters are truly out-of-sample, the latent factors are filtered using in-sample information.

On average, Table IA5 shows that out-of-sample errors can be twice as large as their corresponding in-sample pricing errors. Looking at senior CDS contracts, we find that the model prices out-of-sample 1-year CDS fairly well, but not for longer-maturity CDS contracts. We believe that these relatively large out-of-sample pricing errors are due to the substantial increase in firms' default intensities following the subprime crisis. As illustrated in the top panels of Figure 2, we find that the 1-year default probability starts increasing substantially in 2007, when our out-of-sample period begins.

For subordinate CDS contracts, out-of-sample pricing errors are twice as large as their corresponding in-sample errors. Across all maturities, we find that out-of-sample pricing errors for subordinate contracts are larger than those reported for senior contracts. We believe that the relatively worse out-of-sample pricing performance for subordinate contracts derive from two aspects. The first is the drastic increase in default probability at the beginning of the subprime crisis, which affects both senior and subordinate CDS contracts. The second aspect is the relatively more volatile and steeper slope of expected recovery in the post-crisis period for subordinate contracts relative to those for senior contracts. This is evidenced by the two middle panels in Figure 2, which show that the slope of expected recovery becomes significantly more negative for subordinate CDS contracts after the subprime crisis. These drastic changes in the default probability and in the slope of expected recovery found in subordinate CDS contracts may explain why out-of-sample pricing errors (i.e., crisis/post-crisis period) are larger for subordinate contracts.

Overall, we find that the model's out-of-sample performance is modest, and relatively weak for subordinate contracts. These findings are expected because the out-of-sample period that we use corresponds to a drastically different economic cycle from the period where the model parameters were fitted to CDS data. One implication from this out-of-sample analysis is that it is important to update the model parameters frequently, and using a rolling-window estimation. This is because model parameters that govern the default intensity and recovery rate dynamics are highly sensitive to changes in the economic cycle.

F. Robustness Check: Joint Estimation

We conduct an alternative estimation procedure, i.e., a joint estimation of model parameters across firms, to check if our main results would change substantially. This robustness test affects the second-step estimation procedure only, where the default intensity dynamic and the LGD dynamic are estimated. Instead of estimating default-intensity parameters and loss-given-default (LGD) parameters firm by firm, we estimate them using CDS information on 17 financial firms all at once. We choose to focus on firms in the financial industry because they were most severely affected by the 2008 subprime crisis, which affects their term structure of expected recovery rates. We do not estimate the default intensity and LGD dynamics on all 46 firms in our sample simultaneously because prior studies find that the default probability and LGD dynamics are industry specific (e.g., Chava and Jarrow (2004)). This finding is also supported by our results in Table 5, which show that default probability and expected recovery rate differ across industries.

The joint estimation method yields one set of parameter estimates and time series of latent factors. On the other hand, firm-by-firm estimation yields 17 sets of parameter estimates and time series of latent factors, i.e., one set for each firm. Table IA6 presents results comparing estimates obtained from the joint estimation against those estimated firm by firm. Parameter estimates and their standard errors are reported for the joint estimation. For the firm-by-firm estimation, we report the distribution of each model parameter. We visually present results comparing the recovery rate and default intensity levels obtained from the two estimation methods in Figure IA3.

Looking at the parameters in Table IA6, we find that the joint estimation estimates always lie between the 25th and 75th percentiles of their corresponding parameters' distribution obtained from the firm-by-firm estimations. This finding suggests that parameter estimates obtained using the two different estimation methods are comparable. In other words, we find the joint estimation method yields parameter estimates that are representative of the median firm in the financial industry. Standard errors of joint-estimation parameter estimates are fairly small suggesting that they are precisely estimated. This finding is expected because the number of data that goes into the joint estimation is large, consisting of senior and subordinate CDS spreads of 17 financial firms.

Figure IA3 provides a visual presentation of results obtained from the two estimation methods. Here, we plot daily time series of 1-year expected recovery level, the slope of expected recovery term structure, and the 1-year default probability. The left-column panels plot results from the firm-by-firm estimation; they represent averaged time series across 17 financial firms. The right-column panels plot results from the joint estimation. We find that the 1-year expected recovery time series are very comparable between the two estimation methods.

Looking at the slope of expected recovery and the 1-year default probability, we find that time-series estimates from the joint estimation are able to match the level and the general pattern of the averaged time-series estimates obtained from the firm-by-firm estimation. However, the joint-estimation time series (right-column) are less volatile compared to the averaged results obtained from the firm-by-firm estimation (left-column). We believe this finding is intuitive because joint-estimation results produce one set of estimates that is representative of a firm in the financial industry. On the other hand, in the firm-by-firm estimation, parameter estimates can vary significantly among firms, resulting in a larger fluctuation in their averaged time series of default probability and term structure of expected recovery. Nevertheless, results from the joint estimation of financial firms yield a similar conclusion as those obtained from the firm-by-firm estimation. That is, the level of expected recovery varies significantly through time and worsens during the subprime crisis period. Also, the term structure of expected recovery for financial firms spike above (or close to) 0, during and after the subprime crisis period, indicating the inversion of the term structure of expected recovery levels for financial firms.

G. Monte Carlo Simulation Study: Model Pricing Errors

We evaluate the model performance at fitting the simulated data. Specifically, we examine the model's pricing errors against their "true" values along three dimensions: CDS spreads, expected recovery rates, and binary CDS spreads. Table IA7 reports the results. In Panel A, we report the means and standard deviations of simulated CDS spreads, expected recovery rates, and binary CDS spreads. A binary CDS spread is priced similarly to a standard CDS contract but with a 0 recovery rate when the firm defaults (i.e. the LGD is 100%). There is no recovery risk in binary CDS contracts due to the certainty of 0% recovery, and hence their spreads reflect only the default risk. We use binary CDS spreads as our benchmark for evaluating the robustness of the estimates for the default intensity dynamic. Overall, the simulated samples show an increasing term structure of CDS and binary CDS spreads, while expected recovery rates decrease as the default horizon increases.

Panel B of Table IA7 summarizes the means and standard deviations of absolute percentage pricing errors for CDS spreads, expected recovery rates, and binary CDS spreads grouped by maturity and seniority levels. Results in Panel B show the pricing error is slightly higher for shorter-maturity CDS contracts. Nevertheless, the absolute percentage errors in pricing CDS spreads, on average, are small and fall under 5%. Expected recovery rates and binary CDS spreads are also reasonably well estimated with absolute errors below 11% relative to their true values, respectively. These relatively small pricing errors for expected recovery rates and binary CDS spreads suggest that we can estimate the recovery and intensity dynamic fairly well.

H. Default Intensity and CDS Spreads Ratio

In this section, we show the default intensity dynamic does not affect the relative CDS spreads between senior and subordinate contracts. Thus, the relative term structure of senior to subordinate CDS spreads are identified by their loss-given-default dynamics. From the main paper, we recall that the default intensity dynamic depends on four latent factors.

Its dynamic is given by

$$\lambda_t = (\alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \alpha_4 X_{4,t})^2 \,. \tag{C-12}$$

The first two factors, $X_{1,t}$ and $X_{2,t}$, are estimated from the default-free term structure. The third and fourth factors, $X_{3,t}$ and $X_{4,t}$, are specific to the default intensity and describe the changes in intensity dynamic not already captured by the changes in the default-free term structure.

When pricing CDS spreads, the default intensity behaves as an additional discount factor. Therefore, senior and subordinate CDS spread levels are affected proportionally when there is a shock to the intensity factors. As a result, the ratio of senior to subordinate CDS spreads do not vary as the intensity factors change. This intuition explains why the ratio of senior and subordinate CDS spreads are determined by their loss-given-default dynamic, and not their default intensity dynamic. Figure IA4 illustrates why the intensity factors do not impact the ratio of senior to subordinate CDS spreads. Using the base case parameters in Table 7 of the main text, we simulate 100 sample paths of daily senior and subordinate CDS spreads with five-year maturity. Each simulated sample consists of 1500 days. The results in this figure correspond to daily average values across 100 samples. The middle and bottom panels plot daily cross-sectional averages of five-year senior and subordinate CDS spreads, respectively. The top panel of Figure IA4 plots the ratio of senior to subordinate CDS spreads with five years to maturity. In each panel, we plot the results for three sets of intensity loading parameters. The first is the base line result which uses the base case parameters reported in Table 7. For the two other cases, we simply multiply intensity loadings of all four factors in equation (C-12) by 5 and 10.

