# Internet Appendix for "The Liquidity Effects of Official Bond Market Intervention"

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## A.I Analytical Results

In this section we give the analytical proofs of the propositions in the main text.

*Proof of Lemma 1.* First note that the steady state mass evolutions are unaffected by the introduction of $\lambda_D$ into the model. They are:

$$\begin{aligned}
\dot{\mu}_{ln}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t) \\
\dot{\mu}_{lo}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \\
\dot{\mu}_{hn}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t) \\
\dot{\mu}_{ho}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t).
\end{aligned}$$

Therefore, the first proposition in DGP (2005) applies directly.

Let $\tau_l$ denote the arrival (stopping time) of an intrinsic type shift, $\tau_i$ denote the arrival of a meeting with another agent and $\tau_D$ the arrival of the asset's default. Then the agents' value

functions at time $t$ are given by

$$
\begin{aligned}
\text{(A-1)} \quad V_{lo} &= E_t\Big[\int_t^{\tau_l\wedge\tau_i\wedge\tau_D} e^{-r(u-t)}(1-\delta)du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_i\wedge\tau_D\}} \\
&\qquad + e^{-r(\tau_i-t)}(V_{ln}+P)\mathbf{1}_{\{\tau_i=\tau_l\wedge\tau_i\wedge\tau_D\}} + e^{-r(\tau_D-t)}R(t)\mathbf{1}_{\{\tau_D=\tau_l\wedge\tau_i\wedge\tau_D\}}\Big] \\
V_{ln} &= E_t\Big[e^{-r(\tau_l-t)}V_{hn}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_D\}}\Big] \\
V_{ho} &= E_t\Big[\int_t^{\tau_l\wedge\tau_D} e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_D\}} + e^{-r(\tau_D-t)}R(t)\mathbf{1}_{\{\tau_D=\tau_l\wedge\tau_D\}}\Big] \\
V_{hn} &= E_t\Big[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_i\wedge\tau_D\}} + e^{-r(\tau_i-t)}(V_{ho}-P)\mathbf{1}_{\{\tau_i=\tau_l\wedge\tau_i\wedge\tau_D\}}\Big]
\end{aligned}
$$

where dependence on $t$ has been subsumed. Using Leibniz's Rule, the first-order conditions follow.

The first-order conditions are linear in the value functions and price, hence we have

$$
\begin{pmatrix} 0 \\ \lambda_D R + 1 \\ 0 \\ \lambda_D R + (1-\delta) \\ 0 \end{pmatrix}
=
\begin{bmatrix}
r+\lambda_u+\lambda_d & -\lambda_u & 0 & 0 & 0 \\
0 & 0 & -\lambda_d & \lambda_d+\lambda_D+r & 0 \\
-\lambda_d & r+\lambda_d+2\lambda\mu_{lo} & 0 & -2\lambda\mu_{lo} & 2\lambda\mu_{lo} \\
-2\lambda\mu_{hn} & 0 & r+\lambda_u+2\lambda\mu_{hn}+\lambda_D & -\lambda_u & -2\lambda\mu_{hn} \\
-\tilde{q} & -q & \tilde{q} & q & -1
\end{bmatrix}
\begin{pmatrix} V_{ln} \\ V_{hn} \\ V_{lo} \\ V_{ho} \\ P \end{pmatrix}
$$

where $\tilde{q} \equiv 1-q$. The solution yields (2). $\square$

For completeness, we prove a larger proposition. Because $y_L$ is an increasing function of $L$, $P_f$ is a decreasing function of $\lambda_D$, and $P_f$ is unchanged by $s$, our statements about $L$ directly imply Proposition 1 in the main text.

