Internet Appendix for the paper:

CoMargin

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Online Appendix A: Further Properties of CoMargin

We show in this section that CoMargin exhibits several interesting properties. We consider a simple case with two firms that have normally-distributed P&Ls. For simplicity, we consider an unconditional distribution, with respect to past information, and consequently neglect the time index t. Thus, consider $(V_1, V_2)' \sim N(0, \Sigma)$ where:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{A1}$$

In this setting, the CoMargins of both members, denoted ($B^{1|2}$, $B^{2|1}$), are defined by:

$$\Pr\left(V_i \leq B^{i|j} | V_j \leq -B_j\right) = \alpha \tag{A2}$$

for $i = 1, 2$, $j \neq i$, and where $B_j = -\sigma_j \Phi^{-1}(\alpha)$ denotes the unconditional VaR of firm j and $\Phi(\cdot)$ the cdf of the standard normal distribution. The conditional distribution of V_i given $V_j < c$, $\forall c \in \mathbb{R}$ is a skewed distribution (Horrace, 2005) and is denoted by $g(\cdot)$. The CoMargin for the firm i is the solution to:

$$\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i, \sigma_j, \rho) du = \alpha \tag{A3}$$

$$g\left(u; \sigma_i, \sigma_j, \rho\right) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \Phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right) \tag{A4}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution (Arnold et al., 1993). Notice that contrary to the CoVaR (Adrian and Brunnermeier, 2014), CoMargin has no closed form expression. Using the implicit expression of CoMargin in equation A3, we can illustrate some of its properties:

Property 1: The CoMargin of firm i increases with the variability of its P&L:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} > 0 \tag{A5}$$

Intuitively, it means that since riskier trading portfolios (as measured by their variability) tend to have larger potential losses, more collateral must be collected to guarantee their performance. Or in simple words, riskier clearing members should post higher margins.

Proof: Let $H(B^{i|j}, \sigma_i)$ be a function such that:

$$H(B^{i|j}, \sigma_i) = \int_{-\infty}^{-B^{i|j}} g(u, \sigma_i) du - \alpha = 0 \tag{A6}$$

Then, the CoMargin can be defined as an implicit function $B^{i|j} = h(\sigma_i)$. By the Implicit Function Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j}, \sigma_i)}{H_B(B^{i|j}, \sigma_i)} \tag{A7}$$

The derivative $H_B(B^{i|j}, \sigma_i)$ can be expressed as follows:

$$H_B\left(B^{i|j},\sigma_i\right) = -g\left(-B^{i|j};\sigma_i\right) < 0 \tag{A8}$$

and is negative since $g(u;\sigma_i)$ is a pdf. Thus, the sign of $\frac{\partial B^{i|j}}{\partial \sigma_i}$ is given by the sign of $H_{\sigma_i}(B^{i|j},\sigma_i)$:

$$H_{\sigma_i}\left(B^{i|j},\sigma_i\right) = \frac{\partial}{\partial \sigma_i}\left(\int_{-\infty}^{-B^{i|j}} g(u;\sigma_i)\,du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u;\sigma_i)}{\partial \sigma_i}\,du \tag{A9}$$

For simplicity, let us consider the case where $\rho = 0$:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i} = \frac{\partial}{\partial \sigma_i}\left(\frac{1}{\sigma_i}\phi\left(\frac{u}{\sigma_i}\right)\right) = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right) - \frac{u}{\sigma_i^3}\phi'\left(\frac{u}{\sigma_i}\right) \tag{A10}$$

Since $\phi'(x) = -x\,\phi(x)$, we have:

$$\frac{\partial g(u;\sigma_i)}{\partial \sigma_i} = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)\left(1-\left(\frac{u}{\sigma_i}\right)^2\right) \tag{A11}$$

For any value of u such that $u < -\sigma_i$, we have $\partial g(u;\rho)/\partial \sigma_i > 0$. This condition is satisfied when $u \in \left]-\infty, -B^{i|j}\right]$ since $-B^{i|j} = \sigma_i\Phi^{-1}(\alpha) = -\sigma_i\Phi^{-1}(1-\alpha)$ and $\Phi^{-1}(1-\alpha) > 1$ for most of the considered coverage rates (e.g. 1%, 5%). Consequently, the integral defined in equation A3 is also positive and $H_{\sigma_i}\left(B^{i|j},\sigma_i\right) > 0$. Then, we conclude that:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)} > 0 \tag{A12}$$

A similar result can be obtained when we relax the assumption $\rho = 0$.■

Property 2: The CoMargin of firm i does not depend on the variability of the P&L of firm j:

$$\frac{\partial\, B^{i|j}}{\partial \sigma_j} = 0 \tag{A13}$$

This property turns out to be extremely important. Combined with the previous property, it shows that a firm's CoMargin increases with its P&L dependence but it does not increase with the risk taking of other market participants.

