Risk, Uncertainty, and Expected Returns—Internet Appendix

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A Variance Risk Premium and Empirical Measurement

The central empirical variable of this paper, as a proxy for economic uncertainty, is the market variance risk premium (VRP)—which is not directly observable but can be estimated from the difference between model-free option-implied variance and the conditional expectation of realized variance.

A.1 Variance Risk Premium: Definition and Measurement

In order to define the model-free implied variance, let $C_t(T, K)$ denote the price of a European call option maturing at time T with strike price K, and $B(t, T)$ denote the price of a time t zero-coupon bond maturing at time T. As shown by Carr and Madan (1998) and Britten-Jones and Neuberger (2000), among others, the market's risk-neutral Q expectation of the return variance σ^2_{t+1} conditional on the information set Ω_t, or the implied variance IV_t at time-t, can be expressed in a "model-free" fashion as a portfolio of European calls,

$$IV_t \;\equiv\; \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] \;=\; 2\int_0^\infty \frac{C_t\left(t+1, \frac{K}{B(t,t+1)}\right) - C_t\left(t, K\right)}{K^2}\, dK, \tag{A1}$$

which relies on an ever increasing number of calls with strikes spanning from zero to infinity.[1] This equation follows directly from the classical result in Breeden and Litzenberger (1978), that the second derivative of the option call price with respect to strike equals the risk-neutral density, such that all risk neutral moments payoff can be replicated by the basic option prices (Bakshi and Madan, 2000).

In order to define the actual return variance, let p_t denote the logarithmic price of the asset. The realized variance over the discrete t to $t+1$ time interval can be measured in a "model-free" fashion by

$$RV_{t+1} \equiv \sum_{j=1}^{n}\left[p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}}\right]^2 \longrightarrow \sigma^2_{t+1}, \tag{A2}$$

where the convergence relies on $n \to \infty$; i.e., an increasing number of within period price observations. As demonstrated in the literature (see, e.g., Andersen, Bollerslev, Diebold,

[1] Such a characterization is accurate up to the second order when there are jumps in the underlying asset (Jiang and Tian, 2005; Carr and Wu, 2009), though Martin (2011) has refined the above formulation to make it robust to jumps.

and Ebens, 2001; Barndorff-Nielsen and Shephard, 2002), this "model-free" realized variance measure based on high-frequency intraday data offers a much more accurate ex-post observation of the true (unobserved) return variance than the traditional ones based on daily or coarser frequency returns.

Variance risk premium (VRP) at time t is defined as the difference between the ex-ante risk-neutral expectation and the objective or statistical expectation at time t of the return variance at time $t+1$,

$$VRP_t \equiv \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] - \mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right], \tag{A3}$$

which is not directly observable in practice.[2] To construct an empirical proxy for such a VRP concept, one needs to estimate various reduced-form counterparts of the risk neutral and physical expectations. In practice, the risk-neutral expectation $\mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right]$ is typically replaced by the CBOE implied variance ($\mathrm{VIX}^2/12$) and the true variance σ^2_{t+1} is replaced by realized variance RV_{t+1}.

To estimate the objective expectation, $\mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right]$, we use a linear forecast of future realized variance as $RV_{t+1} = \alpha + \beta IV_t + \gamma RV_t + \epsilon_{t+1}$, with current implied and realized variances. The model-free implied variance from options market is an informationally more efficient forecast for future realized variance than the past realized variance (see, e.g., Jiang and Tian, 2005, among others), while realized variance based on high-frequency data also provides additional power in forecasting future realized variance (Andersen, Bollerslev, Diebold, and Labys, 2003). Therefore, a joint forecast model with one lag of implied variance and one lag of realized variance seems to capture the most forecasting power based on time-t available information (Drechsler and Yaron, 2011).

B DCC Model of Engle (2002)

We estimate the conditional covariances of each equity portfolio with the market portfolio and VRP ($\sigma_{im,t+1}$, $\sigma_{i,VRP,t+1}$) based on the mean-reverting DCC model of Engle (2002).

[2]The difference between option implied and GARCH type filtered volatilities has been associated in existing literature with notions of aggregate market risk aversion (Rosenberg and Engle, 2002; Bakshi and Madan, 2006; Bollerslev, Gibson, and Zhou, 2011).

Engle defines the conditional correlation between two random variables r_1 and r_2 that each has zero mean as

$$\rho_{12,t} = \frac{E_{t-1}\left(r_{1,t} \cdot r_{2,t}\right)}{\sqrt{E_{t-1}\left(r_{1,t}^2\right) \cdot E_{t-1}\left(r_{1,t}^2\right)}}, \tag{A4}$$

where the returns are defined as the conditional standard deviation times the standardized disturbance:

$$\sigma_{i,t}^2 = E_{t-1}\left(r_{i,t}^2\right), \quad r_{i,t} = \sigma_{i,t} \cdot u_{i,t}, \quad i = 1, 2 \tag{A5}$$

where $u_{i,t}$ is a standardized disturbance that has zero mean and variance one for each series. Equations (A4) and (A5) indicate that the conditional correlation is also the conditional covariance between the standardized disturbances:

$$\rho_{12,t} = \frac{E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right)}{\sqrt{E_{t-1}\left(u_{1,t}^2\right) \cdot E_{t-1}\left(u_{1,t}^2\right)}} = E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right). \tag{A6}$$

The conditional covariance matrix of returns is defined as

$$H_t = D_t \cdot \rho_t \cdot D_t, \text{ where } D_t = \text{diag}\left\{\sqrt{\sigma_{i,t}^2}\right\}, \tag{A7}$$

where ρ_t is the time-varying conditional correlation matrix

$$E_{t-1}\left(u_t \cdot u_t'\right) = D_t^{-1} \cdot H_t \cdot D_t^{-1} = \rho_t, \text{ where } u_t = D_t^{-1} \cdot r_t. \tag{A8}$$

Engle (2002) introduces a mean-reverting DCC model:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}}, \tag{A9}$$

$$q_{ij,t} = \bar{\rho}_{ij} + a_1 \cdot \left(u_{i,t-1} \cdot u_{j,t-1} - \bar{\rho}_{ij}\right) + a_2 \cdot \left(q_{ij,t-1} - \bar{\rho}_{ij}\right) \tag{A10}$$

where $\bar{\rho}_{ij}$ is the unconditional correlation between $u_{i,t}$ and $u_{j,t}$. Equation (A10) indicates that the conditional correlation is mean reverting towards $\bar{\rho}_{ij}$ as long as $a_1 + a_2 < 1$.

