Internet Appendix for "Crash Sensitivity and the Cross-Section of Expected Stock Returns"

Abstract

The Internet Appendix consists of four sections. In Section A, we give a short overview on copulas and measures of tail dependence. In Section B, we provide results from additional tests and stability checks for our empirical analysis from Section II and Section III of the main paper. Section C discusses the relation of our theoretical model to other equilibrium models with nonlinear SDFs and Section D provides additional material and proofs for our theory.

A Copulas and Measures of Tail Dependence

Copulas are functions that describe the complete dependence structure between random variables.[1] Every distribution function of two random variables X_1 and X_2 (e.g., an individual asset return and the market return) implicitly contains both, a description of the marginal distribution functions $F_1(x_1)$ and $F_2(x_2)$ and their dependence structure. The copula approach allows us to isolate the description of the dependence structure from the univariate marginal distributions of the bivariate distribution. Sklar (1959) shows that all bivariate distribution functions $F(x_1, x_2)$ can be completely described based on the univariate marginal distributions and a copula C: $[0,1]^2 \rightarrow [0,1]$. Sklar's Theorem explicitly states that all bivariate distributions can be decomposed into copulas and that the marginal distributions and bivariate distributions can be constructed by combining the univariate marginal distributions using copulas (McNeil, Frey, and Embrechts (2005)). Formally,

Sklar's (1959) Theorem *Let F be a bivariate distribution function with margins F_{X_1} and F_{X_2}. Then there exists a copula $C : [0,1]^2 \mapsto [0,1]$ such that, for all x_1, x_2 in $\overline{\mathbb{R}} = [-\infty, \infty]$,*

$$F(x_1, x_2) = C(F_{X_1}(x_1), F_{X_2}(x_2)). \tag{A1}$$

If the margins are continuous, then C is unique. Conversely, if C is a copula and F_{X_1} and F_{X_2} are univariate distribution functions, then the function F defined in (A1) is a bivariate distribution function with margins F_{X_1} and F_{X_2}.

Simple expressions for LTD and UTD in terms of the copula C of the bivariate distribution can be derived based on

$$\text{LTD} = \lim_{u \to 0+} \frac{C(u,u)}{u} \qquad \text{and} \qquad \text{UTD} = \lim_{u \to 1-} \frac{1 - 2u + C(u,u)}{1-u} \tag{A2}$$

if F_1 and F_2 are continuous (McNeil, Frey, and Embrechts (2005)).

The coefficients of tail dependence have closed form solutions for the basic parametric copulas used in this study. The respective formulas can be found in Table IA.I.

[1] A more precise technical, but still accessible, treatment of copula concepts is contained in Nelsen (2006).

Table IA.I: Bivariate Copula Functions with Tail Dependence Coefficients

This table reports the parametric forms of the bivariate copula functions considered in this study in the second column and the corresponding lower and upper tail dependence coefficients, LTD and UTD, in the last two columns. The Clayton copula, the Rotated Joe copula, the Rotated Gumbel copula, and the Rotated Galambos copula exhibit lower tail dependence. The Gauss copula, the Frank copula, the Plackett copula, and the FGM copula are asymptotically independent in both tails. The Joe copula, the Gumbel copula, the Galambos copula, and the Rotated Clayton copula exhibit upper tail dependence. In brackets we assign a label to each basic copula. We define $\overline{u}_1 = 1 - u_1$ and $\overline{u}_2 = 1 - u_2$. Φ denotes the standard normal $N(0,1)$ distribution function, Φ^{-1} the functional inverse of Φ and Φ_θ is the bivariate standard normal distribution function with correlation θ.

Copula	Parametric Form	LTD	UTD
Clayton (1)	$C_{\text{Cla}}(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$2^{-1/\theta}$	–
Rotated-Gumbel (2)	$C_{\text{RGum}}(u_1, u_2) = u_1 + u_2 - 1 + \exp\left(-((-\log(\overline{u}_1))^\theta + (-\log(\overline{u}_2))^\theta)^{1/\theta}\right)$	$2 - 2^{1/\theta}$	–
Rotated-Joe (3)	$C_{\text{RJoe}}(u_1, u_2) = u_1 + u_2 - (u_1^\theta + u_2^\theta - u_1^\theta \cdot u_2^\theta)^{1/\theta}$	$2 - 2^{1/\theta}$	–
Rotated-Galambos (4)	$C_{\text{RGal}}(u_1, u_2) = u_1 + u_2 - 1 + (\overline{u}_1) \cdot (\overline{u}_2) \cdot \exp\left(((-\log(\overline{u}_1))^{-\theta} + (-\log(\overline{u}_2))^{-\theta})^{-1/\theta}\right)$	$2^{-1/\theta}$	–
Gauss (A)	$C_{\text{Gau}}(u_1, u_2; \theta) = \Phi_\theta(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	–	–
Frank (B)	$C_{\text{Fra}}(u_1, u_2; \theta) = -\theta^{-1} \log\left(\frac{1-\exp(-\theta)-(1-\exp(-\theta u_1))(1-\exp(-\theta u_2))}{1-\exp(-\theta)}\right)$	–	–
Plackett (C)	$C_{\text{Pla}}(u_1, u_2; \theta) = \frac{1}{2}(\theta - 1)^{-1}\left\{1 + (\theta - 1)(u_1 + u_2) - [(1 + (\theta - 1)(u_1 + u_2))^2 - 4\theta u_1 u_2]^{1/2}\right\}$	–	–
F-G-M (D)	$C_{\text{Fgm}}(u_1, u_2; \theta) = u_1 u_2(1 + \theta(1 - u_1)(\overline{u}_2))$	–	–
Joe (I)	$C_{\text{Joe}}(u_1, u_2; \theta) = 1 - ((\overline{u}_1)^\theta + (\overline{u}_2)^\theta - (\overline{u}_1)^\theta \cdot (\overline{u}_2)^\theta)^{1/\theta}$	–	$2 - 2^{1/\theta}$
Gumbel (II)	$C_{\text{Gum}}(u_1, u_2; \theta) = \exp\left(-((-\log(u_1))^\theta + (-\log(u_2))^\theta)^{1/\theta}\right)$	–	$2 - 2^{1/\theta}$
Galambos (III)	$C_{\text{Gal}}(u_1, u_2; \theta) = u_1 \cdot u_2 \cdot \exp\left(((-\log(u_1))^{-\theta} + (-\log(u_2))^{-\theta})^{-1/\theta}\right)$	–	$2^{-1/\theta}$
Rotated-Clayton (IV)	$C_{\text{RCla}}(u_1, u_2) = u_1 + u_2 - 1 + ((\overline{u}_1)^{-\theta} + (\overline{u}_2)^{-\theta} - 1)^{-1/\theta}$	–	$2^{-1/\theta}$