The top panel of Figure IA4 shows that the ratios of senior to subordinate CDS contracts are always below one, indicating that the LGD of senior CDS is lower than that of subordinate CDS. More importantly, we find the five-year CDS ratio plots appear almost identical under the three sets of intensity loading parameters. The results from Figure IA4 confirm that shocks to the intensity dynamic affect CDS spreads of different seniorities proportionally. Therefore, the LGDs of senior and subordinate contracts are the primary determinants affecting their CDS spreads ratio.

I. Identifying Time-varying Default Intensity and Recovery

This section discusses the importance of using multiple-seniority CDS contracts to identify the recovery rate and default intensity dynamics. We argue that the term structure of *multiple-seniority* CDS contracts is required to separately identify the time-varying default intensity dynamic from the time-varying recovery rate dynamic.

To support our argument, we estimate a constant recovery model on CDS data generated by the model with stochastic recovery and stochastic default intensity.[29] That is, we estimate a model that misspecifies the recovery rate dynamic but not the default intensity dynamic. Our objective is to show that if we use the term structure of single-seniority CDS contracts in the estimation, the average recovery level can be recovered, but errors from misspecifying the recovery dynamic will cause systematic biases in the default intensity estimates. However, if term structure of multiple-seniority CDS contracts are used, errors from misspecifying the recovery dynamic will not severely impact the default intensity estimates because such estimation approach imposes a strict identification mechanism on the default risk.

Using the CDS data simulated in Section A, we estimate the constant recovery rate model on two distinct samples. The first estimation sample relies on simulated CDS spreads of senior *and* subordinate contracts (multiple seniority). In the second estimation sample, we estimate the model only on simulated senior CDS spreads (single seniority). The simulated data that we use consists of 100 paths each with 1500 observations; see Panel A of Table IA7 for their summary statistics.

Table IA8 reports absolute percentage pricing errors from the two estimation samples. We report the means and standard deviations of pricing errors for senior CDS spreads, binary CDS spreads, and expected recovery rates of senior contracts. We examine pricing errors only for senior contracts to facilitate a fair comparison between the two estimation samples. Absolute pricing error for senior recovery rates is calculated against the time-series averages of their simulated values (Panel A of Table IA7).

As expected, Table IA8 shows the pricing errors for senior CDS spreads are substantially smaller for the estimation sample that uses only senior contracts. This finding is not surprising because the model is optimized to fit only senior CDS spreads. However, this relatively small senior CDS pricing errors come at the expense of systematic biases in the default intensity estimates. Table IA8 shows the errors from pricing binary CDS spreads are much smaller when the model is estimated using both senior and subordinate CDS contracts. Because binary CDS spreads are only affected by default risk, we use them as our benchmark for evaluating the robustness of the default intensity estimates. The absolute pricing errors of binary CDS spreads are between 6.8–12.5% when estimated using multiple-seniority contracts; see estimation sample (1), and 17.7–23.7% when estimated using single-seniority contracts; see estimation sample (2).

[29]Under our modeling framework, the constant recovery rate model is obtained by setting the parameters β_1, β_2 and β_5 in equation (6) to 0.

Table IA8 reports the absolute percentage errors for senior recovery rates calculated against the time-series average of their simulated values, which is 50%. We find the errors are smaller when the model is estimated using both multiple-seniority CDS contracts. Nevertheless, the errors from the two estimation samples are fairly small, suggesting the model can roughly identify the average recovery level even when the true data generating process is a stochastic recovery model.

We next analyze the source of pricing errors for senior CDS spreads in the two estimation samples. Because the model that we estimate misspecifies the recovery rate dynamic to be constant, we expect the model's CDS pricing errors to vary with the time-varying recovery rates, which are absent from the model. Failure to find a strong relationship between the pricing errors and the time-varying recovery rates would suggest that this misspecification is absorbed as biases in other parameter estimates of the model. To test this conjecture, we estimate the following pooled regression of daily log CDS pricing errors on daily log implied LGDs:

$$log(CDS^{\tau}_{i,t}) \; - log(\widehat{CDS^{\tau}_{i,t}}) = a^{\tau} + b \log\left(Implied \;\; LGD^{\tau}_{i,t}\right) + \varepsilon_{i,t}, \tag{C-13}$$

where $CDS^{\tau}_{i,t}$ and $\widehat{CDS^{\tau}_{i,t}}$ are the model-generated and "true" simulated τ-year CDS spreads of simulation path i on day t. The *Implied* $LGD^{\tau}_{i,t}$ is the LGD level for τ-year CDS contracts.

For each of the 100 simulation paths, we estimate the model in equation (C-13) across five CDS maturities.[30] We find the regression adjusted R^2 value for the estimation sample (1) is, on average, 65.5%, while for the estimation sample (2), it is only 2.6%. These results show that when the model is estimated on multiple-seniority CDS spreads, CDS pricing errors correctly reflect time variations in the LGD which is absent from the constant recovery rate model. However, if we estimate the constant recovery model using single-seniority CDS spreads, time variations in the recovery rate is incorrectly picked up by the intensity dynamic, suggesting that the default intensity is severely biased. In this latter case, the intensity dynamic no longer reflects only the default risk, but also the recovery risk,

[30]It is calculated as the ratio of CDS spreads to binary CDS spread with the same maturity. This definition is drawn from the fact that binary CDS assume 0% debt recovery upon default and hence must have bigger spreads relative to CDS contracts of the same maturity. Therefore, the ratio of CDS to binary CDS spreads of the same maturity must be between 0 and 1, which reflects the implied loss rate on the debt value insured by the CDS contract. We use the implied LGD as our instrument that proxies for time-varying recovery dynamics.

which explains why binary CDS spreads in Table IA8 are severely mispriced for the estimation sample (1).

Overall, these results confirm the importance of using the term structure of senior and subordinate CDS spreads when estimating time-varying recovery model. That is, the use of multiple-seniority CDS contracts imposes a stricter identification mechanism on the default intensity, making it less likely to pick up changes in CDS spreads that are due to time-varying recovery rates.

References

[1] Blanco, R.; S. Brennan; and I. Marsh. "An Empirical Analysis of the Dynamic Relation between Investment-grade Bonds and Credit Default Swaps." *Journal of Finance*, 60 (2005), 2255–2281.

[2] Norden, L., and M. Weber. "The Co-movement of Credit Default Swap, Bond, and Stock Markets: An Empirical Analysis." *European Financial Management* 15 (2009), 529–562.

[3] Norden, L., and M. Weber. "When Senior Meets Junior: Information in Credit Default Swap Spreads of Large Banks." Working paper, Rotterdam School of Management (2012).

Table IA1: Expected Recovery, and Default Probability for Individual Firms

This table reports the time-series averages of expected recovery and default probability implied by the CDS valuation model. The results are reported at the firm level. For each firm, we fit the model to a panel of daily CDS spreads. *Data availability* indicates the start and end dates of the CDS data for each firm in our sample. We report the time-series averages of expected recovery for senior and subordinate CDS contracts at 1-,5-, and 10-year horizons. *Default prob.* reports the average default probability for 1-year horizon. The last two columns, labeled *Correlation,* report the time-series correlations between the 1-year expected recovery and 1-year default probability for senior and subordinate contracts. An asterisk next to the firm's name indicates that it is a financial firm. We define financial firms as those categorized under the Finance, Insurance, and Real estate sectors following the classification on Kenneth French's Web site. We report, next to each firm, its official ticker and its corresponding overall credit rating level. We obtain monthly S&P long-term entity credit ratings from Compustat.