Proof: Since $B_j = -\sigma_j\Phi^{-1}(\alpha)$, the pdf $g(.)$ in equation A3 can be rewritten as:

$$g\left(u;\sigma_i,\sigma_j,\rho\right) = \frac{1}{\alpha\sigma_i}\times\phi\left(\frac{u}{\sigma_i}\right)\times\Phi\left[\frac{\Phi^{-1}(\alpha)-\rho u/\sigma_i}{\sqrt{1-\rho^2}}\right] \tag{A14}$$

As $g(.)$ does not depend on σ_j, $\partial B^{i|j}/\partial \sigma_j = 0$. ■

Property 3: The CoMargin of firm i increases with the dependence between its P&L and that of other firms:

$$\frac{\partial B^{i|j}}{\partial \rho} > 0 \tag{A15}$$

The intuition behind this property is that a sound margining system should reflect P&L dependence in order to prevent (or minimize) the occurrence of simultaneous margin-exceeding losses across market

participants.

Proof: Let $F(B^{i|j},\rho)$ *be a function such that:*

$$F(B^{i|j},\rho) = \int_{-\infty}^{-B^{i|j}} g(u;\rho)\,du - \alpha = 0 \tag{A16}$$

The CoMargin can be defined as an implicit function $B^{i|j} = f(\rho)$*. By the Implicit Function Theorem, we have:*

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j},\rho)}{F_B(B^{i|j},\rho)} \tag{A17}$$

where $F_\rho(.)$ *and* $F_B(.)$ *denote respectively the first derivative of the* F *function with respect to* ρ *and* B*. For any function* $H(x)$ *defined as:*

$$H(x) = \int_{-\infty}^{-b(x)} h(t)\,dt \tag{A18}$$

we have $H'(x) = h(b(x))\times \partial b(x)/\partial x$*. Consequently, the derivative* $F_B(B^{i|j},\rho)$ *can be expressed as follows:*

$$F_B\left(B^{i|j},\rho\right) = -g\left(-B^{i|j};\rho\right) < 0 \tag{A19}$$

and is negative since $g(u;\rho)$ *is a pdf. Thus, the sign of* $\partial B^{i|j}/\partial\rho$ *is given by the sign of* $F_\rho(B^{i|j},\rho)$*:*

$$F_\rho\left(B^{i|j},\rho\right) = \frac{\partial}{\partial\rho}\left(\int_{-\infty}^{-B^{i|j}} g(u;\rho)\,du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u;\rho)}{\partial\rho}\,du \tag{A20}$$

Given the expression of the pdf $g(u;\rho)$ *we have:*

$$\begin{aligned}\frac{\partial g(u;\rho)}{\partial\rho} &= \overbrace{-\frac{1}{\alpha\sigma_i}\times\phi\left(\frac{u}{\sigma_i}\right)\times\phi\left(\frac{-B_j/\sigma_j-\rho u/\sigma_i}{\sqrt{1-\rho^2}}\right)}^{A}\\ &\quad\times\left(\frac{-u/\sigma_i\sqrt{1-\rho^2}-(B_j/\sigma_j+\rho u/\sigma_i)\rho(1-\rho^2)^{-1/2}}{1-\rho^2}\right)\\ &= A\times\left(\frac{1}{1-\rho^2}\right)^{3/2}\times\left(\frac{u}{\sigma_i}+\frac{\rho B_j}{\sigma_j}\right)\end{aligned} \tag{A21}$$

This function is positive for any value of u *such that* $u \leq \rho B_i = -\rho\sigma_i\Phi^{-1}(\alpha)$ *with* $-\rho\sigma_i\Phi^{-1}(\alpha) > 0$*. Since* $B^{i|j} \geq 0$ *by definition, this condition is satisfied for the interval* $\left]-\infty, -B^{i|j}\right]$ *and* $F_\rho\left(B^{i|j},\rho\right) > 0$*. Then, we conclude that:*

$$\frac{\partial B^{i|j}}{\partial\rho} = -\frac{F_\rho\left(B^{i|j},\rho\right)}{F_B(B^{i|j},\rho)} > 0 \;\blacksquare \tag{A22}$$

