Supplemental Appendix for

Understanding Portfolio Efficiency with Conditioning Information

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A Preferences Underlying Efficient Returns

The justification of mean-variance preferences under the expected utility paradigm was linked to elliptical distributions by Chamberlain (1983) and Owen and Rabinovitch (1983) in the Markowitz set-up without conditioning information. In that setting, any family of mean-variance preferences can be chosen to explore the entire efficient mean-variance frontier. For instance, the fixed-weight returns that maximize $E(R)-(b/2)\,E(R^2)$ for each real number b lie on the mean-variance frontier and each of these optimal returns also maximizes $E(R)-(\theta/2)\,Var(R)$ for the corresponding real number θ.

However, this appendix shows that this is not the case when investors design portfolio strategies with a nontrivial information set. To study this point, let us map the conditional mean-variance problem (2) into simple criteria based on risk-return trade-offs that are functions of $\mathbf{z}$. The following mean-variance criteria rationalize CE returns (the corresponding proof is available upon request):

1. The optimal return (1) that solves the problem

$$\max_{R} E(R|\mathbf{z}) - \frac{\theta_C}{2} Var(R|\mathbf{z}),$$

   given some positive θ_C depending on $\mathbf{z}$, is a CE return (5) with

$$\omega_C = \frac{1}{\theta_C}.$$

2. The optimal return (1) that solves the problem

$$\max_{R} E(R|\mathbf{z}) - \frac{b_C}{2} E\left(R^2|\mathbf{z}\right),$$

   given some positive b_C depending on $\mathbf{z}$, is a CE return (5) with

$$\omega_C = \left(\frac{1}{b_C} - R_0\right) / \left(1 + \mathcal{S}_C^2\right).$$

Ferson and Siegel (2001) show that the optimal return of an agent with quadratic utility $E\left[R-(b/2)\,R^2|\mathbf{z}\right]$ for some positive real number b is a UE return. In our set-up, from point 2 above and the ω_C of a UE return in (12), the solution of the problem

$$\max_{R} E(R|\mathbf{z}) - \frac{b}{2} E\left(R^2|\mathbf{z}\right) \qquad \text{(A1)}$$

is equal to a UE return with target ν such that

$$\frac{\nu + E\left(R_0 \frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right) - E(R_0)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)} = \frac{1}{b}.$$

Nevertheless, as commented in Section III.B, probably the most common conditional mean-variance preferences in finance theory are

$$\max_R E(R|\mathbf{z}) - \frac{\theta}{2} Var(R|\mathbf{z}) \tag{A2}$$

for some positive real number θ. The criterion (A2) is often justified by the constant absolute risk aversion utility $E[-\exp(-\theta R)|\mathbf{z}]$ plus the conditional normality of R, but none of my results requires this particular utility function or conditional normality.

From point 1 above and the ω_C of an RE return in (16), we can easily characterize the specific subset of CE returns where the solutions of problem (A2) are located. The corresponding optimal return is the RE return with target ν such that

$$\frac{\nu - E(R_0)}{E(\mathcal{S}_C^2)} = \frac{1}{\theta}.$$

Finally, we can also rationalize PE returns, whose ω_C is shown in (9), by means of the problem (A1) with first and second conditional moments in terms of $R - R_0$ instead of R. The solution of such a problem is equal to the PE return with target ν such that

$$\frac{\nu - E(R_0)}{E\left(\frac{\mathcal{S}_C^2}{1+\mathcal{S}_C^2}\right)} = \frac{1}{b}.$$

B Portfolio Efficiency and Asset Pricing Models

Let us decompose the vector of excess returns $\mathbf{r}$ into two vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. From the representation of CE returns (5), any CE return from $(R_0, \mathbf{r})$ is constructed with R_0 and a portfolio of $\mathbf{r}_1$ if and only if $\boldsymbol{\varphi}_C$ has zero entries for $\mathbf{r}_2$. This portfolio efficiency hypothesis is equivalent to zero conditional alphas because we find the equivalent constraints

$$E(\mathbf{r}_2|\mathbf{z}) = Cov(\mathbf{r}_2, \mathbf{r}_1|\mathbf{z}) [Var(\mathbf{r}_1|\mathbf{z})]^{-1} E(\mathbf{r}_1|\mathbf{z}) \tag{B1}$$