**Proposition A1** Assume $s < \frac{\lambda_u}{\lambda_u+\lambda_d}$ and $R = \frac{\zeta}{r}$ for $\zeta \in (0,1)$. Let $\bar{\lambda}_D$ be some constant that depends on model parameters other than $\lambda_D$. The frictionless price $P_f$ is

1. unchanged as $s$, $\lambda_d$ or $\lambda$ varies

2. decreasing in $\lambda_D$

   such that $P_f \xrightarrow[\lambda_D \to 0]{} 1/r$ and $P_f \xrightarrow[\lambda_D \to \infty]{} R$

The liquidity premium $L$ is

3. increasing in $s$

4. increasing in $\lambda_d$

5. decreasing in $\lambda$ for all $\lambda \geq \bar{\lambda}$

   such that $L \xrightarrow[\lambda \to \infty]{} 0$

6. decreasing in $\lambda_D$ for all $\lambda_D \geq \bar{\lambda}_D$

   such that $L \xrightarrow[\lambda_D \to \infty]{} 0$

The price $P$ is

7. decreasing in $s$

8. decreasing in $\lambda_d$

9. increasing in $\lambda$

10. such that $P \xrightarrow[\lambda_D \to 0]{} P_{df}$

    and $P \xrightarrow[\lambda_D \to \infty]{} R$

The effects of $\lambda_D$ on $L$ and $P_f$ hold even when $\lambda$ is a bounded function of $\lambda_D$. If furthermore $\frac{\lambda_d}{1-q} + \lambda\mu_{lo} > (\frac{1}{\zeta} - 1)r$, $L$ is everywhere decreasing in $\lambda_D$.

*Proof of Proposition A1.*Begin by noting that the derivative of $P_f$ with respect to $s, \lambda$, and $\delta$ is 0. Now see that $\partial P_f / \partial \lambda_D = (Rr - 1)(\lambda_D + r)^{-2}$. $Rr - 1$ is negative while $\lambda_D + r$ is positive, hence the derivative is negative.

Turning to $L$, first note that the asset is divided between low-type and high-type owners so that we have the identity $\mu_{ho} = s - \mu_{lo}$. Second, we have the identity $1 - \mu_{ln} - \mu_{ho} - \mu_{lo} = \mu_{hn}$. Third, in steady state the number of low-type agents is given by $\lambda_d(\lambda_d + \lambda_u)^{-1}$. Combining these three statements with the steady state condition $2\lambda\mu_{hn}\mu_{lo} + \lambda_u\mu_{lo} = \lambda_d\mu_{ho}$ we have the quadratic equation determining $\mu_{lo}$ given by $2\lambda\mu_{lo}^2 + (2\lambda\upsilon + \lambda_d + \lambda_u)\mu_{lo} - \lambda_d s = 0$ for $\upsilon = y - s$ and

$y = \lambda_u(\lambda_d + \lambda_u)^{-1}$ as in DGP. Equivalently, we can instead solve for $\mu_{hn} = \upsilon + \mu_{lo}$ from

$$0 = 2\lambda\mu_{hn}^2 + (\lambda_d + \lambda_u - 2\lambda_\upsilon)\mu_{hn} - (\lambda_d + \lambda_u)\upsilon - \lambda_d s, \tag{A-2}$$

which will turn out to be convenient because $\lim_{\lambda\to\infty} \mu_{hn} = \upsilon \neq 0$ whereas $\lim_{\lambda\to\infty} \mu_{lo} = 0$. We will be using the derivatives of (A-2) with respect to both $s$ and $\lambda$:

$$\frac{\partial \mu_{lo}}{\partial s} = \frac{\lambda_d + 2\lambda\mu_{lo}}{4\lambda\mu_{lo} + 2\lambda\upsilon + \lambda_d + \lambda_u} \tag{A-3}$$

$$\frac{\partial \mu_{hn}}{\partial s} = \frac{\partial \mu_{lo}}{\partial s} - 1 = \frac{-\lambda_u - 2\lambda\mu_{hn}}{4\lambda\mu_{hn} - 2\lambda\upsilon + \lambda_d + \lambda_u} \tag{A-4}$$

$$\frac{\partial \mu_{hn}}{\partial \lambda} = -\frac{2\mu_{hn}(\mu_{hn} - \upsilon)}{4\lambda\mu_{hn} - 2\lambda\upsilon + \lambda_d + \lambda_u} \equiv -\gamma. \tag{A-5}$$

