Internet Appendix for Bank Skin in the Game and Loan Contract Design: Evidence from Covenant-Lite Loans

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In this supplementary appendix, we provide additional discussion of the data, extend the model to check for robustness, and provide proofs of the majority of the propositions. It contains the following sections.

C The Covenant-lite Loan

Covenant-lite loans lack traditional financial maintenance covenants. Maintenance covenants contractually oblige the firm to comply with defined financial metrics, such as a maximum debt to EBITDA ratio or a leverage ratio. These covenants are "tested" at specific points in time, most commonly at each quarter end. Violation of the maintenance covenant results in technical default, and is a breach of the loan contract (Miller (2012b)).

Eliminating maintenance tests benefits the issuing firm by reducing the probability of technical default and the consequent loss of shareholder control.[1] Recent research finds that financial covenant violations are neither rare (10-20% of public firms are in technical default in any given year) nor inconsequential. Moreover, in the aftermath of the violation firms face not only increased borrowing costs, but also increased creditor involvement as evidenced by the documented post-violation reductions in acquisitions, capital expenditures, leverage, shareholder payouts, and by increases in CEO turnover (Beneish and Press (1993), Nini, Smith, and Sufi (2012)).

In addition to the lack of maintenance covenants, covenant-lite loans tend to have fewer and looser negative covenants and fewer and looser incurrence covenants. Negative covenants prevent the borrower from taking certain actions, like incurring additional debt, paying dividends, making acquisitions, and/or repaying junior debt (Maxwell and Shenkman (2010)). Incurrence covenants are financial ratio tests that are triggered by these same activities.[2] In standard covenant-heavy loan contracts, actions prohibited by negative and incurrence covenants require creditor approval in addition to a waiver fee, and leave open the possibility of repricing the loan to bring it in line with the increased risk of the firm. These restrictions do not exist or are substantially weaker in covenant-lite loans.

C.1 How Lite is a Covenant-lite Loan?

While the specific covenant-lite loan may lack covenant restrictions, the firm may be bound by covenants elsewhere in its capital structure. First, covenant-lite term loans are typically part of a "loan package" that can include a bank revolver that may contain separate financial covenants.[3] Second, firms may have other covenant-heavy debt, and loan cross-default provisions could provide de facto protection to the covenant-lite debt.

We carefully explore both of these issues. First, to explore the covenant protection of the entire loan package, we hired two senior law students with expertise in contract law to analyze the credit agreements for a random sample of 100 loan packages that contain a covenant-lite loan. The credit agreements are found by searching SEC filings.[4]

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[1]The likelihood of a covenant violation, or a technical default, is substantially lower in the case of covenant-lite loans as they have no financial maintenance covenants to violate.

[2]The key distinction between maintenance and incurrence covenants is when they are tested. Incurrence covenants test only in the event the firm engages in certain activities, whereas maintenance covenants have automatic and recurring testing at regular intervals.

[3]We describe the typical loan package in the data section of the paper.

[4]We randomize the list of covenant-lite contracts in the S&P LCD database using Excel's random number generator. We then go through the list of contracts and search the SEC Edgar database for the credit agreement using Morningstar Document Research's 10kwizard service. We go down the list if a credit agreement is not found until we reach 100 observations.

Our major findings are as follows. We find that 47% of the covenant-lite agreements do not include revolving credit facilities. For the 53% that do have a revolver, we document that 49% of the revolving facilities are "naked revolvers," which is an industry term indicating that the revolver lacks any financial maintenance covenants. An additional 40% have "springing" maintenance covenants which are tested ("spring in") only if the revolver is drawn down beyond a particular threshold (Maugue (2012)). The remaining 11% of the revolvers have standard maintenance covenants, in the form of a leverage ratio.[5]

The credit agreements also indicate that both springing and regular covenants in revolvers provide little protection for other facilities in the loan package. First, covenants are tested at the end of the quarter and borrowers always have the option to pay down the revolving credit before the end of the quarter so that financial covenants go untested (Norris, Barclay and Fanning (2012)). Second, even if a springing covenant in the revolver is violated, the agreements explicitly state that the violation is not an event of default for the covenant-lite facility. Springing covenants in covenant-lite loans are written solely for the benefit of revolving lenders, who retain complete discretion over the terms of the renegotiation. Third, while covenant-lite loans typically include cross-acceleration provisions, such provisions are only triggered if the revolver lender chooses to accelerate payment, and then only after a 30-day grace period. This contrasts with the standard and much stricter cross-default provisions where any event of default in other agreements triggers an immediate event of default in the agreement with the cross-default. In sum, even if the loan package includes regular or springing covenants in the revolver, covenant-lite loan lenders receive minimal spillover protection (Myles (2011), Maugue (2012)).

While other loan and revolver facilities in the loan package do not appear to provide the covenant-lite loan de facto protection, such protection could stem from other firm debt agreements outside the loan package. To see if this is the case, we study the debt instruments and their associated credit agreements for a random sample of 50 firms receiving covenant-lite loans before and after the quarter of covenant-lite loan issuance. We find that covenant-lite loans are large and are designed to replace or refinance existing covenant-heavy loans for the borrower. In cases where other debt exists, we find it is typically in the form of public bonds, which do not include financial maintenance covenants during our sample period, and are thus covenant-lite by construction. Thus, the lack of other covenant-heavy debt makes the issue of cross-acceleration or cross-default moot, and leads us to believe that covenant-lite debt is not de facto protected by covenant-heavy loans, and suggests that covenant-lite loans indeed grant firms much greater flexibility.[6]

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[5]For comparison, we conducted a similar analysis by manually reading credit agreements for a random sample of 100 covenant-heavy loan packages. We found that 31% of covenant-heavy loans did not include a revolving credit facility. Thus, compared to covenant-lite loans, covenant-heavy loan packages were much more likely to include revolving credit facilities.

[6]In the paper we show that covenant-lite loans are issued at a premium over similarly rated covenant-heavy debt. If covenant-lite loans bind the firm to the same obligations as covenant-heavy loans, with the same triggers and risk profiles, firms would not pay a premium for covenant-lite loans. In a competitive loan market, the difference in spread signifies the market's assessment of a differential risk profile.

C.2 Do Loan Credit Default Swaps Replace Covenants in a Covenant-lite Loan?

Another important facet of the loan market to consider is the prevalence of loan credit default swaps (LCDS), and how such instruments may alter a bank's incentive to monitor and to include covenants in the loan. Banks can hedge loan credit risk in two major ways: by buying credit protection or through loan sales (Parlour and Winton (2013)). If the bank hedges its stake in the loan using LCDS then perhaps the usefulness of covenants as an incentive for the bank to monitor diminishes, leading to covenant-lite loans. However, this does not appear to be the case. Bank stakes in covenant-lite loans are often minuscule. We see 92% of covenant-lite loans are bought by institutions, on average, suggesting LCDS use by the bank is unlikely a factor in this market. In fact, bank use of LCDS would be more likely to play a role in covenant-heavy loans, where bank participation is very significant. While bank loan sales and LCDS may act as substitutes, covenants serve a role unlikely to be replaced by LCDS.[7] Loan covenants serve as early-warning tripwires and their mere presence may alter the path to default. Creditor control after a covenant violation or the threat of creditor control prior to bankruptcy may improve outcomes and certainly alters borrower behaviors (see Chava and Roberts (2008), Nini et al (2009), Nini et al (2012), and Roberts and Sufi (2009)).[8] This point is not missed by the rating agencies: *"The pre-eminent risk is that a covenant-lite structure will postpone default, eroding value and recoveries available to creditors when the issuer finally becomes distressed or files for bankruptcy," Moody's said. That risk, however, falls most heavily on subordinated bondholders in companies that have covenant-lite loans. The bondholders' claim is lower in the pecking order of payments in a default than the claim of the loan creditors." Bullock, N., (2011, March 10). Moody's warns on covenant-lite loans. FT.com.*

Last, if the bank's reputation depends on the performance of the loan then LCDS cannot substitute for the effect of influential monitoring on bank reputation.

C.3 Enforcement and Waiving the Covenant

In this section, we review the market practice about covenant enforcement.

Loan participants vote on whether to waive or enforce covenant violations. Decisions to waive or enforce covenants require a simple majority or super majority of the lending syndicate (Sufi (2007), Wight, Cooke and Gray (2009), pg. 482). This majority or super majority threshold for covenant waivers or enforcement mitigates holdout problems by smaller participants, while also allowing for the 'will of the majority'.

In our model the bank has a relative advantage in monitoring and enforcement and thus may be granted complete control over enforcement, even though it has moral hazard problems. This model structure meshes well with

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[7]It is quite possible that these institutions buy credit protection to hedge the loan's risk, which raises the question of who is selling the LCDS, and how they consider the covenant protection, or lack thereof, in the pricing of LCDS.