B Additional Analyses and Stability Checks of Empirical Results

Table IA.II: Statistical Significance of Tail Dependence Coefficients

This table reports results of the non-parametric Poon, Rockinger, and Tawn (2004) test for tail dependence in the distribution between an individual stock return and the market return. In the estimation procedure we use daily returns over non-overlapping intervals of 12 months. We report estimates of the average $\widehat{\overline{\chi}}$-coefficient (as defined in Poon, Rockinger, and Tawn (2004), p. 586) over all individual stock and 12-month periods in the sample as well as the percentage for which tail dependence can/cannot be rejected. Since the estimation framework needs a cutoff (threshold) point u of the tail, we report summary statistics for different u's (e.g., we report results for u being the 10%, 5%, 2%, and 1% percentiles in the case of LTD). Tail dependence is rejected at the 5% significance level, if $\widehat{\overline{\chi}}$ is significantly less than one, i.e., $\widehat{\overline{\chi}} + 1.96 \cdot \sqrt{Var(\widehat{\overline{\chi}})} < 1$. We report the results for statistical significance of LTD. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012.

Lower Tail Dependence (LTD)

Cutoff	10%	5%	2%	1%
Average $\widehat{\overline{\chi}}$	0.336	0.303	0.210	0.224
LTD Rejected	65.15%	48.87%	42.07%	36.42%
LTD Not Rejected	34.85%	51.13%	57.93%	63.58%

Table IA.III: Frequency and Relative Percentage of Copula Selection

This table reports the percentage frequency of the selected parametric copula combinations. The appropriate dependence structure is selected by minimizing the distance between the parametric copulas and the empirical copula via the Integrated Anderson-Darling distance as described in the main text. In columns 1, 3, 6, and 9 we indicate the label of the respective copula combination based on the basic copula labels from Table A.1. The three copula combinations that are most often selected are the Clayton - Gauss - Galambos copula (5.96%, 1-A-III), the Clayton - Gauss - Rotated Clayton copula (5.75%, 1-A-IV), and the Rotated Galambos - Gauss - Rotated Clayton (5.73%, 4-A-IV) (these copula combinations are marked in bold).

Copula	Perc	Copula	Perc	Copula	Perc	Copula	Perc
(1-A-I)	3.60	(2-A-I)	0.68	(3-A-I)	2.46	(4-A-I)	4.05
(1-A-II)	2.17	(2-A-II)	0.46	(3-A-II)	1.17	(4-A-II)	1.94
(1-A-III)	**5.96**	(2-A-III)	1.05	(3-A-III)	3.98	(4-A-III)	3.65
(1-A-IV)	**5.75**	(2-A-IV)	1.12	(3-A-IV)	3.08	(4-A-IV)	**5.73**
(1-B-I)	0.70	(2-B-I)	0.09	(3-B-I)	0.25	(4-B-I)	0.76
(1-B-II)	0.41	(2-B-II)	0.04	(3-B-II)	0.10	(4-B-II)	0.39
(1-B-III)	2.19	(2-B-III)	0.33	(3-B-III)	1.11	(4-B-III)	0.77
(1-B-IV)	1.42	(2-B-IV)	0.22	(3-B-IV)	0.52	(4-B-IV)	1.45
(1-C-I)	1.08	(2-C-I)	0.18	(3-C-I)	0.73	(4-C-I)	1.32
(1-C-II)	0.71	(2-C-II)	0.11	(3-C-II)	0.26	(4-C-II)	0.73
(1-C-III)	2.43	(2-C-III)	0.53	(3-C-III)	1.41	(4-C-III)	1.08
(1-C-IV)	1.84	(2-C-IV)	0.40	(3-C-IV)	0.85	(4-C-IV)	2.09
(1-D-I)	2.28	(2-D-I)	0.29	(3-D-I)	1.87	(4-D-I)	2.17
(1-D-II)	1.09	(2-D-II)	0.07	(3-D-II)	0.66	(4-D-II)	0.80
(1-D-III)	3.29	(2-D-III)	0.30	(3-D-III)	2.29	(4-D-III)	2.29
(1-D-IV)	3.31	(2-D-IV)	0.52	(3-D-IV)	2.22	(4-D-IV)	3.24

Table IA.IV: LTD and UTD in 5-year Subsamples

This table reports means and medians from equal-weighted average lower (upper) tail dependence coefficients, LTD (UTD), between individual stock returns and market returns for 5-year subsamples and over the whole sample period from January 1963 to December 2012. The difference between the average LTD and UTD as well as the median LTD and UTD coefficients are displayed in columns 4 and 7. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

	Mean			Median		
	LTD	UTD	LTD - UTD	LTD	UTD	LTD - UTD
1963 – 1967	0.102	0.052	0.050***	0.077	0.028	0.049***
1968 – 1972	0.093	0.089	0.004***	0.071	0.066	0.005***
1973 – 1977	0.073	0.071	0.002***	0.047	0.045	0.002***
1978 – 1982	0.133	0.054	0.079***	0.117	0.026	0.091***
1983 – 1987	0.092	0.065	0.027***	0.057	0.037	0.020***
1988 – 1992	0.077	0.051	0.026***	0.045	0.025	0.021***
1993 – 1997	0.075	0.039	0.036***	0.051	0.016	0.035***
1998 – 2002	0.090	0.069	0.021***	0.063	0.040	0.023***
2003 – 2007	0.114	0.071	0.043***	0.082	0.039	0.043***
2008 – 2012	0.178	0.114	0.064***	0.152	0.084	0.068***
1963 – 2012	0.100	0.065	0.035***	0.069	0.036	0.041***

Table IA.V: LTD Transition Matrix

This table reports the results of the LTD transition matrix. It shows the relative frequency that a stock is sorted into LTD quintile portfolio i in year t given that it was in LTD quintile portfolio j in year $t-1$. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012.