Below we show that $\gamma > 0 \ \forall \ \lambda$. Moreover, let us introduce the notation

$$L \equiv \frac{\delta}{\Gamma_0}\frac{\Gamma_1}{\Gamma_2}, \quad \Gamma_{10} \equiv \frac{\Gamma_1}{\Gamma_0}, \quad \Gamma_{12} \equiv \frac{\Gamma_1}{\Gamma_2}.$$

We find that

$$\frac{\partial L}{\partial s} = \frac{\delta}{\Gamma_0}\left(\frac{\partial \Gamma_1}{\partial s}\Gamma_2^{-1} - \Gamma_{12}\frac{\partial \Gamma_2}{\partial s}\right).$$

Now see that $\partial\Gamma_1/\partial s = 2\lambda\tilde{q}(\partial\mu_{lo}/\partial s)$ and $\partial\Gamma_2/\partial s = 2\lambda\tilde{q}(\partial\mu_{lo}/\partial s) + 2\lambda q(\partial\mu_{hn}/\partial s)$. Plugging these into $\partial L/\partial s$ and rearranging terms yields

$$\frac{2\delta\lambda}{\Gamma_0\Gamma_2(4\lambda\mu_{lo} + 2\lambda\upsilon + \lambda_d + \lambda_u)}\left[(1 - \Gamma_{12})\tilde{q}\lambda_d + \Gamma_{12}q(\lambda_u + 2\lambda\upsilon) + 2\lambda\mu_{lo}\left(\tilde{q}(1 - \Gamma_{12}) + \Gamma_{12}q\right)\right].$$

To sign this expression, note that

$$\Gamma_{12} = \frac{2\lambda\tilde{q}(\mu_{hn} - \upsilon) + r\tilde{q} + \lambda_d + \lambda_D\tilde{q}}{2\lambda\tilde{q}(\mu_{hn} - \upsilon) + 2\lambda\tilde{q}(\upsilon - \tilde{q}\upsilon) + r + \lambda_d + \lambda_D + \lambda_u}.$$

Because $\mu_{hn} = \upsilon + \mu_{lo}$ and $\mu_{lo} \geq 0$ then it follows that $\mu_{hn} - \upsilon \geq 0$. As a sidenote, this shows that $\gamma \geq 0$. Because $q \in [0, 1]$ then $\tilde{q} \in [0, 1]$ and therefore $\upsilon - \tilde{q}\upsilon \geq 0$. Hence $\Gamma_{12} > 0$. To get a contradiction, assume that $\Gamma_{12} \geq 1$. But then

$$0 \geq 2\lambda\tilde{q}(\upsilon - \tilde{q}\upsilon) + qr + q\lambda_D + \lambda_u \quad \Rightarrow\Leftarrow .$$

Hence $\Gamma_{12} \in (0, 1)$. Therefore $1 - \Gamma_{12}$ is positive and the expression for $\partial L/\partial s$ is positive.

The behavior of $L$ for large $\lambda$ can be directly inferred from DGP's (2007) first proposition and by noting that $P_f$ is unchanged by $\lambda$.

Now see that

$$\frac{\partial L}{\partial \lambda_D} = \frac{\delta}{\Gamma_0\Gamma_2}\left[-\frac{\partial \Gamma_0}{\partial \lambda_D}\Gamma_{10} + \frac{\partial \Gamma_1}{\partial \lambda_D} - \frac{\partial \Gamma_2}{\partial \lambda_D}\Gamma_{10}\Gamma_2^{-1}\right]$$

and

$$\begin{aligned}
\frac{\partial \Gamma_0}{\partial \lambda_D} &= R, \\
\frac{\partial \Gamma_1}{\partial \lambda_D} &= \tilde{q} + 2\lambda\tilde{q}\frac{\partial \mu_{lo}}{\partial \lambda_D} + 2\mu_{lo}\tilde{q}\frac{\partial \lambda}{\partial \lambda_D}, \\
\frac{\partial \Gamma_2}{\partial \lambda_D} &= 1 + 2\lambda\tilde{q}\frac{\partial \mu_{lo}}{\partial \lambda_D} + 2\mu_{lo}\tilde{q}\frac{\partial \lambda}{\partial \lambda_D} + 2\lambda q\frac{\partial \mu_{hn}}{\partial \lambda_D} + 2\mu_{hn}q\frac{\partial \lambda}{\partial \lambda_D}.
\end{aligned}$$