[8]In our theoretical model we abstract from the control elements of covenants: they serve to provide ex ante incentives and to compensate lenders for the additional risk. This is consistent with evidence from Bell and Perry (2013). We thank the referee for pointing this out.

conventional market practice where banks are often implicitly granted complete control over covenant enforcement: *"Although participant lenders have some information about the firm, they generally rely upon the lead arranger for both screening and monitoring duties. For example, almost all syndicated loans contain financial covenants, and the participant lenders rely on the lead arranger to monitor and enforce these covenants."* (Maxwell and Shenkman (2010), p. 90-91).

So, our interpretation of the literature is that (1) contract provisions affect the effective power of the different parties (albeit probabilistically) and (2) the bank has significant advantages in monitoring and enforcement.

Importantly, we also investigate the robustness of our results to our modeling choice. We introduce a second extension of the model in Section G on page 13 of this Internet Appendix. In the extension, the institution has a cost disadvantage in enforcing and it fails in enforcement with some probability (reflecting the bank's expertise in monitoring and enforcement). We show that our main result (theorem 1) still holds.

D   Optimal Enforcement Behavior with Perfect Bank Commitment

Our model relies on the interaction of two frictions – the firm cannot commit to choose action $a = s$ and the lenders cannot commit to the optimal enforcement behavior. In order to disentangle the role of the two frictions, we briefly consider the case when the lender can commit to enforcement and renegotiation behavior. We call this problem the one-sided commitment problem. First, we solve for the optimal enforcement behavior and contract under those circumstances. Second, we check if under some circumstances, the equilibrium outcome can attain the optimal solution.

In particular, we assume that the enforcement strategy E and the repayment function $D'(z, c)$ can be arbitrary (as long as the functions are Borel-measurable). This also implies that the bank share k and the fraction of the value of the second period project captured by the bank β are irrelevant for the bank's enforcement decision.

Consistent with our assumptions about the competitive equilibrium, the optimal contract maximizes the payoff of the firm subject to incentive and break-even constraints. For regulatory and accounting reasons, the bank must break even in expectation and they cannot book future profits when accounting.

(D-1)
$$\max_{(a, D, D', E)} \bar{R} + \bar{c} - \int\int [D + E(z, s)(D'(z, c) - D + c)]h(c)f(z|a^*)dcdz + \mathbf{1}_r(a^*)x$$

subject to (2) if $a = s$

$$\int\int [D + E(z, c)(D'(z, c) - D)]h(c)f(z|a^*)dcdz \geq I + \mathbf{1}_r(a^*)y$$

We call this problem P1. Characterizing the mechanism is tractable since the objective function and the constraints

are integrals of fixed functions, so the problem is convex.

Theorem D.1 *At the optimal solution, $D'(z,c) = R$. There exist positive constants μ and λ such that the covenant is enforced at (z,c) if and only if $z \in [z_a, z^*]$, and $c \in [c_a, c(z)]$, where*

$$c(z) = \frac{(R-D)(\lambda - 1 + \mu g(z))}{1 - \mu g(z)},$$

z^* is implicitly given by $c(z^*) = c_a$, $z^* < c_b$ and $g(z) = f(z|r)/f(z|s) - 1$. The base payment D is such that the break-even constraint holds with equality.

Proof. In Appendix H. ∎

Theorem D.1 shows that for large enough z ($z > z^*$), there will never be an enforcement action. Therefore, the optimal mechanism for problem P1 has a covenant-like structure. Moreover, enforcement depends on the realizations of both the signal z and the relationship rent c.

We combine enforcement policies under different scenarios in Figure 6 (on the following page). Left to right, panel A illustrates the optimal enforcement under commitment, implied by theorem D.1. At the optimum, the covenant is waived if $c > c(z)$ and $c(z)$ is strictly decreasing on some interval $(z_a, z^*]$. Panel B describes enforcement decision consistent with the incentives for the institution, derived in Section B.3. Panel C does the same for the bank. For the bank and the institution (Panels B and C), the value of z does not matter for the waiving of the covenant. *Conditional* on having the right to enforce, the decision of the lender whether to enforce or not depends only on c.

We conclude that the equilibrium contract without commitment cannot replicate the one under an environment where commitment is granted. Therefore, the lenders' lack of commitment is a binding constraint for the competitive equilibrium. Thus it is necessary to analyze the general case of no-commitment.

E   The Model with Endogenous Bank Share k

There are two differences between the basic model and a model with endogenous bank share k. First, the rate of return on the loan is no longer zero and it is different between the two kinds of lenders; second, the bank share k is endogenously chosen.

Let the opportunity cost of funds on *this particular loan* be i_b for banks and i_f for institutions. These are the rates of return that banks or institutions can earn on investments of similar risk characteristics and regulatory requirements to the loan.[9] We adopt a partial equilibrium approach in that we take i_b and i_f to be given. Since bank capital can

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[9]Banks are subject to heavy capital requirement regulations for leveraged loans. This renders their cost of capital for this subset of the loan market relatively high.
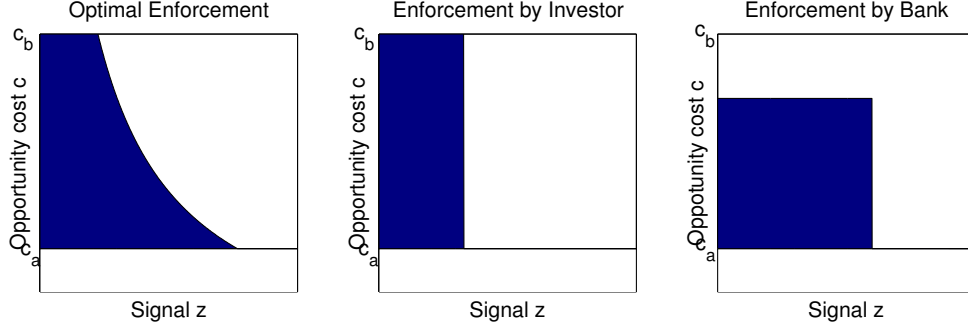
Figure 6: Enforcement under different scenarios

always substitute for institutional capital, but not vice versa, and banks are subject to heavier regulation, we impose the restriction that $i_b \geq i_f$.

In addition to the terms of the debt contract (which remain the same), now we need to determine how the loan is financed and how the cash flow from the firm's repayment of the loan are distributed.

We keep the assumption that the lender's payment cannot be conditioned on its renegotiation activity. The institution and the bank split the proceeds from the firm proportionally, so the payment from the firm to the bank and the institution is pinned down by the bank's share of loan revenues k. This implies that the bank's renegotiating strategy is determined in the same way and hence the incentive constraint for the firm is the same as before.

Secondly, the amount to be raised, I, must be divided up between the bank and the institution. Let M be the amount provided by the bank and $I - M$ provided by the institution.[10]

Let ED denote expected revenues from a contract:

$$ED = D + \int_{\mathbf{C}} \int_{\mathbf{Z}} E(z,c)(D'(z,c) - D)f(z|a)h(c)dzdc.$$

The enforcement and renegotiation functions E and D' depend on the contract, the bank share k, and the party in control.

Then if the bank is in control, we have the following break-even constraints:

(E-1) $$kED \geq M(1 + i_b)$$

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[10]Neither the bank nor the institution can create derivatives on the loan, or $0 \leq M \leq I$.

(E-2)
$$(1-k)ED \geq (I-M)(1+i_f).$$

If the institution is in control, the break-even constraints look similar, but now they must include the cost γ:

(E-3)
$$k[ED - \text{Prob}(A|a)\gamma] \geq M(1+i_b)$$

(E-4)
$$(1-k)[ED - \text{Prob}(A|a)\gamma] \geq (I-M)(1+i_f).$$

The financial markets are competitive, so they maximize the payoff of the firm:

(E-5)
$$\max_{(k,D,z,A,a)} \bar{R} + \bar{c} - D - \int \int E(z,c)(R - D + c)h(c)f(z|a)dcdz + \mathbf{1}_r(a)x,$$

subject to the relevant break-even constraints and, if $a = s$, the firm's incentive constraint.

The problem above is similar to problem (7) with two modifications. First, bank share k is chosen optimally and second the bank share k has an effect on the break-even constraints.

Lemma E.1 *An optimum exists. If the contract is with bank control, at the optimum constraints (E-1) and (E-2) bind. If the contract is with institutional control, at the optimum constraints (E-3) and (E-4) bind.*

Proof. In Appendix H. ∎

The lemma above implies there is one-to-one mapping between bank's share in revenue k and bank's share in financing M/I.[11] This allows us to substitute M as a function of k and derive a consolidated budget constraint for the case with bank control:

(E-6)
$$ED = \frac{I(1+i_b)(1+i_f)}{1+i_b - k(i_b - i_f)}.$$

The left-hand side of the equality is the repayment by the firm and the right-hand side is the cost of capital and the renegotiation costs, adjusted by the capital structure. So we see that lowering k reduces the required rate of return for the loan (since a larger proportion of the loan is financed by the outside institution, which has a lower required rate of return), but increases the agency problems for the bank and increases the renegotiation cost (since renegotiation will take place in more inefficient ways).