using the partitioned inverse in $\boldsymbol{\varphi}_C$. We can also translate the portfolio efficiency hypothesis into Sharpe ratios. Let us use the notation $\mathcal{S}_C$ for the maximum conditional Sharpe ratio from $\mathbf{r}$, $\mathcal{S}_{C1}$ for its counterpart with $\mathbf{r}_1$, and $\mathbf{u}_2$ for the residual

$$\mathbf{u}_2 = \mathbf{r}_2 - Cov\left(\mathbf{r}_2, \mathbf{r}_1|\mathbf{z}\right)\left[Var\left(\mathbf{r}_1|\mathbf{z}\right)\right]^{-1}\mathbf{r}_1.$$

Then we can write

$$\mathcal{S}_C^2 = E\left(\mathbf{r}|\mathbf{z}\right)'\left[Var\left(\mathbf{r}|\mathbf{z}\right)\right]^{-1}E\left(\mathbf{r}|\mathbf{z}\right) = \mathcal{S}_{C1}^2 + E\left(\mathbf{u}_2|\mathbf{z}\right)'\left[Var\left(\mathbf{u}_2|\mathbf{z}\right)\right]^{-1}E\left(\mathbf{u}_2|\mathbf{z}\right). \tag{B2}$$

Therefore, $\mathcal{S}_C^2 = \mathcal{S}_{C1}^2$ if and only if $E\left(\mathbf{u}_2|\mathbf{z}\right) = \mathbf{0}$, which is exactly the portfolio efficiency condition in terms of alphas (B1).

These are well-known results, extending the portfolio efficiency hypothesis from the Markowitz framework to conditional moments. Nevertheless, I can still provide the following novel results:

1. The portfolio efficiency hypothesis is the same regardless of our particular interest in UE, PE, or RE returns because these returns only differ in their choice of ω_C in (5). For instance, any RE return from $(R_0, \mathbf{r})$ is constructed with R_0 and a portfolio of $\mathbf{r}_1$ if and only if $\boldsymbol{\varphi}_C$ has zero entries for $\mathbf{r}_2$.

2. We can translate the portfolio efficiency hypothesis into unconditional and residual Sharpe ratios. For the latter, following equations (17) and (B2), we can write

$$\mathcal{S}_R^2 = E\left(\mathcal{S}_C^2\right) = \mathcal{S}_{R1}^2 + E\left[E\left(\mathbf{u}_2|\mathbf{z}\right)'\left[Var\left(\mathbf{u}_2|\mathbf{z}\right)\right]^{-1}E\left(\mathbf{u}_2|\mathbf{z}\right)\right].$$

   Thus $\mathcal{S}_R^2 = \mathcal{S}_{R1}^2$ if and only if $E\left(\mathbf{u}_2|\mathbf{z}\right) = \mathbf{0}$, the same condition as for $\mathcal{S}_C^2 = \mathcal{S}_{C1}^2$. We can also use equation (10) to develop a similar condition in terms of $\mathcal{S}_U$. Given this connection with $\mathcal{S}_U$ and $\mathcal{S}_R$, we can use the econometric framework of Section IV.A for testing portfolio efficiency. In the notation of that section, the excess returns r_1 and r_2 correspond to the excess returns of RE (or PE) returns from $\mathbf{r}_1$ and the full vector $\mathbf{r}$, respectively.

Importantly, we can relate the portfolio efficiency hypothesis to stochastic discount factors (SDFs). If we define the random variable

$$m^* = \frac{1}{R_0}\left(1 - r_v + E\left(r_v|\mathbf{z}\right)\right), \qquad r_v = \mathbf{r}'\boldsymbol{\varphi}_C,$$

then m^* satisfies

$$E\left(m^* R_0|\mathbf{z}\right) = 1, \qquad E\left(m^*\mathbf{r}|\mathbf{z}\right) = \mathbf{0} \tag{B3}$$

and hence it is a valid SDF. Under the portfolio efficiency hypothesis, m^* depends only on $\mathbf{r}_1$.

The results of Gallant, Hansen and Tauchen (1990) show that m^* is the conditional projection of any random variable m that satisfies the pricing equations (B3) onto the conditional span of R_0 and $\mathbf{r}$. This suggests that we can also use portfolio efficiency tests to test asset pricing models with nontraded factors (e.g., aggregate consumption growth in the consumption CAPM).

