

Real Economic Shocks and Sovereign Credit Risk

Internet Appendix

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Abstract

We provide new empirical evidence that U.S. expected growth and consumption volatility are closely related to the strong co-movement in sovereign spreads. We rationalize these findings in an equilibrium model with recursive utility for CDS spreads. The framework nests a reduced-form default process with country-specific sensitivity to expected growth and macroeconomic uncertainty. Exploiting the high-frequency information in the CDS term structure across 38 countries, we estimate the model and find parameters consistent with preference for early resolution of uncertainty. Our results confirm the existence of time-varying risk premia in sovereign spreads as compensation for exposure to common U.S. macroeconomic risk.

Keywords: CDS, Generalized Disappointment Aversion, Sovereign Risk, Term Structure

JEL Classification: C5, E44, F30, G12, G15

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A-I Introduction

This Internet Appendix contains the mathematical derivations relating to the model developed in the paper *Real Economic Shocks and Sovereign Credit Risk*, as well as additional empirical tests.

Section A-II reports the main model derivations pertaining to the benchmark credit default swap (CDS) model with recursive preferences and a long-run risk economy. In section A-III, we extend the model derivations to accommodate preferences with generalized disappointment aversion (GDA) as in Routledge and Zin (2010). Section A-IV extends the model to a hazard rate with country-specific shocks. In section A-V, we perform a robustness check of the empirical results by running all regressions from the main paper in first differences rather than in levels. In section A-VI, we test whether alternative volatility specifications affect the benchmark regressions.

A-II A CDS Model With Recursive Utility

Section A-II.A reports the derivation for the utility-consumption ratios, the price-consumption ratio, the risk-free rate and the stochastic discount factor. Section A-II.B and A-II.C derive the formulas for, respectively, physical and risk-neutral cumulative default probabilities. In section A-II.D, we derive closed-form solutions for credit default swap (CDS) spreads. Section A-II.E describes the General Method of Moments (GMM) estimation procedure.

A-II.A Asset Prices and the Stochastic Discount Factor

The Markov chain s_t or ζ_t is stationary with ergodic distribution and moments given by

$$(A-1) \quad E[\zeta_t] = \Pi \in \mathbb{R}_+^N, \quad E[\zeta_t \zeta_t^\top] = Diag(\Pi_1, \dots, \Pi_N) \text{ and } Var[\zeta_t] = E[\zeta_t \zeta_t^\top] - \Pi \Pi^\top,$$

where $\text{Diag}(u_1, \dots, u_N)$ is the $N \times N$ diagonal matrix whose diagonal elements are u_1, \dots, u_N , and where Π is the vector of unconditional probabilities of regimes.

To obtain analytic solutions for asset prices such as the price-consumption ratio $P_{c,t}/C_t$ ($P_{c,t}$ is the price of the unobservable portfolio that pays off consumption) and the risk-free return $R_{f,t+1}$, we need expressions for $\mathcal{R}_t(V_{t+1})/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and for V_t/C_t , the ratio of lifetime utility to current consumption. The Markov property of the model is crucial for deriving analytical formulas for these expressions and we adopt the following notations:

$$(A-2) \quad \frac{\mathcal{R}_t(V_{t+1})}{C_t} = \lambda_z^\top \zeta_t, \quad \frac{V_t}{C_t} = \lambda_v^\top \zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_c^\top \zeta_t \quad \text{and} \quad R_{f,t+1} = \frac{1}{\lambda_{1f}^\top \zeta_t}.$$

Solving these ratios amounts to characterizing the vectors λ_z , λ_v , λ_c and λ_{1f} as functions of the parameters of the consumption dynamics and of the recursive utility function. We provide expressions for these ratios below and refer to Bonomo, Garcia, Meddahi, and Tédongap (2011) for formal proofs.

Proposition A-II.1 *Characterization of the Ratios of Utility to Consumption.*

Denote by

$$\frac{\mathcal{R}_t(V_{t+1})}{C_t} = \lambda_z^\top \zeta_t \quad \text{and} \quad \frac{V_t}{C_t} = \lambda_v^\top \zeta_t$$

respectively the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. The components of the vectors λ_z and λ_v are given by

$$(A-3) \quad \lambda_{z,i} = \exp \left(\mu_{g,i} + \frac{1-\gamma}{2} \omega_{g,i} \right) \left(\sum_{j=1}^N p_{ij} \lambda_{v,j}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

$$(A-4) \quad \lambda_{v,i} = \left\{ (1-\delta) + \delta \lambda_{z,i}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \text{ and } \lambda_{v,i} = \lambda_{z,i}^\delta \text{ if } \psi = 1,$$

where the components of the matrix $P^\top = [p_{ij}]_{1 \leq i,j \leq N}$ in (A-3) are defined in equation (9)

in the main manuscript.

Proposition A-II.2 Characterization of Basic Asset Prices. Let respectively be

$$\frac{P_{c,t}}{C_t} = \lambda_c^\top \zeta_t \quad \text{and} \quad R_{f,t+1} = \frac{1}{\lambda_{1f}^\top \zeta_t}$$

the price-consumption ratio and the risk-free rate. The components of the vectors λ_c and λ_{1f} are given by

$$(A-5) \quad \lambda_{c,i} = \delta \left(\frac{1}{\lambda_{z,i}} \right)^{\frac{1}{\psi}-\gamma} \exp \left(\mu_{gg,i} + \frac{\omega_{gg,i}}{2} \right) \left(\lambda_v^{\frac{1}{\psi}-\gamma} \right)^\top P \left(Id - \delta A \left(\mu_{gg} + \frac{\omega_{gg}}{2} \right) \right)^{-1} e_i$$

$$(A-6) \quad \lambda_{1f,i} = \frac{1}{\lambda_{2f,i}} = \delta \exp \left(-\gamma \mu_{g,i} + \frac{\gamma^2}{2} \omega_{g,i} \right) \sum_{j=1}^N p_{ij} \left(\frac{\lambda_{v,j}}{\lambda_{z,i}} \right)^{\frac{1}{\psi}-\gamma}$$

where $\mu_{gg} = (1-\gamma)\mu_g$, $\omega_{gg} = (1-\gamma)^2\omega_g$, where the matrix function $A(u)$ in (A-5) is defined by

$$(A-7) \quad A(u) = Diag \left(\left(\frac{\lambda_{v,1}}{\lambda_{z,1}} \right)^{\frac{1}{\psi}-\gamma} \exp(u_1), \dots, \left(\frac{\lambda_{v,N}}{\lambda_{z,N}} \right)^{\frac{1}{\psi}-\gamma} \exp(u_N) \right) P.$$

Proposition A-II.3 Characterization of the Stochastic Discount Factor. Based on the dynamics of equation (10) in the main manuscript, and using the Euler condition for the claim to aggregate consumption, it can be shown that the stochastic discount factor from equation (12) may be expressed as follows:

$$(A-8) \quad M_{t,t+1} = \exp \left(\zeta_t^\top A \zeta_{t+1} - \gamma g_{t+1} \right),$$

where the components of the $N \times N$ matrix A are given by:

$$(A-9) \quad \begin{aligned} a_{ij} &= \ln \delta + \left(\frac{1}{\psi} - \gamma \right) b_{ij} \\ b_{ij} &= \ln \left(\frac{\lambda_{v,j}}{\lambda_{z,i}} \right). \end{aligned}$$

Observe that the vectors λ_z and λ_v characterize the welfare valuation ratios, for which explicit expressions are provided in equations (A-3) and (A-4).

A-II.B Cumulative Default Probabilities

The time t probability of defaulting between time $t + 1$ and T conditional on not having defaulted prior to $t + 1$, formally $Prob_t(t < \tau \leq T | \tau > t)$, is given by

$$(A-10) \quad Prob_t(t < \tau \leq T | \tau > t) = \frac{Prob_t(t < \tau \leq T)}{Prob_t(\tau > t)} = 1 - E_t \left[\frac{S_T}{S_t} \right].$$

Given the conjecture

$$(A-11) \quad E_t \left[\frac{S_{t+j}}{S_t} \right] = \tilde{\Psi}_j^\top \zeta_t,$$

it can be shown that the solution sequence $\{\tilde{\Psi}_j\}$ satisfies the recursion

$$(A-12) \quad \tilde{\Psi}_j^\top \zeta_t = E_t \left[(1 - h_{t+1}) (\tilde{\Psi}_{j-1}^\top \zeta_{t+1}) \right]$$

with the initial condition $\tilde{\Psi}_0 = e$, and it follows that

$$(A-13) \quad \tilde{\Psi}_j = P^\top \left(\tilde{\Psi}_{j-1} \odot \frac{1}{1 + \lambda} \right).$$

The conditional and unconditional cumulative default probabilities are thus given by

$$(A-14) \quad Prob_t(t < \tau \leq T | \tau > t) = 1 - \left(\tilde{\Psi}_{T-t}^\top \zeta_t \right) \quad \text{and} \quad Prob(t < \tau \leq T | \tau > t) = 1 - \left(\tilde{\Psi}_{T-t}^\top \Pi \right),$$

where the latter simplifies in case of a constant default process to

$$(A-15) \quad Prob(t < \tau \leq T | \tau > t) = 1 - \exp(-\lambda(T - t)) \quad \text{where} \quad \lambda = \exp(\beta_{\lambda 0}).$$