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[11]Since the bank and the institution earn different rates of return on their investment in the loan, there is a distinction between M/I (bank's share of financing) and k (bank's share of cash flows from the loan). There is 1-to-1 mapping between the two. If, for example, $i_b - i_f = 0.02$, the maximum difference between k and M/I is less than half a percentage point.

Similarly, when the institution is in control the consolidated budget constraint becomes:

$$(E\text{-}7) \qquad ED - \text{Prob}(A|a)\gamma = \frac{I(1+i_b)(1+i_f)}{1+i_b - k(i_b - i_f)}.$$

E.1 Endogenous Bank Share k

Contract without covenants The optimal loan package without covenants is the solution to the following problem:

$$\max_{k,D,z,a} \bar{R} + \bar{c} - D + x$$

$$\text{subject to } D \le R, z \in [z_a, z_b], a \in \{r, s\}, k \in [0, 1]$$

$$D \ge \frac{(I+y)(1+i_b)(1+i_f)}{1+i_b - k(i_b - i_f)}.$$

Contract with covenants and bank control The optimal loan package with bank control is the solution to the following problem:

$$\max_{k,D,z,a} \bar{R} + \bar{c} - D - F(z|a)\Delta(D, k) - F(z|a)\int_{c_a}^{k(R-D)/\beta} ch(c)dc + \mathbf{1}_r(a)x$$

$$\text{subject to } D \le R, z \in [z_a, z_b], a \in \{r, s\}, k \in [0, 1]$$

$$\text{constraint } (E-6) \text{ and if } a = s \text{ constraint } (2).$$

Contract with covenants and institutional control Similarly, the optimal loan package with institutional control is the solution to the following problem:

$$\max_{k,D,z,a} \bar{R} + \bar{c} - D - F(z|s)(R - D + \bar{c}) + \mathbf{1}_r(a)x$$

$$\text{subject to } D \le R, z \in [z_a, z_b], a \in \{r, s\}, k \in [0, 1]$$

$$\text{constraint } (E-7) \text{ and if } a = s \text{ constraint } (2).$$

F The Model with Stochastic Cash Flow R

In this section, we develop a richer model of covenants that allows for random cash flows. The benefit of this complication is that it allows us to endogenously derive the benefit of risk taking (x) and its cost to lenders (y).

F.1   Environment

The timeline is the same as before with the modification that the cash flow $\bar{R}$ is stochastic. For tractability, we will not distinguish between R and $\bar{R}$.

The cash flow R is not observable to the lenders directly, but they can learn it if they pay a cost $\delta > 0$. The lenders cannot commit to a verification scheme. Then, as in the classic paper by Townsend (1979), it is optimal to have straight debt – there is a fixed payment D and the firm pays $\min\{R, D\}$ to the bank and institution. In this case verification is interpreted as bankruptcy. The firm will always repay its loan if it can, because it knows that the lenders will force it into bankruptcy.

We assume that if early repayment is demanded (covenant is enforced) or if the firm goes into bankruptcy then the consequent investment opportunity is lost. The assumptions are summarized in the following timeline:

1. I is invested and the loan contract is signed.

2. Firm takes unobservable action a.

3. Signal z is realized according to distribution $F(z|a)$.

4. The value of the next period investment opportunity c is realized according to distribution $H(c)$.

5. Lenders choose whether to demand early repayment (enforce the covenant).

6. Firms and lenders renegotiate.

7. Cash flow is realized according to $G(R|a)$.

8. Firm pays $\min\{R, D\}$.

9. Lenders incur monitoring costs δ if the firm does not pay fully.

10. If the firm defaulted or the covenant is enforced, the firm loses the second period investment opportunity.

We keep the assumptions on F and H. We will have a very simple structure on the cash flow R. It can take three values: $(R_m - \Delta, R_m, R_m + \Delta)$ with probabilities $(p(a), 1 - 2p(a), p(a))$ with $0 < p(s) < p(r) < 1$. So by playing risky, the firm changes the probability distribution of cash flows, keeping the mean constant but increasing the variance. In particular, $E[R|a] = R_m, Var(R|a) = 2p(a)\Delta^2$.

We have assumed that the cash flow R and the signal z are independent, while in a more realistic model they will be positively correlated. This setup will allow for a rich model of debt renegotiation in the event that the firm cannot

repay the face value of the debt. Since the main implication of the model about the relationship between bank share and optimal contract design will not be affected, we do not explore this issue.[12]

F.2 The Loan Without a Covenant

We first consider the case of a loan without a covenant. The loan contract is summarized by the payment D. The firm pays $\min\{R, D\}$ to the lenders.

If $I \leq R_m - \Delta$, then it is optimal to set $D = I$, the firm will never default, so its payoff is just:

$$\pi(a) = \bar{c} + E[R|a] - I = \bar{c} + R_m - I,$$

so the action a is irrelevant. We will ignore this case.

We will make the assumption (made more precise later) that the level of D required for the lenders to break even satisfies $D < R_m$. Also, we assume that $I > R_m - D$ and $D \geq I$, so $D > R_m - \Delta$.

Then the firm's payoff as a function of its action is given by:

$$\pi(a) = p(a)(R_m - \Delta - (R_m - \Delta)) + (1 - 2p(a))(R_m - D + \bar{c}) + p(a)(R_m + \Delta - D + \bar{c}).$$

Rewriting:

$$\pi(a) = R_m - D + \bar{c} + p(a)[D - (R_m - \Delta) - \bar{c}],$$

which implies

(F-1)
$$\pi(r) - \pi(s) = (p(r) - p(s))[D - (R_m - \Delta) - \bar{c}].$$

Then the firm will have an incentive to risk-shift if the payment D is high and the average of the future business opportunity is low.

The lender's monetary return from a contract is:

$$p(a)[R_m - \Delta - \delta] + (1 - p(a))D - I.$$

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[12]This setup will also have an effect on costly state verification part of the model: the conditional distribution of R depends on z. In this case it would be optimal to vary D continuously with z.

Then if an action a is anticipated, the break-even payment $D^*(a)$ is given by:

$$D^*(a) = \frac{1}{1 - p(a)} \left[I - p(a)[R_m - \Delta - \delta] \right].$$

Proposition F.1 *If $D^*(s) \leq R_m - \Delta + \bar{c}$, then the equilibrium contract has payment $D^*(s)$ and the firm plays $a = s$. Otherwise, the equilibrium contract has payment $D^*(r)$ and $a = r$.*

Proof. In Appendix H. ∎

We see that the payoff to the firm from risk-shifting actually depends on the debt contract itself. Nonetheless, we can find the analogues to x and y in the standard model. Using (F-1), we see that the benefit to the firm from risk-shifting is

$$x = (p(r) - p(s))[D^*(r) - (R_m - \Delta) - \bar{c}].$$

The net cost of risk-shifting is the increased probability of verification and of disrupting second period investment:

$$y - x = (p(r) - p(s))(\delta + \bar{c}).$$

Then $y = y - x + x$, so

$$y = (p(r) - p(s))[D^*(r) - (R_m - \Delta) + \delta].$$

F.3 The Contract with Covenants

Next we consider the contract with covenants. We will ignore the issue of commitment to enforcing the covenant for the time being. The contract is again just a covenant trigger $\hat{z}$ and a base repayment D. If the covenant is triggered, the lenders demand repayment of R, that is they extract all the cash flow. Then the firm's payoffs are as follows:

$$
\begin{aligned}
\pi(a) &= F(\hat{z}|a) \times 0 + (1 - F(\hat{z}|a))[p(a) \times 0 + (1 - 2p(a))(R_m - D + \bar{c}) + p(a)(R_m + \Delta - D + \bar{c})] \\
&= (1 - F(\hat{z}|a))[(1 - p(a))(R_m - D + \bar{c}) + p(a)\Delta]
\end{aligned}
$$

Then it is straightforward to see that

$$
\begin{aligned}
\pi(s) - \pi(r) &= (F(\hat{z}|r) - F(\hat{z}|s))[(1 - p(r))(R_m - D + \bar{c}) + p(r)\Delta] \\
&\quad - (1 - F(\hat{z}|s))(p(r) - p(s))[D - (R_m - \Delta) - \bar{c}],
\end{aligned}
$$

so rewriting, we get that the firm will choose action $a = s$ if

(F-2) $\quad (F(\hat{z}|r) - F(\hat{z}|s))[(1 - p(r))(R_m - D + \bar{c}) + p(r)\Delta] \geq (1 - F(\hat{z}|s))(p(r) - p(s))[D - (R_m - \Delta) - \bar{c}].$

We can make several observations: (i) There always exists a contract that satisfies incentive compatibility (F-2); (ii) If $I < R_m - \delta$ (as we will assume from now on), there always exists a contract that satisfies all the constraints; (iii) Decreasing D strengthens the incentive constraint.