Firm names	Ticker	Entity ratings	Data availability	Default prob.(%)	Expected recovery (%)						Correlation (%)	
					Senior			Subordinate			Senior	Subordinate
				1 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	1 Yr
Amkor Tech Inc	AMKR	B	05/03-08/10	5.8	35.8	35.1	35.1	15.8	15.7	15.7	-24.7	-20.7
Bear Stearns Cos Inc*	BSC	A	12/06-07/08	1.7	32.5	34.9	38.6	13.4	14.9	16.8	-26.8	-16.7
Best Buy Co Inc	BBY	BBB	04/03-05/12	1.3	28.2	30.1	32.9	12.4	10.4	8.1	-25.2	-21.2
Boyd Gaming Corp	BYD	BB	12/05-05/12	5.9	5.8	3.9	3.8	3.6	2.4	2.4	-51.7	-58.9
Cap One Bk*	COF	BBB	04/04-05/12	1.2	31.6	27.9	24.9	13.7	7.4	5.4	-28.7	-25.4
Citigroup Inc*	C	AA	02/01-05/12	2.2	59.9	49.4	43.0	40.0	33.8	31.1	-65.1	-60.2
Cox Comms Inc	COX	BBB	03/01-05/12	10.8	57.9	46.5	35.9	52.5	41.1	31.6	23.2	26.5
Ctrywde Finl Corp*	CFC	BBB	12/06-11/08	9.7	62.5	57.2	53.2	52.4	49.5	46.0	-75.8	-76.2
D R Horton Inc	DHI	BB	03/06-01/08	6.0	42.1	15.3	14.9	11.4	9.8	9.8	-47.7	-42.7
Fed Home Ln Mtg Corp*	FRE	AAA	12/02-09/08	0.8	70.7	64.7	61.3	38.3	35.8	34.1	25.3	28.6
Fed Natl Mtg Assn*	FNM	AAA	09/02-09/08	1.0	78.1	70.2	66.5	49.5	43.9	41.7	42.9	44.7
Goldman Sachs Gp Inc*	GS	A	12/06-05/12	1.9	13.3	22.9	38.8	0.0	0.1	0.1	-80.5	-66.1
Harrahs Entmt Inc	HET	BB	07/04-11/10	7.6	38.5	52.3	68.1	34.8	49.6	65.4	-77.2	-72.8
Health Mgmt Assoc Inc	HMA	BB	01/05-05/12	2.2	11.6	9.4	9.2	1.1	0.6	0.6	-20.0	-13.5
Iron Mtn Inc	IRM	BB	05/05-05/12	3.7	36.9	12.7	12.2	31.8	9.9	8.6	-26.7	-16.5
JPMorgan Chase & Co*	JPM	A	01/01-05/12	1.4	59.7	48.3	43.4	45.5	34.6	30.2	-12.4	-7.8
KB Home	KBH	BB	06/03-05/12	3.4	35.3	36.9	39.1	25.1	27.9	30.0	-14.8	-24.8
Kerr Mcgee Corp	KMG	BB	03/06-05/12	0.5	29.9	30.5	30.7	11.8	13.6	16.8	-43.0	-40.1
Kohls Corp	KSS	BBB	01/04-05/12	0.9	41.7	30.4	28.7	35.3	28.8	26.7	39.5	39.6
L 3 Comms Corp	LLL	BBB	08/05-05/12	1.1	23.4	21.3	20.1	23.0	18.5	16.3	21.3	25.7
Lehman Bros Hldgs Inc*	LEH	A	01/07-09/08	2.1	16.1	20.7	22.0	4.1	6.0	6.4	-56.4	-58.6
Loews Corp*	L	A	08/03-05/12	0.4	33.7	35.7	40.4	19.6	22.5	28.6	30.6	17.6
(continued....)												

Table IA1: (Continued...)

Firm names	Ticker	Entity ratings	Data availability	Default prob.(%)	Expected recovery (%)						Correlation (%)	
					Senior			Subordinate			Senior	Subordinate
				1 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	1 Yr
(continued....)												
Merrill Lynch & Co Inc*	MER	A	03/06-05/12	2.4	27.5	26.0	21.9	4.1	3.3	2.5	-38.3	-28.4
MGM Mirage	MGM	BB	06/04-06/10	12.2	29.7	27.1	29.4	21.7	9.7	9.9	-24.4	-27.1
Morgan Stanley*	MS	A	06/04-05/12	4.4	36.3	16.7	13.6	25.4	5.2	3.1	-16.6	-8.9
Navistar Intl Corp	NAV	BB	03/05-08/10	3.4	29.4	35.8	36.7	27.1	33.2	34.1	-54.7	-55.2
Neiman Marcus Gp Inc	NMG	B	01/06-05/12	5.6	22.8	20.7	25.0	14.8	8.4	6.9	-46.5	-27.9
Office Depot Inc	ODP	BB	12/03-05/12	9.9	70.7	34.2	17.0	69.1	31.2	15.2	-17.3	-26.9
Omnicare Inc	OCR	BB	08/04-05/12	1.4	20.4	19.8	19.4	17.8	16.5	15.7	37.2	35.6
Sanmina SCI Corp	SANM	B	01/05-05/12	9.6	64.5	64.8	66.4	47.5	48.7	49.5	-12.1	-28.4
Sinclair Broadcast Gp Inc	SBGI	BB	10/04-05/12	5.6	13.5	10.6	8.9	9.1	7.4	5.2	-25.9	-34.0
Solectron Corp	SLR	BB	04/06-10/07	1.1	51.4	52.5	52.5	19.7	20.4	20.3	9.5	9.9
Sungard Systems	SNDT/SDS	B	08/06-08/10	9.5	23.8	23.1	23.0	0.9	0.6	0.6	-25.0	-8.1
SunTrust Bks Inc*	STI	A	09/04-03/11	1.9	34.4	33.0	30.7	16.4	11.5	9.7	7.6	16.2
Tesoro Corp	TSO	BB	11/04-05/12	11.0	58.3	47.0	42.5	45.9	42.2	37.3	18.0	18.2
Time Warner Inc	TWX	A	04/02-05/12	3.6	56.7	46.3	26.8	47.7	37.2	27.8	43.8	44.8
TJX Cos Inc	TJX	A	08/03-07/11	0.5	59.8	49.9	43.8	41.8	38.1	37.9	9.9	-24.7
Toll Bros Inc	TOL	BBB	04/04-05/12	1.8	39.9	36.6	35.5	27.9	28.6	30.4	3.1	12.4
Triad Hosps Inc	TRI	BB	09/04-03/08	1.6	54.5	50.0	49.4	33.7	32.5	32.4	7.8	8.8
Tribune Co	TRB	BBB	10/03-12/08	6.8	41.0	45.6	60.8	29.7	35.0	58.1	-56.9	-53.5
TRW Automotive Inc	TRW	BBB	10/05-08/10	4.4	27.0	13.2	25.8	0.0	0.0	0.0	-77.9	-51.4
U S Bancorp*	USB	AA	11/04-03/12	0.8	48.4	21.2	18.8	10.4	8.9	7.7	-77.3	-63.2
Utd Rents Inc	URI	BB	07/04-04/11	5.3	48.6	40.6	35.0	31.2	28.2	26.5	-28.2	-28.4
WA Mut Inc*	WM	A	05/04-09/08	3.5	26.8	20.5	18.7	9.3	6.7	5.5	-13.3	-8.8
Wachovia Corp*	WB	A	03/02-12/08	0.7	36.0	29.1	25.1	16.9	13.3	11.6	-22.9	-32.1
Wells Fargo & Co*	WFC	AA	10/03-05/12	1.1	52.1	44.3	39.6	30.7	29.7	29.3	-64.9	-56.5

Table IA2: Accounting for Bid–ask Spreads: Parameter Estimates for Fannie Mae

We estimate the CDS valuation model using time-series data of senior and subordinate CDS contracts written on Fannie Mae with maturities of 1, 3, 5, 7, and 10 years. We report results from two estimation methods. The first method, labeled *Without bid/ask,* does not account for CDS bid and ask spreads. In the second method, labeled *With bid/ask*, we include daily bid and ask information of Fannie Mae when filtering state variables and constructing the optimization function. Daily bid and ask data are collected from Bloomberg. We account for CDS bid ask spreads by assuming that the τ-maturity senior contracts on day t are priced with normally distributed errors with mean 0 and standard deviations $\sigma_u^{SEN}(\tau)\left|Bid_t^{SEN}(t,\tau) - Ask_t^{SEN}(t,\tau)\right|$, where $\sigma_u^{SEN}(\tau)$ is a constant, and $Bid_t^{SEN}(\tau)$ and $Ask_t^{SEN}(\tau)$ refer to day $t's$ bid and ask spreads of the τ-maturity senior contracts. We assume τ-maturity subordinate contracts are priced with normally distributed errors with mean 0 and standard deviations $\sigma_u^{SUB}(\tau)\left|Bid_t^{SUB}(\tau) - Ask_t^{SUB}(\tau)\right|$, where the variables are defined analogously. Panel A reports the factor loadings of the default intensity dynamic, the loss given default (LGD) of senior CDS contracts, and the LGD of junior CDS contracts on the latent factors. See Appendix Table A1 for summary of the model parameters. Panel B reports the structural parameters that drive the factor dynamics of state variables X_3, X_4, and X_5 under the physical and risk-neutral measures. Standard error is reported (in parentheses) beneath each estimate.