Engle (2002) assumes that each asset follows a univariate GARCH process and writes the log likelihood function as:

$$\begin{aligned} L &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|H_t| + r_t' H_t^{-1} r_t\right) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t - u_t' u_t + \log|\rho_t| + u_t' \rho_t^{-1} u_t\right). \end{aligned} \tag{A11}$$

As shown in Engle (2002), letting the parameters in D_t be denoted by θ and the additional parameters in ρ_t be denoted by φ, equation (A11) can be written as the sum of a volatility part and a correlation part:

$$L(\theta, \varphi) = L_V(\theta) + L_C(\theta, \varphi). \tag{A12}$$

The volatility term is

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|D_t|^2 + r_t' D_t^{-2} r_t\right), \tag{A13}$$

and the correlation component is

$$L_C(\theta, \varphi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|\rho_t| + u_t'\rho_t^{-1}u_t - u_t'u_t\right). \tag{A14}$$

The volatility part of the likelihood is the sum of individual GARCH likelihoods:

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log(2\pi) + \log\left(\sigma_{i,t}^2\right) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}\right), \tag{A15}$$

which is jointly maximized by separately maximizing each term. The second part of the likelihood is used to estimate the correlation parameters. The two-step approach to maximizing the likelihood is to find

$$\hat{\theta} = \arg\max\{L_V(\theta)\}, \tag{A16}$$

and then take this value as given in the second stage:

$$\hat{\varphi} = \arg\max\{L_C(\hat{\theta}, \varphi)\}. \tag{A17}$$

C System of Regression Equations

Consider a system of n equations, of which the typical ith equation is

$$y_i = X_i\beta_i + u_i, \tag{A18}$$

where y_i is a $N \times 1$ vector of time-series observations on the ith dependent variable, X_i is a $N \times k_i$ matrix of observations of k_i independent variables, β_i is a $k_i \times 1$ vector of unknown coefficients to be estimated, and u_i is a $N \times 1$ vector of random disturbance terms with mean zero. Parks (1967) proposes an estimation procedure that allows the error term to be both serially and cross-sectionally correlated. In particular, he assumes that the elements of the disturbance vector u follow an AR(1) process:

$$u_{it} = \rho u_{it-1} + \varepsilon_{it};\ \ \rho_i < 1, \tag{A19}$$

where ε_{it} is serially independently but contemporaneously correlated:

$$Cov\left(\varepsilon_{it}\varepsilon_{jt}\right) = \sigma_{ij},\ \text{ for any } i, j,\ \text{ and } Cov\left(\varepsilon_{it}\varepsilon_{js}\right) = 0,\ \text{ for } s \neq t \tag{A20}$$

Equation (A18) can then be written as

$$y_i = X_i\beta_i + P_i u_i, \tag{A21}$$

with

$$P_i = \begin{bmatrix} \left(1-\rho_i^2\right)^{-1/2} & 0 & 0 & ... & 0 \\ \rho_i\left(1-\rho_i^2\right)^{-1/2} & 1 & 0 & ... & 0 \\ \rho_i^2\left(1-\rho_i^2\right)^{-1/2} & \rho & 1 & ... & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ \rho_i^{N-1}\left(1-\rho_i^2\right)^{-1/2} & \rho^{N-2} & \rho^{N-3} & ... & 1 \end{bmatrix}. \tag{A22}$$

Under this setup, Parks (1967) presents a consistent and asymptotically efficient three-step estimation technique for the regression coefficients. The first step uses single equation regressions to estimate the parameters of autoregressive model. The second step uses single equation regressions on transformed equations to estimate the contemporaneous covariances. Finally, the Aitken estimator is formed using the estimated covariance,

$$\hat{\beta} = \left(X^T\Omega^{-1}X\right)^{-1} X^T\Omega^{-1}y, \tag{A23}$$

where $\Omega \equiv E[uu^T]$ denotes the general covariance matrix of the innovation. In our application, we use the aforementioned methodology with the slope coefficients restricted to be the same for all equity portfolios and individual stocks. In particular, we use the same three-step procedure and the same covariance assumptions as in equations (A19) to (A22) to estimate the covariances and to generate the t-statistics for the parameter estimates.

D Robustness Check

In this section, we provide a battery of robustness checks.

D.1 Results from the Generalized Conditional Covariance Model

There appears to be some controversy in the econometrics literature around the consistency of QMLE parameter estimates generated by the DCC models.[3] One may wonder if the lack of consistency in the DCC models affects our main findings. To address this potential concern, we use an alternative econometric methodology and estimate the conditional covariances between excess returns on asset i and the market portfolio m based on the generalized conditional covariance (GCC) specification of Bali (2008):

$$\begin{aligned} &R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\ &R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \\ &E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \\ &E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 \\ &E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \beta_0^{im} + \beta_1^{im} \varepsilon_{i,t}\varepsilon_{m,t} + \beta_2^{im} \sigma_{im,t} \end{aligned} \tag{A24}$$

where $R_{i,t+1}$ and $R_{m,t+1}$ denote the time $(t+1)$ excess return on asset i and the market portfolio m over a risk-free rate, respectively, and $E_t[\cdot]$ denotes the expectation operator conditional on time t information. In the last equation above, one-month-ahead conditional covariance, $\sigma_{im,t+1}$, is defined as a function of the last month's conditional covariance, $\sigma_{im,t}$, and the product of the last month's unexpected shocks to asset i and the market portfolio m $(\varepsilon_{i,t}\varepsilon_{m,t})$.

We estimate the conditional covariances between the excess return on each equity portfolio i and the innovation in the variance risk premia VRP, $\sigma_{i,VRP}$, using an analogous GCC model:

$$\begin{aligned} &R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\ &VRP_{t+1} = \alpha_0^{VRP} + \alpha_1^{VRP} VRP_t + \varepsilon_{VRP,t+1} \\ &E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \\ &E_t\left[\varepsilon_{VRP,t+1}^2\right] \equiv \sigma_{VRP,t+1}^2 = \beta_0^{VRP} + \beta_1^{VRP} \varepsilon_{VRP,t}^2 + \beta_2^{VRP} \sigma_{VRP,t}^2 \\ &E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \beta_0^{i,VRP} + \beta_1^{i,VRP} \varepsilon_{i,t}\varepsilon_{VRP,t} + \beta_2^{i,VRP} \sigma_{i,VRP,t} \end{aligned} \tag{A25}$$

[3] See Aielli (2013), Caporin and McAleer (2013), and the proposed solution in Noureldin, Shephard, and Sheppard (2014).