Transition Matrix	LTD PF 1_t	LTD PF 2_t	LTD PF 3_t	LTD PF 4_t	LTD PF 5_t
LTD PF 1_{t-1}	0.266	0.225	0.202	0.173	0.134
LTD PF 2_{t-1}	0.226	0.214	0.209	0.187	0.164
LTD PF 3_{t-1}	0.209	0.200	0.205	0.205	0.182
LTD PF 4_{t-1}	0.175	0.186	0.201	0.218	0.220
LTD PF 5_{t-1}	0.125	0.156	0.182	0.220	0.316

Table IA.VI: LTD: Predictive Regressions

This table displays the results of multivariate Fama-MacBeth (1973) regressions of LTD in year $t+1$ on different risk- and firm characteristics used in the asset pricing test of the main paper as well as a firm's leverage, cash-flow volatility, distress risk (estimated by a firm's Altman (1968)'s Z-score), and R&D spending. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

	(1) LTD_{t+1}	(2) LTD_{t+1}	(3) LTD_{t+1}	(4) LTD_{t+1}
LTD	0.208^{***}			0.0542^{***}
	(11.46)			(3.79)
β		0.0255^{***}		0.0238^{***}
		(7.16)		(5.27)
SIZE		0.00821^{***}		0.00607^{***}
		(6.47)		(3.05)
BOOK_TO_MARKET		-0.00168		-0.00193
		(-1.30)		(-0.67)
COSKEW		-0.0340^{***}		-0.0259^{***}
		(-5.00)		(-3.28)
ILLIQ		-0.0378^{***}		-0.00475
		(-3.25)		(-0.89)
PAST_RETURN		0.00189		0.00161
		(0.88)		(0.40)
IDIO_VOLA		-0.0139^{*}		-0.0318^{*}
		(-1.93)		(-1.95)
COKURT		0.00332^{*}		0.000877
		(1.74)		(0.42)
MAX		0.00157		-0.0210
		(0.24)		(-0.53)
β_{TAIL}		0.00558^{***}		0.00467^{***}
		(4.12)		(3.65)
LEVERAGE			0.0357^{***}	0.0132^{**}
			(3.94)	(2.03)
CASH_FLOW_VOLA			0.00249^{***}	0.00220
			(3.65)	(0.56)
DISTRESS_RISK			0.00726^{***}	0.00647^{***}
			(5.88)	(4.53)
R&D_SPENDING			0.0919^{***}	-0.00115
			(2.96)	(-0.05)
constant	0.0813^{***}	-0.00950	0.0815^{***}	0.0207
	(16.40)	(-0.73)	(12.30)	(0.85)
R^2	0.053	0.151	0.036	0.174

Table IA.VII: Excess Returns of LTD-Portfolios During Financial Crises

This table reports value-weighted daily excess returns of stocks sorted by past LTD during days of severe financial crises. Each month we rank stocks based on LTD estimated over the previous twelve months. We investigate future value-weighted returns of these portfolios during "Black Monday" (October 19, 1987), the Asian Crisis (October 27, 1997), the burst of the dot-com bubble (April 14, 2000), and the recent Lehman crisis (October 15, 2008). The last column reports average daily returns in excess of the 1-month T-bill rate of the portfolios on the ten worst return days in our sample. The row labeled "Strong – Weak" reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistical significance levels (only last column). The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Daily Returns — Portfolios Sorted By Past LTD

Portfolio	"Black Monday"	Asia Crisis	Dot-Com Bubble Burst	Lehman Crisis	Worst 10 days
1 Weak LTD	−14.03%	−5.50%	−3.82%	−7.93%	−6.35%
2	−17.03%	−6.25%	−4.77%	−8.75%	−6.55%
3	−17.34%	−6.69%	−6.83%	−8.14%	−6.89%
4	−16.79%	−6.86%	−6.23%	−8.51%	−7.82%
5 Strong LTD	−19.15%	−7.48%	−7.73%	−10.80%	−9.25%
Strong – Weak	−5.12%	−1.98%	−3.89%	−2.86%	−2.90%**
					(−2.45)

Table IA.VIII: Temporal Stability: Portfolio Sorts and Fama-MacBeth (1973) Regressions

This tables reports the results of temporal stability checks. Panel A shows results from the same value-weighted univariate sorts based on LTD as in the second column of Panel A in Table 4 of the main text for various subperiods. We report the average future return in excess of the one-month T-bill rate over the next month. Panel B displays the results of Fama-MacBeth (1973) regressions of monthly future excess returns on LTD, firm characteristics and risk characteristics for the different subsamples. We include the full set of independent variables from regression (5) in Table 7 (included in the regression but coefficient estimates suppressed in the table). All independent variables are winsorized at the 1% level and at the 99% level. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Panel A: Univariate Sorts

Portfolio	Jan1927 - Dec1962	Jan1963 - Dec1972	Jan1973 - Dec1982	Jan1983 - Dec1992	Jan1993 - Dec2002	Jan2003 - Dec2012	Jan1927 - Dec1969	Jan1970 Dec2012
1 Weak LTD	0.759%	0.277%	0.156%	0.405%	0.364%	0.480%	0.296%	0.297%
2	0.727%	0.116%	−0.006%	0.570%	0.326%	0.369%	0.140%	0.491%
3	0.851%	0.441%	0.003%	0.567%	0.420%	0.469%	0.284%	0.476%
4	0.747%	0.674%	0.280%	0.694%	0.368%	0.522%	0.530%	0.485%
5 Strong LTD	1.134%	0.649%	0.477%	0.842%	0.487%	0.828%	0.620%	0.693%
Strong - Weak	0.375%***	0.372%**	0.321	0.437%**	0.123	0.347%	0.324%***	0.396%**
	(2.63)	(2.16)	(1.58)	(2.34)	(1.03)	(1.62)	(2.90)	(2.46)

Panel B: Fama-MacBeth (1973) Regressions

	Jan1927 - Dec1962	Jan1963 - Dec1972	Jan1973 - Dec1982	Jan1983 - Dec1992	Jan1993 - Dec2002	Jan2003 - Dec2012	Up Market	Down Market
LTD	0.0189***	0.0171**	0.0075	0.0130**	0.035***	0.0144*	0.0145***	0.0152***
	(3.89)	(1.99)	(1.15)	(2.12)	(3.22)	(1.81)	(3.89)	(4.23)

Table IA.IX: Adjusted Returns and Different Regression Methods

This table present results for regression (5) from Table 7 of the main text for alternative specifications. We only show the coefficient estimate for the impact of LTD. On the left side of the table we show the results when we use industry-adjusted returns. To define industries we use the Fama-French 12 and 48 industry classification as well as SIC 2-, 3-, and 4-digit codes. The right side of the table reports the results when we use DGTW-adjusted returns (Daniel, Grinblatt, Titman, and Wermers (1997)) or apply various regression techniques. In regression R(1) we perform a Fama-MacBeth (1973) regression, but do not cluster standard errors using the Newey-West (1987) adjustment. Regression R(2) displays the results of a Fama-MacBeth (1973) regression with unwinsorized independent variables. In regression R(3) we perform a pooled OLS-regression with time-fixed effects and standard errors clustered by single stocks. Regression R(4) is identical, but we cluster standard errors by Fama-French 12 industry. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

alternative specification	LTD (t-stat)	R^2	alternative specification	LTD (t-stat)	R^2
FF-12	0.113***	0.067	DGTW	0.108**	0.062
	(3.64)			(2.41)	
FF-48	0.109***	0.063	R(1)	0.141***	0.071
	(3.01)			(4.61)	
SIC-2	0.103**	0.063	R(2)	0.147***	0.071
	(2.21)			(4.56)	
SIC-3	0.091**	0.060	R(3)	0.189***	0.067
	(2.00)			(5.92)	
SIC-4	0.079*	0.053	R(4)	0.198***	0.066
	(1.79)			(5.24)	