The lender's break-even constraint is then given by:

(F-3) $\quad\quad\quad\quad F(\hat{z}|s)(R_m - \delta) + (1 - F(\hat{z}|s)((1 - p(s))D + p(s)(R_m - \Delta - \delta)) \geq I.$

Proposition F.2 *Suppose that $D^*(s) > R_m - \Delta + \bar{c}$ and that $I < R_m - \delta$. Then there exists $(\hat{z}, D)$ that satisfy (F-2) and (F-3) and an optimal contract with covenants exists. At the optimal contract with covenants, (F-2) and (F-3) are binding.*

Proof. In Appendix H. ∎

Then the cost of covenants (compared with the first best outcome) is $F(\hat{z}|s)(1 - p(s))(\bar{c} + \delta)$. The cost of risk-shifting is $y - x = (p(r) - p(s))(\delta + \bar{c})$. So clearly, a covenant will be optimal if and only if $F(\hat{z}|s) < (p(r) - p(s))/(1 - p(s))$.

Observation 1 *In this extended model, the incentives for risk-shifting are increasing in the spread. If the firm has claim to the entire value of the project, it will make efficient decisions.*

Observation 2 *If the covenant is enforced, then the debt holders do not suffer from the risk-shift. So in the model, the covenant provides incentives for the firm not to risk-shift, but also fixes problems ex-post.*

Observation 3 *Since the covenant destroys value, if the optimal action is $a = r$, then covenants are not employed, but lenders are compensated with additional spread.*

F.4   The Contract with Covenants and Institutional Control

Next, we start to tackle the case of bank commitment. First, consider the case when the institutions are in charge of enforcing the covenant. The only difference is that now there is an additional cost term that comes from enforcing the covenant: $F(z|s)\gamma$. All the conclusions from the basic case still hold.

F.5 The Contract with Covenants and Lack of Bank Commitment

Finally, we assume that the bank cannot commit to covenant enforcement (as in the main body of the paper). Suppose that the bank holds k fraction of the loan and the institution holds the rest $(1 - k)$. Also, as in the main body of the paper, assume that the bank gets βc if the firm gets c.

First, we need to derive the bank's strategy. The bank does not observe the action directly. Let $\hat{a}$ be its belief about the firm's action.[13]

Given belief $\hat{a}$, the change in expected payment (net of costs δ) from enforcing the covenant is:

$$R_m - \delta - p(\hat{a})(R_m - \delta - \Delta) - (1 - p(\hat{a}))D = p(\hat{a})\Delta + (1 - p(\hat{a}))(R_m - \delta - D).$$

The bank gets a share k of this additional payment. On the other hand, the additional benefit of future business drops from $\beta(1 - p(\hat{a}))c$ to 0. (The firm goes into bankruptcy with probability $p(a)$, in which case the additional investment opportunity is lost.) Then the covenant will be enforced if

(F-4)
$$c \leq \frac{k}{\beta(1 - p(\hat{a}))}[p(\hat{a})\Delta + (1 - p(\hat{a}))(R_m - \delta - D)].$$

Denote the right-hand side of condition (F-4) by $\tilde{c}(\hat{a}, D, k)$. Note that $\tilde{c}$ depends on the bank's *belief* about the firm's action, not the action itself. So the firm does not influence $\tilde{c}$ by its choice of action. The probability that the broken covenant will actually be enforced is $Prob(c \leq \tilde{c}(\hat{a}, D, k)) = H(\tilde{c}(\hat{a}, D, k))$. Some straightforward algebra shows that the firm's payoff is:

$$
\begin{aligned}
\pi(a; \hat{a}) &= (1 - p(a))(R_m - D + \bar{c}) + p(a)\Delta \\
&\quad - F(\tilde{z}|a)H(\tilde{c}(\hat{a}, D, k))[(1 - p(a))(R_m - D + E[c|c \leq \tilde{c}(\hat{a}, D, k)]) + p(a)\Delta].
\end{aligned}
$$

In equilibrium, the bank's belief is correct, so if $a = s$ is induced, $\hat{a} = s$. Then the incentive constraint for the firm is that $\pi(s; s) \geq \pi(r; s)$, which is equivalent to:

$$[F(\tilde{z}|r) - F(\tilde{z}|s)]H(\tilde{c}(D, k))[(1 - p(s))(R_m - D + E(c|c \leq \tilde{c}(D, k))) + p(s)\Delta] \geq$$

$$(p(r) - p(s))F(\tilde{z}|r)H(\tilde{c}(D, k))[D + \Delta - R_m - E(c|c \leq \tilde{c}(D, k))] +$$

(F-5)
$$(p(r) - p(s))[D - (R_m - \Delta) - \bar{c}],$$

---

[13]It is possible that the firm plays r in equilibrium with positive probability in order to provide incentives for the bank, so the bank needs to have some probability distribution over $\hat{a}$. We rule this possibility out.

where $\tilde{c}(D, k) = \tilde{c}(s, D, k)$. It is straightforward, but long, to show that lowering D or β and increasing k strengthens the incentive constraint.

Next, we turn to the break-even constraint for the lenders. The break-even constraint is simply:

(F-6) $\qquad p(s)(R_m - \Delta - \delta) + (1 - p(s))D + F(\hat{z}|s)H(\tilde{c}(D, k))[p(s)\Delta + (1 - p(s))(R_m - \delta - D)] \geq I.$

Lowering the spread D reduces the cash flow during regular operation of the firm, but also increases the probability of the covenant being enforced, which in turn implies that expected repayment may be nonmonotone in D. We can deal with this complication, but for the sake of ease of exposition, we rule this option out.

Assumption F.1 *For all k and β, the left-hand side of (F-6) is strictly increasing in D. Also, the following holds:*

$$1 + h(c)\frac{k}{\beta}(c - \Delta) + H(c) \geq 0, \ \forall c.$$

The assumption allows us to give a clean characterization of the contract with bank control that induces action $a = s$.

Lemma F.1 *An optimal contract exists. At the optimum, (F-5) and (F-6) are binding.*

Finally, we get the analogue of the main result:

Theorem F.1 *There exist cutoffs $0 < \underline{k} \leq \bar{k} \leq 1$ such that if $k < \underline{k}$, the contract is either without covenants or with covenants and institutional control; if $k \geq \underline{k}$, the contract is with covenants and bank control. Covenant tightness (z^*) is strictly decreasing on $(\underline{k}, \bar{k})$ and constant on $(\bar{k}, 1]$.*

   Proof. Analogous to the main result. ∎

G   Enforcement Right Cannot be Allocated

In this extension, we model an extension of the basic model, in which the contract cannot specify which party has the control right to enforce the covenant.

We keep the model exactly as in the basic model of the paper, with one modification: even if it wants to enforce, the institution may fail to do so with probability $1 - \psi$. So the bank and the institution differ along the following dimensions:

1. The bank gets relationship rent βc, while the institution does not.

2. If the bank chooses to enforce the covenant, it succeeds with probability 1; if the institution chooses to enforce the covenant, it succeeds with probability $\psi < 1$.

3. The bank enforces at zero cost; if the institution succeeds in enforcing, it incurs a cost γ.[14]

G.1   Only Institutions Enforce

In order to perform comparative statics on ψ, we first consider a case in which only the institution enforces. The contract consists of a covenant set A and a base payment D. Clearly, the institution will try to enforce whenever the covenant is broken. Let $E(z, c)$ denote the probability of covenant enforcement for signal z and opportunity c. Then:

$$
E(z, c) = \begin{cases} \psi & \text{if } z \in A, \\ 0 & \text{otherwise.} \end{cases}
$$

So applying (2), the firm's incentive constraint is simply:

(G-1)
$$
\psi(Prob(A|r) - Prob(A|s))(R - D + \bar{c}) \geq x
$$

and the break-even constraint is:

(G-2)
$$
D + Prob(A|s)\psi(R - D - \gamma) \geq I
$$

and, finally, the firm's payoff is:

(G-3)
$$
\bar{c} + \bar{R} - D - Prob(A|s)\psi(R - D + \bar{c}) \geq I.
$$

Lemma G.1 *The covenant set is $A = [z_a, z]$ and the incentive and promise-keeping constraints bind.*

Proof. As in the basic case. ∎

Next, we show what is the effect of introducing ψ and γ.

Lemma G.2 *The firm's payoff is strictly increasing in ψ and strictly decreasing in γ. There exists $\underline{\psi} > 0$ such that if $\psi < \underline{\psi}$, either no feasible contract with covenants exists, or it is dominated by the no-covenant contract.*

Proof. Suppose that some contracts inducing s are feasible for ψ_1 and ψ_2, $\psi_1 < \psi_2$; let (z_i, D_i) be the corresponding optimal contract.