We estimate the conditional covariances of each equity portfolio with the market portfolio and with the variance risk premia using the maximum likelihood method described in Bali (2008). Once we generate the conditional covariances, we estimate the system of equations given in equations (23)-(24) of the main text using the SUR methodology described in Section C of the internet appendix.

Table II of the internet appendix reports the parameter estimates and the t-statistics of the system of equations for the 10 size, 10 book-to-market, 10 momentum, and 10 industry portfolios (total of 40 portfolios) for the sample period January 1990 to December 2012. As shown in the first two rows of Table II, the risk aversion and the uncertainty aversion coefficients are estimated to be positive and highly significant for the pooled dataset: $A = 2.86$ with a t-statistic of 4.78 and $B = 0.0026$ with a t-statistic of 4.50, indicating a significantly positive market price of risk and uncertainty. Similar to our earlier findings from the DCC model, the Wald_1 and Wald_2 statistics reported in Table II indicate that the two-factor model with risk and uncertainty provides both statistical and economic success in explaining stock market anomalies, except momentum.

D.2 DCC with Asymmetric GARCH

Because the conditional variance and covariance of stock market returns are not observable, different approaches and specifications used in estimating the conditional variance and covariance could lead to different conclusions. We have so far used the bivariate GARCH(1,1) model of Bollerslev (1986) in equations (13)-(14) and (19)-(20) to obtain conditional variance and covariance estimates. In this section, we investigate whether changing these specifications influences our main findings.

The current volatility in the GARCH(1,1) model is defined as a symmetric, linear function of the last period's unexpected news and the last period's volatility. Since, in a symmetric GARCH process, positive and negative information shocks of the same magnitude produce the same amount of volatility, the symmetric GARCH model cannot cope with the skewness of stock return distribution. If a negative return shock causes more volatility than a positive return shock of the same size, the symmetric GARCH model underpredicts the amount of volatility following negative shocks and overpredicts the amount of volatility following positive shocks. Furthermore, if large return shocks cause more volatility than a quadratic function allows, then the symmetric GARCH model underpredicts volatility after a large return shock and overpredicts volatility after a small return shock.

In this section we use an asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) that explicitly takes account of skewed distributions and allows good news and bad news to have different impacts on the conditional volatility forecasts. To test whether such variations in the variance forecasting specification alter our conclusion, we re-estimate the DCC-based conditional covariances using the following alternative specification:

$$
\begin{aligned}
&R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\
&R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \\
&VRP_{t+1} = \alpha_0^{VRP} + \alpha_1^{VRP} VRP_t + \varepsilon_{VRP,t+1} \\
&E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 + \beta_3^i \varepsilon_{i,t}^2 D_{i,t}^- \\
&E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 + \beta_3^m \varepsilon_{m,t}^2 D_{m,t}^- \\
&E_t\left[\varepsilon_{VRP,t+1}^2\right] \equiv \sigma_{VRP,t+1}^2 = \beta_0^{VRP} + \beta_1^{VRP} \varepsilon_{VRP,t}^2 + \beta_2^{VRP} \sigma_{VRP,t}^2 + \beta_3^{VRP} \varepsilon_{VRP,t}^2 D_{VRP,t}^- \\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \rho_{i,VRP,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{VRP,t+1} \\
&E_t\left[\varepsilon_{m,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{m,VRP,t+1} = \rho_{m,VRP,t+1} \cdot \sigma_{m,t+1} \cdot \sigma_{VRP,t+1}
\end{aligned}
\tag{A26}
$$

where $D_{i,t}^-$, $D_{m,t}^-$, and $D_{VRP,t}^-$ are indicator functions that equals one when $\varepsilon_{i,t+1}$, $\varepsilon_{m,t+1}$, and $\varepsilon_{VRP,t+1}$ are negative and zero otherwise. The indicator function generates an asymmetric GARCH effect between positive and negative shocks. $\rho_{im,t+1}$, $\rho_{i,VRP,t+1}$, and $\rho_{m,VRP,t+1}$ are the time-t expected conditional correlations estimated using the mean-reverting DCC model of Engle (2002).

A notable point in Table III is that the main findings from an asymmetric GARCH specification of the conditional covariances are very similar to those reported in Table 1. Specifically, the risk aversion coefficients are estimated to be positive and highly significant for all equity portfolios; A is in the range of 2.53 to 3.54 with the t-statistics ranging from 2.58 to 3.11, implying a significantly positive link between expected return and risk. Similar to our results from GARCH(1,1) specification, asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) yields positive and significant coefficient estimates on the covariance between equity portfolios and the variance risk premia. Specifically, the uncertainty aversion coefficients (B) are in the range of 0.0054 to 0.0075 with the t-statistics between 2.68 and 3.30. These results show that equity portfolios that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or lowly correlated with VRP.

With this alternative covariance specification, we also examine the empirical validity of the conditional asset pricing model by testing the joint hypothesis. As shown in Table III, the Wald_1 statistics for the size, book-to-market, and industry portfolios are, respectively, 16.91, 7.89, and 14.41 with the corresponding p-values of 0.11, 0.72, and 0.21. The significantly positive risk and uncertainty aversion coefficients and the insignificant Wald_1 statistics indicate that the two-factor model explains the time-series and cross-sectional variation in equity portfolios. Finally, we investigate whether the model with asymmetric GARCH specification explains the return spreads between Small and Big; Value and Growth; and HiTec and Telcm portfolios. The last row in Table III reports Wald_2 statistics from testing the equality of conditional alphas for high-return and low-return portfolios ($H_0 : \alpha_1 = \alpha_{10}$). For the size, book-to-market, and industry portfolios, the Wald_2 statistics provide no evidence for a significant conditional alpha for "Small-Big", "Value-Growth", and "HiTec-Telcm" arbitrage portfolios. Overall, the DCC-based conditional covariances from the asymmetric GARCH model captures the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios and generates significantly positive risk-return and uncertainty-return tradeoffs.

D.3 Results from Larger Cross-Section of Industry Portfolios

Given the positive risk-return and positive uncertainty-return coefficient estimates from the three data sets and the success of the conditional asset pricing model in explaining the industry, size, and value premia, we now examine how the model performs when we use a larger cross-section of equity portfolios.