Table IA.X: Accounting for Time-Varying Volatility in LTD Computation

This table reports results for the impact of LTD on monthly future excess returns where we calculate LTD based on the residuals from different time series models of daily individual stock and market returns. As time series models we use the ARCH(1) model of Engle (1982), the GARCH(1, 1) model of Bollerslev (1986), and the EGARCH(1, 1) model of Nelson (1991). Each time series model is estimated using normally distributed- and t-student distributed error terms, respectively. In Panel A, we report the results of value-weighted portfolio sorts based on LTD. We present the future monthly return difference between the quintile with the strongest LTD stocks and the quintile with the weakest LTD stocks (Strong-Weak) as in the second column of Panel A in Table 4 from the main text. Panel B reports results for the estimate of the influence of LTD from Fama-MacBeth (1973) regressions of future monthly excess returns over the risk-free rate on LTD and the full set of controls as in regression (5) from Table 7 in the main text (included in the regression but coefficient estimates suppressed in the table). The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Panel A: Residual LTD: Univariate Portfolio Sorts

Specification	ARCH(1)		GARCH(1, 1)		EGARCH(1, 1)	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
Strong - Weak	0.341%***	0.356%***	0.321%***	0.317%***	0.318%***	0.305%**
	(3.67)	(3.54)	(3.07)	(2.98)	(2.89)	(2.54)

Panel B: Residual LTD: Fama-MacBeth (1973) Regressions

	ARCH(1)		GARCH(1, 1)		EGARCH(1, 1)	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
LTD	0.131***	0.136***	0.132***	0.137***	0.089***	0.095**
	(4.23)	(3.92)	(4.21)	(3.63)	(2.93)	(2.20)

Table IA.XI: Equal-weighted Predictive Portfolio Sorts

Panel A of this table reports results from univariate portfolio sorts based on LTD. In each month, we rank stocks into quintiles (1-5) and form portfolios based on LTD. We report the average future excess return ('Return'), Sharpe (1964)'s 'CAPM-Alpha', Carhart (1997)'s 'CAR-Alpha', and Fama and French (2015)'s 'FF-Alpha' over the following month. The row labeled 'Strong – Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistical significance levels. Panels B – F show results of equal-weighted average future monthly excess returns over the one-month T-Bill rate of portfolios double-sorted on LTD and beta (Panel B), downside beta (Panel C), coskewness (Panel D), cokurtosis (Panel E), and tail risk beta (Panel F), respectively. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Panel A: Lower Tail Dependence (LTD)

Portfolio	LTD	Return	CAPM-Alpha	CAR-Alpha	FF5-Alpha
1 Weak LTD	0.00	0.460%	−0.020%	−0.140%*	−0.276%***
2	0.03	0.872%	+0.357%**	+0.271%**	+0.034%
3	0.06	0.702%	+0.196%	+0.085%	−0.076%
4	0.12	0.833%	+0.284%**	+0.212%*	+0.048%
5 Strong LTD	0.22	0.944%	+0.344%***	+0.283%***	+0.295%**
Strong - Weak	0.22***	0.484%***	0.364%***	0.423%***	0.571%***
		(4.10)	(3.31)	(3.76)	(5.87)

Panel B: Beta (β) and Lower Tail Dependence (LTD)

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	0.40%	0.51%	0.60%	0.37%	0.12%	0.40%
5 Strong LTD	0.88%	0.92%	0.93%	0.92%	0.74%	0.88%
Strong - Weak	0.48%***	0.41%***	0.32%**	0.55%***	0.62%***	0.48%***
	(4.80)	(3.20)	(2.13)	(4.69)	(4.18)	(3.87)

Panel C: Downside Beta (β^{-}) and Lower Tail Dependence (LTD)

Portfolio	1 Low β^{-}	2	3	4	5 High β^{-}	Average
1 Weak LTD	0.39%	0.45%	0.45%	0.34%	0.05%	0.33%
5 Strong LTD	0.83%	0.93%	0.91%	0.84%	0.70%	0.84%
Strong - Weak	0.44%***	0.48%***	0.46%***	0.50%***	0.66%***	0.51%***
	(4.12)	(3.98)	(3.87)	(4.81)	(3.74)	(3.84)

Table IA.XI: Continued

Panel D: Coskewness (COSKEW) and Lower Tail Dependence (LTD)

Portfolio	1 Low COSKEW	2	3	4	5 High COSKEW	Average
1 Weak LTD	0.66%	0.63%	0.55%	0.35%	0.30%	0.50%
5 Strong LTD	0.94%	0.90%	1.00%	0.89%	0.71%	0.89%
Strong - Weak	0.28%	0.27%*	0.45%***	0.54%***	0.41%***	0.39%**
	(1.13)	(1.88)	(3.65)	(3.79)	(3.14)	(2.12)

Panel E: Cokurtosis (COKURT) and Lower Tail Dependence (LTD)

Portfolio	1 Low COKURT	2	3	4	5 High COKURT	Average
1 Weak LTD	0.39%	0.38%	0.45%	0.37%	0.31%	0.38%
5 Strong LTD	0.91%	0.93%	1.02%	0.91%	0.71%	0.90%
Strong - Weak	0.53%***	0.54%***	0.57%***	0.54%***	0.40%***	0.52%***
	(4.77)	(4.15)	(4.51)	(4.24)	(3.21)	(4.13)

Panel F: Tail Risk Beta (β_{tail}) and Lower Tail Dependence (LTD)

Portfolio	1 Low β_{tail}	2	3	4	5 High β_{tail}	Average
1 Weak LTD	0.35%	0.31%	0.40%	0.37%	0.29%	0.34%
5 Strong LTD	0.87%	0.89%	0.95%	0.87%	0.81%	0.88%
Strong - Weak	0.52%***	0.58%***	0.55%***	0.50%***	0.52%***	0.53%***
	(3.66)	(5.15)	(4.51)	(4.45)	(3.88)	(4.32)

C Relation to Other Equilibrium Models with Nonlinear SDFs

In this section, we discuss how distinct the SDF (14) from Theorem 1 in the main text is compared to recent equilibrium models from the asset pricing literature in which the SDFs is also a function of non-linear payoffs such as option payoffs. To make it easy for readers, we keep the same notations as in the articles mentioned below.