---

[14]We can assume that the cost γ is always incurred. This will strengthen the results.

Since the break-even constraint is binding, the payoff to the firm is given by $\bar{R} + \bar{c} - I - \psi_i F(z_i|s)(\bar{c} + \gamma)$. Therefore we need to show that $\psi_2 F(z_2|s) < \psi_1 F(z_1|s)$ to prove the first statement.

First, we show that $z_1 > z_2$. Suppose not: $z_1 \leq z_2$. Since (z_1, D_1) satisfies all the constraints for $\psi = \psi_1$, it satisfies all constraints for $\psi = \psi_2$ with strict inequality. Then there exists some (z', D') with $z' < z_1 \leq z_2$ such all the constraints bind for $\psi = \psi_2$. The new contract dominates (z_2, D_2), which is a contradiction.

Suppose, by contradiction, that $\psi_1 F(z_1|s) \leq \psi_2 F(z_2|s)$. Since both break-even constraints are binding, $D_1 \geq D_2$. Since the incentive constraints are binding, $D_1 \geq D_2$ implies that $\psi_1(F(z_1|r) - F(z_1|s)) \geq \psi_2(F(z_2|r) - F(z_2|s))$. Then $F(z_1|s)/[F(z_1|r) - F(z_1|s)] \leq F(z_2|s)/[F(z_2|r) - F(z_2|s)]$. By lemma H.1 $F(z|s)/[F(z|r) - F(z|s)]$ is strictly increasing, so $z_1 \leq z_2$. This contradicts the fact that $z_1 > z_2$.

The proof that the payoff is strictly decreasing in γ is analogous.

Now we prove the last claim. Let $m = \max_{z \in [z_a, z_b]} F(z|r) - F(z|s)$. Then from the incentive constraint,

$$D \leq R + \bar{c} - \frac{x}{\psi m}.$$

.

On the other hand, the break-even constraint implies that $D \geq I - \psi R$. Clearly for all ψ low enough, the two inequalities cannot be satisfied at the same time. ∎

G.2 Bank and Institution Enforce

Now we consider the case when both the bank and the institution can enforce the covenant if they so choose.

This implies that for given contract (D, A), the enforcement function is as follows:

$$E(z, c) = \begin{cases} 1 & \text{if } z \in A \text{ and } c \leq \frac{k}{\beta}(R - D), \\ \psi & \text{if } z \in A \text{ and } c > \frac{k}{\beta}(R - D) \\ 0 & \text{otherwise.} \end{cases}$$

Note that $E(z, c) = \psi E_I(z, c) + (1 - \psi) E_B(z, c)$, where E_I and E_B are the enforcement functions of the institution and the bank in the basic model.

Then it is easy to apply (2), and get the incentive contstraint:

(G-4) $\qquad (Prob(A|r) - Prob(A|s))[\psi(R - D + \bar{c}) + (1 - \psi)(\Delta(D, k) + ce(D, k))] \geq x,$

where all the notation is as in the basic model.

Similarly, the break-even constraint is:

(G-5)
$$D + Prob(A|s)(\psi(R - D) + (1 - \psi)\Delta(D, k) - (1 - H(\tilde{c}(D, k)))\psi\gamma) \geq I.$$

Thus the additional enforcement costs γ are incurred *only* if the bank refuses to enforce. and, finally, the firm's payoff is:

(G-6)
$$\bar{c} + \bar{R} - D - Prob(A|s)(\psi(R - D + \bar{c}) + (1 - \psi)(\Delta(D, k) + ce(D, k))) \geq I.$$

Lemma G.3 *The covenant set is $A = [z_a, z]$ and the incentive and promise-keeping constraints bind.*

Proof. As in the basic case. ∎

Next, we show that the main result of the paper still holds under the new scenario.

Proposition G.1 *There exist cutoffs $0 \leq \underline{k}(\psi) < \bar{k}(\psi) \leq 1$ such that if $k > \bar{k}(\psi)$, the bank always enforces and the contract (z, D) is independent of k. Covenant tightness is decreasing on $(\underline{k}, \bar{k})$ and for $k < \underline{k}(\psi)$, the contract is without covenant. Covenant tightness is decreasing in ψ.*

Proof. Let $(z(k, \psi), D(k, \psi))$ be the optimal contract for (k, ψ).

The first three statements are proved analogously to the proof of theorem 1.

Next, as before, define

$$M(D, z; k, \psi) = D + F(z|s)(\psi(R - D) + (1 - \psi)\Delta(D, k) - (1 - H(\tilde{c}(D, k)))\psi\gamma) - I$$

and

$$N(D, z; k, \psi) = (F(z|r) - F(z|s))[\psi(R - D + \bar{c}) + (1 - \psi)(\Delta(D, k) + ce(D, k))] - x.$$

As before, the optimal contract (D^*, z^*) minimizes (z) subject to the constraint that $M(D, z; k, \psi) \geq 0$ and $N(D, z; k, \psi) \geq 0$. M and N are increasing in ψ; moreover they are strictly increasing if $k \in (\underline{k}, \bar{k})$. Let $\psi_1 < \psi_2$. Then $M(D(k, \psi_1), z(k, \psi_1); k, \psi_2) \geq M(D(k, \psi_1), z(k, \psi_1); k, \psi_1) = 0$ and $N(D(k, \psi_1), z(k, \psi_1); k, \psi_2) \geq N(D(k, \psi_1), z(k, \psi_1); k, \psi_1)$ 0. So, similar to the proof of theorem 1, $z(k, \psi_2) \leq z(k, \psi_1)$. ∎

Giving the bank a control right is equivalent to setting $\psi = 0$. Thus when both bank and institution enforce, it may be optimal to have low ψ, since the institution displace the more cost-effective enforcement by the bank.

Finally, we want to explore what is the optimal ψ - the probability that the institution will be able to force enforcement action, even if the bank objects. Increasing ψ will lead to tighter incentive constraint (a positive), but will

increase inefficient enforcement (enforcement when c is high) and will impose additional costs. We investigate this question in a numerical simulation and find that for some parameter values (distribution of c, γ, k) it is optimal to reduce ψ as much as possible.

H   Proofs

Proof of Theorem D.1.   First, we show that without loss of generality, $D'(z, c) = R$. Suppose not. Then increase $D'(z, c)$ to R and decrease D to keep the break-even constraint. The incentive constraint is strengthened and the objective function is unchanged.

Relax the constraint that $E(z, c) \in \{0, 1\}$ to $E(z, c) \in [0, 1]$. Then P1 is a convex maximization problem with a nonempty interior. Then by Theorem 1 in Luenberger (1969), page 217 there exist constants $\lambda \geq 0$, $\mu \geq 0$ such that the optimal solution to P1 maximizes

$$
\begin{aligned}
L(D, E(c, z); \lambda, \mu) \;=\; & \bar{R} + \bar{c} - \int \int [D + E(z, s)(R - D + c)]h(c)f(z|s)dcdz \\
& + \lambda \left(\int \int [D + E(z, s)(R - D)]h(c)f(z|a^*)dcdz - I \right) \\
& + \mu \left(\int \int E(z, s)(R - D + c)h(c)(f(z|r) - f(z|s))dcdz - x \right).
\end{aligned}
$$

Taking Gateaux derivatives, the following conditions are necessary:

$$
D_{E(z,c)}L(D, E(z, c); \lambda, \mu)
\begin{cases}
\geq 0 & \text{if } E(z, c) = 1 \\
= 0 & \text{if } E(z, c) \in (0, 1) \\
\leq 0 & \text{if } E(z, c) = 0
\end{cases}.
$$

Then if we define $c(z) = (R - D)(\lambda - 1 + \mu g(z))/[1 - \mu g(z)]$, $D_{E(z,c)}L > 0$ if and only if $c < c(z)$; $D_{E(z,c)}L < 0$ if and only if $c > c(z)$. Therefore $E(z, c) \in \{0, 1\}$ almost surely. This concludes the proof.

∎

Lemma H.1 $v(z) = F(z|s)/[F(z|r) - F(z|s)]$ *is strictly increasing in z.*

Proof.

$$
v'(z) = \frac{f(z|s)F(z|r) - F(z|s)f(z|r)}{(F(z|r) - F(z|s)^2}
$$

. Then it is sufficient to show that $f(z|s)F(z|r) - F(z|s)f(z|r) > 0$.

$$f(z|s)F(z|r) - F(z|s)f(z|r) = f(z|s) \int_{z_a}^{z} \left[\frac{f(w|r)}{f(w|s)} - \frac{f(z|r)}{f(z|s)} \right] f(w|s)dw > 0.$$

∎

Proof of Lemma 1. Let z^* be defined by $F(z^*|s) = Prob(A_z|s)$. By Lemma A.1 in Elkamhi et al (2012), $F(z^*|r) - F(z^*|s) \geq Prob(A_z|r) - Prob(A_z|s)$ with strict inequality if $Prob(A_z\Delta[z_a, z^*]|s) > 0$. Then setting the covenant set to $A = [z_a, z^*]$ does not affect the firm's payoff or the break-even constraint and strengthens the incentive constraint. ∎

Proof of lemma A.1.