Online Appendix B: CoMargin with *N-1* Conditioning Firms

With $N-1$ conditioning firms, the CoMargin for firm i, $B_t^{i|j\neq i}$, is defined by:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j\neq i}\right) \cap \mathbf{C}\left(V_{j\neq i,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{j\neq i,t+1}\right)\right]} = \alpha \tag{B1}$$

where the conditioning event is that at least one of the $N-1$ other firms is in financial distress:

$$\Pr\left[\mathbf{C}\left(V_{j\neq i,t+1}\right)\right] = \Pr\left[\bigcup_{j=1,j\neq i}^{N} \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \tag{B2}$$

Note that only $N-1$ firms are considered in the conditioning event, since by definition, firm i is excluded. Then, the estimation method of the CoMargin is similar to the one used with two firms in Section 4.2. Given the simulated sample of P&Ls $\left\{v_{1i,t+1}^s, .., v_{N,t+1}^s\right\}_{s=1}^S$, a simple estimate of the joint probability given in equation B2, denoted $\hat{P}_t^{j\neq i}$, is given by:

$$\hat{P}_t^{j\neq i} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(\bigcup_{j=1,j\neq i}^{N} \left(v_{j,t+1}^s \leq -B_{j,t}\right)\right) \tag{B3}$$

where $v_{j,t+1}^s$correspond to the sth simulated P&L of firms j. In a similar way, conditionally on $B_t^{i|j\neq i}$, a simple estimator of the numerator in equation B1, denoted $\hat{P}_t$, is defined as:

$$\hat{P}_t = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j\neq i}\right) \times \mathbf{I}\left(\bigcup_{j=1,j\neq i}^{N} \left(v_{j,t+1}^s \leq -B_{j,t}\right)\right) \tag{B4}$$

For each time t and for each firm i, we look for the value $B_t^{i|j\neq i}$ that minimizes the distance between the ratio $\hat{P}_t/\hat{P}_t^{j\neq i}$ and α:

$$\hat{B}_t^{i|j\neq i} = \arg\min_{\left\{B_t^{i|j\neq i}\right\}} \left(\frac{\hat{P}_t}{\hat{P}_t^{j\neq i}} - \alpha\right)^2 \tag{B5}$$

Thus, for each firm i, we end up with a time series of CoMargins $\left\{\hat{B}_t^{i|j\neq i}\right\}_{t=1}^T$ for which confidence bounds can be bootstrapped.

Table A1: Scenarios used in the SPAN system

Scenario	Underlying Asset Price Change	Volatility Change	Time to Expiration
1	0	+ volatility range	$-1/252$
2	0	- volatility range	$-1/252$
3	$+1/3$ x price range	+ volatility range	$-1/252$
4	$+1/3$ x price range	- volatility range	$-1/252$
5	$-1/3$ x price range	+ volatility range	$-1/252$
6	$-1/3$ x price range	- volatility range	$-1/252$
7	$+2/3$ x price range	+ volatility range	$-1/252$
8	$+2/3$ x price range	- volatility range	$-1/252$
9	$-2/3$ x price range	+ volatility range	$-1/252$
10	$-2/3$ x price range	- volatility range	$-1/252$
11	$+3/3$ x price range	+ volatility range	$-1/252$
12	$+3/3$ x price range	- volatility range	$-1/252$
13	$-3/3$ x price range	+ volatility range	$-1/252$
14	$-3/3$ x price range	- volatility range	$-1/252$
15	Positive extreme change	0	$-1/252$
16	Negative extreme change	0	$-1/252$

Note: The table shows the sixteen scenarios used to determine the contract family charge in the SPAN system. Price and volatility ranges usually cover 99% of the data points over a rolling historical estimation window. Positive and negative extreme changes are designed to assess the effect of deep out of the money options.

Table A2: Description of the data used in the empirical analysis

Item	Number	Comments
Clearing members	48	There is entry and exit in the sample, so the number of clearing members varies over time.
Trading Days	2,066	The sample period is from January 2, 2003 to March 31, 2011.
Underlying Assets	3	The three underlying assets are: 1. Yield on the three-month Canadian bankers' acceptance. 2. Yield on the ten-year Government of Canada Bond Futures 3. Level of the S&P/TSX 60 Index
Three-Month Canadian Bankers' Acceptance Futures Contracts (BAX)	45	Delivery dates range from January 2003 to December 2013.
Ten-Year Government of Canada Bond Futures Contracts (CGB)	34	Delivery dates range from March 2003 to June 2011.
S&P/TSX 60 Index Standard Futures Contracts (SXF)	34	Delivery dates range from March 2003 to June 2011.
Total futures contracts	113	These represent all the futures contracts (i.e., all delivery dates) written on the three underlying assets during the sample period.
Active firm accounts	21	We report results only for this type of account.
Active client accounts	23	
Active omnibus accounts	16	

Note: The table presents an overview of the dataset used in the empirical analysis, which was obtained from the Canadian Derivatives Clearing Corporation. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset.