The robustness of our findings is investigated using the monthly excess returns on the value-weighted 17-, 30-, 38-, 48-, and 49-industry portfolios. Table IV reports the common slope estimates (A, B), their t-statistics in parentheses, and the Wald_1 and Wald_2 statistics along with their p-values in square brackets. For the industry portfolios, the risk aversion coefficients (A) are estimated to be positive, in the range of 2.20 to 2.78, and highly significant with the t-statistics ranging from 2.31 to 3.34. Consistent with our earlier findings from the 10 size, 10 book-to-market, and 10 industry portfolios, the results from the larger cross-section of industry portfolios (17 to 49) imply a positive and significant relation between expected return and market risk. Again similar to our findings from 10 decile portfolios, the uncertainty aversion coefficients are estimated to be positive, in the range of 0.0036 to 0.0041, and highly significant with the t-statistics ranging from 2.44 to 4.21. These results provide evidence for a significantly positive market price of uncertainty and show that assets with higher correlation with the variance risk premia generate higher returns next month.

Not surprisingly, the Wald_1 statistics for all industry portfolios have p-values in the range of 0.20 to 0.75, indicating that the two-factor asset pricing model explains the time-series and cross-sectional variation in larger number of equity portfolios. The last row shows that the Wald_2 statistics from testing the equality of conditional alphas on the high-return and low-return industry portfolios have p-values ranging from 0.44 to 0.80, implying that there is no significant risk-adjusted return difference between the extreme portfolios of 17, 30, 38, 48, and 49 industries. The differences in conditional alphas are both economically and statistically insignificant, showing that the two-factor model introduced in the paper provides success in explaining industry effects.

D.4 Controlling for Macroeconomic Variables

A series of papers argue that the stock market can be predicted by financial and/or macroeconomic variables associated with business cycle fluctuations. The commonly chosen variables include default spread (DEF), term spread (TERM), dividend price ratio (DIV), and the de-trended riskless rate or the relative T-bill rate (RREL).[4] We define DEF as the difference between the yields on BAA- and AAA-rated corporate bonds, and TERM as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. RREL is defined as the difference between 3-month T-bill rate and its 12-month backward moving

[4]See, e.g., Campbell (1987), Fama and French (1989), and Ferson and Harvey (1991) who test the predictive power of these variables for expected stock returns.

average.[5] We obtain the aggregate dividend yield using the CRSP value-weighted index return with and without dividends based on the formula given in Fama and French (1988). In addition to these financial variables, we use some fundamental variables affecting the state of the U.S. economy: Monthly inflation rate based on the U.S. Consumer Price Index (INF); Monthly growth rate of the U.S. industrial production (IP) obtained from the G.17 database of the Federal Reserve Board; and Monthly US unemployment rate (UNEMP) obtained from the Bureau of Labor Statistics.

According to Merton's (1973) ICAPM, state variables that are correlated with changes in consumption and investment opportunities are priced in capital markets in the sense that an asset's covariance with those state variables affects its expected returns. Merton (1973) also indicates that securities affected by such state variables (or systematic risk factors) should earn risk premia in a risk-averse economy. Macroeconomic variables used in the literature are excellent candidates for these systematic risk factors because innovations in macroeconomic variables can generate global impact on firm's fundamentals, such as their cash flows, risk-adjusted discount factors, and/or investment opportunities. Following the existing literature, we use the aforementioned financial and macroeconomic variables as proxies for state variables capturing shifts in the investment opportunity set.

We now investigate whether incorporating these variables into the predictive regressions affects the significance of the market prices of risk and uncertainty. Specifically, we estimate the portfolio-specific intercepts and the common slope coefficients from the following panel regression:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1} \end{aligned}$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF), growth rate of industrial production (IP), and unemployment rate (UNEMP). The common

[5] The monthly data on 10-year T-bond yields, 3-month T-bill rates, BAA- and AAA-rated corporate bond yields are available from the Federal Reserve statistics release website.

slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios.

As presented in Table V, after controlling for a wide variety of financial and macroeconomic variables, our main findings remain intact for all equity portfolios. The common slope estimates on the conditional covariances of equity portfolios with the market factor (A) remain positive and highly significant, indicating a positive and significant relation between expected return and market risk. Similar to our earlier findings, the common slopes on the conditional covariances of equity portfolios with the uncertainty factor (B) remain significantly positive as well, showing that assets with higher correlation with the variance risk premium generate higher returns next month. Among the control variables, the growth rate of industrial production is the only variable predicting future returns on equity portfolios; λ_{IP} turns out to be positive and significant—especially for the industry portfolios. The positive relation between expected stock returns and innovations in output makes economic sense. Increases in real economic activity (proxied by the growth rate of industrial production) increase investors' expectations of future growth. Overall, the results in Table V indicate that after controlling for variables associated with business conditions, the time-varying exposures of equity portfolios to the market and uncertainty factors carry positive risk premiums.[6]

D.5 Results from Individual Stocks

We have so far investigated the significance of risk, uncertainty, and return tradeoffs using equity portfolios. In this section, we replicate our analyses using individual stocks trading at NYSE, AMEX, and NASDAQ. First, we generate a dataset for the largest 500 common stocks (share code = 10 or 11) traded at NYSE/AMEX/NASDAQ. Following Shumway (1997), we adjust for stock de-listing to avoid survivorship bias.[7] Firms with missing observations on

[6] We also used "expected business conditions" variable of Campbell and Diebold (2009) and our main findings remain intact for all equity portfolios. To save space, we do not report these results in the paper. They are available upon request.

[7] Specifically, the last return on an individual stock used is either the last return available on CRSP, or the de-listing return, if available. Otherwise, a de-listing return of -100% is included in the study, except that the deletion reason is coded as 500 (reason unavailable), 520 (went to OTC), 551-573, 580 (various reason), 574 (bankruptcy), and 584 (does not meet exchange financial guidelines). For these observations, a return

beginning-of-month market cap or monthly returns over the period January 1990 – December 2010 are eliminated. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, we obtain more than 500 firms over the period 1990-2010. Specifically, the largest 500 firms are determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. There are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms are determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample (as of December 2010). We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010.