Before proceeding, note that (A-2) implies that

$$\begin{aligned}
\mu_{hn} &= \frac{-(\lambda_d+\lambda_u-2\lambda\upsilon)+\sqrt{(\lambda_d+\lambda_u-2\lambda\upsilon)^2+4(2\lambda)([\lambda_d+\lambda_u]\upsilon+\lambda_d s)}}{2(2\lambda)} \\
&= \frac{\lambda^{-1}(-\lambda_d-\lambda_u)+2\upsilon+\sqrt{4\upsilon^2+R_1(\lambda^{-1})+R_2(\lambda^{-2})}}{4} \\
&\xrightarrow[\lambda\to\infty]{} \frac{4\upsilon}{4}=\upsilon.
\end{aligned}$$

where the second line follows from multiplying by $1=\lambda^{-1}/\lambda^{-1}$ where $R_1$ is $O(\lambda^{-1})$ and $R_2$ is $O(\lambda^{-2})$. Limits are therefore finite and the result follows. We can therefore write

$$\text{(A-6)} \quad \frac{\partial L}{\partial \lambda_D} = \frac{\delta}{\Gamma_0\Gamma_2}\left[\tilde{q}-\Gamma_{10}R-\Gamma_{12}+2\left(\mu_{hn}-\upsilon\right)\tilde{q}\eta\left(1-\Gamma_{12}\right)-2\lambda\eta\tilde{q}\gamma\left(1-\Gamma_{12}\right)-2\Gamma_{12}q\eta\left(\mu_{hn}-\lambda\gamma\right)\right]$$

which is difficult to sign in general when $\eta$ is nonzero since the fifth and sixth terms are of the opposite sign as the fourth term regardless of the value of $\eta$. In the main part of Proposition A1 we have $\eta=0$ and therefore the sign is dictated by $\tilde{q}-\Gamma_{10}R-\Gamma_{12}$. We know that $\Gamma_{12}$ is positive. However, note

$$\text{(A-7)} \qquad \Gamma_{10}R = \frac{\tilde{q}\lambda_D+r\tilde{q}+\lambda_d+2\lambda\mu_{lo}\tilde{q}}{\lambda_D+1/R}=\tilde{q}\frac{\lambda_D+r+\lambda_d/\tilde{q}+2\lambda\mu_{lo}}{\lambda_D+1/R}$$

and this is not generally-speaking above or below $\tilde{q}$. Hence our inability to sign $\partial L/\partial\lambda_D$ absent further parametric restrictions. However, it is possible to look directly at $L$ and note that it is of the form $O(\lambda)/O(\lambda^2)=O(\lambda^{-1})$ and therefore $L \xrightarrow[\lambda\to\infty]{} 0$.

When $\lambda(\lambda_D)$ is bounded, then $L$ is still of the form $O(\lambda)/O(\lambda^2)=O(\lambda^{-1})$ which is what we needed above in our calculation of $\lambda_D$'s effects on $L$ and $P_f$. Those results therefore hold even when $\lambda$ is a bounded function of $\lambda_D$.