Panel A: Estimates of the factor loading parameters for Fannie Mae

Default intensity (λ_t)			Senior loss given default			Subordinate loss given default			
Constant	X_1	X_2	Constant	X_1	X_2	Constant	X_1	X_2	X_5
$\alpha_0 \times 100$	$\alpha_1 \times 100$	$\alpha_2 \times 100$	β_0^{sen}	β_1^{sen}	β_2^{sen}	β_0^{sub}	β_1^{sub}	β_2^{sub}	β_5^{sub}
Without bid/ask spread									
-0.18	-3.36	-37.34	1.22	-7.72	-1.70	0.77	-0.22	-11.40	0.67
(1.2E-03)	(1.9E-02)	(4.5E-02)	(6.6E-03)	(3.2E-02)	(2.6E-02)	(6.6E-02)	(6.3E-02)	(7.8E-02)	(5.0E-02)
With bid/ask spread									
-0.19	-3.73	-32.57	1.22	-7.67	-2.02	0.76	-0.24	-12.85	0.62
(6.9E-03)	(6.8E-02)	(3.3E-02)	(2.2E-03)	(2.6E-02)	(2.2E-02)	(8.1E-02)	(7.0E-02)	(6.5E-02)	(3.2E-02)

Panel B: Estimates of factor dynamics of X_3, X_4, and X_5 for Fannie Mae

Intensity factor X_3				Intensity factor X_4				Recovery factor X_5			
ρ_3	ρ_3^P	$\mu_3\times100$	$\sigma_3\times100$	ρ_4	ρ_4^P	$\mu_4\times100$	$\sigma_4\times100$	ρ_5	ρ_5^P	$\mu_5\times100$	$\sigma_5\times100$
Without bid/ask											
0.9900	0.9899	0.0001	0.0124	0.9976	0.9956	-0.0001	0.0088	0.9992	0.9872	-0.0321	1.7490
(2.4E-06)	(2.0E-01)	(3.0E-06)	(5.5E-05)	(1.0E-04)	(6.7E-03)	(1.2E-05)	(3.2E-05)	(4.5E-05)	(2.0E-02)	(6.6E-04)	(1.5E-02)
Without bid/ask											
0.9900	0.9787	0.0001	0.0104	0.9981	0.9934	-0.0001	0.0114	0.9991	0.9958	-0.0359	1.7506
(2.1E-06)	(1.2E-01)	(3.0E-06)	(7.8E-05)	(5.8E-05)	(1.2E-02)	(7.2E-06)	(6.9E-05)	(5.7E-05)	(1.4E-02)	(4.8E-04)	(1.3E-02)

Table IA3: Price Discovery Between Senior and Subordinate CDS Spreads

This table reports the estimation results from the second stage of a Vector Error Correction Model (VECM) for CDS contracts with 1, 3, 5, 7, and 10 years to maturities. In the first stage, we estimate the long-run relationship between senior and subordinate CDS spreads according to

$$CDS^{SUB}_{i,\tau,t} = \alpha_\tau + \rho_1 CDS^{SEN}_{i,\tau,t} + \rho_2 CRISIS_t + E_{i,\tau,t},$$

where $CDS^{SEN}_{i,\tau,t}$ and $CDS^{SUB}_{i,\tau,t}$ are market-observed senior and subordinate spreads of firm i with maturity τ on day t. $CRISIS_t$ is a time-series dummy equal to one from Dec. 2007 to June 2009. In the second stage, we estimate the following regression model.

$$\Delta CDS^{SEN}_{i,\tau,t} = \delta_{1\tau} + \lambda_1 E_{i,\tau,t-1} + \beta_1 \Delta CDS^{SEN}_{i,\tau,t-1} + \gamma_1 \Delta CDS^{SUB}_{i,\tau,t-1} + \varepsilon_{i,\tau,t}$$
$$\Delta CDS^{SUB}_{i,\tau,t} = \delta_{2\tau} + \lambda_2 E_{i,\tau,t-1} + \beta_2 \Delta CDS^{SEN}_{i,\tau,t-1} + \gamma_2 \Delta CDS^{SUB}_{i,\tau,t-1} + \varepsilon_{i,\tau,t},$$

where $\Delta CDS^{SEN}_{i,\tau,t}$ and $\Delta CDS^{SUB}_{i,\tau,t}$ denote the change in senior and subordinate CDS spreads, respectively. We report regression results for two periods. The first is the full sample which we have data available from 2001 to 2012; see data availability in Table 1 of the main text. The sample consists of firms described in Table 1. We report regression results for two periods. The first is the full sample for which we have data available from 2001 to 2012; see data availability in Table 1. The second period spans from Dec. 2007 to June 2009, representing the subprime crisis. The crisis period is defined following the NBER's business cycle dating committee. The table also reports adjusted R-squared, number of observations, and clustering for each regression. We include maturity fixed effects in the regressions. Robust standard error clustered at the firm and maturity levels is reported (in bracket) beneath each estimate. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Full sample		Crisis period (Dec 07—Jun 09)	
	Dependent variable		Dependent variable	
	Δ CDS$^{SEN}_{t}$	Δ CDS$^{SUB}_{t}$	Δ CDS$^{SEN}_{t}$	Δ CDS$^{SUB}_{t}$
	(1)	(2)	(3)	(4)
E_{t-1}	0.044***	-0.052***	0.051***	-0.061***
	(0.0129)	(0.0178)	(0.0178)	(0.0224)
Δ CDS$^{SEN}_{t-1}$	-0.202***	0.083***	-0.205***	0.080***
	(0.0286)	(0.0187)	(0.0308)	(0.0210)
Δ CDS$^{SUB}_{t-1}$	0.153***	-0.114***	0.040***	-0.109**
	(0.0347)	(0.0419)	(0.0556)	(0.0437)
Fixed effects	Yes	Yes	Yes	Yes
Adj. R-squared	0.069	0.038	0.073	0.041
No. observations	371820	371800	79005	78990
No. clusters	230	230	220	220

Table IA4: Subperiod Analysis of Parameter Estimates

This table reports results examining the stability of model parameters using subperiod estimations. For each of 46 firms in our sample, we re-estimate their CDS valuation model on two non-overlapping periods. We report the average deviation of subperiod estimates (in %) *relative* to their corresponding values estimated over the full-sample period (Jan. 2001 to May 2012). The percentage deviation of each model parameter is first calculated firm by firm. For instance, if θ_s and θ_f are the two parameters that we estimate from the subsample and full-sample periods, respectively, we calculate its percentage deviation as $(\theta_s - \theta_f)/\theta_f \times 100$. We then report the average deviation of each parameter (in %) across 46 firms. We consider two subperiods. The first subperiod is from Jan. 2001 to June 2007, which is before the subprime crisis period. The second subperiod is from July 2007 to May 2012, corresponding to the start of the subprime crisis to the end of our sample. Panel A reports factor loadings of the default intensity, the senior LGD, and the subordinate LGD on the latent factors. In Panel B, we report the distribution of parameter estimates that drive the factor dynamic of X_3, X_4, and X_5. See Appendix Table A1 for summary of the model parameters.

Panel A: Deviation in factor loadings *relative* to the full-sample estimates

Default intensity (λ_t)			Senior loss given default			Subordinate loss given default			
Constant	X_1	X_2	Constant	X_1	X_2	Constant	X_1	X_2	X_5
α_0	α_1	α_2	β_0^{sen}	β_1^{sen}	β_2^{sen}	β_0^{sub}	β_1^{sub}	β_2^{sub}	β_5^{sub}
Before the crisis: January 2001 to June 2007									
-5.53%	-6.15%	-1.37%	-2.15%	-0.20%	-0.03%	-2.56%	-1.27%	-0.84%	1.78%
During and post-crisis: July 2007 to May 2012									
-4.26%	-1.32%	-2.30%	-2.23%	-5.01%	1.57%	-1.95%	-5.59%	-4.42%	3.00%

Panel B: Deviation in factor dynamic of X_3, X_4, and X_5 *relative* to the full-sample estimates

Intensity factor X_3				Intensity factor X_4				Recovery factor X_5			
ρ_3	ρ_3^P	μ_3	σ_3	ρ_4	ρ_4^P	μ_4	σ_4	ρ_5	ρ_5^P	μ_5	σ_5
Before the crisis: January 2001 to June 2007											
0.10%	0.04%	-19.94%	4.46%	0.27%	0.00%	-9.45%	7.17%	0.12%	0.33%	-6.02%	-0.82%
During and post-crisis: July 2007 to May 2012											
0.22%	0.05%	-9.21%	19.13%	0.36%	0.23%	-6.53%	18.33%	0.15%	0.27%	-7.62%	4.51%

Table IA5: Out-of-sample CDS Pricing

This table reports average CDS pricing errors for a subperiod from July 2007 to May 2012, which we refer to as the "crisis/post-crisis" period. We report pricing errors calculated using in-sample parameter estimates and out-of-sample parameter estimates. In-sample pricing errors are calculated using parameters that are fitted to CDS data from July 2007 to May 2012. Out-of-sample pricing errors are calculated using parameters fitted to CDS data from Jan. 2001 to June 2007, which we refer to as the "pre-crisis" period. We report two measures of average in-sample fit. Panel A reports the relative root-mean-squared-error (RMSE). Panel B reports the mean of absolute percentage error (APE).The relative RMSE for maturity h contracts is calculated as

$$\text{Relative RMSE} = \sqrt{\frac{1}{G}\sum_{j=1}^{G}\left(\frac{CDS(j,h) - CDS^M(j,h)}{CDS(j,h)}\right)^2},$$

where G is the number of observations used, and $CDS(j,h)$ and $CDS^M(j,h)$ are the market-observed and model-implied spreads. Similarly, the absolute percentage error is calculated as

$$\text{APE} = \frac{1}{G}\sum_{j=1}^{G}\frac{\left|CDS(j,h) - CDS^M(j,h)\right|}{CDS(j,h)}.$$

We report the average values of relative RMSE and APE in percentage terms for five maturities: 1-year, 3-year, 5-year, 7-year, and 10-year.