Discussion of Nonlinear SDFs. Vanden (2004) derives a SDF that depends on the market return and the return on a call option by imposing non-negative wealth constraints on a group of agents that have linear risk tolerance. More specifically, the SDF in Vanden (2004), Equations (17)-(19) on pages 215-216, takes the form

$$M_{t+1} = \lambda_1^{-1}\tau_1 - \lambda_1^{-1}S_{t+1} - \sum_{j=2}^{I}\left(\frac{1}{\overline{\lambda}_j} - \frac{1}{\overline{\lambda}_{j-1}}\right)(S_{t+1} - K_j)^+ , \tag{C1}$$

where τ_1, λ_1, K_j, $\overline{\lambda}_j s$ are constant parameters and I is the number of agents in the economy. The SDF (C1) is a special case of (14).

Under the assumption of heterogeneous investors with piecewise risk tolerance functions, Vanden (2006), page 1280, further shows that the SDF has the form

$$M_{t+1} = b_0 + b_1 R_{Mt+1} + b_2 R_{Ot+1} + b_3 R_{Mt+1}^2 + b_4 R_{Ot+1}^2 + b_5 R_{Mt+1} R_{Ot+1}, \tag{C2}$$

where R_{Ot+1} is the traded option return. Recognizing that the option return R_{Ot+1} is a function of $(R_{Mt+1} - k)^+$, it can be shown that the SDF in (C2) is a nonlinear function of the market return. The spanning formula of Lemma 1 (in the main text) can, therefore, be used to show that (C2) can also be seen as a special case of (14).

Recently, Routledge and Zin (2010) characterize Generalized Disappointment Aversion (GDA) risk preferences that have the key feature to overweight lower-tail outcomes relative to expected utility. Disappointment-averse preferences overweight outcomes that are "disappointing" which is defined as being below the certainty equivalent. In the GDA model, the SDF takes the form

$$M_{t+1} = \frac{z_{t+1}^{\alpha}\left(R_{Mt+1}\right)^{-1}\left(1 + \theta \mathbf{I}_{z_{t+1}<\delta}\right)}{1 + \theta\delta^{\alpha} E_t\left[\mathbf{I}_{z_{t+1}<\delta}\right]}, \tag{C3}$$

where θ is the disappointment aversion, δ is the disappointment threshold, and α is the risk aversion parameter. $\mathbf{I}$ is an indicator function. The random variable z_{t+1} in (C3) is defined as

$$z_{t+1} = \left(\beta x_{t+1}^{\rho-1} R_{Mt+1}\right)^{1/\rho}, \tag{C4}$$

where β is the time-preference, the elasticity of intertemporal substitution is $1/(1-\rho)$ and x_{t+1} is the consumption growth. The special case $\rho = 1$ leads to a Stochastic Discount Factor of the form

$$M_{t+1} = \frac{\beta R_{Mt+1}^{\alpha-1}\left(1+\theta \mathbf{I}_{R_{Mt+1}<\delta/\beta}\right)}{1+\theta\delta^{\alpha}E_t\left[\mathbf{I}_{R_{Mt+1}<\delta/\beta}\right]}. \tag{C5}$$

Applying Lemma 1 (in our main text) to $R_{Mt+1}^{\alpha-1}$ when $\alpha \neq 1$, and replacing the result in (C5) shows that the SDF in (C5) can also be interpreted as a special case of (14).

D Proofs

Proof of Theorem 1. Since the first and second derivative of $u^{'}[.]$ exist, application of Lemma 1 to $H[S_{t+1}] = u^{'}[S_{t+1}/S_t]$ with $\overline{S} = S_t$ yields

$$\begin{aligned} u^{'}[S_{t+1}/S_t] &= u^{'}[1] + (S_{t+1} - S_t)\frac{1}{S_t}u^{''}[1] \\ &+ \int_{S_t}^{\infty}\frac{1}{S_t^2}u^{'''}[K/S_t](S_{t+1} - K)^{+}dK \\ &+ \int_0^{S_t}\frac{1}{S_t^2}u^{'''}[K/S_t](K - S_{t+1})^{+}dK \end{aligned} \tag{D1}$$

which simplifies to

$$\begin{aligned} u^{'}[S_{t+1}/S_t] &= u^{'}[1] + (R_{M,t+1} - 1)u^{''}[1] \\ &+ \int_1^{\infty}u^{'''}[k](R_{M,t+1} - k)^{+}dk \\ &+ \int_0^1 u^{'''}[k](k - R_{M,t+1})^{+}dk \end{aligned} \tag{D2}$$

with $k = \frac{K}{S_t}$. We can, therefore, write the SDF (13) as

$$\begin{aligned} M_{t+1} &= \frac{1}{R_{f,t}}\frac{u^{'}[1]}{u'[a]} + \frac{1}{R_{f,t}}\frac{u^{''}[1]}{u'[a]}(R_{M,t+1} - 1) \\ &+ \int_1^{\infty}\frac{1}{R_{f,t}}\frac{u^{'''}[k]}{u'[a]}(R_{M,t+1} - k)^{+}dk \\ &+ \int_0^1\frac{1}{R_{f,t}}\frac{u^{'''}[k]}{u'[a]}(k - R_{M,t+1})^{+}dk. \end{aligned} \tag{D3}$$

Now, we denote by $k_{\max}$ the maximum value of k, and rewrite (D3) as

$$\begin{aligned} M_{t+1} &= \frac{1}{R_{f,t}}\frac{u^{'}[1]}{u'[a]} + \frac{1}{R_{f,t}}\frac{u^{''}[1]}{u'[a]}(R_{M,t+1} - 1) \\ &+ \int_1^{k_{\max}}\frac{1}{R_{f,t}}\frac{u^{'''}[k]}{u'[a]}(R_{M,t+1} - k)^{+}dk \\ &+ \int_0^1\frac{1}{R_{f,t}}\frac{u^{'''}[k]}{u'[a]}(k - R_{M,t+1})^{+}dk. \end{aligned} \tag{D4}$$

This ends the proof. □

Proof of Theorem 2. We expand the Euler equation (15),

$$E[R_i] - R_f = -R_f Cov(R_i, M). \tag{D5}$$

We then replace the SDF (14) in (D5) and decompose the excess return into many components.