To see the sufficiency of the condition $\frac{\lambda_d}{1-q} + \lambda\mu_{lo} > (\frac{1}{\zeta} - 1)r$, consider the derivative $\partial L/\partial\lambda_D$ in (A-6) and the expression (A-7). $\Gamma_{10}R$ is greater than $\tilde{q}$ when $(\lambda_D + r + \lambda_d\tilde{q} + 2\lambda\mu_{lo})(\lambda_D + 1/R)^{-1}$ is greater than unity. This implies

$$\begin{aligned} r + \frac{\lambda_d}{\tilde{q}} + 2\lambda\mu_{lo} &> 1/R \\ \frac{\lambda_d}{\tilde{q}} + 2\lambda\mu_{lo} &> (\frac{1}{\zeta} - 1)r \end{aligned} \tag{A-8}$$

Therefore, when (A-8) holds and $\eta = 0$, (A-6) has a sign dictated by

$$-\tilde{q}(\Gamma_{10}R - 1) - \Gamma_{12},$$

that $\Gamma_{10}R - 1$ and $\Gamma_{12}$ are positive, and therefore $\partial L/\partial\lambda_D < 0$.

The results for the price $P$ follow directly from the above results for $P_f$ and $L$. $\square$

Now let us try to better understand the condition $\frac{\lambda_d}{1-q} + \lambda\mu_{lo} > (\frac{1}{\zeta} - 1)r$. For our data, risk-free rates have been quite low and inflation tame. Hence $r$ will be no more than an annual rate of 0.05. Rules of thumb for bonds' recovery value hover around 40% of par, which implies that $\zeta$ is around 0.4. These values would imply the right hand side of the condition is 0.125.

The term $\lambda\mu_{lo}(\lambda)$ is $O(\lambda^{1/2})$ by the following argument. DGP's (2005) proof of their first proposition (characterizing $\mu_{lo}$ as the positive root of a quadratic equation) along with the identity $\mu_{hn} = \upsilon + \mu_{lo}$ gives us

$$\begin{aligned} \mu_{hn} &= \frac{-(\lambda_d + \lambda_u - 2\lambda\upsilon) + \sqrt{(\lambda_d + \lambda_u + 2\lambda\upsilon)^2 + 8\lambda\lambda_d s}}{4\lambda} \\ &= \frac{-\lambda_d - \lambda_u}{4\lambda} + \frac{\upsilon}{2} + \frac{\lambda_d + \lambda_u}{4\lambda} + \frac{\upsilon}{2} + \frac{R_1(\sqrt{\lambda})}{4\lambda}, \end{aligned}$$

where $R_1(\sqrt{\lambda})$ is $O(\sqrt{\lambda})$. Combining this with the terms gives us

$$\mu_{hn} = \upsilon + R_2(\lambda^{-1/2}),$$

where, considering $R_1$, we can see that $\sqrt{\lambda}R_2(\lambda^{-1/2}) \neq 0$. Therefore, $\mu_{hn} = \upsilon + O(\lambda^{-1/2})$ and then $\mu_{lo} = O(\lambda^{-1/2})$. Thus $\lambda\mu_{lo}$ is $O(\lambda^{1/2})$ and nonzero. Hence the condition is satisfied for even extremely low values of $\lambda$ like, e.g. 1, where the expected time between matches is about $250/(2 \times 1) = 125$ days. In every simulation shown in the paper (in all figures) we find $\lambda\mu_{lo}$ to be greater than unity.

## A.II Discussion of Model Simulations

In this section we discuss our modeling and simulation choices.

In the main text we stated that the ECB must pay a price higher than the private market equilibrium. The equilibrium price is typically below $V_{ho}$ while *ho* must be offered at least $V_{ho}$ to be indifferent to selling. But in order to purchase the amounts of bonds we see in the data, the official purchases must be from both *lo* and *ho* bondholders. We could instead (as we did in previous paper drafts of the paper) model the ECB purchases as happening only at the equilibrium price and therefore only from *lo*. This type of purchase has a *stronger* effect on the liquidity premium because it essentially clears out the low-type bondholders each time. But, at our calibrations, the *lo* pool is smaller than the amount of bonds we saw purchased in the data, and so in the model this type of $s$-shock purchase cannot mimic the size of the SMP purchases we saw in practice.