	Senior					Subordinate				
	1 yr	3 Yr	5 Yr	7 Yr	10 Yr	1 yr	3 Yr	5 Yr	7 Yr	10 Yr
Panel A. Relative RMSE (%)										
In-sample pricing	15.1	11.4	9.2	8.4	9.9	14.9	12.6	9.1	8.8	10.1
Out-of-sample pricing	19.2	18.2	13.7	14.7	18.3	29.8	20.0	13.8	14.7	18.7
Panel B. Absolute Percentage Error (%)										
In-sample pricing	11.4	8.4	7.0	6.2	7.4	11.9	9.1	6.7	6.5	7.7
Out-of-sample pricing	14.8	15.8	11.3	11.8	15.0	24.3	16.7	11.2	11.9	15.4

Table IA6: Joint Estimation Results using Financial Firms: A Robustness Check

We report parameter estimates of the CDS valuation model on 17 financial firms obtained using two estimation methods: (1) Firm-by-firm estimation, and (2) Joint estimation. Firm-by-firm estimation is the main method that we use in the paper. The credit-risk parameters are estimated for each firm individually, resulting in 17 sets of parameter estimates. For this estimation method, we report the distribution of parameter estimates. In the joint estimation, we fit the model to all 17 financial firms at once using one set of parameters. For this method, we report the parameter estimates and their standard errors in parentheses. Panel A reports loading coefficients of the default intensity, the senior LGD, and the subordinate LGD on latent state variables. In Panel B, we report the distribution of parameter estimates that drive the dynamic of X_3, X_4, and X_5. See Appendix Table A1 for summary of the model parameters. The sample period is from Jan. 2001 to May 2012. The 17 financial firms in our sample are denoted with an asterisk in Table 1.

Panel A: Factor loading parameters

	Default intensity (λ_t)			Senior loss given default			Subordinate loss given default			
	Constant	X_1	X_2	Constant	X_1	X_2	Constant	X_1	X_2	X_5
	$\alpha_0 \times 100$	$\alpha_1 \times 100$	$\alpha_2 \times 100$	β_0^{sen}	β_1^{sen}	β_2^{sen}	β_0^{sub}	β_1^{sub}	β_2^{sub}	β_5^{sub}
Estimates from a *joint estimation* across 17 financial firms										
Estimate	0.097	-1.615	-1.815	0.747	-0.977	-1.949	0.488	-0.131	-0.395	0.766
Std errors	(0.001)	(0.023)	(0.079)	(0.029)	(0.011)	(0.013)	(0.043)	(0.002)	(0.013)	(0.149)
Distribution of parameter estimates from *firm-by-firm estimations* on 17 financial firms										
25%	0.007	-5.753	-6.508	0.611	-1.578	-3.578	0.207	-2.558	-8.353	0.235
50%	0.093	-2.016	-3.204	0.804	-0.567	-1.797	0.343	-0.665	0.047	0.670
75%	0.308	-0.455	4.216	1.055	0.156	-0.614	0.773	0.904	1.168	0.925
Mean	0.147	-5.155	1.122	0.903	-1.454	-1.586	0.480	-0.757	-1.597	0.600
Std deviation	0.537	11.315	26.342	0.455	2.643	4.438	0.531	3.857	6.098	0.408

Panel B: Parameters in the factor dynamics of X_3, X_4, and X_5

	Intensity factor X_3				Intensity factor X_4				Recovery factor X_5			
	ρ_3	ρ_3^P	$\mu_3 \times 100$	$\sigma_3 \times 100$	ρ_4	ρ_4^P	$\mu_4 \times 100$	$\sigma_4 \times 100$	ρ_5	ρ_5^P	$\mu_5 \times 100$	$\sigma_5 \times 100$
Estimates from a *joint estimation* across 17 financial firms												
Estimate	0.9574	0.9998	-0.0011	0.0205	0.9989	0.9989	0.0001	0.0093	0.9996	0.9982	-0.0107	1.4376
Std errors	(0.0037)	(0.0028)	(0.0005)	(0.0007)	(0.0001)	(0.0012)	(0.0000)	(0.0001)	(0.0018)	(0.0007)	(0.0004)	(0.0209)
Distribution of parameter estimates from *firm-by-firm estimations* on 17 financial firms												
25%	0.9606	0.9994	-0.0055	0.0101	0.9943	0.9994	-0.0001	0.0086	0.9992	0.9978	-0.0321	1.3866
50%	0.9977	0.9995	0.0001	0.0141	0.9986	0.9996	0.0001	0.0102	0.9996	0.9982	-0.0098	1.5651
75%	0.9996	0.9998	0.0007	0.0255	0.9998	0.9998	0.0003	0.0112	0.9997	0.9991	0.0021	2.2386
Mean	0.9857	0.9993	-0.0011	0.0339	0.9495	0.9988	-0.0001	0.0100	0.9979	0.9983	-0.0200	1.9663
Std deviation	0.0192	0.0011	0.0087	0.0680	0.1756	0.0034	0.0006	0.0035	0.0061	0.0011	0.0455	1.1465

Table IA7: Monte Carlo Simulation Study: Pricing Errors

We simulate 100 paths of daily CDS and binary CDS spreads over 1500 days using the "true" base case parameters reported in Table 7. Panel A reports the means and standard deviations of simulated CDS spreads, as well as their corresponding values of expected recovery rates and binary CDS spreads. CDS spreads data are simulated with normally distributed noise. We assume the noise term's standard deviation is five percent of the model-implied spread level. CDS are priced using the valuation model shown in Section D. Binary CDS are priced similarly to standard CDS contracts with the exception of 0% recovery when the firm defaults. Consequently, only the default risk is priced in binary CDS spreads. All the beta coefficients in equation (6) are set to 0 such that $LGD = 1$ when we price binary CDS contracts. In Panel B, we summarize pricing errors from estimating the full stochastic recovery model on each of the 100 simulated CDS samples. The estimation exercise yields 100 sets of parameter estimates. Using these parameter estimates, we price senior and subordinate CDS contracts and calculate their absolute percentage pricing errors with respect to their "true" values implied by the base case parameters. We report the means and standard deviations of absolute percentage errors for CDS spreads, expected recovery rates, and binary CDS spreads.