A-II.C Risk-Neutral Cumulative Default Probabilities

We denote probabilities under the risk-neutral measure with the \mathbb{Q} subscript and we define $Z_{t,t+1} = M_{t,t+1}R_{f,t+1}$. The T -year conditional cumulative default probability under the risk-neutral measure is defined by

$$(A-16) \quad \text{Prob}_t^{\mathbb{Q}} [t < \tau \leq T \mid \tau > t]$$

and can be rewritten as

$$(A-17) \quad \begin{aligned} \text{Prob}_t^{\mathbb{Q}} [t < \tau \leq T \mid \tau > t] &= \frac{\text{Prob}_t^{\mathbb{Q}} (\tau > t) - \text{Prob}_t^{\mathbb{Q}} (\tau > T)}{\text{Prob}_t^{\mathbb{Q}} (\tau > t)} = 1 - \frac{\text{Prob}_t^{\mathbb{Q}} (\tau > T)}{\text{Prob}_t^{\mathbb{Q}} (\tau > t)} \\ &= 1 - E_t^{\mathbb{Q}} \left[\frac{S_T}{S_t} \right] = 1 - E_t \left[Z_{t,T} \frac{S_T}{S_t} \right]. \end{aligned}$$

Given the conjecture

$$(A-18) \quad E_t \left[Z_{t,t+j} \frac{S_{t+j}}{S_t} \right] = \left(\tilde{\Psi}_j^{\mathbb{Q}} \right)^{\top} \zeta_t,$$

it turns out the sequence $\left\{ \tilde{\Psi}_j^{\mathbb{Q}} \right\}$ satisfies the recursion

$$(A-19) \quad \left(\tilde{\Psi}_j^{\mathbb{Q}} \right)^{\top} \zeta_t = E_t \left[Z_{t,t+1} (1 - h_{t+1}) \left(\left(\tilde{\Psi}_{j-1}^{\mathbb{Q}} \right)^{\top} \zeta_{t+1} \right) \right]$$

with the initial condition $\tilde{\Psi}_0^{\mathbb{Q}} = e$, and it follows that

$$(A-20) \quad \tilde{\Psi}_j^{\mathbb{Q}} = \text{diagonal of } \left(\tilde{M} \odot \left(\lambda_{2f} \left(\left(\tilde{\Psi}_{j-1}^{\mathbb{Q}} \right) \odot \frac{1}{1+\lambda} \right)^{\top} \right) \right) P.$$

Thus the $(T - t)$ -year horizon conditional and unconditional cumulative default probability

are

$$(A-21) \quad \begin{aligned} Prob_t^Q [t < \tau \leq T | \tau > t] &= 1 - \left(\left(\tilde{\Psi}_{T-t}^Q \right)^\top \zeta_t \right) \\ Prob^Q [t < \tau \leq T | \tau > t] &= 1 - \left(\left(\tilde{\Psi}_{T-t}^Q \right)^\top \Pi^Q \right). \end{aligned}$$

A-II.D Credit Default Swap Spreads

Recall that the hazard rate h_t and the associated default intensity λ_t are given by

$$(A-22) \quad h_t = \frac{\lambda_t}{1 + \lambda_t} \quad \text{where} \quad \lambda_t = \exp(\beta_{\lambda 0} + \beta_{\lambda x} x_t + \beta_{\lambda \sigma} \sigma_t),$$

and that the loss rate L_t is constant over time. Dividing both the numerator and the denominator of the expression in equation (7) of the main manuscript by S_t , computing the CDS spread is equivalent to deriving the individual expressions

$$(A-23) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right].$$

To compute these expressions, we conjecture that

$$(A-24) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] = (\Psi_j^*)^\top \zeta_t \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right] = (\Psi_j)^\top \zeta_t.$$

Given our conjecture, both sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ satisfy the same recursion

$$(A-25) \quad \begin{aligned} (\Psi_j^*)^\top \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) \left((\Psi_{j-1}^*)^\top \zeta_{t+1} \right) \right] \\ (\Psi_j)^\top \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) \left((\Psi_{j-1})^\top \zeta_{t+1} \right) \right] \end{aligned}$$

but with different initial conditions given by

$$(A-26) \quad (\Psi_1^*)^\top \zeta_t = E_t [M_{t,t+1}] \quad \text{and} \quad (\Psi_0)^\top \zeta_t = 1.$$

It can be shown that

$$(A-27) \quad E_t [M_{t,t+1} \mid \zeta_m, m \in \mathbb{Z}] = \zeta_t^\top \tilde{M} \zeta_{t+1},$$

where the components of the matrix \tilde{M} are given by

$$(A-28) \quad \tilde{m}_{ij} = \exp \left(a_{ij} - \gamma \mu_{g,i} + \frac{1}{2} \gamma^2 \omega_{g,i} \right).$$

It follows that the initial conditions in A-26 are determined by

$$(A-29) \quad \Psi_1^* = \lambda_{1f} \quad \text{and} \quad \Psi_0 = e,$$

where e denotes the $N \times 1$ vector with all components equal to one.

The solution for the recursion (A-25) satisfied by the solution sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ is

$$(A-30) \quad \begin{aligned} \Psi_j^* &= \text{diagonal of } \left(\tilde{M} \odot \left(e \left(\Psi_{j-1}^* \odot \frac{1}{1+\lambda} \right)^\top \right) \right) P \\ \Psi_j &= \text{diagonal of } \left(\tilde{M} \odot \left(e \left(\Psi_{j-1} \odot \frac{1}{1+\lambda} \right)^\top \right) \right) P. \end{aligned}$$

Proposition A-II.4 *Characterization of the Price of the CDS.*

$$(A-31) \quad CDS_t(K) = \lambda_s(K)^\top \zeta_t$$

The components of the vectors $\lambda_s(K)$ are functions of the consumption dynamics and of the recursive utility function defined above, and its components are given by

$$(A-32) \quad \lambda_{i,s}(K) = \frac{\sum_{j=1}^{KJ} L [\Psi_{i,j}^* - \Psi_{i,j}]}{\sum_{k=1}^K \Psi_{i,kJ} + \sum_{j=1}^{KJ} (\frac{j}{J} - \lfloor \frac{j}{J} \rfloor) [\Psi_{i,j}^* - \Psi_{i,j}]},$$

where e is the vector with all components equal to one, L the vector of conditional loss rates, and where the sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ are given by the recursion (A-30), with initial conditions (A-29).

A-II.E Estimation Procedure

We obtain analytical moments of the form

$$(A-33) \quad \mu_{CDS^j}(K, n) = E \left[(CDS_t^j(K))^n \right],$$

$n \in \{1, 2\}$, $K \in \{1Y, 2Y, 3Y, 5Y, 7Y, 10Y\}$, $j \in \{AAA, AA, A, BBB, BB, B\}$. All moments are functions of the parameter vector θ , which contains the preference parameters and the default parameters of the six rating groups. We thus have 72 moments to estimate 23 parameters. Let

$$(A-34) \quad g_t(\theta) = \left[(CDS_t^j(K))^n - \mu_{CDS^j}(K, n) \right]_{j,K,n}$$

denote the 72×1 vectors of the chosen moments. We have $E[g_t(\theta)] = 0$ and we define the sample counterpart of this moment condition as

$$(A-35) \quad \widehat{g}(\theta) = \widehat{E} \left[\left[(CDS_t^j(K))^n - \mu_{CDS^j}(K, n) \right]_{j,K,n} \right].$$

Given the 72×72 matrix \widehat{W} used to weight the moments, the GMM estimator $\widehat{\theta}$ of the parameter vector is given by

$$(A-36) \quad \widehat{\theta} = \arg \min_{\theta} T \left(\widehat{g}(\theta)^{\top} \widehat{W} \widehat{g}(\theta) \right),$$

where T is the sample size. The heteroskedasticity and autocorrelation (HAC) estimator of the variance-covariance matrix of $g_t(\theta)$ is simply that of the variance-covariance matrix of

$$(A-37) \quad \left[(CDS_t^j(K))^n \right]_{j,K,n},$$

which does not depend on the vector of parameters θ . This is an advantage, since with a nonparametric empirical variance-covariance matrix of moment conditions, the optimal GMM procedure can be implemented in one step. It is important to note that two different preference models can be estimated via the same moment conditions and weighting matrix. Only the model-implied moments

$$(A-38) \quad [\mu_{CDS^j}(K, n)]_{j,K,n}$$

differ from one preference model to another in this estimation procedure. In this case, the minimum value of the GMM objective function itself is a criterion for comparison of the alternative preference models, since it represents the distance between the model-implied and actual moments. Moments are weighted using the inverse of the diagonal of their long-run variance-covariance matrix

$$(A-39) \quad \widehat{W} = \left\{ Diag \left(\widehat{Var}[g_t] \right) \right\}^{-1}.$$

This matrix is nonparametric and puts more weight on moments with low magnitude. If the number of moments to match is large, as in our case, then inverting the long-run variance-

covariance matrix of moments will be numerically unstable. Using the inverse of the diagonal instead of the inverse of the long-run variance-covariance matrix itself allows for numerical stability if the number of moments to match is large, as inverting a diagonal matrix is equivalent to taking the diagonal of the inverse of its diagonal elements. The distance to minimize reduces to

$$(A-40) \quad \sum_j \sum_K \sum_n \left(\frac{\widehat{E} \left[(CDS_t^j(K))^n \right] - E \left[(CDS_t^j(K))^n \right]}{\widehat{\sigma} \left[(CDS_t^j(K))^n \right] / \sqrt{T}} \right)^2,$$

where observed moments are denoted with a hat and the model-implied theoretical moment without.

A-III A CDS Model With Generalized Disappointment Aversion

In section A-III.A, we define the preferences and the stochastic discount factor when the agent has GDA preferences. Using this set up, section A-III.B follows with a derivation of the formulas of the stochastic discount factor and basic asset prices. Section A-III.C derives the new valuation for CDS spreads.