Table VI presents the common slope estimates (A, B) and their t-statistics for the individual stocks in the aforementioned data sets. The risk aversion coefficient is estimated to be positive and highly significant for all stock samples considered in the paper: $A = 6.42$ with the t-statistic of 8.04 for the first dataset containing 738 stocks (largest 500 stocks as of the end of each month from January 1990 to December 2010); $A = 6.80$ with the t-statistic of 8.70 for the second dataset containing largest 500 stocks as of the end of December 2010; and $A = 6.02$ with the t-statistic of 6.79 for the last dataset containing 318 stocks with non-missing monthly return observations for the period 1990-2010. Confirming our findings from equity portfolios, the results from individual stocks imply a positive and significant relation between expected return and market risk. Similarly, consistent with our earlier findings from equity portfolios, the uncertainty aversion coefficient is also estimated to be positive and highly significant for all data sets: $B = 0.0043$ with the t-statistic of 3.61 for the first dataset, $B = 0.0044$ with the t-statistic of 3.67 for the second dataset, and $B = 0.0046$ with the t-statistic of 3.52 for the last dataset. These results indicate a significantly positive market price of uncertainty for large stocks trading in the U.S. stock market.

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of -30% is assigned.

D.6 Controlling for Market Illiquidity and Default Risk

Elevated variance risk premia during economic recessions and market downturns often correspond to the periods in which market illiquidity and default risk are both higher. Thus, it is natural to think that the conditional covariances of equity portfolios with market illiquidity and credit risk factors are positively linked to expected returns. In this section, we test whether the covariances with VRP_{t+1}^{shock} could be picking up covariances with illiquidity and default risk.

Following Amihud (2002), we measure market illiquidity in a month as the average daily ratio of the absolute market return to the dollar trading volume within the month:

$$ILLIQ_t = \frac{1}{n}\sum_{d=1}^{n}\frac{|R_{m,d}|}{VOLD_m}$$

where $R_{m,d}$ and $VOLD_{m,d}$ are, respectively, the daily return and daily dollar trading volume for the S&P 500 index on day d, and n is the number of trading days in month t.

First, we generate the DCC-based conditional covariances of portfolio returns with market illiquidity and then estimate the common slope coefficients (A, B_1, B_2) from the following panel regressions:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{m,t+1} \end{aligned}$$

where $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right)$ are the time-t expected conditional covariance between the change in market illiquidity and the excess return on portfolio i and market portfolio m, respectively.

Table VII, Panel A, presents the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the 10 size, book-to-market, and industry portfolios. The slope on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is found to be positive but statistically insignificant for all equity portfolios considered in the paper. A notable point in Table VII is that the slopes on $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ remain positive and highly significant after controlling for the covariances of equity portfolios with market illiquidity.

Next, we test whether the variance risk premium is proxying for default or credit risk. We use the TED spread as an indicator of credit risk and the perceived health of the banking system. The TED spread is the difference between the interest rates on interbank loans and short-term U.S. government debt (T-bills). TED is an acronym formed from T-Bill and ED, the ticker symbol for the Eurodollar futures contract.[8] The size of the spread is usually denominated in basis points (bps). For example, if the T-bill rate is 5.10% and ED trades at 5.50%, the TED spread is 40 bps. The TED spread fluctuates over time but generally has remained within the range of 10 and 50 bps (0.1% and 0.5%) except in times of financial crisis. A rising TED spread often presages a downturn in the U.S. stock market, as it indicates that liquidity is being withdrawn. The TED spread is an indicator of perceived credit risk in the general economy. This is because T-bills are considered risk-free while LIBOR reflects the credit risk of lending to commercial banks. When the TED spread increases, that is a sign that lenders believe the risk of default on interbank loans (also known as counterparty risk) is increasing. Interbank lenders therefore demand a higher rate of interest, or accept lower returns on safe investments such as T-bills. When the risk of bank defaults is considered to be decreasing, the TED spread decreases.

We first estimate the DCC-based conditional covariances of portfolio returns with the TED spread and then estimate the common slope coefficients from the following SUR re-

[8]Initially, the TED spread was the difference between the interest rates for three-month U.S. Treasuries contracts and the three-month Eurodollars contract as represented by the London Interbank Offered Rate (LIBOR). However, since the Chicago Mercantile Exchange dropped T-bill futures, the TED spread is now calculated as the difference between the three-month T-bill interest rate and three-month LIBOR.

gressions:

$$\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}$$

where $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ are the time-t expected conditional covariance between the changes in TED spread and the excess returns on portfolio i and market portfolio m, respectively.

Table VII, Panel A, shows the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the size, book-to-market, and industry portfolios. The slope on $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is found to be positive for the size and book-to-market portfolios, and negative for the industry portfolios. Aside from yielding an inconsistent predictive relation with future returns, the slopes on the conditional covariances with the change in TED spread are statistically insignificant for all equity portfolios. Similar to our earlier findings, the slopes on the conditional covariances with the market risk and uncertainty factors remain positive and highly significant after controlling for the covariances with default risk.

Finally, we investigate the significance of risk and uncertainty coefficients after controlling for liquidity and credit spread simultaneously:

$$\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}$$

As shown in Panel A of Table VII, for the extended specification above, the common slope coefficient, B_2 on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is estimated to be positive and marginally significant for the book-to-market and industry portfolios, whereas B_2 is insignificant for the size portfolios. The covariances of equity portfolios with the change in TED spread do not predict future returns as B_3 is insignificant for all equity portfolios. Controlling for the market illiquidity and credit risk does not affect our main findings: the market risk-return and uncertainty-return coefficients (A and B_1) are both positive and highly significant for all equity portfolios. Equity portfolios that are highly correlated with VRP_{t+1}^{shock} carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP_{t+1}^{shock}.

We have so far provided evidence from the individual equity portfolios (10 size, 10 book-to-market, and 10 industry portfolios). We now investigate whether our main findings remain intact if we use a joint estimation with all test assets simultaneously (total of 30 portfolios). Panel B of Table VII reports the parameter estimates and the t-statistics that are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the error terms. As shown in the first row of Panel B, the risk aversion coefficient is estimated to be positive and highly significant for the pooled dataset: $A = 2.31$ with the t-statistic of 2.64, implying a positive and significant relation between expected return and market risk. Similar to our earlier findings, the uncertainty aversion coefficient is also estimated to be positive and highly significant for the joint estimation: $B = 0.0053$ with the t-statistic of 3.72. These results indicate a significantly positive market price of uncertainty when all portfolios are combined together. Equity portfolios with higher sensitivity to increases in VRP are expected to generate higher returns next period.

