Internet Appendix for
"Time-Varying Margin Requirements
and Optimal Portfolio Choice"

This Appendix describes the implementation of the projection method that I use to solve the nonlinear differential equations for $H(v)$ stated in Proposition 1. The key idea of the method is to represent solution as a linear combination of orthogonal polynomials (Judd (1998)). As an orthogonal basis, I use Chebyshev polynomials of the first kind. Because Chebyshev polynomials form an orthogonal basis in $L^2([-1,1])$, the variable $v \in (0,\infty)$ is rescaled to fit this interval. In particular, I introduce another variable z that is related to v as

$$z = \frac{v - \bar{v}}{v + \bar{v}}. \tag{IA-1}$$

Equation (IA-1) defines a continuous and monotonic map of the interval $(0,\infty)$ into the interval $(-1,1)$ such that the boundary $z = -1$ corresponds to $v = 0$ and z approaches 1 as $v \to \infty$. The middle of the interval $z = 0$ corresponds to $v = \bar{v}$. The inverse map is

$$v = \bar{v}\frac{1+z}{1-z}.$$

The change in the variables affects the derivatives appearing in the differential equation. The derivatives with respect to v and z are related as

$$\frac{\partial}{\partial v} = \frac{(1-z)^2}{2\bar{v}}\frac{\partial}{\partial z}, \qquad \frac{\partial^2}{\partial v^2} = \frac{(1-z)^4}{4\bar{v}^2}\frac{\partial^2}{\partial z^2} - \frac{(1-z)^3}{2\bar{v}^2}\frac{\partial}{\partial z}.$$

After applying the transformation (IA-1), the function $H(z)$ may not be in $L^2([-1,1])$ because $|H(z)|$ grows too fast as $z \to 1$. To address this issue, I look for $H(z)$ in the form $H(z) = H_0(z) + \tilde{H}(z)$, where $H_0(z)$ is a chosen function and $\tilde{H}(z) \in L^2([-1,1])$. The function $H_0(z)$ is determined by the asymptotic behavior of $H(z)$ as $z \to 1$ ($v \to \infty$). When volatility is large, margin requirements are very tight and $\omega^*(v) = 1$. In the case $\psi = 1$, equation (12) with $\omega^*(v) = 1$ has a closed-form solution $H_0(v) = A_0 + B_0 v$, where the constants A_0 and B_0 are determined by the following equations:

$$\frac{1}{2}\bar{\sigma}_v^2 B_0^2 - B_0\left(\phi_v + \frac{Q_v\lambda}{\bar{v}}\right) + (1-\gamma)\left(\frac{\mu_S - \lambda Q_S}{\bar{v}} + B_0\bar{\sigma}_v\rho_{vS} - \frac{\gamma}{2}\right) - \beta B_0 + \frac{\lambda}{\bar{v}}\left[(1+Q_S)^{1-\gamma}\exp(B_0 Q_v) - 1\right] = 0,$$

$$A_0 = \frac{1}{\beta}(B_0\phi_v\bar{v} + (1-\gamma)(r - \beta + \beta\log(\beta))).$$

Thus, it is natural to set $H_0(z) = A_0 + B_0\bar{v}(1+z)/(1-z)$, and this choice ensures that $\tilde{H}(z) \to 0$ as $z \to 1$. In the case $\psi \neq 1$, equation (9) does not have an analytical solution even when $\omega^*(v) = 1$ and I use the technique developed in Zhou and Zhu (2012) to find an approximation for $H_0(z)$, which also appears to have a linear form.

Plugging $H(z) = H_0(z) + \tilde{H}(z)$ into equation (12) (or equation (9) when $\psi \neq 1$), I obtain a differential equation for $\tilde{H}(z)$, which is solved by the projection method. In particular, I look for a solution $\tilde{H}(z)$ in the following form:

(IA-2) $$\tilde{H}(z) = \sum_{j=0}^{N} a_j T_j(z),$$

where $\{T_j(z),\ j = 0, \ldots, N\}$ are Chebyshev polynomials of the first kind and N denotes their highest order. The projection method prescribes to find the coefficients a_j such that $\tilde{H}(z)$

minimizes the deviation of the left hand side of the differential equation from zero. The deviation is measured as a sum of squared errors computed at the points of a uniform grid $\{z_l, l = 0, \ldots, L\}$, where $L > N$. Because the number of the points exceeds the polynomial degree, the solution to the differential equation exists if the value of the minimized deviation is close to zero. In the practical implementation of the projection method, I use $L = 200$ and $N = 50$. For these parameters, the error of approximation is typically of order 10^{-7} and 10^{-6} in the models without and with jumps, respectively. Moreover, the results are stable with respect to the polynomial degree and the number of the points in the grid. This provides additional evidence that the numerical approximation converges to the exact solution $\tilde{H}(z)$.

References

Judd, K. L. *Numerical Methods in Economics.* Cambridge, MA: MIT Press (1998).

Zhou, G., and Y. Zhu. "Volatility Trading: What Is the Role of the Long-Run Volatility Component?" *Journal of Financial and Quantitative Analysis*, 47 (2012), 273–307.