A-III.A Preferences and stochastic discount factor

An investor with generalized disappointment aversion (GDA) preferences of Routledge and Zin (2010) derives utility V_t from consumption recursively

$$(A-41) \quad V_t = \begin{cases} (1 - \delta) C_t^{1-\frac{1}{\psi}} + \delta [\mathcal{R}_t(V_{t+1})]^{1-\frac{1}{\psi}} \end{cases}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \\ = C_t^{1-\delta} [\mathcal{R}_t(V_{t+1})]^\delta \quad \text{if } \psi = 1. \end{math>$$

The current period lifetime utility V_t is a combination of current consumption C_t and $\mathcal{R}_t(V_{t+1})$, a certainty equivalent of next period lifetime utility. Both components are weighted by the subjective discount factor δ . The parameter ψ defines the elasticity of intertemporal substitution (EIS), which can be disentangled from the coefficient of relative risk aversion γ through this form of utility. With GDA preferences the risk-adjustment function $\mathcal{R}(.)$ is implicitly defined by

$$(A-42) \quad \frac{\mathcal{R}^{1-\gamma} - 1}{1 - \gamma} = \int_{-\infty}^{\infty} \frac{V^{1-\gamma} - 1}{1 - \gamma} dF(V) - \left(\frac{1}{\alpha} - 1 \right) \int_{-\infty}^{\kappa\mathcal{R}} \left(\frac{(\kappa\mathcal{R})^{1-\gamma} - 1}{1 - \gamma} - \frac{V^{1-\gamma} - 1}{1 - \gamma} \right) dF(V),$$

where $0 < \alpha \leq 1$ and $0 < \kappa \leq 1$. When α is equal to one, the certainty equivalent function \mathcal{R} reduces to the Kreps and Porteus's preferences (Kreps and Porteus (1978)), henceforth KP, where the investor cares only about systematic risk, while V_t represents Epstein and Zin (1989) recursive utility. When $\alpha < 1$, the agent values downside risk and the certainty

equivalent decreases as outcomes below the threshold $\kappa\mathcal{R}$ receive an additional weight. Thus, α characterizes disappointment aversion, while κ reflects the fraction of the certainty equivalent \mathcal{R} below which outcomes become disappointing.¹ Formula (A-42) emphasizes that state-probabilities are redistributed when disappointment kicks in, the threshold of disappointment being endogenously time-varying.

Hansen, Heaton, and Li (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption when preferences are KP as

$$(A-43) \quad M_{t,t+1}^* = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\frac{1}{\psi}-\gamma}.$$

If $\gamma = 1/\psi$, equation (A-43) corresponds to the stochastic discount factor of an investor with time-separable utility and constant relative risk aversion. Alternatively, if $\gamma > 1/\psi$, Bansal and Yaron (2004) for instance show that a premium for long-run consumption risk is added by the ratio of future utility V_{t+1} to its certainty equivalent $\mathcal{R}_t(V_{t+1})$. For GDA preferences, downside consumption risks enter through an additional term capturing disappointment aversion

$$(A-44) \quad M_{t,t+1} = M_{t,t+1}^* \left(\frac{1 + (1/\alpha - 1) I(V_{t+1} < \kappa\mathcal{R}_t(V_{t+1}))}{1 + (1/\alpha - 1) \kappa^{1-\gamma} E_t[I(V_{t+1} < \kappa\mathcal{R}_t(V_{t+1}))]} \right).$$

Hence, disappointing outcomes obtain relatively higher importance and require higher risk premia.

Based on the endowment dynamics and the recursion (A-41), we show in section (A-III.B) that the stochastic discount factor (A-44) may be expressed as

$$(A-45) \quad M_{t,t+1} = \exp \left(\zeta_t^\top A \zeta_{t+1} - \gamma g_{t+1} \right) \left[1 + \left(\frac{1}{\alpha} - 1 \right) I(g_{t+1} < -\zeta_t^\top B \zeta_{t+1} + \ln \kappa) \right],$$

¹The certainty equivalent is decreasing in γ , increasing in α (for $0 < \alpha \leq 1$) and decreasing in κ (for $0 < \kappa \leq 1$). Thus, α and κ characterize also measures of risk aversion, but they are of a different nature than γ .

where the components of the $N \times N$ matrices A and B are described in equation (A-55).

A-III.B Asset Prices and the Stochastic Discount Factor

The Markov chain s_t or ζ_t is stationary with ergodic distribution and moments given by

$$(A-46) \quad E[\zeta_t] = \Pi \in \mathbb{R}_+^N, \quad E[\zeta_t \zeta_t^\top] = \text{Diag}(\Pi_1, \dots, \Pi_N) \quad \text{and} \quad \text{Var}[\zeta_t] = E[\zeta_t \zeta_t^\top] - \Pi \Pi^\top,$$

where $\text{Diag}(u_1, \dots, u_N)$ is the $N \times N$ diagonal matrix whose diagonal elements are u_1, \dots, u_N , and where Π is the vector of unconditional state probabilities.

To obtain analytic solutions for asset prices such as the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off consumption) and the risk-free return $R_{f,t+1}$, we need expressions for $\mathcal{R}_t(V_{t+1})/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and for V_t/C_t , the ratio of lifetime utility to current consumption. The Markov property of the model is crucial for deriving analytical formulas for these expressions and we adopt the following notations:

$$(A-47) \quad \frac{\mathcal{R}_t(V_{t+1})}{C_t} = \lambda_z^\top \zeta_t, \quad \frac{V_t}{C_t} = \lambda_v^\top \zeta_t, \quad \frac{P_{c,t}}{C_t} = \lambda_c^\top \zeta_t \quad \text{and} \quad R_{f,t+1} = \frac{1}{\lambda_{1f}^\top \zeta_t}.$$

Solving these ratios amounts to characterizing the vectors λ_z , λ_v , λ_c and λ_{1f} as functions of the parameters of the consumption dynamics and of the recursive utility function defined above. We here provide expressions for these ratios and refer to Bonomo, Garcia, Meddahi, and Tédongap (2011) for formal proofs.

Proposition A-III.1 *Characterization of the Ratios of Utility to Consumption.*

Denote by

$$\frac{\mathcal{R}_t(V_{t+1})}{C_t} = \lambda_z^\top \zeta_t \quad \text{and} \quad \frac{V_t}{C_t} = \lambda_v^\top \zeta_t$$

respectively the ratio of the certainty equivalent of future lifetime utility to current consumption and the ratio of lifetime utility to consumption. The components of the vectors λ_z and

λ_v are given by

$$(A-48) \quad \lambda_{z,i} = \exp \left(\mu_{g,i} + \frac{1-\gamma}{2} \omega_{g,i} \right) \left(\sum_{j=1}^N p_{ij}^* \lambda_{v,j}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

$$(A-49) \quad \lambda_{v,i} = \left\{ (1-\delta) + \delta \lambda_{z,i}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \text{ if } \psi \neq 1 \text{ and } \lambda_{v,i} = \lambda_{z,i}^\delta \text{ if } \psi = 1,$$

where the components of the matrix $P^{*\top} = [p_{ij}^*]_{1 \leq i,j \leq N}$ in (A-48) and (A-49) are given by

$$(A-50) \quad p_{ij}^* = p_{ij} \frac{1 + \left(\frac{1}{\alpha} - 1\right) \Phi(q_{ij} - (1-\gamma) \sqrt{\omega_{g,i}})}{1 + \left(\frac{1}{\alpha} - 1\right) \kappa^{1-\gamma} \sum_{j=1}^N p_{ij} \Phi(q_{ij})},$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution function and where the expression for the components q_{ij} is given in equation (A-56).

Proposition A-III.2 Characterization of Basic Asset Prices. Let respectively be

$$\frac{P_{c,t}}{C_t} = \lambda_c^\top \zeta_t \quad \text{and} \quad R_{f,t+1} = \frac{1}{\lambda_{1f}^\top \zeta_t}$$

the price-consumption ratio and the risk-free rate. The components of the vectors λ_c and λ_{1f} are given by

$$(A-51) \quad \lambda_{c,i} = \delta \left(\frac{1}{\lambda_{z,i}} \right)^{\frac{1}{\psi}-\gamma} \exp \left(\mu_{gg,i} + \frac{\omega_{gg,i}}{2} \right) \left(\lambda_v^{\frac{1}{\psi}-\gamma} \right)^\top P^* \left(Id - \delta A^* \left(\mu_{gg} + \frac{\omega_{gg}}{2} \right) \right)^{-1} e_i$$

$$(A-52) \quad \lambda_{1f,i} = \frac{1}{\lambda_{2f,i}} = \delta \exp \left(-\gamma \mu_{g,i} + \frac{\gamma^2}{2} \omega_{g,i} \right) \sum_{j=1}^N \tilde{p}_{ij}^* \left(\frac{\lambda_{v,j}}{\lambda_{z,i}} \right)^{\frac{1}{\psi}-\gamma}$$

where $\mu_{gg} = (1-\gamma) \mu_g$, $\omega_{gg} = (1-\gamma)^2 \omega_g$, where the matrix function $A^*(u)$ in (A-51) is defined by

$$(A-53) \quad A^*(u) = Diag \left(\left(\frac{\lambda_{v,1}}{\lambda_{z,1}} \right)^{\frac{1}{\psi}-\gamma} \exp(u_1), \dots, \left(\frac{\lambda_{v,N}}{\lambda_{z,N}} \right)^{\frac{1}{\psi}-\gamma} \exp(u_N) \right) P^*,$$

and where the components of the matrix $\tilde{P}^{*\top} = [\tilde{p}_{ij}^*]_{1 \leq i,j \leq N}$ in (A-52) are given by

$$\tilde{p}_{ij}^* = p_{ij} \frac{1 + (\frac{1}{\alpha} - 1) \Phi(q_{ij} + \gamma \sqrt{\omega_{g,i}})}{1 + (\frac{1}{\alpha} - 1) \kappa^{1-\gamma} \sum_{j=1}^N p_{ij} \Phi(q_{ij})}.$$

Proposition A-III.3 Characterization of the Stochastic Discount Factor. Based on the endowment dynamics and using the Euler condition for the claim to aggregate consumption, it can be shown that the stochastic discount factor (A-44) may be expressed as follows:

$$(A-54) \quad M_{t,t+1} = \exp(\zeta_t^\top A \zeta_{t+1} - \gamma g_{t+1}) \left[1 + \left(\frac{1}{\alpha} - 1 \right) I(g_{t+1} < -\zeta_t^\top B \zeta_{t+1} + \ln \kappa) \right],$$

where the components of the $N \times N$ matrices A and B are given by:

$$(A-55) \quad a_{ij} = \ln \delta + \left(\frac{1}{\psi} - \gamma \right) b_{ij} - \ln \left[1 + \left(\frac{1}{\alpha} - 1 \right) \kappa^{1-\gamma} \sum_{j=1}^N p_{ij} \Phi(q_{ij}) \right]$$

$$b_{ij} = \ln \left(\frac{\lambda_{v,j}}{\lambda_{z,i}} \right),$$

respectively, and where

$$(A-56) \quad q_{ij} = \frac{-b_{ij} + \ln \kappa - \mu_{g,i}}{\sqrt{\omega_{g,i}}}.$$

Observe that the vectors λ_z and λ_v characterize the welfare valuation ratios, for which explicit expressions are provided in equations (A-48) and (A-49).