As mentioned in the main text, we use an admittedly simplistic way of modeling ECB purchases. It is useful for two chief reasons. One: Since the ECB is buy-and-hold, if the ECB is

"in the market" for bonds (even stochastically) then the model does not have a non-degenerate steady state. Two: Since SMP purchase sizes are large enough to necessitate buying from low- and high-types, the ECB's higher purchase price must enter into the equilibrium price's determination unless future ECB purchases are unexpected. Since Table 2 did not find large changes in the point estimates when we used only after-first SMP purchases, we do not see the data arguing strongly against our stark model assumption that all SMP purchases are surprises.

An anonymous referee pressed us on this issue, and so we considered the following enriched model: the ECB enters the search market, announces how many bonds will be purchased in total, and searches with its own matching intensity $\lambda_E(t)$ which deterministically decays such that purchases taper as in the data. Technically, this makes ECB purchasing an imhomogeneous Poisson process. Fortunately this process has analytical properties like the Poisson processes already in the paper, just with the weighted average $\int_t^\infty \lambda_E(s)ds$ of future search intensities playing the same role that the fixed $\lambda$ plays for market participants. Simulations of the enriched model gave us the same qualitative features as our simpler model here. There are two main reasons why. First, the search-and-matching frictions and constant deadweight loss for sellers prevent the equilibrium price from instantly dropping to the new steady state *that will be obtained* after the ECB completes its operation. The ECB must pay at least $V_{ho}$ to buy sufficient bonds, and that is always above the equilibrium price (which is certainly rising as $s$ decreases, and which is certainly higher than it would be were ECB purchases not deterministically known), so anyone who matches with the ECB sells. But sales also exist in the private market as *lo* meet *hn* agents, and it is still optimal for the *lo* to sell then (at a higher-than-otherwise equilibrium price) rather than hold out and hope to match with the ECB, paying the deadweight loss in the meantime. Second, the ECB matches with sufficiently many bonds every week to create overshooting in the liquidity premium.

## A.III Estimating Cumulative Default Probabilities from CDS Spreads

In this section we describe our estimation approach for extracting probabilities of default from sovereign CDS spreads. Unlike Pan and Singleton (2008), who simultaneously combine information from both the cross-section and the time-series dimension, we extract our estimates of cumulative default by fitting the cross section of sovereign CDS prices for each day separately. In our fitting procedure we follow Jarrow and Turnbull (1995) and account for the presence of a default event arrival process by means of a Poisson counting process. A default event is characterized as the first event of this counting process that occurs at some time $t^*$ with a probability defined as

$$P[t^* < t + dt | t^* \geq t] = \lambda_{D,t}(dt)dt \tag{A-9}$$

This implies that the probability of the default event, occurring within the time interval $[t,\ t + dt)$ and conditional on no default having occurred till time $t$, is proportional to the hazard rate function $\lambda_{D,t}(dt)$ and the length of that time interval, $dt$. In particular, the (discrete) probability of not experiencing a default from time $t$ to a future time point $t^*$, $P[t^* > t]$, equals $\exp\left(\sum_{\tau=t}^{t^*} \lambda_{D,t}(\tau)\Delta\tau\right)$, while the probability of defaulting between time $t_1^*$ and $t_2^*$ equals $P[t_2^* > t] - P[t_1^* > t]$.

We assume that the hazard rate is a continuous and horizon-*dependent* function.[27] Nothing changes if we instead assume a piece-wise flat step function for the hazard rate for CDS

[27] Although we make the hazard rate horizon-dependent, we do assume that it is independent of interest rates and recovery rates.

spreads, as is common also. While an easier approach would be to assume that the hazard rate is constant over any horizon (i.e., $\lambda_{D,t}(\tau) \equiv \lambda_{D,t}$), we deem such an assumption unnecessarily restrictive, and counterintuitive. One can think of obvious examples why agents would think it is more likely that a country is to default between, say, 2 and 3 years from now compared to between today and a year from now, for example if the country has large bond redemption obligations due in 2 years time and it seems unlikely that the government will be able to meet those at that time, as was the case during the euro-area debt crisis.