Panel A: Data simulated using the base case parameters

Simulated data	Senior contracts					Subordinate contracts				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
CDS spreads (bps)										
Mean	18.9	30.5	39.3	46.0	53.3	29.3	45.3	57.0	65.7	75.1
Std deviation	10.1	11.1	11.3	11.1	10.6	15.4	16.4	16.4	16.0	15.0
Recovery rates (%)										
Mean	54.6	51.0	49.1	48.1	47.2	31.0	29.5	28.6	28.1	27.6
Std deviation	0.9	0.5	0.4	0.2	0.1	0.6	0.4	0.3	0.2	0.1
Binary CDS spreads (bps)										
Mean	42.6	65.0	81.0	92.7	105.0					
Std deviation	22.0	23.3	23.2	22.5	21.1					

Panel B: Pricing errors from estimating the full stochastic recovery model

Absolute percentage error	Senior contracts					Subordinate contracts				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
CDS spreads (bps)										
Mean	7.8%	4.2%	2.8%	1.9%	2.1%	3.1%	3.0%	2.9%	1.9%	2.1%
Std deviation	1.9%	2.0%	1.9%	1.1%	0.9%	2.0%	1.0%	1.0%	1.0%	1.0%
Recovery rates (%)										
Mean	10.3%	8.4%	6.2%	4.2%	2.9%	10.7%	9.1%	7.2%	6.6%	4.9%
Std deviation	4.1%	3.0%	3.4%	1.9%	1.0%	4.0%	3.1%	3.2%	3.0%	2.0%
Binary CDS spreads (bps)										
Mean	10.1%	7.1%	5.2%	5.7%	5.5%					
Std deviation	4.0%	3.4%	2.9%	3.1%	3.4%					

Table IA8: Robust Identification of the Recovery Level and Default Risk: A Simulation Study

This table illustrates the importance of using multiple-seniority CDS term structures to robustly identify the dynamics of recovery rate and default risk. We report errors from estimating a constant recovery model on CDS data simulated by a stochastic recovery model. We simulated 100 samples of CDS data, each with 1500 daily observations, using the base case parameters reported in Table 7. CDS spreads data are simulated with normally distributed noise. We assume the noise term's standard deviation is five percent of the model-implied spread. The means and standard deviations of the simulated data are summarized in Panel A of Table IA7. We estimate a constant recovery rate model on simulated CDS data using two different samples. In the first estimation sample, we use simulated CDS spreads for senior and subordinate contracts. In the second estimation sample, only CDS spreads for senior contracts are used. The constant recovery rate model that we use is obtained by setting parameters β_1, β_2 and β_5 in equation (6) to 0. Our simulated data consists of 100 sample paths, and therefore each estimation sample yields 100 sets of parameter estimates. Using the parameters estimated from these two estimation samples, we price senior CDS and binary CDS contracts and calculate their absolute percentage pricing errors with respect to "true" values implied by the base case parameters. Binary CDS are priced similarly to standard CDS contracts with the assumption of 0% recovery on the underlying security when the firm defaults (i.e., LGD is 100%). We report the absolute percentage pricing errors for senior CDS spreads, binary CDS spreads, and expected recovery rates of senior contracts.

	Estimation sample (1)					Estimation sample (2)				
Absolute percentage error	Senior and subordinate CDS spreads					Senior CDS spreads only				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
Senior CDS spreads										
Mean	13.2%	6.1%	4.7%	4.0%	5.4%	11.1%	5.2%	3.8%	3.4%	2.9%
Std deviation	4.0%	2.1%	1.6%	1.5%	1.3%	3.7%	2.4%	1.8%	1.6%	1.5%
Binary CDS spreads										
Mean	12.5%	9.0%	7.3%	7.3%	6.8%	23.7%	20.7%	19.1%	18.5%	17.7%
Std deviation	4.9%	4.3%	3.6%	3.6%	3.8%	8.0%	10.8%	13.0%	14.3%	14.9%
Recovery of senior contracts										
Mean			2.7%					7.1%		
Std deviation			2.9%					6.2%		

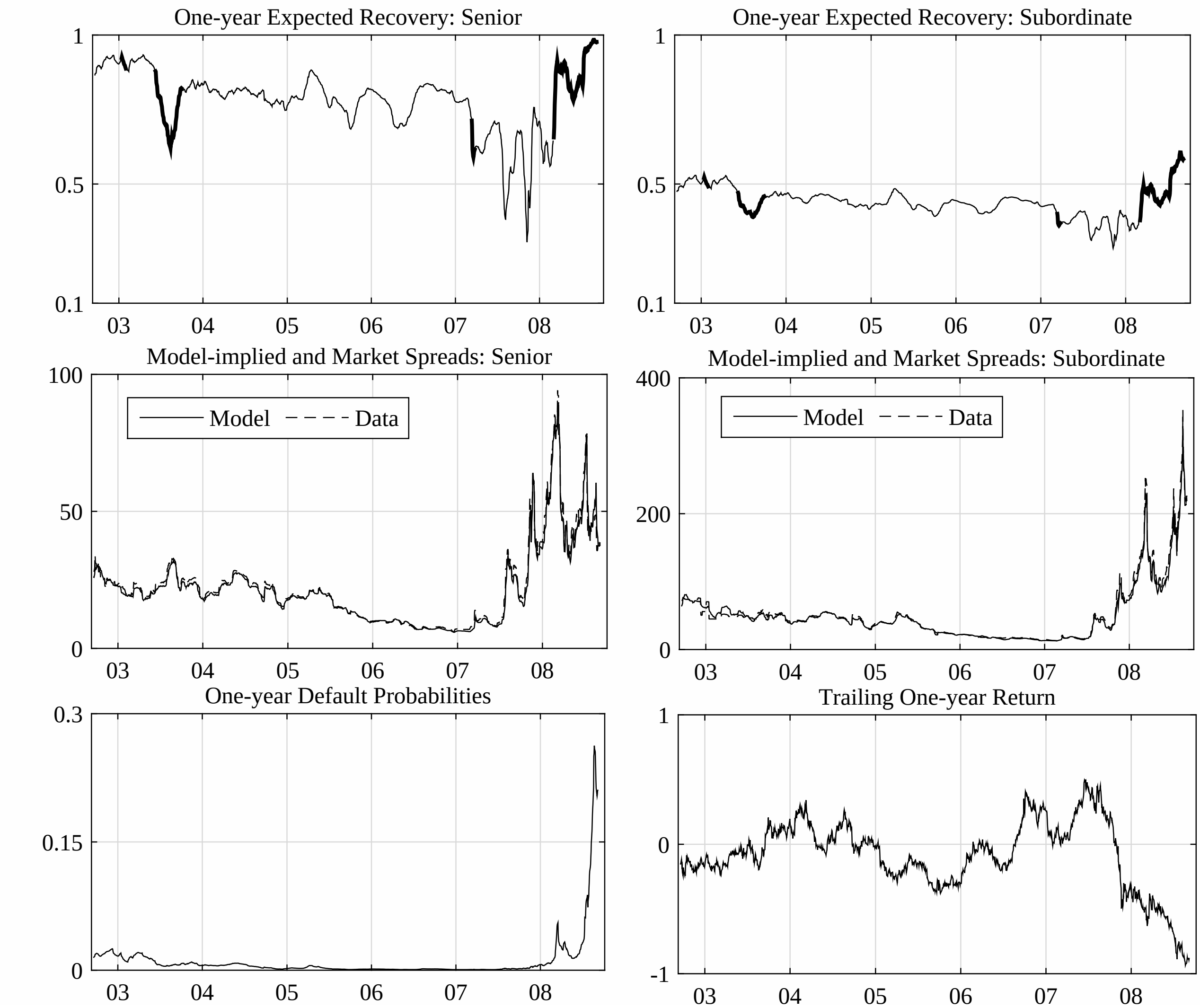

Figure IA1: Fannie Mae: A case study

This figure plots various time-series properties and estimates for Fannie Mae. The top two panels plot time series of expected recovery at the 1-year horizon for senior and subordinate contracts. Panels in the second row plot the market observed (in gray) and model-implied CDS spreads (in black) for senior and subordinate contracts with five-year maturity. Panels in the third row plot the model-implied 1-year default probability and the trailing 1-year cumulative return of the firm. The bottom-left panel plots the time series of option-implied volatilities calculated using at-the-money put options with 30 days to maturity. The bottom-right panel plots the trailing 1-year cumulative return of the firm. We highlight certain portions of the plot in the top two panels using a dark-colored line to indicate the events discussed in the text.