Table A3: Specifications of the contracts included in the empirical analysis

	S&P/TSX 60 Index Standard Futures (SXF)	Three-Month Canadian Bankers' Acceptance Futures (BAX)	Ten-Year Government of Canada Bond Futures (CGB)
Underlying Interest	The S&P/TSX 60 Index C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Expiration Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index: 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Settlement	Cash settlement	Cash settlement	Physical delivery of eligible Government of Canada Bonds.
Trading Hours (EST)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Table A4: Clearing members included in the empirical analysis

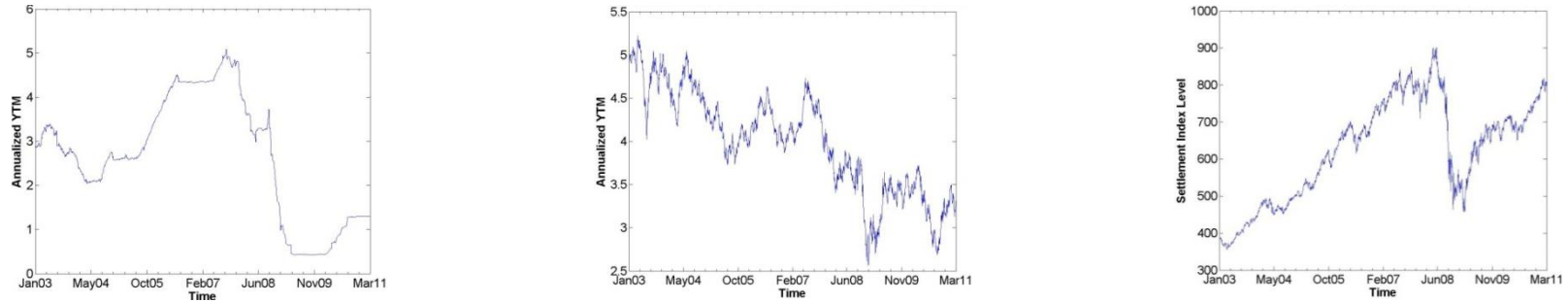
Number	Name	Number	Name
1	Newedge Canada Inc.	25	Morgan Stanley Canada LTD.
2	RBC Dominion Securities Inc.	26	CFG Financial Group Inc.
3	Union Securities LTD.	27	MF Global Canada Co.
4	T.D. Securities Inc.	28	Haywood Securities Inc.
5	BMO Nesbitt Burns LTD.	29	Goldman Sachs Canada Inc.
6	Macquarie Private Wealth Inc.	30	Timber Hill Canada Co.
7	UBS Securities Canada Inc.	31	Credit Suisse Securities
8	Desjardins Securities Inc.	32	CIBC World Markets Inc.
9	Macquarie Capital Markets Inc.	33	NBCN Clearing Services Inc.
10	Name not reported	34	HSBC Securities (Canada) Inc.
11	Merrill Lynch Canada Inc.	35	Mackie Research Capital Corporation
12	Odlum Brown LTD.	36	Benson-Quinn GMS Inc.
13	Penson Financial Services Inc.	37	Scotia Capital Inc.
14	Dundee securities corporation	38	E*trade Canada Securities Corporation
15	Daex Commodities Inc.	39	Raymond Kames LTD.
16	Canaccord Capital Corporation	40	Lévesque Beaubien Geoffrion Inc.
17	Friedberg Mercantile Group LTD.	41	TD Waterhouse Canada Inc.
18	W.D. Latimer Co. LTD.	42	Citigroup Global Markets Canada Inc.
19	Canadian Imperial Bank of Commerce (CIBC)	43	National Bank of Canada
20	Jones, Gable & Co. LTD.	44	J.P. Morgan Securities Canada Inc.
21	Name not reported	45	Merrill Lynch Canada Inc.
22	Timber Hill Canada Company	46	Name not reported
23	Laurentian Bank Securities Inc.	47	Fidelity Clearing Canada ULC
24	Deutsche Bank Securities LTD.	48	Maple Securities Canada LTD.