Next, we replace (D4) in (D5) and obtain

$$\begin{aligned} E[R_i] - R_f &= -\frac{u''[1]}{u'[a]} Cov(R_i, R_M) \\ &\quad - \int_1^{k_{\max}} \frac{u'''[k]}{u'[a]} Cov\left(R_i, (R_M - k)^+\right) dk \\ &\quad - \int_0^1 \frac{u'''[k]}{u'[a]} Cov\left(R_i, (k - R_M)^+\right) dk. \end{aligned} \tag{D6}$$

Now, observe that

$$(R_i - k)^+ - (k - R_i)^+ = R_i - k \text{ for any given } k. \tag{D7}$$

We replace the exact relation (D7) in the last two terms of the RHS of (D6) and express (D6) as

$$\begin{aligned} E[R_i] - R_f &= -\frac{u''[1]}{u'[a]} Cov(R_i, R_M) \\ &\quad - \int_1^{k_{\max}} \frac{u'''[k]}{u'[a]} Cov\left((R_i - k)^+ - (k - R_i)^+, (R_M - k)^+\right) dk \\ &\quad - \int_0^1 \frac{u'''[k]}{u'[a]} Cov\left((R_i - k)^+ - (k - R_i)^+, (k - R_M)^+\right) dk, \end{aligned} \tag{D8}$$

which expands to

$$\begin{aligned} E[R_i] - R_f &= -\frac{u''[1]}{u'[a]} Cov(R_i, R_M) \\ &\quad - \int_1^{k_{\max}} \frac{u'''[k]}{u'[a]} Cov\left((R_i - k)^+, (R_M - k)^+\right) dk \\ &\quad + \int_1^{k_{\max}} \frac{u'''[k]}{u'[a]} Cov\left((k - R_i)^+, (R_M - k)^+\right) dk \\ &\quad - \int_0^1 \frac{u'''[k]}{u'[a]} Cov\left((R_i - k)^+, (k - R_M)^+\right) dk, \\ &\quad + \int_0^1 \frac{u'''[k]}{u'[a]} Cov\left((k - R_i)^+, (k - R_M)^+\right) dk. \end{aligned} \tag{D9}$$

It is particularly useful to observe that

$$\begin{aligned} & Cov\left((k - R_M)^+, (R_M - k)^+\right) \\ = \; & E\left[(k - R_M)^+ (R_M - k)^+\right] - E\left[(k - R_M)^+\right] E\left[(R_M - k)^+\right] \\ = \; & 0 - E\left[(k - R_M)^+\right] E\left[(R_M - k)^+\right] < 0. \end{aligned} \tag{D10}$$

We then exploit (D10) and write (D9) as:

$$\begin{aligned}
E\left[R_i\right]-R_f &= -\frac{u^{''}\left[1\right]}{u'\left[a\right]}Var\left[R_M\right]\frac{Cov\left(R_i,R_M\right)}{Var\left[R_M\right]} \\
&\quad -\int_1^{k_{\max}}\frac{u^{'''}\left[k\right]}{u'\left[a\right]}Var\left[\left(R_M-k\right)^+\right]\frac{Cov\left(\left(R_i-k\right)^+,\left(R_M-k\right)^+\right)}{Var\left[\left(R_M-k\right)^+\right]}dk \\
&\quad -\int_1^{k_{\max}}\frac{u^{'''}\left[k\right]}{u'\left[a\right]}\left(\mu_M^u\left[k\right]\mu_M^d\left[k\right]\right)\frac{Cov\left(\left(k-R_i\right)^+,\left(R_M-k\right)^+\right)}{Cov\left(\left(k-R_M\right)^+,\left(R_M-k\right)^+\right)}dk \\
&\quad +\int_0^1\frac{u^{'''}\left[k\right]}{u'\left[a\right]}\left(\mu_M^u\left[k\right]\mu_M^d\left[k\right]\right)\frac{Cov\left(\left(R_i-k\right)^+,\left(k-R_M\right)^+\right)}{Cov\left(\left(R_M-k\right)^+,\left(k-R_M\right)^+\right)}dk \\
&\quad +\int_0^1\frac{u^{'''}\left[k\right]}{u'\left[a\right]}Var\left[\left(k-R_M\right)^+\right]\frac{Cov\left(\left(k-R_i\right)^+,\left(k-R_M\right)^+\right)}{Var\left[\left(k-R_M\right)^+\right]}dk.
\end{aligned} \tag{D11}$$

The above expression (D11) simplifies to:

$$\begin{aligned}
E\left[R_i\right]-R_f &= \lambda\beta_i \\
&\quad +\int_1^{k_{\max}}\lambda^{uu}\left[k\right]\delta_i^{uu}\left[k\right]dk+\int_1^{k_{\max}}\lambda^{du}\left[k\right]\delta_i^{du}\left[k\right]dk \\
&\quad +\int_0^1\lambda^{ud}\left[k\right]\delta_i^{ud}\left[k\right]dk+\int_0^1\lambda^{dd}\left[k\right]\delta_i^{dd}\left[k\right]dk.
\end{aligned} \tag{D12}$$

This ends the proof. □

Proof of "In the limiting case, the expected excess return changes disproportionally". Suppose that the regularity conditions mentioned earlier are satisfied. We examine the impact of $\delta_i^{dd}\left[\varepsilon\right]$ ($\delta_i^{uu}\left[\varepsilon\right]$) on the expected excess return when ε approaches 0 from the right (k_{max} from the left).

(i) Consider a constant $k_0>\varepsilon$. Since $u^{''''}\left[.\right]<0$, we have

$$u^{'''}\left[k_0\right]<u^{'''}\left[\varepsilon\right] \tag{D13}$$

and hence

$$u^{'''}\left[k_0\right]<\lim_{\varepsilon\to0^+}u^{'''}\left[\varepsilon\right]. \tag{D14}$$

Assume the same level of risk quantities:

$$Cov\left(\left(k_0-R_i\right)^+,\left(k_0-R_M\right)^+\right)=\lim_{\varepsilon\to0^+}Cov\left(\left(\varepsilon-R_i\right)^+,\left(\varepsilon-R_M\right)^+\right). \tag{D15}$$

Multiplying (D14) by (D15) yields

$$\lambda^{dd}\left[k_0\right]\delta_i^{dd}\left[k_0\right]<\lim_{\varepsilon\to0^+}\lambda^{dd}\left[\varepsilon\right]\delta_i^{dd}\left[\varepsilon\right]. \tag{D16}$$

Inequality (D16) shows that, everything else being equal, the expected excess return (D12) increases disproportionally with $\delta_i^{dd}[\varepsilon]$ when ε approaches 0 from the right.

(ii) Consider a constant $k_0 < \varepsilon$. Since $u''''[.] < 0$, we have

$$u'''[k_0] > \lim_{\varepsilon \to k_{\max}^-} u'''[\varepsilon]. \tag{D17}$$

Assume the same level of risk quantities:

$$Cov\left((R_i - k_0)^+, (R_M - k_0)^+\right) = \lim_{\varepsilon \to k_{\max}^-} Cov\left((R_i - \varepsilon)^+, (R_M - \varepsilon)^+\right). \tag{D18}$$

Multiplying (D17) by (D18) yields

$$\lambda^{uu}[k_0]\,\delta_i^{u.u}[k_0] > \lim_{\varepsilon \to k_{\max}^-} \lambda^{uu}[\varepsilon]\,\delta_i^{uu}[\varepsilon]. \tag{D19}$$

Everything else being equal, the expected excess return (D12) decreases disproportionally with $\delta_i^{uu}[\varepsilon]$ when ε approaches $k_{\max}$ from the left.