Define the problem M as follows:

(H-1)
$$\max_{D,z,\tilde{c}} \bar{c} + \bar{R} - D - F(z|s)H(\tilde{c})(R - D + E[c|c \leq \tilde{c}])$$

(H-2)
$$\text{s.t. } [F(z|r) - F(z|s)]H(\tilde{c})(R - D + E[c|c \leq \tilde{c}]) \geq x$$

(H-3)
$$D + F(z|s)H(\tilde{c})(R - D) \geq I$$

Here $\tilde{c}$ is optimally chosen and not a function of k and D.

Clearly increasing D relaxes the break-even constraint; reducing D relaxes the incentive constraint, so as in the main body of the text, the two constraints are binding.

Let $z(\tilde{c}), D(\tilde{c})$ be the optimal choices for any given $\tilde{c}$. We know that for any c, the incentive and break-even constraints are binding. Then it is easy to show that the firm's payoff $v(\tilde{c})$ is:

$$v(\tilde{c}) = \bar{R} + \bar{c} - I - F(z(\tilde{c})|s)H(\tilde{c})E[c|c \leq \tilde{c}]$$

Let $c_0 < c_b$ be arbitrary. Define $z^*(\tilde{c})$ by $[F(z^*(\tilde{c})|r) - F(z^*(\tilde{c})|s)]H(\tilde{c})(R - D(c_0) + E[c|c \leq \tilde{c}]) = x$. It is defined on some neighborhood of c_0. For $\tilde{c} > c_0$, the contract $(z^*(\tilde{c}), D(c_0))$ satisfies the IC constraint with equality and the BE with strict inequality, which implies that $z(\tilde{c}) < z^*(\tilde{c})$ for $\tilde{c} > c_0$. This implies that for $\tilde{c} > c_0$, $v(\tilde{c}) > \hat{v}(\tilde{c}) \equiv \bar{R} + \bar{c} - I - F(z^*(\tilde{c})|s)H(\tilde{c})E[c|c \leq \tilde{c}]$.

Using the implicit function theorem and the fact that the IC binds,

$$z_c^*(\tilde{c}) = -\frac{(F(z^*(\tilde{c})|r) - F(z^*(\tilde{c})|s))^2(R - D + \tilde{c})h(\tilde{c})}{(f(z^*(\tilde{c})|r) - f(z^*(\tilde{c})|s))x} \leq -\frac{(F(z^*(\tilde{c})|r) - F(z^*(\tilde{c})|s))h(\tilde{c})}{(f(z^*(\tilde{c})|r) - f(z^*(\tilde{c})|s))H(\tilde{c})}.$$

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From the definition, it follows that

$$
\begin{aligned}
\hat{v}'(\tilde{c}) &= -f(z|s)z_c^*(\tilde{c})\int_{c_a}^{\tilde{c}} h(c)dc - F(z^*(\tilde{c})|s)\tilde{c}h(\tilde{c}) \\
&\geq f(z^*(\tilde{c})|s)\frac{(F(z^*(\tilde{c})|r) - F(z^*(\tilde{c})|s))h(\tilde{c})}{(f(z^*(\tilde{c})|r) - f(z^*(\tilde{c})|s))H(\tilde{c})}\int_{c_a}^{\tilde{c}} h(c)dc - F(z^*(\tilde{c})|s)\tilde{c}h(\tilde{c}) \\
&= h(\tilde{c})E[c|c \leq \tilde{c}]\left[f(z^*(\tilde{c})|s)\frac{(F(z^*(\tilde{c})|r) - F(z^*(\tilde{c})|s))}{(f(z^*(\tilde{c})|r) - f(z^*(\tilde{c})|s))} - F(z^*(\tilde{c})|s)\frac{\tilde{c}}{E[c|c \leq \tilde{c}]}\right] \\
&\geq 0
\end{aligned}
$$

∎

Lemma H.2 *For any contract satisfying constraints (A-2) and (A-3) and at least one of the constraints slack, there exists a contract with $A = [z_a, z^*]$, both constraints are binding and the firm's payoff is strictly larger.*

Proof. By Lemma 1, we can set $A = [z_a, z^*]$, which tightens the incentive constraint without affecting the payoff of the firm. Let $M(D, z; k) = D + F(z|s)(R - D - \gamma) - I$. The break-even constraint is $M(D, z; k) \geq 0$. Similarly, let $N(D, z; k) = (F(z|r) - F(z|s))(R - D - \bar{c}) - x$. The incentive constraint is $N(D, z; k) \geq 0$. The objective function is $f(D, z; k) = \bar{R} + \bar{c} - D - F(z|s)(R - D + \bar{c})$.

Suppose that both constraints are slack. Then there exists $z' \in (z_a, z^*)$ such that one of the constraints is binding. The new contract $(D, [z_a, z'])$ increases the firm's payoff strictly.

Suppose that the break-even constraint is slack and the incentive constraint holds. Let $\hat{D}(z) = (I - F(z|s)R)/(1 - F(z|s))$. Set $D' = \hat{D}(z^*) < D$. Clearly, $M(D', z^*; k) = 0$, $N(D', z^*; k) > 0$ and $f(D', z^*; k) > f(D, z^*; k)$.

Suppose that the incentive constraint is slack, but the break-even constraint is binding. It can be shown that for z small enough, $N(\hat{D}(z), z; k)$ is strictly decreasing in z and for some $z' < z^*$, $N(\hat{D}(z'), z'; k) = 0$. Since $D^* = \hat{D}(z^*)$,

$$
f(\hat{D}(z'), z'; k) = \bar{R} + \bar{c} - I - F(z'|s)\bar{c} > \bar{R} + \bar{c} - I - F(z'|s)\bar{c} = f(D^*, z^*; k).
$$

∎

Proof of Lemma A.2. We will ignore the constraint $R - D \geq \gamma$ in the first step of the proof. First, we show that a maximum exists. Clearly, the payoff from any contract that satisfies (A-2) and (A-3) is bounded by above from $\bar{R} + \bar{c} - I$. Suppose that the constraint set is nonempty. Let the sup of the payoffs be B. Let (z_n, D_n) be a sequence of contracts that satisfies the constraints (A-2) and (A-3) and the payoff of contract n is larger than $B - 1/n$. By Lemma H.2, there exist contracts $(\hat{D}(z_n'), z_n')$ that also satisfy (A-2) and (A-3), the break-even constraint is binding,

and their payoff is larger than $B - 1/n$. Since the sequence z'_n lies in a compact set, there exists a subsequence z'_{n_k} that converges to some z^*. Then by continuity $\hat{D}(z'_{n_k})$ converge to $\hat{D}(z^*)$, and the contract $(\hat{D}(z^*), z^*)$ satisfies (A-2) and (A-3) and has payoff B. Thus a maximum exists.

The rest of the lemma is implied by the fact that a maximum exists and by Lemma H.2. Finally, we need to check that at the optimum $R - D \geq \gamma$. But since the break-even constraint is binding, $D = [I - F(z|s)(R - \gamma)]/[1 - F(z|s)] \leq I$. Then $\gamma \leq R - I \leq R - D$. $\blacksquare$

Lemma H.3 *For any contract satisfying constraints (A-6) and (A-7) and at least one of the constraints slack, there exists a contract with $A = [z_a, z^*]$, both constraints are binding and the firm's payoff is strictly larger.*

Proof. By Lemma 1, we can set $A = [z_a, z]$, which tightens the incentive constraint without affecting the payoff of the firm.

If $z = z_a$, or $z = z_b$, $Prob([z_a, z]|r) - Prob([z_a, z]|s) = F(z|r) - F(z|s) = 0$, or the incentive constraint will not be satisfied. Therefore $z \in (z_a, z_b)$.

Let $M(D, z; k) = D + F(z|s)\Delta(D, k) - I$ and $N(D, z; k) = (F(z|r) - F(z|s))(\Delta(D, k) + \int_{c_a}^{k(R-D)/\beta} ch(c)dc) - x$. These functions evaluate the break-even and the incentive constraints. Also let $f(D, z; k) = \bar{R} + \bar{c} - D - F(z|s)(\Delta(D, k) + \int_{c_a}^{k(R-D)/\beta} ch(c)dc$ be the firm's payoff.

Suppose that both constraints are slack. Then there exists $z' \in (z_a, z^*)$ such that one of the constraints is binding. The new contract $(D, [z_a, z'])$ increases the firm's payoff strictly.