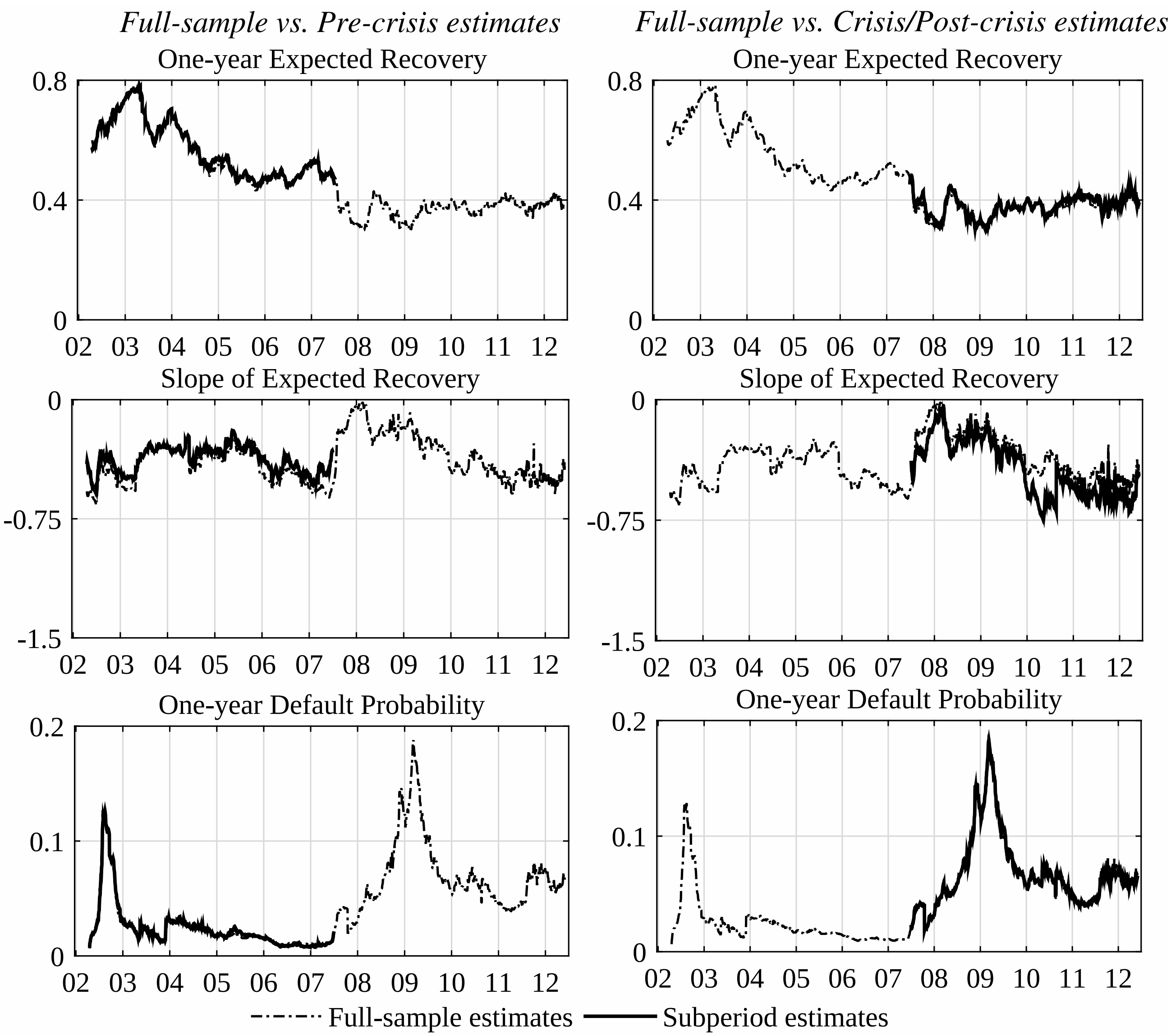

Figure IA2: Subperiod analysis of model estimates

This figure reports results examining the stability of model estimates in subsample periods. For each of the 46 firms in our sample, we re-estimate its CDS valuation model on two non-overlapping periods and compare their results to those obtained using the full-period estimation (Jan. 2001 to May 2012). The left-column panels compare the full-period estimates to the subperiod estimate from Jan. 2001 to June 2007, which corresponds to the "pre-crisis" period. The right-column panels compare the full-period estimate to the subperiod estimate from July 2007 to May 2012, which corresponds to the "crisis/post-crisis" period. We report the time-series averages of 1-year expected recovery of senior contracts (top panels), slope of expected recovery for senior contracts (middle panels), and 1-year default probability (bottom panels). In each panel, results from the subperiod estimation are plotted using bold line, while the full-period results are plotted using normal black line.

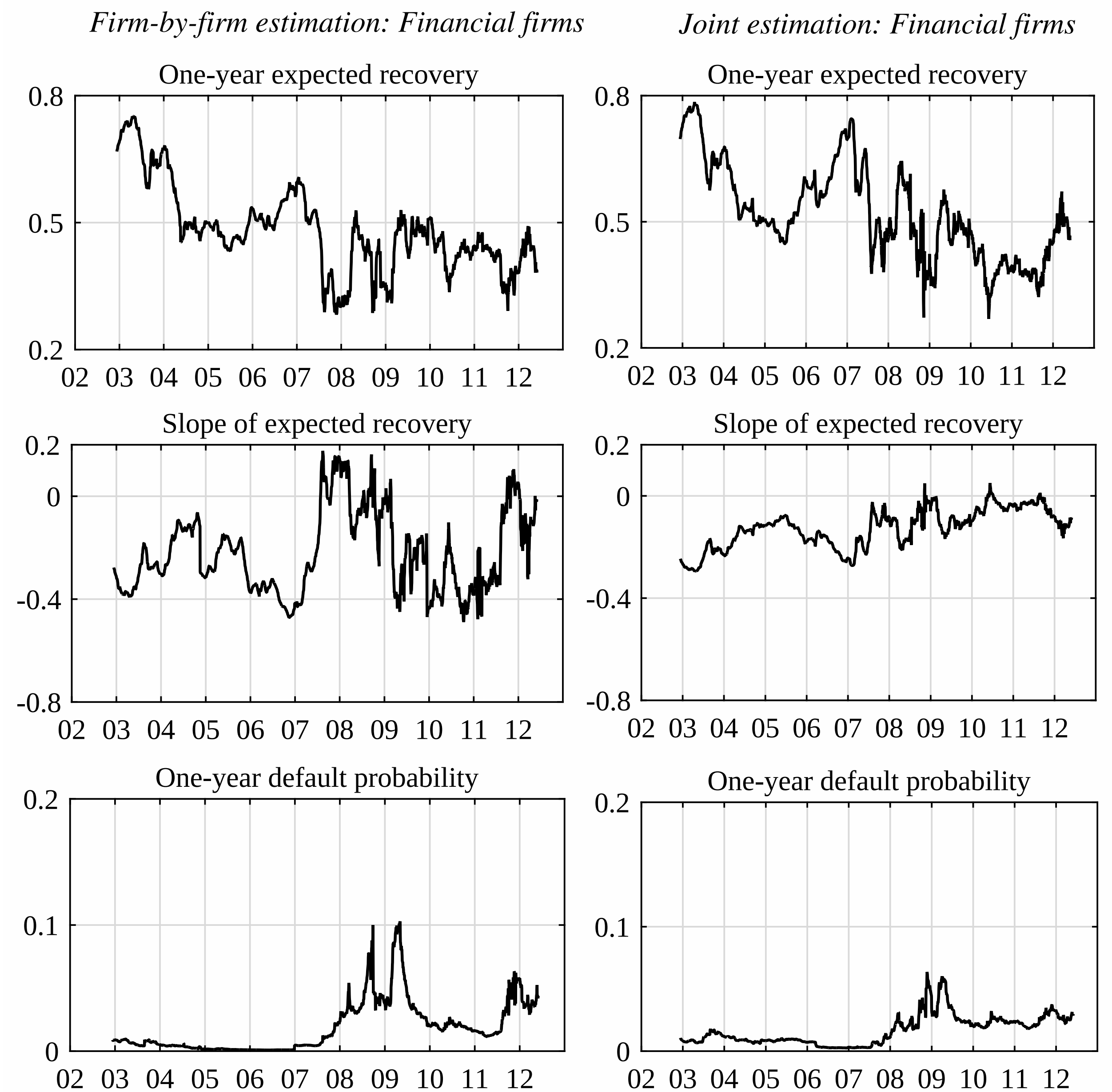

Figure IA3: Firm-by-firm vs Joint estimations: Evidence from financial firms

We plot time series of 1-year expected recovery, slope of expected recovery, and 1-year default probability for 17 financial firms obtained using the firm-by-firm estimation method (left-column panels) and the joint estimation method (right-column panels). The financial firms in our sample are denoted with an asterisk in Table 1. Firm-by-firm estimation is the method that we use in the main paper. Here, the credit-risk parameters are estimated for each firm individually. This method yields 17 sets of parameter estimates, and 17 sets of daily time series of default probability and recovery rates. Time-series plots for the firm-by-firm estimation represent the average values across financial firms. In the joint estimation method, we estimate the model using all 17 financial firms at once. This yields one set of parameter estimates, and one set of daily times-series default probability and recovery rates.