A-III.C Credit Default Swap Spreads

Lemma A-III.1 *Let $\Phi_\rho(\cdot, \cdot)$ be the bivariate normal cumulative distribution function with correlation parameter ρ . Then, if*

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right),$$

$$\begin{aligned} & E[\exp(\sigma_1 \varepsilon_1) I(\varepsilon_1 < q_1) \times \exp(\sigma_2 \varepsilon_2) I(\varepsilon_2 < q_2)] \\ &= \exp\left(\frac{1}{2}(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2)\right) \Phi_\rho(q_1 - \sigma_1 - \rho\sigma_2, q_2 - \sigma_2 - \rho\sigma_1). \end{aligned}$$

Recall that the hazard rate h_t and the associated default intensity λ_t are given by

$$(A-57) \quad h_t = \frac{\lambda_t}{1 + \lambda_t} \quad \text{where} \quad \lambda_t = \exp(\beta_{\lambda 0} + \beta_{\lambda x} x_t + \beta_{\lambda \sigma} \sigma_t),$$

and that the loss rate L_t is constant over time. Dividing both the numerator and the denominator of the expression in equation (7) of the main paper by S_t , computing the CDS spread is equivalent to deriving the individual expressions

$$(A-58) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right].$$

To compute these expressions, we conjecture that

$$(A-59) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] = (\Psi_j^*)^\top \zeta_t \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right] = (\Psi_j)^\top \zeta_t.$$

Given our conjecture, both sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ satisfy the same recursion

$$(A-60) \quad \begin{aligned} (\Psi_j^*)^\top \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) \left((\Psi_{j-1}^*)^\top \zeta_{t+1} \right) \right] \\ (\Psi_j)^\top \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) \left((\Psi_{j-1})^\top \zeta_{t+1} \right) \right] \end{aligned}$$

but with different initial conditions given by

$$(A-61) \quad (\Psi_1^*)^\top \zeta_t = E_t [M_{t,t+1}] \quad \text{and} \quad (\Psi_0)^\top \zeta_t = 1.$$

Using Lemma A-III.1, it can be shown that

$$(A-62) \quad E_t [M_{t,t+1} \mid \zeta_m, m \in \mathbb{Z}] = \zeta_t^\top \tilde{M} \zeta_{t+1},$$

where the components of the matrix \tilde{M} are given by

$$(A-63) \quad \tilde{m}_{ij} = \exp \left(a_{ij} - \gamma \mu_{g,i} + \frac{1}{2} \gamma^2 \omega_{g,i} \right) \left[1 + \left(\frac{1}{\alpha} - 1 \right) \Phi \left(q_{ij} + \gamma \sqrt{\omega_{g,i}} \right) \right].$$

It follows that the initial conditions in A-61 are determined by

$$(A-64) \quad \Psi_1^* = \lambda_{1f} \quad \text{and} \quad \Psi_0 = e,$$

where e denotes the $N \times 1$ vector with all components equal to one.

The solution for the recursion (A-60) satisfied by the solution sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ is

$$(A-65) \quad \begin{aligned} \Psi_j^* &= \text{diagonal of } \left(\tilde{M} \odot \left(e \left(\Psi_{j-1}^* \odot \frac{1}{1+\lambda} \right)^\top \right) \right) P \\ \Psi_j &= \text{diagonal of } \left(\tilde{M} \odot \left(e \left(\Psi_{j-1} \odot \frac{1}{1+\lambda} \right)^\top \right) \right) P. \end{aligned}$$

Proposition A-III.4 Characterization of the Price of the CDS.

$$(A-66) \quad CDS_t(K) = \lambda_s(K)^\top \zeta_t$$

The components of the vectors $\lambda_s(K)$ are functions of the consumption dynamics and of the recursive utility function defined above, and its components are given by

$$(A-67) \quad \lambda_{i,s}(K) = \frac{\sum_{j=1}^{KJ} L [\Psi_{i,j}^* - \Psi_{i,j}]}{\sum_{k=1}^K \Psi_{i,kJ} + \sum_{j=1}^{KJ} \left(\frac{j}{J} - \left\lfloor \frac{j}{J} \right\rfloor \right) [\Psi_{i,j}^* - \Psi_{i,j}]},$$

where e is the vector with all components equal to one, L the vector of conditional loss rates, and where the sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ are given by the recursion (A-65), with initial conditions (A-64).

A-IV Default Risk With Idiosyncratic Shocks

In this section of the Internet Appendix, we provide additional model derivations where we allow for heterogeneity across rating classes both through their sensitivities to aggregate risk factors and through idiosyncratic shocks. This specification assumes that the hazard rate h_t and the associated default process λ_t are given by

$$(A-68) \quad h_t = \frac{\lambda_t}{1 + \lambda_t} \text{ where } \lambda_t = \exp(\beta_{\lambda 0} + \beta_{\lambda x} x_t + \beta_{\lambda \sigma} \sigma_t + u_t) = \zeta_t^{*\top} \lambda^* \lambda^\top \zeta_t,$$

where the idiosyncratic shock $u_t = \eta^\top \zeta_t^*$ is an independent two-state Markov chain with mean zero, variance σ_u^2 , persistence ϕ_u and zero excess kurtosis, and where

$$(A-69) \quad \lambda = \exp(\beta_{\lambda 0} + \beta_{\lambda x} \mu_g + \beta_{\lambda \sigma} \sqrt{\omega_g}) \quad \text{and} \quad \lambda^* = \exp(\eta).$$

The vector η contains the state values of the idiosyncratic chain u_t . We first derive in section (A-IV.A) the risk-neutral dynamics of the states, which simplifies the model derivations. This is followed by the solutions to the default probabilities under the physical and risk-neutral measure in sections (A-IV.B) and (A-IV.C) respectively. The CDS spread is derived in section (A-IV.D). Estimation results and asset pricing implications are reported in tables (A-1) and (A-2) respectively.

A-IV.A Risk Neutral Dynamics of the States

Henceforth, dynamics under the risk-neutral (\mathbb{Q}) measure will be represented with a \mathbb{Q} superscript. Note that

$$\begin{aligned}
 (A-70) \quad E_t^{\mathbb{Q}} [\zeta_{t+1}] &= E_t [M_{t,t+1} R_{f,t+1} \zeta_{t+1}] \\
 &= \dots \\
 &= E_t [\zeta_{t+1} \zeta_{t+1}^\top] \left(\tilde{M} \odot (\lambda_{2f} e^\top) \right)^\top \zeta_t \\
 &= \left(\text{Diag}(e_1^\top P \zeta_t, \dots, e_N^\top P \zeta_t) \right) \left(\tilde{M} \odot (\lambda_{2f} e^\top) \right)^\top \zeta_t \\
 &= \dots \\
 &= \mathcal{E} \left(\left(\tilde{M} \odot (\lambda_{2f} e^\top) \right)^\top \otimes P \right) \mathcal{E}^\top \zeta_t,
 \end{aligned}$$

where \mathcal{E} is the $N \times N^2$ matrix such that the i th row is the vector $(e_i \otimes e_i)^\top$, where the components of the matrix \tilde{M} are given by

$$(A-71) \quad \tilde{m}_{ij} = \exp \left(a_{ij} - \gamma \mu_{c,i} + \frac{1}{2} \gamma^2 \omega_{c,i} \right) [1 + \ell \Phi(q_{ij} + \gamma \sqrt{\omega_{c,i}})],$$

and where $\lambda_{2f} = 1/\lambda_{1f}$. It follows that, under the risk-neutral measure, the Markov chain s_t follows the transition probability matrix

$$(A-72) \quad P^{\mathbb{Q}} = \mathcal{E} \left(\left(\tilde{M} \odot (\lambda_{2f} e^\top) \right)^\top \otimes P \right) \mathcal{E}^\top.$$

Also note that

$$\begin{aligned}
(A-73) \quad E_t^{\mathbb{Q}} [\zeta_{t+1}^*] &= E_t [M_{t,t+1} R_{f,t+1} \zeta_{t+1}^*] \\
&= E_t [M_{t,t+1} R_{f,t+1} \zeta_{t+1}^*] \\
&= E_t [\zeta_{t+1}^* E_t [M_{t,t+1} R_{f,t+1} | \zeta_{t+1}^*]] \\
&= E_t [\zeta_{t+1}^* E_t [M_{t,t+1} R_{f,t+1}]] \\
&= E_t [\zeta_{t+1}^* \cdot 1] \\
&= P^* \zeta_{t+1}^*.
\end{aligned}$$

It follows that the idiosyncratic Markov chain s_t^* has the same transition probability matrix under both the real-world and the risk-neutral measures.