To clarify the use of Sharpe ratios in such a context, let us think of an SDF that is affine in a vector of non-traded factors $\mathbf{f}$ and prices the vector of excess returns $\mathbf{r}$, with fewer factors than returns. That is, there is a vector of risk prices $\boldsymbol{\tau}$ depending on $\mathbf{z}$ such that

$$m = 1 - [\mathbf{f} - E(\mathbf{f}|\mathbf{z})]' \boldsymbol{\tau}$$

satisfies $E(m\mathbf{r}|\mathbf{z}) = \mathbf{0}$ or, equivalently,

$$E(\mathbf{r}|\mathbf{z}) = Cov(\mathbf{r}, \mathbf{f}|\mathbf{z})\, \boldsymbol{\tau}.$$

If the asset pricing model holds, the mimicking portfolios

$$\mathbf{f}_v = Cov(\mathbf{r}, \mathbf{f}|\mathbf{z}) [Var(\mathbf{r}|\mathbf{z})]^{-1} \mathbf{r}$$

satisfy

$$\mathbf{f}_v' [Var(\mathbf{f}_v|\mathbf{z})]^{-1} E(\mathbf{f}_v|\mathbf{z}) = r_v$$

and hence the portfolios $\mathbf{f}_v$ provide the same maximum residual Sharpe ratio as the full vector $\mathbf{r}$. Therefore, we can test the asset pricing model by means of the difference between the maximum residual Sharpe ratios obtained with $\mathbf{f}_v$ and $\mathbf{r}$. We can also use unconditional Sharpe ratios in a similar way.

Finally, it is important to clarify that if the vector $\mathbf{r}_1$ in the portfolio efficiency hypothesis is a single excess return, then the return $R_1 = r_1 + R_0$ is CE but not necessarily UE, PE, or RE. In this context, the representation of CE returns (5) becomes

$$R_C = R_0 + \omega_C [E(r_1|\mathbf{z}) / Var(r_1|\mathbf{z})] r_1$$

where ω_C can be a function of $\mathbf{z}$. Therefore, the choice $\omega_C = Var(r_1|\mathbf{z}) / E(r_1|\mathbf{z})$ is compatible with CE returns and R_1 is CE. However, UE, PE, and RE returns require a particular structure in ω_C, as shown in (12), (9), and (16) respectively. These efficiencies require the following additional constraints: The return R_1 is PE when

$$Var\left[E\left(r_1|\mathbf{z}\right)/E\left(r_1^2|\mathbf{z}\right)\right]=0, \tag{B4}$$

it is RE when

$$Var\left[E\left(r_1|\mathbf{z}\right)/Var\left(r_1|\mathbf{z}\right)\right]=0, \tag{B5}$$

and it is UE when

$$Var\left[R_0+E\left(r_1^2|\mathbf{z}\right)/E\left(r_1|\mathbf{z}\right)\right]=0. \tag{B6}$$

In each case, the corresponding efficient returns are constructed by fixed-weight portfolios of r_1 and R_0.

For instance, when we test the conditional CAPM, the null hypothesis is that the market portfolio is CE. A different null hypothesis would be that the market portfolio is UE, PE, or RE itself, not only CE. If investors have preferences such that their optimal choices are RE returns (see Appendix A) and the safe asset return is in zero net supply, then the market portfolio must be RE in equilibrium. Equation (B5) characterizes the additional predictability constraint on the market return that this stronger hypothesis would require. Equations (B4) and (B6) characterize the cases of the market portfolio being PE or UE instead.

C The Tangency Portfolio

This appendix studies the connection between mean-variance frontiers with and without a conditionally riskless return R_0. The excess returns r_e and r_v defined in (20) have the following important property: r_e is the unique excess return that satisfies $E\left(r_e r|\mathbf{z}\right)=E\left(r|\mathbf{z}\right)$ and r_v is the unique excess return that satisfies $Cov(r_v,r|\mathbf{z})=E\left(r|\mathbf{z}\right)$ for every excess return r. In this appendix their counterparts when the safe asset is not available are denoted $\mathfrak{r}_e$ and $\mathfrak{r}_v$, respectively. Similarly, I define R_e as the return with minimum $E\left(R^2|\mathbf{z}\right)$ and R_v as the return with minimum $Var\left(R|\mathbf{z}\right)$ when the safe asset is not available. Their counterparts are $R_0\left(1-r_e\right)$ and R_0, respectively, when the safe asset is available.