□

Proof of Theorem 3. To prove Theorem 3, we will use the following lemma:

Lemma 2 *Consider two constants a and b, and discretize the interval $[a, b]$ into N+1 grid points (1, 2, ..., N+1), where the grid spacing is h=(b-a)/N. The approximation to the integral $\int_a^b f(x)\,dx$ becomes*

$$\int_a^b f[x]\,dx = \frac{(b-a)}{N}\left[\frac{f[a] + f[b]}{2} + \sum_{j=1}^{N-1} f[a + j((b-a)/N)]\right]. \tag{D20}$$

Proof. See Atkinson (1989). □

We first discretize Theorem 2. To discretize the integrals in this theorem, we use Lemma 2.

(i) First, we consider two constants 0 and 1, and discretize the interval $[0, 1]$ into N+1 grid points (1, 2, ..., N+1), where the grid spacing is h=(1-0)/N. We then approximate the integrals $\int_0^1 \lambda^{ud}[k]\,\delta_i^{ud}[k]\,dk$ and $\int_0^1 \lambda^{dd}[k]\,\delta_i^{dd}[k]\,dk$.

(ii) Second, we consider two constants 1 and k_{max}, and discretize the interval $[1, k_{max}]$ into N+1 grid points (1, 2, ..., N+1), where the grid spacing is h=(k_{max}-1)/N.

For characterizations to follow, we define the limiting tail-based co-moment risks

$$\begin{aligned} \delta_i^{uu}[1] &= \lim_{\varepsilon \to 1^+} \delta_i^{uu}[\varepsilon], & \delta_i^{dd}[1] &= \lim_{\varepsilon \to 1^-} \delta_i^{dd}[\varepsilon], \\ \delta_i^{uu}[k_{\max}] &= \lim_{\varepsilon \to k_{\max}^-} \delta_i^{uu}[\varepsilon], & \delta_i^{dd}[0] &= \lim_{\varepsilon \to 0^+} \delta_i^{dd}[\varepsilon], \end{aligned}$$

and their prices of risks

$$\begin{aligned} \lambda^{uu}[1] &= \lim_{\varepsilon\to 1^+} \lambda^{uu}[\varepsilon] & \lambda^{dd}[1] &= \lim_{\varepsilon\to 1^-} \lambda^{dd}[\varepsilon], \\ \lambda^{uu}[k_{\max}] &= \lim_{\varepsilon\to k_{\max}^-} \lambda^{uu}[\varepsilon] & \lambda^{dd}[0] &= \lim_{\varepsilon\to 0^+} \lambda^{dd}[\varepsilon]. \end{aligned}$$

We further define the limiting tail-based co-moment risks

$$\begin{aligned} \delta_i^{du}[1] &= \lim_{\varepsilon\to 1^+} \delta_i^{du}[\varepsilon], & \delta_i^{ud}[1] &= \lim_{\varepsilon\to 1^-} \delta_i^{ud}[\varepsilon], \\ \delta_i^{du}[k_{\max}] &= \lim_{\varepsilon\to k_{\max}^-} \delta_i^{du}[\varepsilon], & \delta_i^{ud}[0] &= \lim_{\varepsilon\to 0^+} \delta_i^{ud}[\varepsilon], \end{aligned}$$

and their prices of risks

$$\begin{aligned} \lambda^{du}[1] &= \lim_{\varepsilon\to 1^+} \lambda^{du}[\varepsilon] & \lambda^{ud}[1] &= \lim_{\varepsilon\to 1^-} \lambda^{ud}[\varepsilon], \\ \lambda^{du}[k_{\max}] &= \lim_{\varepsilon\to k_{\max}^-} \lambda^{du}[\varepsilon] & \lambda^{ud}[0] &= \lim_{\varepsilon\to 0^+} \lambda^{ud}[\varepsilon]. \end{aligned}$$

Since we are interested on the impact of limiting tail dependence measures on expected excess returns, we apply without loss of generality the discretization in Lemma 2 to the integrals

$$\int_1^{k_{max}} \lambda^{du}[k]\,\delta_i^{du}[k]\,dk,\ \int_1^{k_{max}} \lambda^{uu}[k]\,\delta_i^{uu}[k]\,dk, \tag{D21}$$

$$\int_0^1 \lambda^{ud}[k]\,\delta_i^{ud}[k]\,dk, \text{ and } \int_0^1 \lambda^{dd}[k]\,\delta_i^{dd}[k]\,dk \tag{D22}$$

by setting $N = 1$ in (D12) and get

$$\begin{aligned} E[R_i] - R_f &= \lambda\beta_i \\ &+ \frac{1}{2}\lambda^{dd}[0]\,\delta_i^{dd}[0] + \frac{1}{2}\lambda^{ud}[0]\,\delta_i^{ud}[0] \\ &+ \frac{1}{2}(k_{\max}-1)\,\lambda^{uu}[k_{\max}]\,\delta_i^{uu}[k_{\max}] \\ &+ \frac{1}{2}(k_{\max}-1)\,\lambda^{du}[k_{\max}]\,\delta_i^{du}[k_{\max}] \\ &+ \frac{1}{2}(k_{\max}-1)\,\lambda^{du}[1]\,\delta_i^{du}[1] + \frac{1}{2}(k_{\max}-1)\,\lambda^{uu}[1]\,\delta_i^{uu}[1] \\ &+ \frac{1}{2}\lambda^{ud}[1]\,\delta_i^{ud}[1] + \frac{1}{2}\lambda^{dd}[1]\,\delta_i^{dd}[1]. \end{aligned} \tag{D23}$$