Suppose that the incentive constraint is slack, but the break-even constraint is binding, i.e. $M(D^*, z^*) = 0$. Let $\hat{D}(z)$ be an implicit function, given by $M(\hat{D}(z), z; k) = 0$. By Assumption 2, $\hat{D}(z)$ is well-defined and decreasing in z. It can be shown that for z small enough, $N(\hat{D}(z), z; k)$ is strictly decreasing in z and for some $z' < z^*$, $N(\hat{D}(z'), z'; k) = 0$. Since $D^* = \hat{D}(z^*)$, we see that

$$\bar{R} + \bar{c} - \hat{D}(z') - F(z'|s)\Delta(\hat{D}(z'), k) - F(z'|s)\int_{z_a}^{k(R-\hat{D}(z'))/\beta} ch(c)dc >$$
$$\bar{R} + \bar{c} - I - F(z^*|s)\int_{z_a}^{k(R-\hat{D}(z^*))/\beta} ch(c)dc =$$
$$\bar{R} + \bar{c} - \hat{D}(z^*) - F(z^*|s)\Delta(\hat{D}(z^*), k) - F(z^*|s)\int_{z_a}^{k(R-\hat{D}(z^*))/\beta} ch(c)dc,$$

so the payoff of $(\hat{D}(z'), z')$ is strictly higher.

Now suppose that the break-even constraint is slack. By the same reasoning as above, there exists a strictly decreasing continuous function $\tilde{D}(z)$ such that $N(\tilde{D}(z), z; k) = 0$ and $D^* = \tilde{D}(z^*)$. Then there exists $z' < z^*$ such that $M(\tilde{D}(z'), z'; k) = 0$. Since $N(\tilde{D}(z), z; k) = 0$, we know that $\int_{c_a}^{k(R-\tilde{D}(z))/\beta} ch(c)dc = x/[F(z|r) - F(z|s)] -$

$\Delta(\tilde{D}(z), z)$. Then we have that

$$\bar{R} + \bar{c} - \tilde{D}(z') - F(z'|s)\Delta(\tilde{D}(z'), k) - F(z'|s)\int_{z_a}^{k(R-\tilde{D}(z'))/\beta} ch(c)dc =$$

$$\bar{R} + \bar{c} - \tilde{D}(z') - F(z'|s)\frac{x}{F(z'|r) - F(z'|s)} >$$

$$\bar{R} + \bar{c} - \tilde{D}(z^*) - F(z^*|s)\frac{x}{F(z^*|r) - F(z^*|s)} =$$

$$\bar{R} + \bar{c} - \tilde{D}(z^*) - F(z^*|s)\Delta(\tilde{D}(z^*), k) - F(z^*|s)\int_{z_a}^{k(R-\tilde{D}(z^*))/\beta} ch(c)dc,$$

where we used the fact that $\tilde{D}(z)$ and $F(z|s)/[F(z|r) - F(z|s)]$ are strictly decreasing functions (lemma H.1). Therefore the payoff of $(\hat{D}(z'), z')$ is strictly higher. ∎

Proof of Lemma A.3. By the same proof as for Lemma A.2, we establish the fact that a maximum exists. The rest of the lemma is implied by the fact that a maximum exists and by Lemma H.3. The last claim follows from the fact that the break-even constraint is binding and Assumption 2. ∎

Proof of Theorem 1. Let $B_2 = \{(D, z, k) : \text{constraints } (A-6) \text{ and } (A-7) \text{ are satisfied.}\}$. By feasibility $z \in [z_a, z_b]$, $k \in [0, 1]$, and by Lemma H.3, $D \in [0, R]$.

Suppose that B_2 is nonempty. B_2 is bounded. Moreover, since the constraints are continuous, B_2 is closed and hence compact. Then by the Weierstrass extreme value theorem there exists (D_*, z_*, k_*) such that $k_* \le k$ for all k such that $(D, z, k) \in B_2$. Since for all $(D, z, k) \in B_2$ $k > 0$, therefore $k_* > 0$. If $B_2 = \emptyset$, then set $k_* = 1$. Then for all $k \in [0, k_*)$, no contract with covenant is feasible.

Since, $\partial M(D, z, k)/\partial k \ge 0$ and $\partial N(D, z, k)/\partial k \ge 0$, increasing k relaxes the constraints, so if bank control is feasible for some k, it is feasible for all $k' \ge k$.

Let $(\hat{D}, \hat{z})$ be the optimal rectangular contract. Define $\bar{k} = \beta c_b/(R - \hat{D})$. Clearly, for all $k \ge \bar{k}$, $(D^*(k), z^*(k)) = (\hat{D}, \hat{z})$ and for all $k_* \le k < \bar{k}$, $k(R - D^*) < \beta c_b$.

Let $k_* \le k_1 < k_2 \le k^*$ and let (D_i^*, z_i^*) be the corresponding optimal contract. From lemma A.3, $D_i^* = D(z_i^*, k_i)$ and z_i^* is the smallest z such that $M(D(z, k_i), z; k_i) \ge 0$. By the implicit function theorem, $\partial D(z, k)/\partial k > 0$. Since M is increasing in D (Assumption 2), z and k, it follows that $z_1^* > z_2^*$.

Finally, by the proof of lemma A.1, the firm's payoff is increasing in k if the bank is in control and is constant in k if the institution is in control. Therefore there exists some $\underline{k} \ge k_*$ such that the firm's payoff is higher with bank control if and only if $k \ge \underline{k}$. ∎

Proof of Proposition 1. As we have shown in the proof of Theorem 1, if $k \ge \bar{k}$, the optimal contract $(D^*(k), z^*(k)) = (\hat{D}, \hat{z})$, where $(\hat{D}, \hat{z})$ is the optimal rectangular contract.

Suppose that $\underline{k} \leq k_1 < k_2 \leq \bar{k}$. We want to show that $D(k_1) < D(k_2)$. Suppose not: $D(k_1) \geq D(k_2)$. Therefore:

$$
\begin{aligned}
D(k_1) + F(z(k_1)|s)\Delta(D(k_1), k_1) \quad &> \quad D(k_1) + F(z(k_2)|s)\Delta(D(k_1), k_1) \\
&\geq \quad D(k_2) + F(z(k_2)|s)\Delta(D(k_2), k_1) \\
&\geq \quad D(k_2) + F(z(k_2)|s)\Delta(D(k_2), k_2) = I,
\end{aligned}
$$

where we used the fact that $z(k)$ is strictly decreasing on $[\underline{k}, \bar{k}]$, Assumption 2 and the fact that all the constraints bind at the optimal contract (Lemma A.3). Then the break-even constraint is slack for $(D(k_1), z(k_1))$, which is a contradiction. ∎

Proof of Proposition 2. Suppose that for some k and $\beta_1 > 0$, the optimal contract $(D(k), z(k))$ is with covenants and bank control. Then for all $\beta_2 < \beta_1$, the contract $(D(k), z(k))$ is feasible and all the constraints are slack. Then by Lemma H.3 and A.1 there exists a feasible contract that gives the firm strictly higher payoff than under $(D(k), z(k))$ and $\beta = \beta_1$. This implies that for a fixed k, the set of β such that investor control or no covenants is preferred has the form $(\bar{\beta}, 1]$.

Let $\beta = 1$. By Theorem 1, there exists some $\underline{k} > 0$ such that the contract is without bank control if $k < \underline{k}$. Then for all $k < \underline{k}$, $\bar{\beta} < 1$. ∎

Proof of Lemma E.1. Any feasible contract for $a = r$ is weakly dominated by the contract $k = 0$, $z = z_a$, $D = I(1 + i_f) + y$. Then this is an optimal contract, conditional on $a = r$.

Let $B = \{(D, z, k) :$ constraints $E - 1, E - 2$ and 2 are satisfied$\}$. As in the proof of Proposition 1, we can show that B is compact. Therefore there exists a contract that maximizes the firm's value if $a = s$ and banks are in control.

Similarly, let $B_1 = \{(D, z, k) :$ constraints $E - 3, E - 4$ and 2 are satisfied.$\}$ As in the proof of proposition 1, we can show that B_1 is compact. Therefore there exists a contract that maximizes the firm's value conditional on $a = s$ and institutions are in control.

Comparing the three contracts, we can find the optimal contract.

Suppose that the optimal contract is with bank control. If $M = 0$ or $M = I$, then the constraints must be binding by the argument in lemma H.3.

Suppose that both (E-1) and (E-2) are slack. Then by the same variation as described in the proof of Lemma H.3, we can increase the firm's objective function, which is a contradiction.

Suppose that (E-1) is slack and (E-2) is binding. Then it is feasible to increase M marginally, without affecting the firm's value. This will make both constraints slack, which is a contradiction. The case when (E-2) is slack, but (E-1) is binding, is analogous.