The last three rows in Panel B of Table VII provide evidence for a positive and marginally significant relation between $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and future returns, indicating that the conditional covariances of equity portfolios with the market illiquidity are positively linked to expected returns. However, the insignificant relation between $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ and portfolio returns remains intact for the joint estimation as well. A notable point in Panel B is that controlling for the market illiquidity and default risk individually and simultaneously does not influence the significant predictive power of the conditional covariances of portfolio returns with the market risk and VRP factors.

D.7 Relative Performance of the Conditional Asset Pricing Model with Risk and Uncertainty

We now assess the relative performance of the newly proposed model in predicting the cross-section of expected returns on equity portfolios. Specifically, we test whether the conditional asset pricing model with the market and uncertainty factors outperforms the conditional CAPM with the market factor in terms of statistical fit. The goodness of fit of an asset pricing model describes how well it fits a set of realized return observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the values expected under the model in question. Hence, we focus on the cross-section of realized average returns on equity portfolios (as a benchmark) and the portfolios' expected returns implied by the two competing models.

Using equation (23), we compute the expected excess return on equity portfolios based on the estimated prices of risk and uncertainty (A, B) and the sample averages of the conditional covariance measures, $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$:

$$E_t\left[R_{i,t+1}\right] = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right). \tag{A27}$$

Table VIII of the online appendix presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the Conditional CAPM and the two-factor conditional asset pricing models. Clearly the newly proposed model with risk and uncertainty provides much more accurate estimates of expected returns on the size, book-to-market, and industry portfolios. Especially for the size and industry portfolios, expected returns implied by the two-factor model with the market and VRP factors are almost identical to the realized average returns. The last row in Table VIII reports the Mean Absolute Percentage Errors (MAPE) for the two competing models:

$$MAPE = \frac{|\text{Realized} - \text{Expected}|}{\text{Expected}}, \tag{A28}$$

where "Realized" is the realized monthly average excess return on each equity portfolio and "Expected" is the expected excess return implied by equation (A27). For the conditional

CAPM with the market factor, MAPE equals 5.20% for the size portfolios, 5.37% for the book-to-market portfolios, and 6.32% for the industry portfolios. Accounting for the variance risk premium improves the cross-sectional fitting significantly: MAPE reduces to 0.61% for the size portfolios, 1.66% for the book-to-market portfolios, and 0.55% for the industry portfolios.

Figure 1 of the internet appendix provides a visual depiction of the realized and expected returns for the size, book-to-market, and industry portfolios. It is clear that the two-factor model with uncertainty nails down the realized returns of the size, book-to-market, and industrial portfolios, while the conditional CAPM systematically over-predicts these portfolio returns. Overall, the results indicate superior performance of the conditional asset pricing model introduced in the paper.

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Table I Monthly Raw Returns and CAPM Alphas for the Long-Short Equity Portfolios

This table presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios. The results are reported for the size, book-to-market (BM), and industry portfolios for the sample periods July 1926 – December 2012, July 1963 – December 2012, and January 1990 – December 2012. The OLS t-statistics are reported in parentheses. The Newey-West t-statistics are given in square brackets.

	July 1926 – December 2012				July 1963 – December 2012				January 1990 – December 2012		
Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha
10 Size	Small-Big	0.0056	0.0025	10 Size	Small-Big	0.0033	0.0025	10 Size	Small-Big	0.0031	0.0025
		(2.39)	(1.11)			(1.67)	(1.25)			(1.02)	(0.84)
		[2.46]	[1.28]			[1.33]	[1.00]			[0.99]	[0.80]
10 BM	Value-Growth	0.0053	0.0025	10 BM	Value-Growth	0.0051	0.0050	10 BM	Value-Growth	0.0023	0.0021
		(2.62)	(1.32)			(2.63)	(2.61)			(0.77)	(0.69)
		[2.67]	[1.28]			[2.29]	[2.17]			[0.69]	[0.54]
10 MOM	Winner-Loser	0.0120	0.0153	10 MOM	Winner-Loser	0.0136	0.0147	10 MOM	Winner-Loser	0.0105	0.0133
		(4.83)	(6.53)			(4.71)	(5.15)			(2.05)	(2.67)
		[5.01]	[7.80]			[4.26]	[5.28]			[1.91]	[2.82]
10 Industry	Durbl-Telcm	0.0024	-0.0013	10 Industry	Enrgy-Utils	0.0025	0.0013	10 Industry	HiTech-Telcm	0.0045	0.0020
		(1.27)	(-0.81)			(1.35)	(0.71)			(1.35)	(0.63)
		[1.26]	[-0.78]			[1.41]	[0.71]			[1.49]	[0.60]

Table II Results from the Generalized Conditional Covariance

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1}$$

where the conditional variance of the market and the conditional covariances are estimated with the generalized conditional covariance (GCC) specification of Bali (2008). The parameters in the panel regression and their t-statistics are estimated using monthly excess returns on the market portfolio and the pooled datasets of ten decile size, book-to-market, momentum, and industry portfolios (total of 40 equity portfolios) for the sample period from January 1990 to December 2012. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. Table show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

	A	2.8562
		(4.78)
	B	0.0026
		(4.50)
Size	Wald_1	9.22
		[51.11%]
Small vs. Big	Wald_2	0.88
		[34.85%]
Book-to-Market	Wald_1	4.46
		[92.43%]
Value vs. Growth	Wald_2	0.78
		[37.60%]
Momentum	Wald_1	19.67
		[3.25%]
Winner vs. Loser	Wald_2	5.35
		[2.07%]
Industry	Wald_1	11.39
		[32.80%]
HiTec vs. Telcm	Wald_2	0.33
		[56.38%]

Table III Results from Asymmetric GARCH Model

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{m,t+1}$$

where the conditional variance and covariances are estimated using the asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993). The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