For characterizations to follow, consider the following notations:

$$\begin{aligned} \psi_i^{dd}[k] &= E[(k-R_i)(k-R_M)\,|R_M<k, R_i<k]\,P[R_M<k] > 0, \\ \psi_i^{uu}[k] &= E[(R_i-k)(R_M-k)\,|R_M>k, R_i>k]\,P[R_M>k] > 0, \end{aligned} \tag{D24}$$

and

$$\begin{aligned} \sigma_M^d &= \sqrt{Var\left[(k-R_M)^+\right]}, \\ \sigma_M^u &= \sqrt{Var\left[(R_M-k)^+\right]}, \end{aligned}$$

and

$$
\begin{aligned}
\varphi_{i1}^{dd}[k] &= \frac{\psi_i^{dd}[k]}{\left(\sigma_M^d\right)^2} \quad \text{and} \quad \varphi_{i2}^{dd}[k] = \frac{\mu_i^d[k]\mu_M^d[k]}{\left(\sigma_M^d\right)^2}, \\
\varphi_{i1}^{uu}[k] &= \frac{\psi_i^{uu}[k]}{\left(\sigma_M^u\right)^2} \quad \text{and} \quad \varphi_{i2}^{uu}[k] = \frac{\mu_i^u[k]\mu_M^u[k]}{\left(\sigma_M^u\right)^2},
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_i^d[k] &= E\left[(k-R_i)^+\right], \\
\mu_i^u[k] &= E\left[(R_i-k)^+\right], \\
\mu_M^d[k] &= E\left[(k-R_M)^+\right], \\
\mu_M^u[k] &= E\left[(R_M-k)^+\right].
\end{aligned}
$$

The tail-based co-moment risks in equations (18) and (19) (of the main text) can be decomposed as:

$$
\begin{aligned}
\delta_i^{dd}[k] &= \varphi_{i1}^{dd}[k]\, T_i^{dd}[k] - \varphi_{i2}^{dd}[k], \\
\delta_i^{uu}[k] &= \varphi_{i1}^{uu}[k]\, T_i^{uu}[k] - \varphi_{i2}^{uu}[k],
\end{aligned}
$$

with:

$$
\begin{aligned}
T_i^{dd}[k] &= P\left[r_i < k-1 | r_M < k-1\right], \\
T_i^{uu}[k] &= P\left[r_i > k-1 | r_M > k-1\right],
\end{aligned}
$$

where $r_i = R_i - 1$ and $r_M = R_M - 1$. The above expressions can be alternatively expressed as

$$
\begin{aligned}
T_i^{dd}[k] &= P\left[r_i < F_i^{-1}(\nu_i)\,| r_M < F_M^{-1}(\nu)\right], \\
T_i^{uu}[k] &= P\left[r_i > F_i^{-1}(\nu_i)\,| r_M > F_M^{-1}(\nu)\right],
\end{aligned}
$$

with

$$
\nu = F_M[k-1] \text{ and } \nu_i = F_i[k-1], \tag{D25}
$$

where $F_i[.]$ and $F_M[.]$ are cumulative probability distributions of asset i's return and the market return, respectively. We further define the limiting tail dependence measures as

$$
\begin{aligned}
T_i^{uu}[1] &= \lim_{\varepsilon\to 1^+} T_i^{uu}[\varepsilon], & T_i^{dd}[1] &= \lim_{\varepsilon\to 1^-} T_i^{dd}[\varepsilon], \\
T_i^{uu}[k_{\max}] &= \lim_{\varepsilon\to k_{\max}^-} T_i^{uu}[\varepsilon], & T_i^{dd}[0] &= \lim_{\varepsilon\to 0^+} T_i^{dd}[\varepsilon].
\end{aligned} \tag{D26}
$$

Among the limiting tail dependence measures in (D26), we focus on the most interesting extreme tail dependence measures $T_i^{dd}[0]$ and $T_i^{uu}[k_{\max}]$.

To proceed, we express the tail dependence measures (LTD, UTD) as:

$$
\begin{aligned}
T_i^{dd}[0] &= \lim_{q\to 0^+} P\left[r_i < F_i^{-1}(q)\,| r_M < F_M^{-1}(q)\right] = LTD_i, \\
T_i^{uu}[k_{\max}] &= \lim_{q\to 1^-} P\left[r_i > F_i^{-1}(q)\,| r_M > F_M^{-1}(q)\right] = UTD_i.
\end{aligned} \tag{D27}
$$

The limiting tail-based co-moment risks are linearly related to the tail dependence measures:

$$\begin{aligned} \delta_i^{dd}[0] &= LTD_i \varphi_{i1}^{dd}[0] - \varphi_{i2}^{dd}[0], \\ \delta_i^{uu}[k_{\max}] &= UTD_i \varphi_{i1}^{uu}[k_{\max}] - \varphi_{i2}^{uu}[k_{\max}], \end{aligned} \tag{D28}$$

where LTD_i and UTD_i are defined in (D27). The partial derivatives of the expected excess return (in equation (26) of the main text) with respect to the tail dependence measures (LTD, UTD) are

$$\frac{\partial (E[R_i] - R_f)}{\partial LTD_i} = \frac{\partial (E[R_i] - R_f)}{\partial \delta_i^{dd}[0]} \times \frac{\partial \delta_i^{dd}[0]}{\partial LTD_i}, \tag{D29}$$

$$\frac{\partial (E[R_i] - R_f)}{\partial UTD_i} = \frac{\partial (E[R_i] - R_f)}{\partial \delta_i^{uu}[k_{\max}]} \times \frac{\partial \delta_i^{uu}[k_{\max}]}{\partial UTD_i}. \tag{D30}$$

Next, we find each partial derivative that appears in the RHS of (D29) and (D30). First, the partial derivative of the expected excess return (26) with respect to the tail-based co-moment risks are

$$\frac{\partial\left(E[R_i]-R_f\right)}{\partial \delta_i^{dd}[0]} = \frac{1}{2}\lambda^{dd}[0] \quad \text{and} \quad \frac{\partial\left(E[R_i]-R_f\right)}{\partial \delta_i^{uu}[k_{\max}]} = \frac{1}{2}(k_{\max} - 1)\lambda^{uu}[0]. \tag{D31}$$

Second, the partial derivative of the tail-based co-moment risks with respect to the tail dependence measures (LTD, UTD) are

$$\frac{\partial \delta_i^{dd}[0]}{\partial LTD_i} = \lim_{k \to 0^+} \varphi_{i1}^{dd}[k] > 0 \quad \text{and} \quad \frac{\partial \delta_i^{uu}[k_{\max}]}{\partial UTD_i} = \lim_{k \to k_{\max}^-} \varphi_{i1}^{uu}[k] > 0. \tag{D32}$$

Equations in (D31) jointly with equations in (D32) allow to write (D29) and (D30) as

$$\begin{aligned} \frac{\partial\left(E[R_i]-R_f\right)}{\partial LTD_i} &= \frac{1}{2}\lambda^{dd}[0]\left(\lim_{k \to 0^+} \varphi_{i1}^{dd}[k]\right) > 0, \\ \frac{\partial\left(E[R_i]-R_f\right)}{\partial UTD_i} &= \frac{1}{2}(k_{\max} - 1)\lambda^{uu}[0]\left(\lim_{k \to k_{\max}^-} \varphi_{i1}^{uu}[k]\right) < 0. \end{aligned}$$

The partial derivatives show that LTD (UTD) is positively (negatively) related to expected excess returns on risky assets. This ends the proof. □

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