A-IV.B Cumulative Default Probabilities

We recall that the hazard rate h_t and the associated default process λ_t are given by

$$(A-74) \quad h_t = \frac{\lambda_t}{1 + \lambda_t} \text{ where } \lambda_t = \exp(\beta_{\lambda 0} + \beta_{\lambda x} x_t + \beta_{\lambda \sigma} \sigma_t + u_t) = \zeta_t^{*\top} \lambda^* \lambda^\top \zeta_t,$$

where the idiosyncratic shock $u_t = \eta^\top \zeta_t^*$ is an independent two-state Markov chain with mean zero, variance σ_u^2 , persistence ϕ_u and zero excess kurtosis, and where

$$(A-75) \quad \lambda = \exp(\beta_{\lambda 0} + \beta_{\lambda x} \mu_g + \beta_{\lambda \sigma} \sqrt{\omega_g}) \text{ and } \lambda^* = \exp(\eta).$$

The vector η contains the state values of the idiosyncratic chain u_t . We also assume that the loss rate L_t is constant over time. The coefficients $\beta_{\lambda x}$ are non-positive, and the coefficients $\beta_{\lambda \sigma}$ are non-negative, so that default intensities tend to increase when forecasts of macroeconomic growth are negative or when macroeconomic uncertainty increases.

The time t cumulative default probability of defaulting between time $t+1$ and T , con-

ditional on no default prior to $t + 1$, $\text{Prob}_t(t < \tau \leq T | \tau > t)$, is given by

$$(A-76) \quad \text{Prob}_t(t < \tau \leq T | \tau > t) = \frac{\text{Prob}_t(t < \tau \leq T)}{\text{Prob}_t(\tau > t)} = 1 - E_t \left[\frac{S_T}{S_t} \right].$$

Using the conjecture

$$(A-77) \quad E_t \left[\frac{S_{t+j}}{S_t} \right] = \zeta_t^{*\top} \tilde{\Psi}_j \zeta_t,$$

we show that the solution sequence $\{\tilde{\Psi}_j\}$ satisfies the recursion

$$(A-78) \quad \zeta_t^{*\top} \tilde{\Psi}_j \zeta_t = E_t \left[M_{t,t+1} R_{f,t+1} (1 - h_{t+1}) (\zeta_{t+1}^{*\top} \tilde{\Psi}_{j-1} \zeta_{t+1}) \right]$$

with the initial condition

$$(A-79) \quad \tilde{\Psi}_0 = e^* e^\top.$$

It follows that

$$(A-80) \quad \tilde{\Psi}_j = P^{*\top} \left(\left(\frac{1}{1 + \lambda^* \lambda^\top} \right) \odot \tilde{\Psi}_{j-1} \right) P.$$

The conditional and unconditional cumulative default probabilities are thus given by

$$(A-81) \quad \begin{aligned} \text{Prob}_t[t < \tau \leq T | \tau > t] &= 1 - (\zeta_t^{*\top} \tilde{\Psi}_{T-t} \zeta_t) \\ \text{Prob}[t < \tau \leq T | \tau > t] &= 1 - (\Pi^{*\top} \tilde{\Psi}_{T-t} \Pi). \end{aligned}$$

With a constant default process, the unconditional cumulative default probability simplifies to

$$(A-82) \quad \text{Prob}(t < \tau \leq T | \tau > t) = 1 - \exp(-\lambda(T - t)) \quad \text{where } \lambda = \exp(\beta_{\lambda 0}).$$

A-IV.C Risk-Neutral Cumulative Default Probabilities

Define $Z_{t,t+1} = M_{t,t+1}R_{f,t+1}$. The T -year cumulative default probability under the risk-neutral measure is defined by

$$Prob_t^{\mathbb{Q}} [t < \tau \leq T \mid \tau > t]$$

and can be rewritten as

$$\begin{aligned} (A-83) \quad Prob_t^{\mathbb{Q}} [t < \tau \leq T \mid \tau > t] &= \frac{Prob_t^{\mathbb{Q}} (\tau > t) - Prob_t^{\mathbb{Q}} (\tau > T)}{Prob_t^{\mathbb{Q}} (\tau > t)} \\ &= 1 - \frac{Prob_t^{\mathbb{Q}} (\tau > T)}{Prob_t^{\mathbb{Q}} (\tau > t)} \\ &= 1 - E_t^{\mathbb{Q}} \left[\frac{S_T}{S_t} \right] \\ &= 1 - E_t \left[Z_{t,T} \frac{S_T}{S_t} \right]. \end{aligned}$$

Using the conjecture

$$(A-84) \quad E_t \left[Z_{t,t+j} \frac{S_{t+j}}{S_t} \right] = \zeta_t^{*\top} \tilde{\Psi}_j^{\mathbb{Q}} \zeta_t$$

we show that the sequence $\{\tilde{\Psi}_j^{\mathbb{Q}}\}$ satisfies the recursion

$$(A-85) \quad \zeta_t^{*\top} \tilde{\Psi}_j^{\mathbb{Q}} \zeta_t = E_t \left[Z_{t,t+1} (1 - h_{t+1}) \left(\zeta_{t+1}^{*\top} \tilde{\Psi}_{j-1}^{\mathbb{Q}} \zeta_{t+1} \right) \right]$$

with the initial condition:

$$(A-86) \quad \tilde{\Psi}_0^{\mathbb{Q}} = e^* e^\top.$$

It follows that

$$(A-87) \quad \tilde{\Psi}_j^{\mathbb{Q}} = P^{*\top} \left(\left(\frac{1}{1 + \lambda^* \lambda^\top} \right) \odot \tilde{\Psi}_{j-1}^{\mathbb{Q}} \right) P^{\mathbb{Q}}.$$

Thus, the conditional and unconditional risk-neutral cumulative default probability over a $(T - t)$ -year horizon are given by

$$(A-88) \quad \text{Prob}_t^Q [t < \tau \leq T \mid \tau > t] = 1 - \left(\zeta_t^{*\top} \tilde{\Psi}_{T-t}^{\mathbb{Q}} \zeta_t \right)$$

$$(A-89) \quad \text{Prob}^Q [t < \tau \leq T \mid \tau > t] = 1 - \left(\Pi^{*\top} \tilde{\Psi}_{T-t}^{\mathbb{Q}} \Pi^{\mathbb{Q}} \right).$$

A-IV.D Credit Default Swap Spreads

We have the following lemma.

Lemma A-IV.1 *If*

$$(A-90) \quad \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right),$$

then

$$\begin{aligned} E [\exp(\sigma_1 \varepsilon_1) I(\varepsilon_1 < q_1) \times \exp(\sigma_2 \varepsilon_2) I(\varepsilon_2 < q_2)] \\ = \exp \left(\frac{1}{2} (\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2) \right) \Phi_\rho(q_1 - \sigma_1 - \rho\sigma_2, q_2 - \sigma_2 - \rho\sigma_1), \end{aligned}$$

where $\Phi_\rho(\cdot, \cdot)$ is the bivariate normal cumulative distribution function with correlation ρ .

Dividing both the numerator and the denominator of the expression in equation (7) of the main paper by S_t , computing the price of the CDS is equivalent to computing the expressions

$$(A-91) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right].$$

We conjecture that

$$(A-92) \quad E_t \left[M_{t,t+j} \frac{S_{t+j-1}}{S_t} \right] = \zeta_t^{*\top} \Psi_j^* \zeta_t \quad \text{and} \quad E_t \left[M_{t,t+j} \frac{S_{t+j}}{S_t} \right] = \zeta_t^{*\top} \Psi_j \zeta_t.$$

Given our conjecture, it turns out that both sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ satisfy the same recursion

$$(A-93) \quad \begin{aligned} \zeta_t^{*\top} \Psi_j^* \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) (\zeta_{t+1}^{*\top} \Psi_{j-1}^* \zeta_{t+1}) \right] \\ \zeta_t^{*\top} \Psi_j \zeta_t &= E_t \left[M_{t,t+1} (1 - h_{t+1}) (\zeta_{t+1}^{*\top} \Psi_{j-1} \zeta_{t+1}) \right], \end{aligned}$$

but with different initial conditions:

$$(A-94) \quad \begin{aligned} \zeta_t^{*\top} \Psi_1^* \zeta_t &= E_t [M_{t,t+1}] \\ \zeta_t^{*\top} \Psi_0 \zeta_t &= 1. \end{aligned}$$

To derive an explicit solution for the first initial condition in (A-94), we need to compute the expectation $E_t [M_{t,t+1} \mid \zeta_m, m \in \mathbb{Z}]$. Using Lemma A-IV.1, we show that

$$(A-95) \quad E_t [M_{t,t+1} \mid \zeta_m, m \in \mathbb{Z}] = \zeta_t^\top \tilde{M} \zeta_{t+1},$$

where the components of the matrix \tilde{M} are given by

$$(A-96) \quad \tilde{m}_{ij} = \exp \left(a_{ij} - \gamma \mu_{g,i} + \frac{1}{2} \gamma^2 \omega_{g,i} \right) \left[1 + \left(\frac{1}{\alpha} - 1 \right) \Phi(q_{ij} + \gamma \sqrt{\omega_{g,i}}) \right].$$

It follows that

$$(A-97) \quad \begin{aligned} \Psi_1^* &= e^* \lambda_{1f}^\top \\ \Psi_0 &= e^* e^\top, \end{aligned}$$

$$(A-98)$$

where e denotes the $N \times 1$ vector with all components equal to one and e^* denotes the $N^* \times 1$ vector with all components equal to one.