The case when institutions are in control is analogous. ∎

Proof of Theorem 2. Let $\underline{k}(i_b)$ be the smallest k such that a contract with bank control exists and $\bar{k}(i_b)$ be the largest k such that a contract with bank control exists. It is immediate that $\underline{k}(i_b)$ $(\bar{k}(i_b))$ exist and are weakly increasing (decreasing) in i_b and for any $k \in [\underline{k}(i_b), \bar{k}(i_b)]$, a feasible contract with bank control exists.

Let $v_b(k, i_b)$ be the firm's payoff if the bank is in control, has share k and its interest rate is i_b. Let $v_b(i_b) = \max_{k \in [\underline{k}(i_b), \bar{k}(i_b)]}\{v_b(k, i_b)\}$.

Suppose that $i_b < i_b'$, $k' \in [\underline{k}(i_b'), \bar{k}(i_b')]$. Let (D, z) be the optimal contract for k' and i_b'. Then (D, z) is feasible for k' and i_b and the break-even constraint is slack. Then by Lemma H.3, there exists (D^*, z^*) that is feasible and has strictly higher payoff. Therefore $v_b(i_b) \geq v_b(k', i_b) > v_b(k', i_b') = v_b(i_b')$. So v_b is strictly decreasing in i_b.

Let $v_n(k, i_b)$ be the firm's payoff in the covenant-lite case and $v_f(k, i_b)$ be the firm's payoff if institutions are in control; $v_n(i_b)$ and $v_f(i_b)$ are defined similarly to $v_b(i_b)$. Clearly, $v_n(i_b) = v_n(0, i_b)$ is independent of i_b and similarly for $v_f(i_b)$.

Bank control is chosen if $v_b(i_b) > \max\{v_n(i_b), v_f(i_b)\}$. Since $v_b(i_b)$ is strictly decreasing in i_b the conclusion of the theorem follows. ∎

Proof of Proposition F.1. We start with the first statement. We will show that this contract maximizes the firm's payoff subject to incentive and break-even constraints. Clearly, this contract induces action s, breaks even and is the best amongst contracts with these properties. The payoff of $D^*(s)$ is $R_m + \bar{c} - I - p(s)[\delta + \bar{c}]$. For any contract that induces $a = r$ and breaks even, the firm payoff is at most $R_m + \bar{c} - I - p(r)[\delta + \bar{c}] < R_m + \bar{c} - I - p(s)[\delta + \bar{c}]$. Then the contract $D^*(s)$ maximizes the firm's payoff.

If $D^*(s) > R_m - \Delta + \bar{c}$, then there exists no contract that breaks even and induces action s. Clearly, out of all the contracts that induce action r and break even, $D^*(r)$ maximizes the firm's payoff. ∎

Proof of Proposition F.2. Since (F-3) is linear in D, for any $z < z_b$, there exists some $D(z)$ such that (F-3) is binding. $D(z)$ is continuous, strictly decreasing, $D(z_a) = D^*(s)$ and $\lim_{z \to z_b} D(z_b) = -\infty$. The break-even constraint (for $z < z_b$) is equivalent to $D \geq D(z)$.

First, we show that $z = z_b$ is not optimal. Let $\hat{z}$ be defined (uniquely) by $D(\hat{z}) = 0$. Direct evaluation shows that the contract $(\hat{z}, 0)$ satisfies all the constraints and has a better payoff than (z_b, D).

Second, we show that the break-even constraint is binding. Define $h(z, D)$ by

$$
\begin{aligned}
h(z, D) &\equiv (F(\hat{z}|r) - F(\hat{z}|s))[(1 - p(r))(R_m - D + \bar{c}) + p(r)\Delta] \\
&\quad - (1 - F(\hat{z}|s))(p(r) - p(s))[D - (R_m - \Delta) - \bar{c}]
\end{aligned}
$$

The incentive constraint is equivalent to $h(z, D) \geq 0$; h is decreasing in D. We know that $z < z_b$. Suppose that $D > D(z)$. Then lowering D to $D(z)$ tightens the incentive constraint and increases the payoff of the firm.

Finally, we show that the incentive constraint is binding and that an optimal contract exists. Define $v(z) \equiv h(z, D(z))$. For a contract of the form $(z, D(z))$, the incentive constraint is equivalent to $v(z) \geq 0$. Let $M = \{z \in [z_a, z_b) : v(z) \geq 0\}$. This is the set of values of z, for which feasible contracts exist. Clearly, $M \neq \emptyset$. By direct evaluation, we see that the payoff of the contract $(z, D(z))$ is strictly decreasing in z. Then showing that an optimal contract exists is equivalent to showing that $z^* = \min M$ exists; showing that the incentive constraint binds is equivalent to showing that $v(z^*) = 0$.

Define $M' = \{z \in [z_a, \hat{z}] : v(z) \geq 0\}$. By construction, $v(z) \geq 0$ for all $z \geq \hat{z}$, so $\inf M' = \inf M$. Since v is continuous, then M' is compact, and $\min M'$ exists. Since $M' \subseteq M$, then $z^* = \min M$ exists.

Finally, suppose that $v(z^*) > 0$. Since $v(z_a) < 0$ and v is continuous, there exists some $z' < z^*$ such that $v(z') = 0$, which is a contradiction. ∎

Proof of Lemma F.1. Let $A = \{(z, D) : \text{(F-2) and (F-3) hold}\}$. Suppose $A \neq \emptyset$. The constraint set is nonempty. Since the constraint functions are continuous, A is closed. Since $A \subseteq [z_a, z_b] \times [0, R_m]$ is bounded, it is compact, so an optimal contract exists. Next, we show that both constraints bind. Since an optimal contract exists, it is sufficient to show that if one or both of the constraints are slack it would be possible to improve the contract.

Let $m(D, k) = H(\tilde{c}(D, k))[(1 - p(s)(R_m - D) + p(s)\Delta]$ and $ce(D, k) = \int_0^{\tilde{c}(D,k)} ch(c)dc$. Then the incentive constraint is

$$
\begin{aligned}
(F(z|r) - F(z|s))(m(D, k) + ce(D, k)) \geq\ & (p(r) - p(s))\{F(\tilde{z}|r)H(\tilde{c}(D, k))[D + \Delta - R_m] \\
& - F(\tilde{z}|r)ce(D, k) + [D - (R_m - \Delta) - \bar{c}]\}.
\end{aligned}
$$

The break-even constraint is:

$$
p(s)(R_m - \Delta) + (1 - p(s))D + F(\hat{z}|s)m(D, k) \geq I + p(s)\delta + F(\hat{z}|s)H(\tilde{c}(D, k))(1 - p(s))\delta.
$$

The firm's payoff is:

$$
\pi = R_m + \bar{c} - p(s)(R_m - \Delta) - (1 - p(s))D - F(z|s)[m(D, k) + ce(D, k)].
$$

First, suppose that both constraints are slack; $z = z_a$ violates (F-2), so it is feasible to reduce z and keep both constraints satisfied. Then by inspection we see that the firm's payoff is strictly increased.

Next, suppose that the incentive constraint is binding, but the break-even constraint is slack. Similar to the case in the basic model, there exist (z', D'), $z' < z^*$, $D' < D^*$ such that both constraints bind. Assumption F.1 implies that $(p(r) - p(s))\{F(\tilde{z}|r)H(\tilde{c}(D, k))[D + \Delta - R_m] - F(\tilde{z}|r)ce(D, k) + [D - (R_m - \Delta) - \bar{c}]\}$ is lower evaluated at (z', D') than at (z^*, D^*). Then since the incentive constraint is binding at (z^*, D^*) and (z', D')

$$(F(z'|r) - F(z'|s))(m(D', k) + ce(D', k)) < (F(z^*|r) - F(z^*|s))(m(D^*, k) + ce(D^*, k)).$$

Then the fact that $F(z|s)/(F(z|r) - F(z|s))$ is increasing implies that

$$F(z'|s)(m(D', k) + ce(D', k)) < F(z^*|s)(m(D^*, k) + ce(D^*, k)).$$

The inequality above and the fact that $D' < D$ implies that the firm's payoff is higher for (z', D').

Finally suppose that the incentive constraint is slack, but the break-even constraint is binding. Again as described in the main body of the paper there exist (z', D'), $z' < z^*$, $D' > D^*$ such that both constraints bind. Then we have

$$p(s)(R_m - \Delta) + (1 - p(s))D^* + F(z^*|s)m(D^*, k) =$$
$$I + p(s)\delta + F(z|s)H(\tilde{c}(D^*, k))(1 - p(s))\delta >$$
$$I + p(s)\delta + F(z|s)H(\tilde{c}(D^*, k))(1 - p(s))\delta =$$
$$p(s)(R_m - \Delta) + (1 - p(s))D' + F(z'|s)m(D', k).$$

Also $F(z'|s)ce(D', k) < F(z^*|s)ce(D^*, k)$. So, by plugging in the firm's payoff function, we see that (z', D') gives the firm a strictly better payoff. ∎

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