Size	α_i, α_m	*BM*	α_i, α_m	*Industry*	α_i, α_m
Small	0.0052	Growth	0.0035	NoDur	0.0051
	(1.23)		(0.87)		(1.94)
2	0.0037	2	0.0047	Durbl	0.0028
	(0.85)		(1.35)		(0.57)
3	0.0040	3	0.0052	Manuf	0.0055
	(0.99)		(1.55)		(1.61)
4	0.0030	4	0.0064	Enrgy	0.0064
	(0.75)		(1.85)		(1.85)
5	0.0038	5	0.0056	HiTec	0.0029
	(0.97)		(1.71)		(0.52)
6	0.0037	6	0.0050	Telcm	0.0004
	(1.05)		(1.48)		(0.11)
7	0.0041	7	0.0057	Shops	0.0036
	(1.19)		(1.76)		(1.04)
8	0.0034	8	0.0058	Hlth	0.0043
	(0.97)		(1.74)		(1.37)
9	0.0036	9	0.0066	Utils	0.0042
	(1.11)		(1.92)		(1.58)
Big	0.0012	Value	0.0081	Other	0.0030
	(0.38)		(1.88)		(0.81)
Market	0.0018	Market	0.0033	Market	0.0028
	(0.57)		(1.20)		(0.82)
A	3.2927	A	2.5303	A	3.5369
	(3.11)		(2.62)		(2.58)
B	0.0054	B	0.0060	B	0.0075
	(3.12)		(2.68)		(3.30)
Wald_1	16.91	Wald_1	7.89	Wald_1	14.41
	[0.11]		[0.72]		[0.21]
Wald_2	1.48	Wald_2	1.99	Wald_2	0.46
	[0.22]		[0.16]		[0.50]

Table IV Results from Larger Cross-Section of Industry Portfolios

This table presents the common slope estimates (A, B) from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP^{shock}_{t+1}), $Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and VRP^{shock}_{t+1}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 17, 30, 38, 48, and 49 industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

17-industry portfolios		30-industry portfolios		38-industry portfolios		48-industry portfolios		49-industry portfolios	
A	2.6399	A	2.1975	A	2.2988	A	2.3271	A	2.7840
	(2.31)		(2.52)		(2.47)		(2.97)		(3.34)
B	0.0041	B	0.0036	B	0.0035	B	0.0041	B	0.0041
	(2.44)		(2.98)		(2.45)		(3.47)		(4.21)
Wald_1	16.41	Wald_1	35.11	Wald_1	30.89	Wald_1	57.20	Wald_1	52.04
	[0.56]		[0.28]		[0.75]		[0.20]		[0.39]
Wald_2	0.58	Wald_2	0.06	Wald_2	0.32	Wald_2	0.53	Wald_2	0.13
	[0.44]		[0.80]		[0.57]		[0.47]		[0.72]

Table V Controlling for Macroeconomic Variables

This table presents the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1}$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF), growth rate of industrial production (IP), and unemployment rate (UNEMP). The common slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios for the sample period January 1990 to December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last two rows the Wald$_1$ statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald$_2$ statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald$_1$ and Wald$_2$ statistics are given in square brackets.

	Size	Book-to-Market	Industry
A	4.2630	2.5763	4.0421
	(3.32)	(2.40)	(2.74)
B	0.0057	0.0051	0.0066
	(2.85)	(2.25)	(2.96)
λ_{DEF}	-0.3804	-0.0739	0.6243
	(-0.50)	(-0.09)	(1.02)
λ_{TERM}	-0.1964	-0.5366	-0.5405
	(-0.64)	(-1.69)	(-2.17)
λ_{RREL}	0.2330	0.1834	0.0104
	(0.68)	(0.52)	(0.04)
λ_{DIV}	0.0489	0.0228	0.0314
	(1.33)	(0.60)	(1.05)
λ_{INF}	0.0270	0.7158	-0.1862
	(0.04)	(0.93)	(-0.31)
λ_{IP}	0.7433	0.8689	1.1941
	(1.77)	(2.01)	(3.51)
λ_{UNEMP}	0.0031	0.0047	0.0026
	(1.13)	(1.61)	(1.15)
Wald$_1$	16.96	7.97	14.78
	[0.11]	[0.72]	[0.19]
Wald$_2$	1.46	1.63	0.67
	[0.23]	[0.20]	[0.41]

Table VI Results from Individual Stocks

This table presents the common slope estimates (A, B) from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia VRP_{t+1}^{shock}, $Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the VRP_{t+1}^{shock}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the largest 500 stocks trading at NYSE, AMEX, and NASDAQ, and 318 stocks in the S&P 500 index for the sample period from January 1990 to December 2010. First, the largest 500 firms is determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, there are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms is determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample. We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios.

Largest 500 Stocks		*Largest 500 Stocks*		*Largest 500 Stocks*	
end-of-month		*as of December 2010*		*S&P 500 Index*	
A	6.4237	A	6.8014	A	6.0243
	(8.04)		(8.70)		(6.79)
B	0.0043	B	0.0044	B	0.0046
	(3.61)		(3.67)		(3.52)

Table VII Controlling for Market Illiquidity and Default Risk

This table presents the common slope estimates (A, B_1, B_2, B_3) from the following panel regression:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP_{t+1}^{shock}), $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in market illiquidity ($\Delta ILLIQ_{t+1}$), $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in TED spread (ΔTED_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. In Panel A, the parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 10 decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. In Panel B, the results are generated using a joint estimation with all test assets simultaneously (total of 30 portfolios). The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the portfolios.

Panel A. Results from 10 Equity Portfolios

10 Equity Portfolios	A	B_1	B_2	B_3
Size	6.2227	0.0069	1.2423	
	(2.47)	(3.07)	(1.29)	
Size	3.6465	0.0052		0.6372
	(2.84)	(2.09)		(0.91)
Size	5.7826	0.0057	0.4347	1.1582
	(2.48)	(2.12)	(0.69)	(1.17)
Book-to-Market	5.3065	0.0062	2.2003	
	(2.66)	(2.65)	(1.34)	
Book-to-Market	2.5695	0.0056		0.3148
	(2.24)	(2.37)		(0.54)
Book-to-Market	6.4767	0.0079	2.8237	0.3247
	(2.13)	(2.90)	(1.69)	(0.61)
Industry	7.8266	0.0080	2.5677	
	(2.35)	(3.16)	(1.52)	
Industry	3.1868	0.0071		-0.7625
	(2.17)	(2.88)		(-1.11)
Industry	9.2805	0.0102	3.5064	-1.0014
	(2.69)	(3.49)	(1.99)	(-1.43)

Table VII (continued)

Panel B. Results from 30 Equity Portfolios

A	B_1	B_2	B_3
2.3110	0.0053		
(2.64)	(3.72)		
3.2552	0.0060	0.6796	
(2.82)	(4.03)	(1.94)	
2.1153	0.0055		-0.0477
(2.41)	(3.49)		(-0.11)
3.0967	0.0062	0.6497	-0.0844
(2.72)	(3.78)	(1.95)	(-0.20)

Table VIII Relative Performance of the Two-Factor Model with VRP

This table presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the conditional CAPM with the market factor and the two-factor conditional asset pricing model with the market and VRP factors. The last row reports the Mean Absolute Percentage Errors (MAPE) for the two competing models.