Using this notation, the CE returns without a safe asset can be represented as

$$R_C=R_e+\omega_e\mathfrak{r}_e=R_v+\omega_v\mathfrak{r}_v,$$

where ω_e and ω_v are functions of $\mathbf{z}$. The UE returns can be represented as

$$R_U=R_e+\omega_U\mathfrak{r}_e,$$

the PE returns as

$$R_P = (R_e + R_0\mathfrak{r}_e) + \omega_P \mathfrak{r}_e,$$

where their associated risk measure is still $Var(R - R_0)$ but we cannot invest on the safe return, and the RE returns as

$$R_R = R_v + \omega_R \mathfrak{r}_v,$$

where ω_U, ω_P, and ω_R are real numbers. We can also develop the associated beta-pricing results. The proofs are available upon request.

The CE frontier without a safe asset is a hyperbola in the $[\sqrt{Var(R|\mathbf{z})}, E(R|\mathbf{z})]$ space for a particular value of $\mathbf{z}$. Generally, like in the Markowitz set-up, there is a unique optimal portfolio that is shared by the CE frontier with and without a safe asset, which is called the tangency portfolio. I characterize this portfolio in the following result (the corresponding proof is available upon request): If $E(R_v|\mathbf{z}) \neq R_0$, then there is a tangency portfolio between the CE frontiers with and without a safe asset with return

$$R_v + \frac{Var(R_v|\mathbf{z})}{E(R_v|\mathbf{z}) - R_0}\mathfrak{r}_v = R_e + \left[\frac{E(x^2|\mathbf{z})}{E(x|\mathbf{z})} + R_0\right]\mathfrak{r}_e, \qquad x = R_e - R_0(1 - \mathfrak{r}_e). \tag{C1}$$

The UE, PE, and RE frontiers without a safe asset are hyperbolas in their respective spaces. In contrast with the Markowitz set-up, there is generally no tangency in any of these frontiers. Peñaranda and Sentana (2016) point out this fact for the UE frontier. Following (C1) and the previous expressions of efficient returns without a safe asset, we can characterize the special cases where there are tangencies:

1. The tangency portfolio with return (C1) is RE when

$$Var\left[\frac{Var(R_v|\mathbf{z})}{E(R_v|\mathbf{z}) - R_0}\right] = 0.$$

   In this case, we can span the RE frontier with a safe asset by means of fixed-weight portfolios in the safe asset and the tangency portfolio.

2. The tangency portfolio with return (C1) is PE when

$$Var\left[\frac{E(x^2|\mathbf{z})}{E(x|\mathbf{z})}\right] = 0.$$

   In this case, we can span the PE frontier with a safe asset by means of fixed-weight portfolios in the safe asset and the tangency portfolio.

3. The tangency portfolio with return (C1) is UE when

$$Var\left[\frac{E\left(x^2|\mathbf{z}\right)}{E\left(x|\mathbf{z}\right)}+R_0\right]=0.$$

Nevertheless, we cannot span the UE frontier with a safe asset by means of fixed-weight portfolios in the safe asset and the tangency portfolio unless additionally $Var\left(R_0\right)=0$. In that case, following Corollary 1, UE and PE returns are equivalent.

Finally, let us interpret these results in the simple case of a single risky asset with return R and excess return $r=R-R_0$. Then we have $R_e=R_v=R$, $\mathfrak{r}_e=\mathfrak{r}_v=0$, and $x=r$. The CE, RE, PE, and UE frontiers without a safe asset are equal and given by a single point, R itself. However, these frontiers are different with a safe asset. The return R is also on the CE frontier with a safe asset, being the tangency portfolio return; but R is not necessarily on the RE, PE, or UE frontiers with a safe asset. These tangencies require the conditions stated in points 1-3 above. If we apply these equations to the case of a single risky asset, then the conditions in points 1, 2, and 3 coincide with the constraints (B5), (B4), and (B6), respectively, in Appendix B.

D Empirical Application: Additional Results

Annual Returns

The return frequency is another relevant dimension for the differences across UE, PE, and RE returns. Unfortunately, there are not many observations at low frequencies and hence the annual computations in this appendix should be interpreted with care.