Before we go into the details of the estimation, we first address the issue of the recovery rate. Since the default risk premium in CDS depends on both the probability of default and the recovery rate conditional upon default, we can only identify default probabilities up to a scaling constant. Throughout the paper we assume the recovery rate to be equal to 40%. However, the bottom four panels of Figure A1 show the time-series of the 5-year cumulative default probability that we would estimate for each of the countries in our sample under alternative recovery rate assumptions of 20% and 60% (the black and dark gray (blue) lines, respectively). High recovery rates are associated with high default probabilities and vice versa. Choosing a different recovery rate seems to mainly level shift default probabilities and does not impact our regression results in the main text.

**Extracting Default Probabilities** We estimate the term structure of default probabilities from CDS spreads by assuming a flexible functional form for the continuous and time-varying hazard rate function. In particular, we specify the time-varying hazard rate, $\lambda_{D,t}$, using the same Nelson-Siegel parametric form as used by Nelson and Siegel (1987) to estimate zero-coupon yield curves, but we take the exponent to ensure that it is positive everywhere on its domain;

$\lambda_{D,t}(\tau) = \exp\left(\lambda^*_{D,t}(\tau)\right)$ with[28]

$$\lambda^*_{D,t}(\tau) = \delta_1 + \delta_2 \left[ \frac{1 - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right)}{\left(\frac{\tau}{c_{\lambda,t}}\right)} \right] + \delta_3 \left[ \frac{1 - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right)}{\left(\frac{\tau}{c_{\lambda,t}}\right)} - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right) \right]. \tag{A-10}$$

We then minimize the sum of squared pricing errors for CDS spreads for each day separately by optimizing over the parameters of the hazard rate. For each CDS maturity that we include, the pricing error is the difference between the time-$t$ present values of the CDS premium leg and the protection leg. The *premium* leg of a CDS contract is the series of payments of the CDS spread, $\text{SPR}_{\text{CDS}}(\tau)$, made until maturity $\tau$ of the contract or until default occurs. The *protection* leg is the payment of $(1 - R)$ percent of the face value $F$ of the bond upon default, with $R$ the recovery rate. At time $t$ we therefore set

$$P_{\text{PREMIUM_LEG},t} = P_{\text{PROTECTION_LEG},t} \tag{A-11}$$

with

$$\begin{aligned} P_{\text{PREMIUM_LEG},t} &= F \times \text{SPR}_{\text{CDS}}(\tau) \times \sum_{k=t}^{\tau-1} B(t,k) \times P[t^* > k], \\ P_{\text{PROTECTION_LEG},t} &= (1 - R) \times F \times \sum_{k=t}^{\tau-1} B(t,k) \times (P[t^* > k] - P[t^* > k+1]), \end{aligned}$$

[28]Estimating the constant $c_{\lambda,t}$ alongside the parameters adds additional nonlinearity to the hazard rate curve and complicates the estimation procedure. Therefore, similar as in Diebold and Li (2006) for estimating zero-coupon yield curves for the United States, we fix $c_{\lambda,t}$ to a pre-determined constant such that the curvature factor in the hazard rate function reaches it maximum loading at a 18-month horizon. Our results are robust to alternative choices for this horizon.

where $B(t,k)$ is the time-$t$ discount factor for a $k$-period horizon. We use euro-area swap rates to construct $B(t,k)$ as CDS protection on a European sovereign bond position provided by a typical dealer would arguably reduce the credit risk on such a bond position to roughly that of lending in the swap market. Because of the credit risk of the counterparty dealer selling protection, the default risk on the CDS-protected bond position would presumably not be as as low as that of sovereign bonds. We estimate the CDS-implied hazard rate function by minimizing the sum of squared pricing errors in (A-11) across CDS maturities under the assumption of a quarterly CDS payment frequency.