Note: The table provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period (January 2, 2003 to March 31, 2011). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Notice that this list includes more clearing members than those currently affiliated with the Canadian Derivatives Clearing Corporation (CDCC) because some of them entered and exited the market during the sample period.

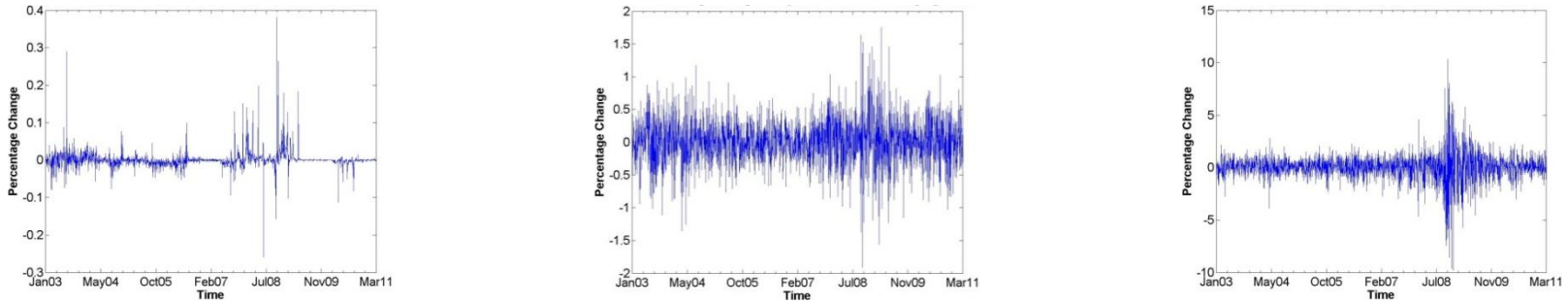
Figure A1: Underlying assets and futures contracts used in the empirical analysis

Three-month Canadian Bankers' Acceptance (BAX) | **Ten-year Government of Canada Bond (CGB)** | **S&P/TSX 60 Index Standard (SXF)**