It is well known that the coefficients of determination R^2 in annual predictive regressions are much higher than in monthly regressions, due to the persistence in the predictors. The data show the following values and patterns. The R^2 of the market portfolio clearly decreases from 0.372 in the first period to 0.082 in the second one. However, this is not the case for the other two Fama-French factors, where R^2 goes from 0.102 to 0.231 for SMB and from 0.119 to 0.118 for HML.

The intercept and slope of the CE frontier show the following features. The rate of return of the safe asset has a mean of 5.332 and a standard deviation of 3.217 in the first period. Both statistics decrease in the second period to 4.183 and 2.621, respectively. These standard deviations are much higher than their annualized monthly counterparts, due to the high persistence in interest rates. This suggests a larger gap between UE and PE returns at the annual frequency. When we work with the three Fama-French factors and their predictive regressions, the maximum conditional Sharpe ratio has a mean of

1.052 and a standard deviation of 0.636 in the first period. Both statistics decrease in the second period to 0.914 and 0.415, respectively.

Table D1 is the counterpart of Tables 2 and 3 with annual returns. I do not consider GARCH effects in the construction of efficient returns and I only report the results for the three Fama-French factors. The performance gaps are even larger for the extended investment sets. The statistical significance with asymptotic p-values is indicated by the symbol *, while I use + for bootstrap p-values. I compute the p-values for two and four lags in the Newey-West standard errors (the cubic root of the sample size is around three). The bootstrap experiment resamples individual residuals instead of blocks because we do not expect strong GARCH effects in annual returns. Table D1 only reports the statistical significance for two lags, the significance is similar for four lags.

<Table D1>

Interestingly, Table D1 shows that the performance gap of PE returns with respect to UE returns can be larger than that with respect to FE returns. The difference in unconditional Sharpe ratios with respect to UE returns with mean target 6% is 0.782 in the first period and 0.352 in the second. The difference with respect to UE returns with mean target 10% is also high, but only statistically significant for bootstrap p-values in the second period.

RE returns only provide a large and statistically significant performance gap in the first period with respect to UE returns with mean target 6%. A researcher comparing RE against FE returns would miss the gains from conditioning information that we find with PE returns. This suggests that it is good empirical practice to compute the performance of both PE and RE returns. In the second period the performance gaps of RE returns are large with respect to FE returns and UE returns with mean target 10% (0.462 and 0.238, respectively), albeit only the former is statistically significant with bootstrap p-values.

We can conclude that the differences across the subsets of CE returns are stronger with annual returns. UE returns are clearly different from PE or RE returns even for small investment set.

Out-of-Sample Analysis

We may be worried that the in-sample gains from conditioning information cannot be obtained out-of-sample. To study this issue, I perform an out-of-sample analysis during the second period (1984–2012) where we find, in-sample, lower market predictability and higher SMB predictability.

Each month, I use only past data on returns and predictors to estimate the conditional mean and variance of **r**, with the same models that are considered in Section IV.C. I use three rolling windows at each monthly estimation (10, 20, and 30 years of data). With these estimations, I construct the portfolio weights associated with PE and RE returns and store the return obtained when the new observation of **r** is realized. This exercise provides a time-series for PE and RE returns for the entire second period, and I can then compute their performance.

I compare the performance of PE and RE returns for each one of the Fama-French factors against simply holding that factor in Table D2. I also compare the performance of both efficient returns for the three Fama-French factors and the six and 25 sorted portfolios against the corresponding equally weighted portfolios. The PE and RE returns are constructed for a constant and a time-varying conditional variance (denoted C and V, respectively).

<Table D2>

Given the low market predictability that we find in-sample during the second period, it is not surprising that dynamic strategies perform badly for the market, much worse than simply holding this portfolio. On the other hand, I do not consider estimation risk when computing the optimal portfolios. Johannes, Korteweg and Polson (2014) find gains from exploiting market predictability from 1927 to 2007 for a Bayesian investor with constant relative risk aversion utility.

Dynamic strategies on HML perform better than on the market but are still not better than simply holding HML. It is more interesting that the in-sample predictability that we find for SMB is robust to an out-of-sample analysis. Moreover, considering GARCH effects in the construction of optimal portfolios increases the performance considerably. The intermediate rolling window (20 years of data) performs better than the other two windows.