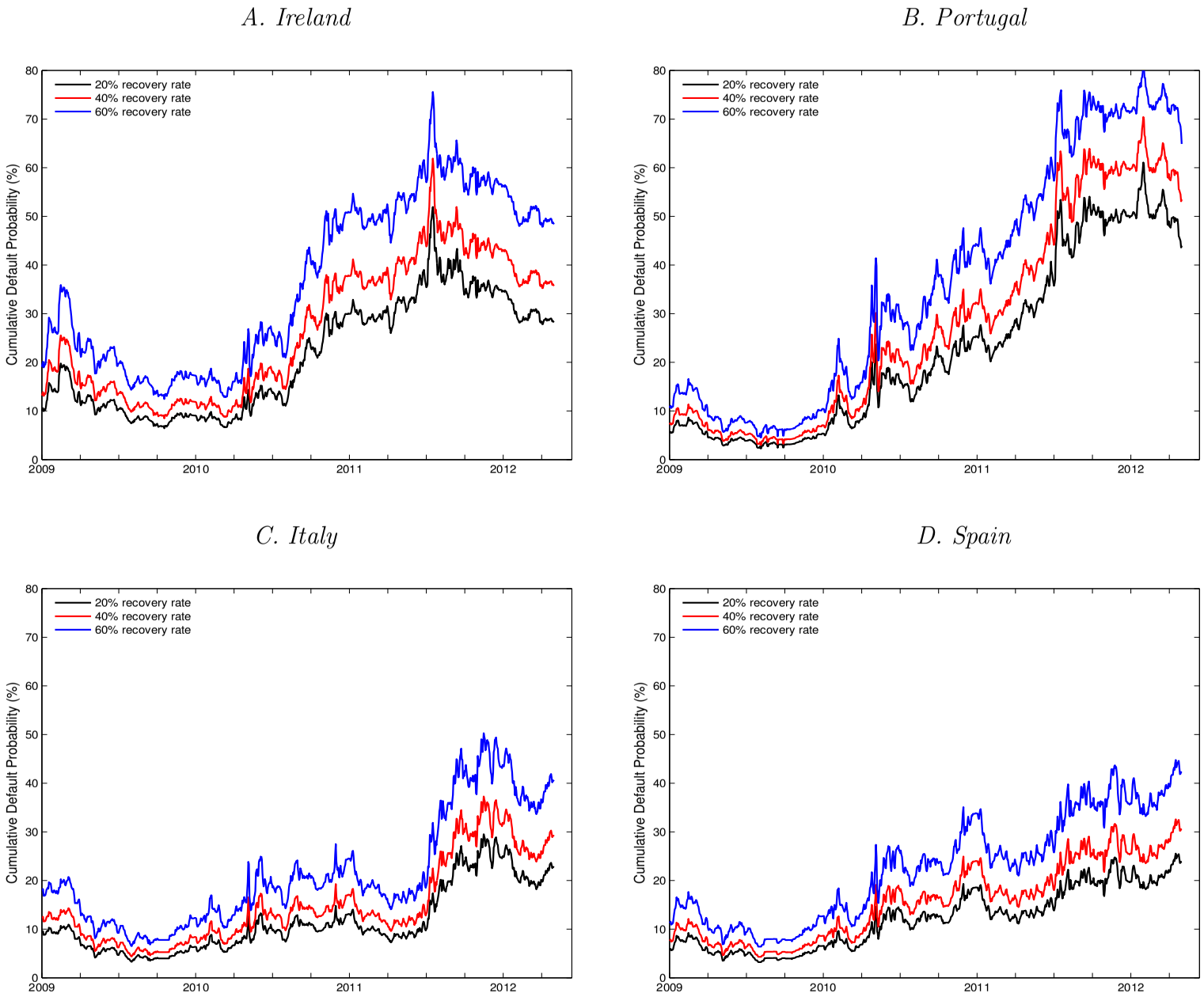


Figure A1: Cumulative Default Probabilities under Various Recovery Rates

Daily time-series estimates (estimated from CDS spreads of the cumulative probability, by country) that the sovereign will default within the 5 years amidst different assumptions on the recovery rate upon default (a 20% recovery rate in black, a 40% recovery rate in light gray (red), and a 60% recovery rate in dark gray (blue)).