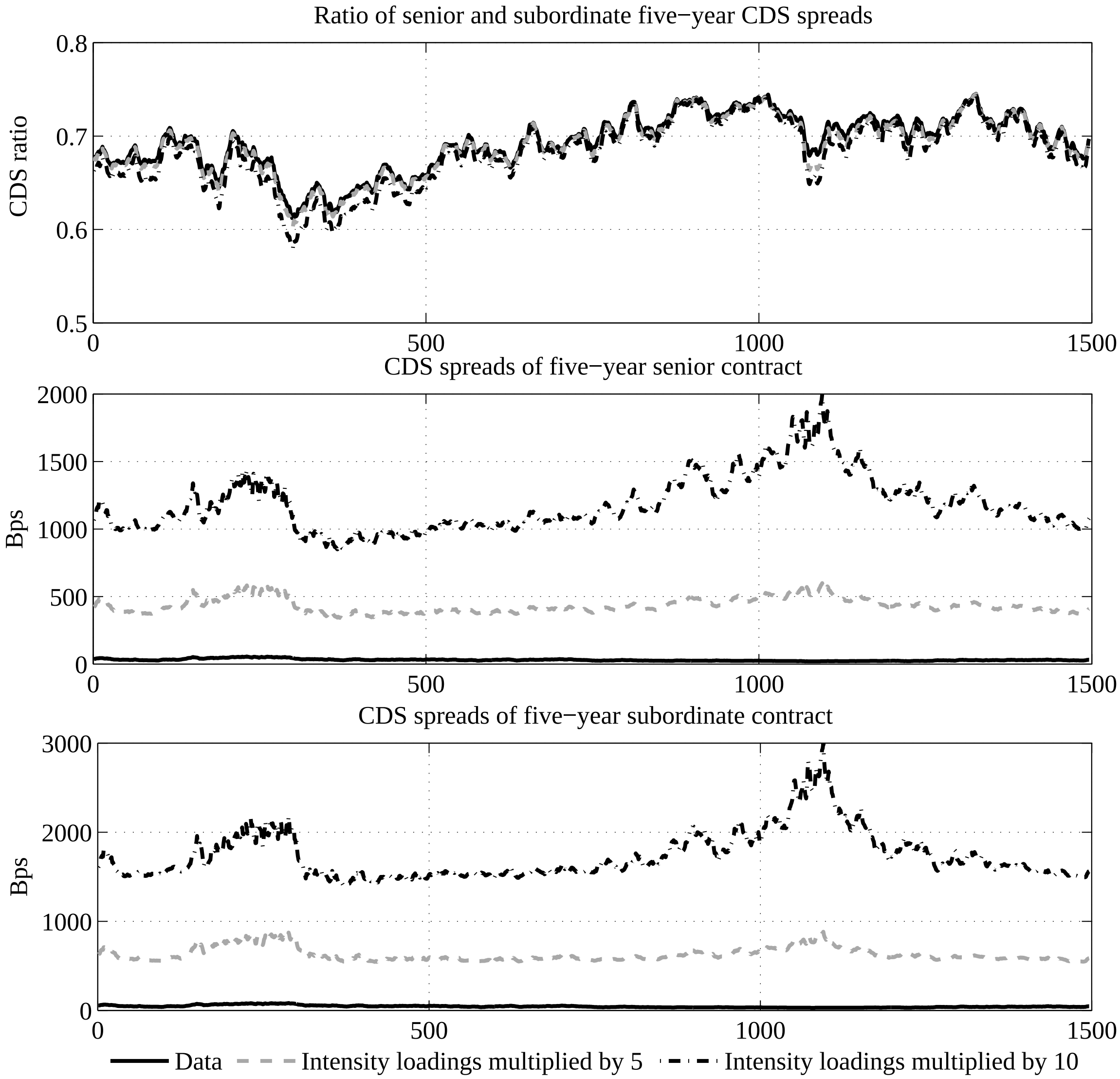

Figure IA4: Default intensity loading and CDS spreads ratio

This figure illustrates the impact of default intensity loading on the CDS spreads of senior and subordinate contracts. Using the base case parameters in Table 7, we simulate 100 samples of 5-year seniorvand subordinate CDS spreads. Each simulated sample consists of 1500 days. The results in this figure correspond to daily average values across 100 samples. The middle and bottom panels plot daily sample averages of five-year senior and subordinate CDS spreads, respectively. The top panel plots the ratio of senior to subordinate CDS spreads. In each panel, we plot the results for three sets of intensity loading parameters. The solid black line labeled *Data* plots the results generated using the base case parameters as reported in Table 7. The grey-dashed line plots the results generated when multiplying the loadings of all factors by 5. The grey-dashed-dotted line plots the results generated when multiplying the loadings of all factors by 10.

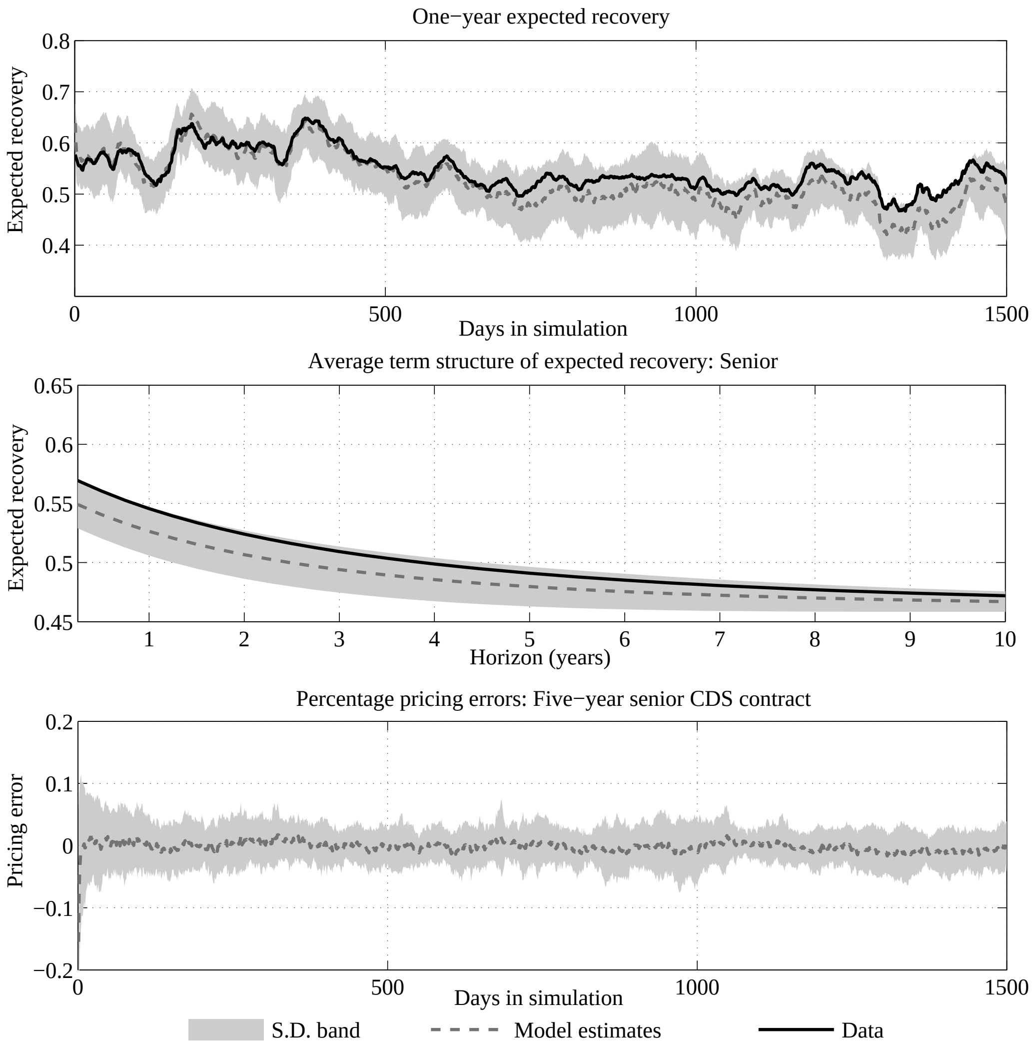

Figure IA5: Simulation study results

We plot results from estimating the term structure of CDS spreads simulated using parameters reported in Table 7. We simulate 100 sample paths of daily CDS spreads with 1, 3, 5, 7, and 10 years to maturity. CDS data are simulated with normally distributed noise. We set the noise term's standard deviation equal to five percent of the model-implied spread level. We estimate the model on each simulation path using the method described in Section A of the main text. The top panel reports results for the expected 1-year recovery, and the middle panel reports results for the average term structure of recovery. We plot the mean of the model estimates using a grey-dashed line. One standard deviation band (S.D band) is plotted on top of the mean of model estimates. Solid lines in the top two panels plot the mean values of 1-year expected recovery and term structure of expected recovery, calculated using the "true" simulated data. We do not plot the standard deviation band around the results calculated using the simulated data to avoid visual clustering. The bottom panel plots the sample mean and standard deviation of daily percentage pricing errors for five-year senior contract.