Internet Appendix to
"Risk Premium Information from Treasury Bill Yields"

January 10, 2016

1 GMM Estimation

State variable estimation

$$\delta_t = y_t^{(1y)} - \begin{pmatrix} 1 & \tau_t \end{pmatrix} \boldsymbol{\beta}^{(\delta)}, \tag{1}$$

$$rpl_t = \overline{y}_t - \begin{pmatrix} 1 & \tau_t & \delta_t \end{pmatrix} \boldsymbol{\beta}^{(rpl)}, \tag{2}$$

$$rps_t = -y_t^{(3m)} + \begin{pmatrix} 1 & \tau_t & \delta_t & rpl_t \end{pmatrix} \boldsymbol{\beta}^{(rps)}, \tag{3}$$

where $\overline{y}_t$ denotes the average of 2- to 5-year T-bond yields, $\overline{y}_t = \frac{1}{4}\left(y_t^{(2y)} + y_t^{(3y)} + y_t^{(4y)} + y_t^{(5y)}\right)$. The superscripts of βs are determined by the corresponding dependent variables.

Risk premium forecasts

$$exr_{t,t+1}^{(n)} = \begin{pmatrix} 1 & rpl_t & rps_t \end{pmatrix} \boldsymbol{\beta}^{(n)} + \epsilon_{t+1}^{(n)}, \tag{4}$$

for n = 2, 3, 5, 7, 10 and 15 years. The superscripts of β's denote the maturities of the dependent variables. The sample spans from November 1971 since 15-year T-bonds were not available earlier.

GMM moments

$$
g_T(\boldsymbol{\beta}) = E_T \begin{bmatrix} \delta_t \otimes \begin{pmatrix} 1 & \tau_t \end{pmatrix}^\top \\ rpl_t \otimes \begin{pmatrix} 1 & \tau_t & \delta_t \end{pmatrix}^\top \\ rps_t \otimes \begin{pmatrix} 1 & \tau_t & \delta_t & rpl_t \end{pmatrix}^\top \\ \boldsymbol{\epsilon}_{t+1} \otimes \begin{pmatrix} 1 & rpl_t & rps_t \end{pmatrix}^\top \end{bmatrix} \in \mathbb{R}^{27}, \tag{5}
$$

where $\boldsymbol{\beta}$ is a vector of 27 parameters,

$$
\begin{aligned}
\boldsymbol{\beta} \equiv \Big[\, & \beta_0^{(\delta)} \quad \beta_1^{(\delta)} \quad \beta_0^{(rpl)} \quad \beta_1^{(rpl)} \quad \beta_2^{(rpl)} \quad \beta_0^{(rps)} \quad \beta_1^{(rps)} \quad \beta_2^{(rps)} \quad \beta_3^{(rps)} \\
& \beta_0^{(2y)} \quad \beta_1^{(2y)} \quad \beta_2^{(2y)} \quad \cdots \quad \beta_0^{(15y)} \quad \beta_1^{(15y)} \quad \beta_2^{(15y)} \Big]^\top .
\end{aligned} \tag{6}
$$

The GMM moments are exactly identified, thus the GMM criterion is zero at the OLS estimates. The standard errors of $\boldsymbol{\beta}$ are computed by combining the spectral density matrix with the delta method,

$$
\operatorname{cov}\left(\hat{\boldsymbol{\beta}}\right) = \frac{1}{T}\left(d^\top S^{-1} d\right)^{-1}. \tag{7}
$$

The spectral density matrix, S, is estimated as

$$
\hat{S} \equiv \sum_{j=-k}^{k} \left(\frac{k-|j|}{k}\right) E_T\left(\hat{u}_t\, \hat{u}_{t+j}^\top\right), \tag{8}
$$

where $\hat{u}_t$ denotes the GMM moment residuals. The higher-order correlations are down-weighted as suggested by Newey and West (1987). I choose $k = 2$ since measurement errors may create mechanical autocorrelations between adjacent time periods.