We now derive an explicit solution for the recursion (A-93), that is satisfied by the solution sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$. We show that

$$(A-99) \quad \begin{aligned} \Psi_j^* &= (e^* \lambda_{1f}^\top) \odot \left(P^{*\top} \left(\left(\frac{1}{1 + \lambda^* \lambda^\top} \right) \odot \Psi_{j-1}^* \right) P^Q \right). \\ \Psi_j &= (e^* \lambda_{1f}^\top) \odot \left(P^{*\top} \left(\left(\frac{1}{1 + \lambda^* \lambda^\top} \right) \odot \Psi_{j-1} \right) P^Q \right). \end{aligned}$$

Proposition A-IV.1 *Characterization of the Price of the CDS.*

$$(A-100) \quad CDS_t(K) = \zeta_t^{*\top} [CDS(K)] \zeta_t$$

The components of the matrix $CDS(K)$ are functions of the consumption dynamics and of the recursive utility function defined above, and its components are given by

$$(A-101) \quad CDS_{il}(K) = \frac{\sum_{j=1}^{KJ} L (\Psi_{il,j}^* - \Psi_{il,j})}{\sum_{k=1}^K \Psi_{il,kJ} + \sum_{j=1}^{KJ} \left(\frac{j}{J} - \left\lfloor \frac{j}{J} \right\rfloor \right) (\Psi_{il,j}^* - \Psi_{il,j})},$$

where L is the loss given default, and where the sequences $\{\Psi_j^*\}$ and $\{\Psi_j\}$ are given by the recursion (A-65), with initial conditions (A-97).

A-V Difference Regressions

We emphasize that our analysis focuses on spread levels rather than differences, which is consistent with Doshi, Ericsson, Jacobs, and Turnbull (2013) and references therein.² As the authors point out, there is no consensus in the literature, but economic intuition suggests that spreads are mean-reverting and stationary, as opposed to trending stock prices. In addition, first differencing comes at the cost of less efficient statistical estimates, and measurement errors may reduce the signal-to-noise ratio more for difference regressions. We therefore advocate the use of levels. Irrespectively of our motivation though, a concern may be that the results are spurious because of high persistence in spreads, expected growth and consumption volatility. Therefore, we report results using difference regressions in Tables A-3 and A-4. Consistent with the literature, R^2 statistics are significantly smaller for the difference regressions, ranging around 9%. But we maintain statistical significance, in particular for the regressions with the slope factor. Importantly, we run a Dickey-Fuller test on the residuals of the regressions in levels and we strictly reject the presence of a unit root (see last row in Tables 5 and 6 of the main manuscript). Regressions are thus not spurious. Even if the factors and consumption variables were integrated of order one, these results would suggest that there exists a cointegrating relationship. We don't pursue such tests as they are not the focus of our study. However, if we were to find evidence in favor of a long-run equilibrium relationship, this would still not undue our message that there exists a strong relationship between the common information in the term structure of spreads and macroeconomic risk.

²Also Campbell and Taksler (2003) use spread levels as opposed to changes, likewise Benzoni, Collin-Dufresne, Goldstein, and Helwege (2012) advocate regressions in levels.

A-VI Regressions With Alternative Volatility Specifications

We also test whether our base regression results are affected by the functional form specification of the process for consumption volatility. Thus, we adopt other GARCH models commonly adopted in the financial literature: the standard GARCH(1,1) model (Bollerslev (1986)), the EGARCH model (Nelson (1991)) and the GJR GARCH model (Glosten, Jagannathan, and Runkle (1993)). In these models, the dynamics for the variance of aggregate consumption growth are summarized as follows:

$$(A-102) \quad \begin{aligned} GARCH(1,1) : \sigma_{t+1}^2 &= (1 - \phi_\sigma) \mu_\sigma + \phi_\sigma \sigma_t^2 + \nu_\sigma \sigma_t^2 (\epsilon_{c,t+1}^2 - 1) \\ EGARCH : \ln \sigma_{t+1}^2 &= (1 - \phi_\sigma) \ln \mu_\sigma + \phi_\sigma \ln \sigma_t^2 + \nu_\sigma \left(|\epsilon_{c,t+1}| - \sqrt{2/\pi} \right) + \lambda_\sigma \epsilon_{c,t+1} \\ GJR : \sigma_{t+1}^2 &= (1 - \phi_\sigma) \mu_\sigma + \phi_\sigma \sigma_t^2 + \nu_\sigma \sigma_t^2 (\epsilon_{c,t+1}^2 - 1) + \lambda_\sigma \sigma_t^2 \left(\epsilon_{c,t+1}^2 I(\epsilon_{c,t+1} < 0) - \frac{1}{2} \right), \end{aligned}$$

where the additional leverage parameter λ_σ in the EGARCH and GJR-GARCH specifications allows for asymmetric effects of positive and negative shocks to volatility. The parameter estimates of the different GARCH specifications are reported in Panel B of Table A-6. Interestingly, these alternative specifications are comparatively more persistent, with estimates for ϕ_σ ranging from 0.9779 for the GARCH(1,1,) specification to 0.9907 for the EGARCH specification. Replications of the benchmark regression results, using these different volatility specifications, are reported in the external appendix because of space restrictions. Both the univariate and the multivariate tests illustrate that a different functional form of the volatility process does not alter our conclusion of a strong relationship between the first two principal components and US expected consumption growth and volatility.

Table A-5 reports the univariate tests and illustrates that a different functional form of the volatility process does not alter our conclusion that there is a strong relationship between the first two principal components and US expected consumption growth and volatility. We reach the same conclusion for the multivariate specifications, which are reported in Tables

A-7, A-8 and A-9.

Table A-1: Default Parameters - Time-Varying Hazard Rate With Idiosyncratic Shocks

The table reports the estimation results for the parameters of the default process for the rating categories AAA to B, as well as their t-statistics (in parentheses), where the default process h_t is defined as $h_t = \lambda_t(1 + \lambda_t)$, where $\lambda_t = \exp(\beta_{\lambda 0} + \beta_{\lambda x}x_t + \beta_{\lambda \sigma}\sigma_t + u_t)$. The idiosyncratic factor u_t defines an independent Markov chain, for which we estimate the volatility σ_u and the persistence ϕ_u . The estimation is carried out via the Generalized Method of Moments using the historical observed time series of credit default swap spreads over the sample period 9 May 2003 through 19 August 2010. The moments in the estimation are the expectations of the CDS spreads and their squared values. The weighting matrix is the inverse of the diagonal of the spectral density matrix. Coefficient estimates are reported together with Newey-West standard errors. The last two lines report the J statistics for the test of overidentifying restrictions and the corresponding p-values. The estimation uses the preference parameters obtained from the extended model with generalized disappointment averse preferences.

	δ 0.9989	γ 4.9081	ψ 1.4874	α 0.2486	κ 0.8470	
AAA	-17.47	-19.10	-17.70	-14.83	-22.07	-64.57
(s.e.)	(0.77)	(1.39)	(3.14)	(2.53)	(43.85)	(127.84)
$\beta_{\lambda x}$	-6,496.60	-6,350.04	-8,544.41	-8,352.39	-16,127.72	-4,145.55
(s.e.)	(413.01)	(510.81)	(839.62)	(959.62)	(3005.42)	(298.38)
$\beta_{\lambda \sigma}$	1,818.83	1,392.87	1,141.58	799.11	483.07	590.70
(s.e.)	(240.41)	(87.64)	(62.50)	(78.91)	(75.61)	(30.93)
σ_u	0.76	2.11	1.91	1.38	4.58	22.54
(s.e.)	(0.14)	(0.64)	(1.26)	(1.04)	(17.83)	(52.20)
ϕ_u	0.34	0.95	0.99	1.00	0.34	0.99
(s.e.)	(0.20)	(0.25)	(0.00)	(0.00)	(1.94)	(0.00)
J -test	2.26	0.65	0.99	0.91	2.24	3.89
p-value	0.9438	0.9987	0.9949	0.9962	0.9453	0.7924

Table A-2: Model-Implied and Observed Term Structure of CDS Spreads AAA-B: Downside Risk Aversion and Idiosyncratic Shocks

This table reports observed and model-implied unconditional means, standard deviations (in basis points), skewness, kurtosis and first-order autocorrelation coefficients for CDS spreads for maturities 1 to 10 at the aggregated level for the rating categories AAA-B when the hazard rate contains idiosyncratic shocks. The column labeled *RMSE* reports the Root Mean Squared Errors in basis points for the model fit. The recovery rate is constant and exogenously set at 25%. Preference parameters are those of an investor with Generalized Disappointment Aversion. We use the estimated preference parameters obtained with the extended model with generalized disappointment averse preferences.

	MODEL					DATA								
	1	2	3	5	7	10	RMSE	1	2	3	5	7	10	
AAA														
Mean	13	16	18	21	23	26	0.74	14	16	18	22	23	25	
Volatility	24	26	27	29	30	31	1.13	23	25	27	31	31	31	
Skewness	2	2	2	2	1	1	0.17	2	2	2	2	2	2	
Kurtosis	10	7	5	4	3	3	1.43	7	7	6	5	5	4	
AC1	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.0044	0.9930	0.9944	0.9960	0.9970	0.9970	0.9971	
AA														
Mean	24	28	31	37	40	45	0.54	24	28	31	38	41	45	
Volatility	37	39	41	44	46	46	1.16	38	40	42	45	45	44	
Skewness	3	2	2	2	1	1	0.20	2	2	2	2	2	2	
Kurtosis	10	7	5	4	3	3	1.75	6	6	5	5	4	4	
AC1	0.9996	0.9998	0.9999	0.9999	0.9999	0.9999	0.0034	0.9956	0.9961	0.9965	0.9968	0.9968	0.9967	
A														
Mean	35	43	49	58	65	72	0.94	35	42	48	60	65	71	
Volatility	51	53	56	60	63	64	1.90	51	55	58	63	62	61	
Skewness	4	3	2	2	2	1	0.46	3	2	2	2	2	2	
Kurtosis	19	11	7	5	4	3	4.18	10	9	8	7	7	7	
AC1	0.9991	0.9996	0.9997	0.9999	0.9999	0.9999	0.0022	0.9973	0.9974	0.9974	0.9978	0.9976	0.9976	
BBB														
Mean	81	96	109	127	140	153	1.54	79	95	108	130	141	152	
Volatility	101	103	104	105	105	104	4.67	106	108	107	104	101	97	
Skewness	3	3	2	2	2	2	0.61	2	2	2	2	2	2	
Kurtosis	19	13	9	6	5	4	5.55	7	7	7	6	6	7	
AC1	0.9993	0.9994	0.9996	0.9997	0.9998	0.9998	0.0026	0.9973	0.9973	0.9971	0.9969	0.9969	0.9967	
BB														
Mean	129	171	203	249	279	308	3.64	129	168	202	255	281	303	
Volatility	149	136	129	123	121	119	3.98	141	138	133	127	122	118	
Skewness	4	3	3	2	2	2	0.25	3	3	3	2	2	2	
Kurtosis	18	16	14	10	7	5	2.89	15	14	13	11	11	10	
AC1	0.9990	0.9992	0.9993	0.9996	0.9997	0.9998	0.0050	0.9954	0.9951	0.9948	0.9943	0.9939	0.9937	
B														
Mean	436	473	502	543	570	597	11.13	416	472	510	558	572	593	
Volatility	305	290	287	287	286	284	20.93	320	312	302	287	265	248	
Skewness	2	2	2	2	1	1	0.48	2	2	2	2	2	2	
Kurtosis	8	6	5	4	4	3	4.22	10	9	9	8	9	9	
AC1	0.9986	0.9993	0.9996	0.9998	0.9998	0.9999	0.0064	0.9931	0.9933	0.9934	0.9930	0.9924	0.9924	