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Size	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Small	0.8464%	0.8461%	0.8742%
2	0.7737%	0.7677%	0.8110%
3	0.7690%	0.7647%	0.8093%
4	0.6632%	0.6637%	0.7032%
5	0.7525%	0.7550%	0.7943%
6	0.7055%	0.7025%	0.7406%
7	0.7409%	0.7379%	0.7749%
8	0.6837%	0.6810%	0.7221%
9	0.6670%	0.6643%	0.7000%
Big	0.4479%	0.4598%	0.4789%
MAPE		**0.61%**	**5.20%**

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Book-to-Market	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Growth	0.5286%	0.5327%	0.5645%
2	0.5614%	0.5658%	0.5961%
3	0.6140%	0.6039%	0.6488%
4	0.6752%	0.6559%	0.6960%
5	0.6119%	0.6017%	0.6423%
6	0.5439%	0.5547%	0.5803%
7	0.6014%	0.5979%	0.6360%
8	0.5885%	0.5956%	0.6233%
9	0.6827%	0.6666%	0.7133%
Value	0.8221%	0.7994%	0.8564%
MAPE		**1.66%**	**5.37%**

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Industry	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Telcm	0.2727%	0.2747%	0.3280%
Utils	0.4712%	0.4727%	0.4965%
Other	0.4965%	0.4910%	0.5366%
Durbl	0.5313%	0.5315%	0.5513%
Shops	0.5954%	0.5912%	0.6247%
Hlth	0.6138%	0.6088%	0.6478%
NoDur	0.6110%	0.6152%	0.6534%
Manuf	0.7172%	0.7206%	0.7474%
Enrgy	0.7606%	0.7643%	0.7824%
HiTec	0.8358%	0.8350%	0.8466%
MAPE		**0.55%**	**6.32%**

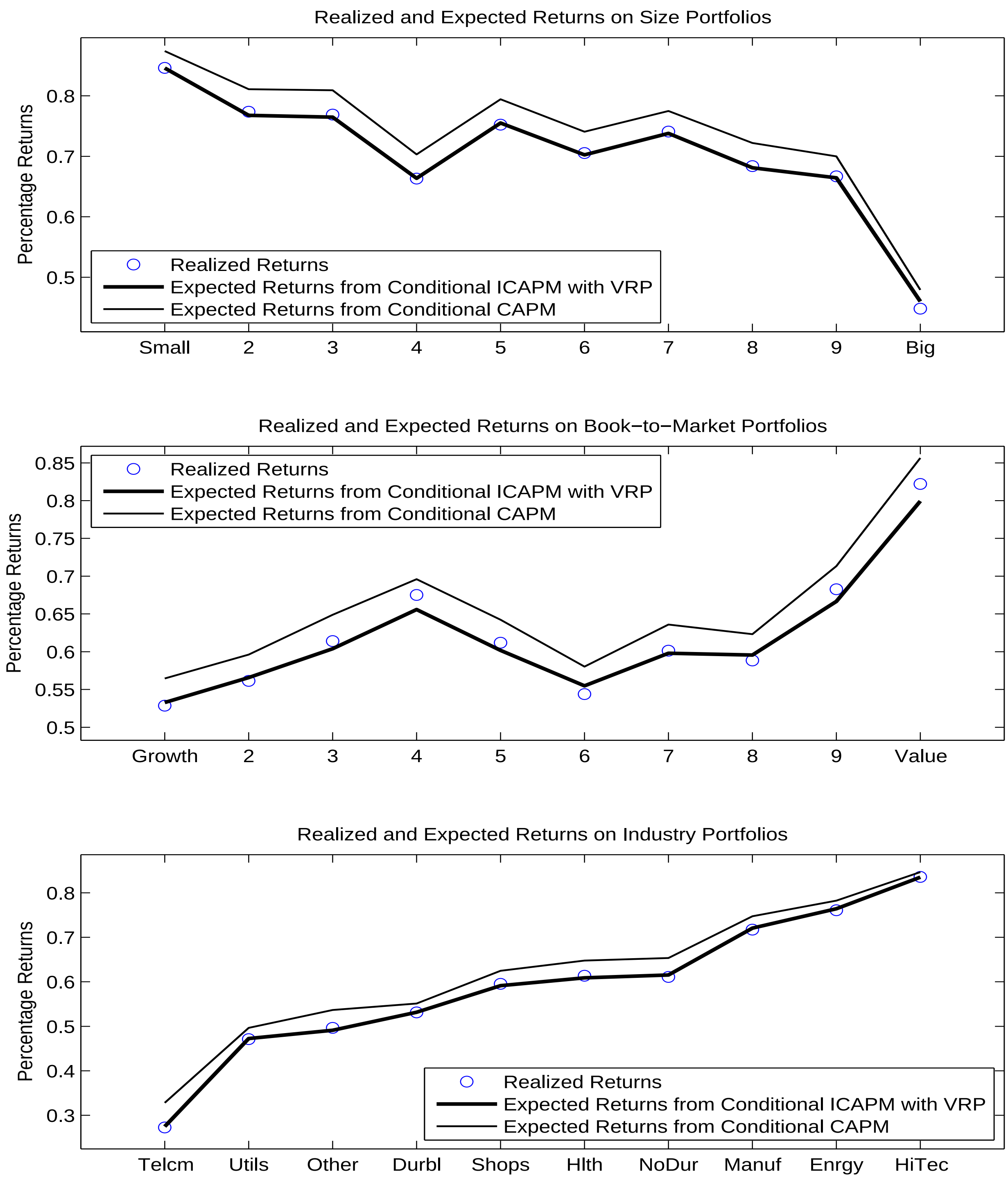

Figure 1 Relative Performance of the Conditional ICAPM with Uncertainty

This figure plots the realized monthly average excess returns on the size (top panel), book-to-market (middle panel), and industry portfolios (bottom panel) and the cross-section of expected excess returns generated by the Conditional CAPM with the market factor and the Conditional ICAPM with the market and VRP factors. The results indicate superior performance of the conditional asset pricing model introduced in the paper.