If we jointly consider the three Fama-French factors then we can still find an improvement in PE returns with respect to an equally weighted portfolio. The best outcome is obtained from the GARCH model for 20 years of data. Obviously, the performance improvement is not as strong as when we invest on SMB only, since the out-of-sample predictability of the market and HML is weak.

Regarding the six and 25 sorted portfolios, dynamic strategies can provide unconditional Sharpe ratios around one. There is a clear improvement of PE and RE strategies with respect to equally-weighted portfolios. On the other hand, the Sharpe ratios of the latter portfolios are not far from holding the market and are lower than the Sharpe ratio of an equally weighted portfolio for the three Fama-French factors (the factors use short positions to exploit the size and value effects). Considering GARCH effects is especially useful with RE returns.

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Table D1
Unconditional and residual Sharpe ratios for annual returns

	1954–1983		1984–2012	
	Unconditional ratio	Residual ratio	Unconditional ratio	Residual ratio
PE and RE	1.024	0.961	0.973	1.137
FE	0.714	0.814	0.637	0.675
PE-FE and RE-FE	0.310	0.148	0.337	0.462
	**		***, ++	**, +
UE6	0.242	0.321	0.621	0.679
PE-UE6 and RE-UE6	0.782	0.640	0.352	0.458
	***, ++	***. ++	***, +++	***. ++
UE10	0.841	0.911	0.866	0.899
PE-UE10 and RE-UE10	0.183	0.051	0.107	0.238
	***		**, ++	**

Note: This table shows Sharpe ratios from the annual excess returns on the investment set given by the three Fama-French factors. The table reports the unconditional Sharpe ratios of PE, FE, and UE returns and the residual Sharpe ratios of RE, FE, and UE returns. The UE returns are reported for mean targets of 6% and 10%. The PE, RE, and UE returns are constructed with a conditional mean of excess returns given by the predictive regressions and a constant conditional variance. The differences in Sharpe ratios and their statistical significance with Newey-West standard errors are also displayed (*, ** and *** indicate significance with asymptotic *p*-values at 10%, 5%, and 1%, respectively; +, ++ and +++ indicate significance with bootstrap *p*-values at 10%, 5%, and 1%, respectively).

Table D2
Out-of-sample unconditional and residual Sharpe ratios 1984–2012

		10 years		20 years		30 years	
Panel A. Unconditional Sharpe ratios							
	EW	PE C	PE V	PE C	PE V	PE C	PE V
MMR	0.444	0.059	-0.062	0.083	0.081	0.099	0.035
SMB	0.075	0.141	0.242	0.297	0.386	0.080	0.176
HML	0.331	0.109	0.212	0.157	0.313	0.202	0.202
FF3	0.580	0.399	0.418	0.520	0.624	0.328	0.380
FF6	0.482	0.805	0.815	0.929	0.943	0.854	0.870
FF25	0.491	1.028	1.000	1.112	1.006	0.998	0.944
Panel B. Residual Sharpe ratios							
	EW	RE C	RE V	RE C	RE V	RE C	RE V
MMR	0.445	0.123	-0.030	0.084	0.080	0.116	0.053
SMB	0.076	0.150	0.241	0.278	0.350	0.076	0.179
HML	0.333	0.096	0.207	0.146	0.304	0.194	0.192
FF3	0.584	0.301	0.390	0.337	0.524	0.306	0.375
FF6	0.483	0.645	0.774	0.686	0.823	0.806	0.856
FF25	0.492	0.743	0.905	0.762	0.871	0.799	0.805

Note: This table shows Sharpe ratios from the monthly excess returns on each Fama-French factor (MMR, SMB, and HML) and three investment sets: the three Fama-French factors (FF3) and the six and 25 Fama-French portfolios (FF6 and FF25, respectively). Panel A reports the unconditional Sharpe ratios of PE returns constructed with a constant and a time-varying conditional variance of excess returns (PE C and PE V, respectively). Panel B reports the residual Sharpe ratios of RE returns constructed with a constant and a time-varying conditional variance of excess returns (RE C and RE V, respectively). In both panels, the performance of the corresponding equally-weighted (EW) portfolio is also displayed. The out-of-sample analysis is based on a monthly re-estimation of the conditional moments with rolling windows of 10, 20, and 30 years of past data.