The derivative matrix, $d \equiv \frac{\partial g_T(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^\top}$, can be derived by combining the following derivatives. First, the derivatives of state variables are

$$
\begin{aligned}
&\frac{\partial\, \delta_t}{\partial\, \beta_0^{(\delta)}} = -1, && \frac{\partial\, \delta_t}{\partial\, \beta_1^{(\delta)}} = -\tau_t, && \\
&\frac{\partial\, rpl_t}{\partial\, \beta_0^{(\delta)}} = \beta_2^{(rpl)}, && \frac{\partial\, rpl_t}{\partial\, \beta_1^{(\delta)}} = \beta_2^{(rpl)}\, \tau_t, && \\
&\frac{\partial\, rpl_t}{\partial\, \beta_0^{(rpl)}} = -1, && \frac{\partial\, rpl_t}{\partial\, \beta_1^{(rpl)}} = -\tau_t, && \frac{\partial\, rpl_t}{\partial\, \beta_2^{(rpl)}} = -\delta_t,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial\, rps_t}{\partial\, \beta_0^{(\delta)}} = -\beta_2^{(rps)} + \beta_3^{(rps)}\beta_2^{(rpl)}, &&\frac{\partial\, rps_t}{\partial\, \beta_1^{(\delta)}} = \left\{-\beta_2^{(rps)} + \beta_3^{(rps)}\beta_2^{(rpl)}\right\}\tau_t, && \\
&\frac{\partial\, rps_t}{\partial\, \beta_0^{(rpl)}} = -\beta_3^{(rps)}, &&\frac{\partial\, rps_t}{\partial\, \beta_1^{(rpl)}} = -\beta_3^{(rps)}\,\tau_t, &&\frac{\partial\, rps_t}{\partial\, \beta_2^{(rpl)}} = -\beta_3^{(rps)}\,\delta_t, \\
&\frac{\partial\, rps_t}{\partial\, \beta_0^{(rps)}} = 1, &&\frac{\partial\, rps_t}{\partial\, \beta_1^{(rps)}} = \tau_t, &&\frac{\partial\, rps_t}{\partial\, \beta_2^{(rps)}} = \delta_t, \\
&\frac{\partial\, rps_t}{\partial\, \beta_3^{(rps)}} = rpl_t.
\end{aligned}
$$

Second, the derivatives of excess return forecast residuals are

$$
\begin{aligned}
&\frac{\partial\, \epsilon_{t+1}^{(n)}}{\partial\, \beta_i^{(x)}} = -\beta_1^{(n)}\,\frac{\partial\, rpl_t}{\partial\, \beta_i^{(x)}} - \beta_2^{(n)}\,\frac{\partial\, rps_t}{\partial\, \beta_i^{(x)}} \quad \text{for} \quad i = 0,1,2,3 \text{ and } x = \delta, \tau, rpl, rps, \\
&\frac{\partial\, \epsilon_{t+1}^{(n)}}{\partial\, \beta_0^{(n)}} = -1, \qquad \frac{\partial\, \epsilon_{t+1}^{(n)}}{\partial\, \beta_1^{(n)}} = -rpl_t, \qquad \frac{\partial\, \epsilon_{t+1}^{(n)}}{\partial\, \beta_2^{(n)}} = -rps_t.
\end{aligned}
$$

d is estimated by plugging these derivatives into the following column vector,

$$
d = E_T \begin{bmatrix}
\partial\delta_t/\partial\boldsymbol{\beta}^\top \\
\partial\delta_t/\partial\boldsymbol{\beta}^\top \cdot \tau_t \\
\\
\partial rpl_t/\partial\boldsymbol{\beta}^\top \\
\partial rpl_t/\partial\boldsymbol{\beta}^\top \cdot \tau_t \\
\partial rpl_t/\partial\boldsymbol{\beta}^\top \cdot \delta_t + rpl_t \cdot \partial\delta_t/\partial\boldsymbol{\beta}^\top \\
\\
\partial rps_t/\partial\boldsymbol{\beta}^\top \\
\partial rps_t/\partial\boldsymbol{\beta}^\top \cdot \tau_t \\
\partial rps_t/\partial\boldsymbol{\beta}^\top \cdot \delta_t + rps_t \cdot \partial\delta_t/\partial\boldsymbol{\beta}^\top \\
\partial rps_t/\partial\boldsymbol{\beta}^\top \cdot rpl_t + rps_t \cdot \partial rpl_t/\partial\boldsymbol{\beta}^\top \\
\\
\partial\epsilon_{t+1}^{(n)}/\partial\boldsymbol{\beta}^\top \\
\partial\epsilon_{t+1}^{(n)}/\partial\boldsymbol{\beta}^\top \cdot rpl_t + \epsilon_{t+1}^{(n)} \cdot \partial rpl_t/\partial\boldsymbol{\beta}^\top \\
\partial\epsilon_{t+1}^{(n)}/\partial\boldsymbol{\beta}^\top \cdot rps_t + \epsilon_{t+1}^{(n)} \cdot \partial rps_t/\partial\boldsymbol{\beta}^\top
\end{bmatrix}. \tag{9}
$$

References

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.