Table A-3: Difference Regressions - Macroeconomic and Financial Risk

Regression results from the regression of the factors extracted from a principal component analysis on the term structure of monthly spread changes onto changes in conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price-earnings ratio (PE), as well as the U.S. investment-grade (AAA.BBB) and high-yield (BBB.BB) bond spreads. Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the PE from Kenneth French's website, the VIX, AAA.BBB and BBB.BB from the FRED H15 report. Robust standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$\begin{aligned} F_{i,t}^d = & a_0 \cdot i + a_{1,i} \times \Delta \hat{x}_{t|t} + a_{2,i} \times \Delta \hat{\sigma}_t + a_{3,i} \times \Delta VRP_t + a_{4,i} \times \Delta VIX_t + a_{5,i} \times \Delta USret_t \\ & + a_{6,i} \times \Delta PE_t + a_{7,i} \times \Delta AAA_BBB_t + a_{8,i} \times \Delta BBB_BB_t + \epsilon_t, \end{aligned}$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}^d$ denote the principal components, $\hat{x}_{t|t}$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility.

VARIABLES	(1) F1	(2) F2	(3) F1	(4) F2	(5) F1	(6) F2	(7) F1	(8) F2	(9) F1	(10) F2	(11) F1	(12) F2	(13) F1	(14) F2
$\Delta \hat{x}_{t t}$	-2.50 (2.58)	-1.52* (0.88)												
$\Delta \sigma_t$	6.03 (3.71)	-3.33** (1.42)												
ΔVRP		0.01 (0.02)	0.00* (0.00)		0.26** (0.11)	0.03** (0.01)		-0.05 (0.06)	-0.01 (0.02)	-0.18* (0.09)	-0.18* (0.09)	4.42*** (1.53)	0.04 (0.25)	3.47** (1.02)
ΔVIX														
$\Delta USret$														
ΔPE														
ΔAAA_BBB														
ΔBBB_BB														
Constant	0.11 (0.27)	-0.24*** (0.08)	0.13 (0.27)	-0.23*** (0.08)	0.12 (0.24)	-0.23*** (0.08)	0.13 (0.27)	-0.23*** (0.08)	-0.24*** (0.08)	0.09 (0.22)	-0.23*** (0.09)	0.09 (0.22)	0.04 (0.25)	0.17 (0.23)
Observations	87	87	87	87	87	87	87	87	87	87	87	87	87	87
R-squared	0.03	0.05	0.03	0.03	0.24	0.04	0.01	0.00	0.04	0.36	0.00	0.33	0.01	0.32
adj.R2	0.08	0.03	0.02	0.02	0.23	0.03	0.00	-0.01	0.03	0.35	-0.01	0.32	-0.00	

Table A-4: Difference Regressions - Macroeconomic and Financial Risk

Regression results from the regression of the factors extracted from a principal component analysis on the term structure of monthly CDS spread changes onto changes in conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price-earnings ratio (PE), as well as the U.S. investment-grade (AAA.BBB) and high-yield (BBB.BB) bond spreads. Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the USret on Kenneth French's website, the PE from Robert Shiller's website, and the VIX, AAA.BBB and BBB.BB from the FRED H15 report. Robust standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$\begin{aligned}
 F_{i,t}^d = & a_{0,i} + a_{1,i} \times \Delta \hat{x}_{t|t} + a_{2,i} \times \Delta \hat{\sigma}_t + a_{3,i} \times \Delta VRP_t + a_{4,i} \times \Delta VIX_t + a_{5,i} \times \Delta USret_t \\
 & + a_{6,i} \times \Delta PE_t + a_{7,i} \times \Delta AAA_BBB_t + a_{8,i} \times \Delta BBB_BB_t + \epsilon_t,
 \end{aligned}$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}$ denote the principal components, $\hat{x}_{t|t}$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility.

VARIABLES	(1) F1	(2) F2	(3) F1	(4) F2	(5) F1	(6) F2	(7) F1	(8) F2	(9) F1	(10) F2	(11) F1	(12) F2	(13) F1	(14) F2	
$\Delta \hat{x}_{t t}$	-3.35 (2.89)	-1.69* (0.87)	-3.31 (2.86)	-1.60* (0.84)	-4.03 (3.30)	-1.66* (0.94)	0.97 (2.17)	-1.25 (0.88)	-0.51 (2.46)	-1.46 (0.88)	-1.47* (2.97)	1.05 (0.85)	-1.13 (2.50)		
$\Delta \hat{\sigma}_t$	8.11** (3.40)	-2.90** (1.41)	-2.90** (3.83)	-2.90** (1.34)	-3.23 (3.73)	-3.25** (1.43)	4.21 (3.23)	-3.48** (1.42)	0.82 (3.88)	-3.48** (1.45)	6.51 (4.16)	-3.11** (1.39)	-2.77** (3.12)	(1.33)	
ΔVRP	0.02 (0.02)	0.00 (0.00)	0.00 (0.10)	0.03*** (0.01)	-0.08 (0.07)	-0.01 (0.02)						0.01* (0.01)	0.00 (0.01)	0.00 (0.01)	
ΔVIX												0.03 (0.07)	0.02 (0.03)		
$\Delta USret$												0.05 (0.06)	0.02 (0.03)		
ΔPE												-1.64** (0.29)	-0.17 (0.16)		
ΔAAA_BBB												1.21 (1.08)	-0.24 (0.37)		
ΔBBB_BB												1.08 (0.72)	0.01 (0.32)		
Constant	0.10 (0.26)	-0.24*** (0.08)	0.08 (0.23)	-0.24*** (0.08)	0.09 (0.26)	-0.24*** (0.09)	0.04 (0.17)	-0.24*** (0.08)	0.09 (0.22)	-0.24*** (0.08)	0.11 (0.22)	3.45*** (1.04)	0.15 (0.20)	-0.24*** (0.05)	
Observations	87	87	87	87	87	87	87	87	87	87	87	87	87	87	
R-squared	0.09 adj.R2	0.08 0.05	0.31 0.28	0.09 0.06	0.06 0.03	0.06 0.02	0.63 0.62	0.09 0.06	0.36 0.34	0.06 0.02	0.33 0.33	0.03 0.03	0.69 0.66	0.12 0.03	

Table A-5: Regression Analysis - Macroeconomic and Financial Risk

Regression results from the regression of the factors extracted from a principal component analysis onto conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price earnings ratio (PE), as well as the U.S. investment-grade (AAA_BBB) and high-yield (BBB_BB) bond spreads. Factor scores are first averaged at the end of each month and then projected onto the explanatory variables. Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the USret on Kenneth French's website, the PE from Robert Shiller's website, and the VIX, AAA_BBB and BBB_BB from the FRED H15 report. Block-bootstrapped standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$F_{i,t} = a_{0,i} + a_{1,i} \times \hat{x}_{t|t} + a_{2,i} \times \hat{\sigma}_t + a_{3,i} \times VRP_t + a_{4,i} \times VIX_t + a_{5,i} \times USret_t + a_{6,i} \times PE_t + a_{7,i} \times AAA_BBB_t + a_{8,i} \times BBB_BB_t + \epsilon_t,$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}$ denote the principal components, $\hat{x}_{t|t}$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility. The regressions are run for three different specifications of the process for the volatility of aggregate consumption growth: The simple GARCH(1,1) (Bollerslev 1986), the exponential GARCH (Nelson 1991) and the GJR-GARCH model (Glosten, Jagannathan and Runkle 1993).

VARIABLES	(1)			(2)			(3)			(4)			(5)			(6)			(7)			(8)		
	F1	GARCH(1,1)	F2	-	F1	-	F2	-	F3	-	F1	-	F2	-	F3	-	F1	-	F2	-	F3	-		
$\hat{x}_{t t}$	-81.27 (51.55)	277.92*** (34.40)	64.96*** (18.12)		-256.03*** (55.23)		166.82*** (25.34)		37.99*** (16.47)		-125.47** (49.65)		244.29*** (31.10)		51.88*** (18.87)									
$\hat{\sigma}_t$	531.82*** (73.34)	355.67*** (53.00)	111.75*** (20.98)		328.32*** (56.51)		238.32*** (44.21)		100.66*** (16.28)		479.17*** (76.91)		312.03*** (50.23)		322.84*** (21.38)									
Constant	-1.37*** (0.17)	-0.83*** (0.13)	-0.27*** (0.05)		-0.84*** (0.12)		-0.51*** (0.10)		-0.22*** (0.04)		-1.22*** (0.18)		-0.70*** (0.12)		-0.21*** (0.05)									
Observations	88	88	88		88		88		88		88		88		88		88		88		88			
R-squared	0.75 0.74	0.62 0.61	0.14 0.12		0.70 0.69		0.53 0.52		0.20 0.18		0.75 0.74		0.60 0.59		0.12 0.10									
adj.R2																								

Table A-6: Kalman Filter Estimates

This table reports the Kalman Filter estimates for the parameters of the conditional expectation of consumption growth and conditional consumption volatility. Standard errors are given in parentheses. Panel A reports the result for the base specification, where the GARCH-like stochastic volatility is defined as in Heston and Nandi (2000), henceforth HN. Panel B reports the estimated coefficients and standard errors in parentheses for alternative volatility specifications: the GARCH(1,1) model of Bollerslev (1986), the EGARCH model of Nelson (1991) and the GJR GARCH model of Glosten, Jagannathan, and Runkle (1993).