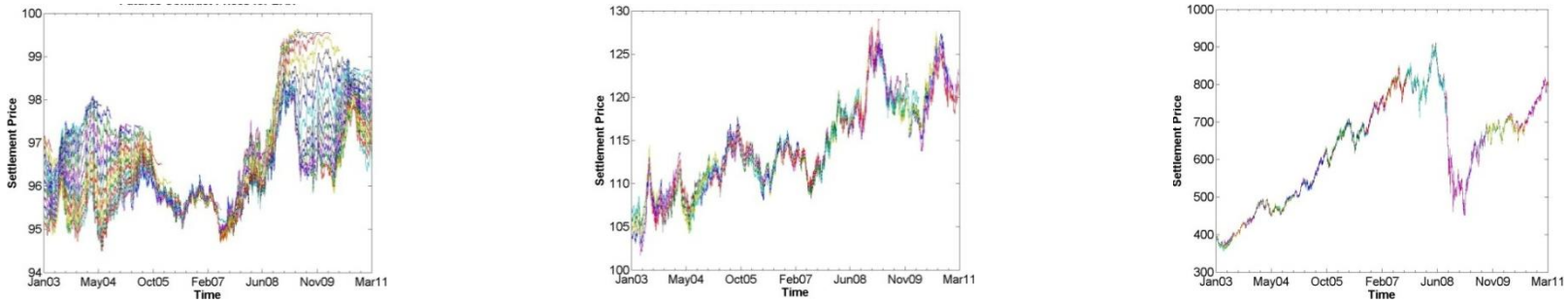
Panel A: Underlying Asset Prices



Panel B: Underlying Asset Returns

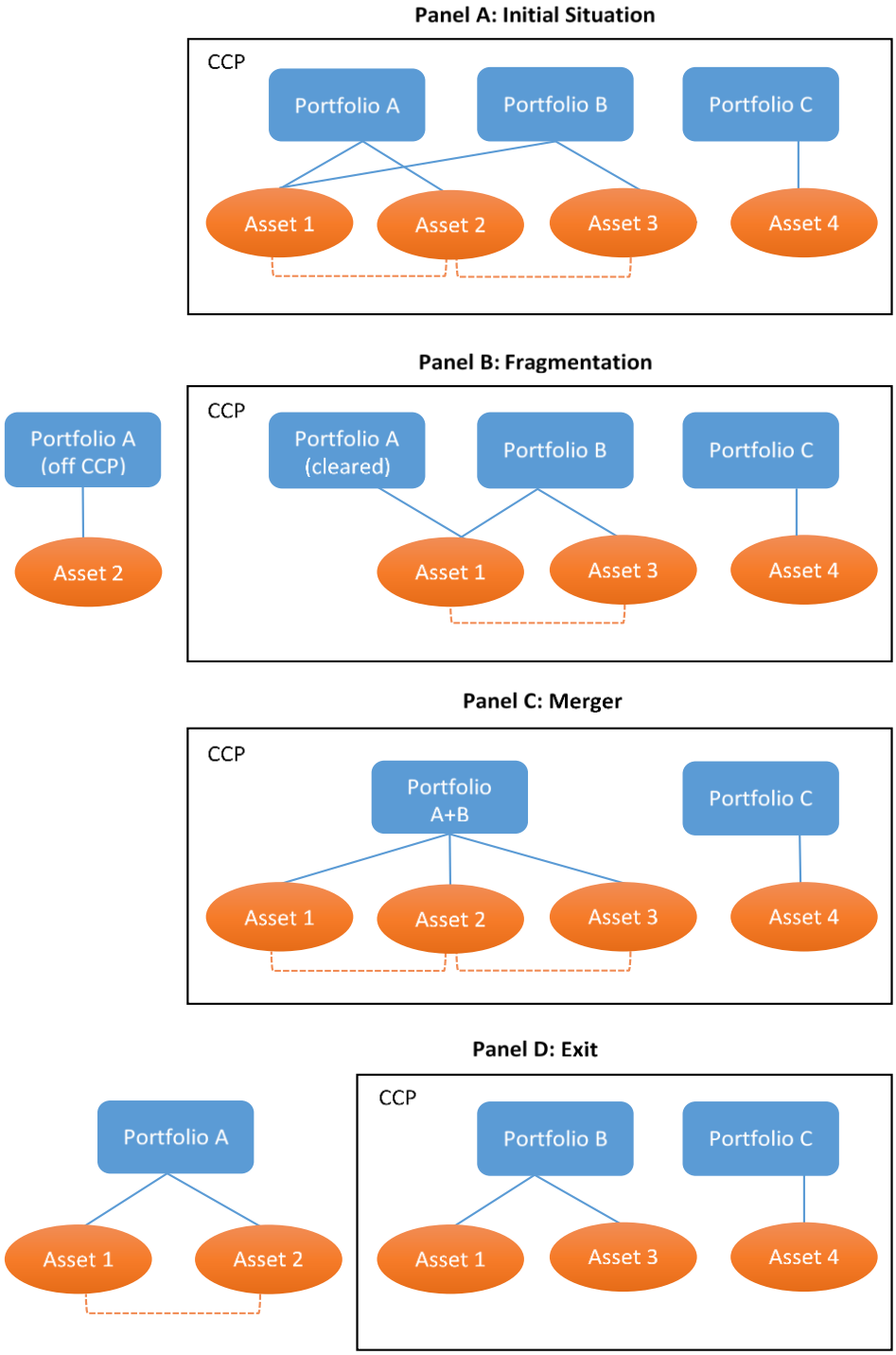


Panel C: Futures Prices (All Maturities)



Note: Panel A presents the daily annualized settlement yield for the Three-month Canadian Bankers' Acceptance, the annualized yield on the Ten-year Government of Canada Bond and the settlement level of the S&P/TSX 60 Index. Panel B shows the daily returns (i.e., percentage changes) of the variables presented in Panel A. Panel C presents the daily settlement futures prices for the futures contracts (BAX, CGB, and SXF) written on the variables presented in Panel A and traded in the Montreal Exchange. Lines in different colours represent different delivery dates. The sample period is from January 2, 2003 to March 31, 2011. Source: Bloomberg.

Figure A2: Potential Clearing Member Reactions to CoMargin



Note: This figure displays the various designs considered in Section 6. A clearing member may engage in some form of margin arbitrage by (1) fragmenting its portfolio and clearing some positions on another CCP, (2) merging its trading portfolio with the one of another member who exhibits correlated trading positions, or (3) moving all its positions off the CCP. We consider a subset of three clearing members who can invest in four assets. Assets 1 and 2 are assumed to be negatively correlated and assets 2 and 3 positively correlated. The lines between portfolios and assets denote positions whereas dotted underbraces symbolize correlation among assets.