Panel A		μ_x	ϕ_x	ν_x	μ_σ	ϕ_σ	ν_σ
HN		0.001785 (0.000235)	0.955642 (0.033936)	0.058611 (0.028885)	1.372177e-05 (1.541653e-06)	0.9610790 (0.013410)	7.5528e-007 (1.7537e-007)
Panel B							
	GARCH(1,1)		μ_σ 9.0536e-06 (1.0199e-05)	ϕ_σ 0.97791 (0.00364)	ν_σ 0.03598 (0.00840)	λ_σ – (–)	
	EGARCH		1.5787e-05 (4.5563e-06)	0.99068 (0.00460)	0.09589 (0.02338)	0.02269 (0.00969)	
	GJR		1.3293e-05 (1.6063e-05)	0.98495 (0.00554)	0.07086 (0.01731)	-0.03128 (0.01736)	

Table A-7: Multivariate Regression Analysis - Macroeconomic and Financial Risk - GARCH

Regression results from the regression of the factors extracted from a principal component analysis onto conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price-earnings ratio (PE), as well as the U.S. investment-grade (AAA_BBB) and high-yield (BBB_BB) bond spreads. Factor scores are first averaged at the end of each month and then projected onto the explanatory variables. The process for the volatility of aggregate consumption growth follows a simple GARCH(1,1) volatility process as defined by Bollerslev(1986). Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the USret on Kenneth French's website, the PE from Robert Shiller's website, and the VIX, AAA_BBB and BBB_BB from the FRED HI5 report. Block-bootstrapped standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$F_{i,t} = a_{0,i} + a_{1,i} \times \hat{x}_t|_t + a_{2,i} \times \hat{\sigma}_t + a_{3,i} \times VRP_t + a_{4,i} \times VIX_t + a_{5,i} \times USret_t + a_{6,i} \times PE_t + a_{7,i} \times AAA_BBB_t + a_{8,i} \times BBB_BB_t + \epsilon_t,$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}$ denote the principal components, $\hat{x}_{t|t}$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility.

VARIABLES	(1) F1	(2) F2	(3) F1	(4) F2	(5) F1	(6) F2	(7) F1	(8) F2	(9) F1	(10) F2	(11) F1	(12) F2	(13) F1	(14) F2
$\hat{x}_t _t$	-75.39 (63.86)	280.63*** (36.23)	33.19 (61.03)	323.04*** (35.48)	-60.96 (50.02)	282.23*** (33.18)	277.90*** (43.48)	363.29*** (46.13)	201.76*** (41.12)	363.91*** (41.14)	122.03*** (41.12)	337.64*** (37.11)	272.79*** (37.11)	370.17*** (44.15)
$\hat{\sigma}_t$	526.15*** (69.64)	353.20*** (53.21)	516.38*** (61.44)	349.59*** (50.36)	566.71*** (75.46)	363.07*** (51.55)	398.32*** (56.76)	323.94*** (55.85)	410.23*** (54.58)	318.73*** (53.02)	477.28*** (47.99)	339.65*** (47.43)	417.96*** (48.11)	314.80*** (54.96)
VRP	3.35 (15.18)	1.54 (4.84)	0.69* (0.40)	0.27*** (0.08)	-0.64* (0.36)	-0.14 (0.16)	-0.14 (0.16)	-0.14 (0.16)	-0.05*** (0.01)	-0.01*** (0.00)	16.78*** (2.60)	5.10*** (1.20)	12.25*** (1.86)	3.60*** (0.87)
VIX500														
USret														
PE														
IG_AAA_BBB														
HY_BBBBB														
Constant	-1.36*** (0.17)	-0.82*** (0.13)	-1.44*** (0.15)	-0.85*** (0.12)	-1.45*** (0.18)	-0.84*** (0.12)	0.23 (0.31)	-0.14** (0.20)	-1.22*** (0.13)	-0.78*** (0.12)	-1.39*** (0.12)	-0.83*** (0.11)	3.60*** (0.87)	3.71* (0.23)
Observations	88	88	88	88	88	88	88	88	88	88	88	88	88	88
R-squared	0.75	0.62	0.81	0.67	0.77	0.62	0.66	0.66	0.91	0.70	0.69	0.69	0.94	0.73
adj R2	0.74	0.61	0.80	0.66	0.77	0.61	0.87	0.64	0.91	0.69	0.90	0.68	0.94	0.70

Table A-8: Multivariate Regression Analysis - Macroeconomic and Financial Risk - EGARCH

Regression results from the regression of the factors extracted from a principal component analysis onto conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price-earnings ratio (PE), as well as the U.S. investment-grade (AAA_BBB) and high-yield (BBB_BB) bond spreads. Factor scores are first averaged at the end of each month and then projected onto the explanatory variables. The process for the volatility of aggregate consumption growth follows a simple exponential GARCH volatility process as defined by Nelson (1991). Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the USret on Kenneth French's website, the PE from Robert Shiller's website, and the VIX, AAA_BBB and BBB_BB from the FRED HI5 report. Block-bootstrapped standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$F_{i,t} = a_{0,i} + a_{1,i} \times \hat{x}_t|_t + a_{2,i} \times \hat{\sigma}_t + a_{3,i} \times VRP_t + a_{4,i} \times VIX_t + a_{5,i} \times USret_t + a_{6,i} \times PE_t + a_{7,i} \times AAA_BBB_t + a_{8,i} \times BBB_BB_t + \epsilon_t,$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}$ denote the principal components, $\hat{x}_t|_t$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility.

VARIABLES	(1) F1	(2) F2	(3) F1	(4) F2	(5) F1	(6) F2	(7) F1	(8) F2	(9) F1	(10) F2	(11) F1	(12) F2	(13) F1	(14) F2
$\hat{x}_t _t$	-242.45*** (72.35)	174.10*** (28.50)	-109.56* (62.50)	230.46*** (29.84)	-242.04*** (53.17)	170.40*** (26.01)	204.38** (45.07)	303.55*** (45.07)	105.40** (43.01)	290.01*** (36.99)	2.81 (41.48)	253.37*** (31.56)	202.42*** (67.77)	316.88*** (44.84)
$\hat{\sigma}_t$	327.40*** (54.63)	237.83*** (44.02)	344.82*** (48.54)	245.50*** (40.12)	366.10*** (63.89)	248.00*** (45.39)	272.22*** (47.22)	221.83*** (40.64)	279.93*** (40.28)	221.83*** (39.09)	335.95*** (37.08)	240.38*** (37.55)	296.35*** (37.55)	220.15*** (38.74)
VRP	5.36 (16.21)	2.87 (5.40)	0.80* (0.43)	0.35*** (0.09)	-0.69 (0.44)	-0.18 (0.21)	-0.18 (0.44)	-0.06*** (0.01)	-0.06*** (0.01)	-0.02*** (0.00)	18.25*** (2.69)	6.22*** (1.26)	13.57*** (2.10)	4.54*** (0.95)
VIX500														
USret														
PE														
IG_AAA_BBB														
HY_BBBBB														
Constant	-0.84*** (0.12)	-0.51*** (0.10)	-1.00*** (0.12)	-0.58*** (0.09)	-0.92*** (0.14)	-0.53*** (0.10)	0.73*** (0.30)	-0.04 (0.16)	-0.88*** (0.09)	-0.52*** (0.08)	-1.02*** (0.09)	-0.57*** (0.09)	-0.57*** (0.09)	-0.45* (0.24)
Observations	88	88	88	88	88	88	88	88	88	88	88	88	88	88
R-squared	0.71	0.54	0.62	0.73	0.54	0.87	0.62	0.90	0.67	0.88	0.65	0.94	0.71	0.71
adj R2	0.70	0.53	0.77	0.60	0.72	0.53	0.86	0.60	0.66	0.88	0.64	0.93	0.68	0.68

Table A-9: Multivariate Regression Analysis - Macroeconomic and Financial Risk - GJR-GARCH

Regression results from the regression of the factors extracted from a principal component analysis onto conditional expected consumption growth, conditional consumption volatility and the Variance Risk Premium (VRP), the CBOE S&P500 volatility index (VIX), the excess return on the CRSP value-weighted portfolio (USret), the US price-earnings ratio (PE), as well as the U.S. investment-grade (AAA_BBB) and high-yield (BBB_BB) bond spreads. Factor scores are first averaged at the end of each month and then projected onto the explanatory variables. The process for the volatility of aggregate consumption growth follows a GJR-GARCH volatility process as defined by Glosten, Jagannathan and Runke (1993). Data for real per capita consumption is taken from the FRED database of the Federal Reserve Bank of St.Louis from January 1959 until August 2010. The data for the VRP is taken from Hao Zhou's webpage, for the USret on Kenneth French's website, the PE from Robert Shiller's website, and the VIX, AAA_BBB and BBB_BB from the FRED H15 report. Block-bootstrapped standard errors are reported in brackets. ***, ** and * indicate significance at the 1%, 5% and 10% respectively.

$$F_{i,t} = a_{0,i} + a_{1,i} \times \hat{x}_t|_t + a_{2,i} \times \hat{\sigma}_t + a_{3,i} \times VRP_t + a_{4,i} \times VIX_t + a_{5,i} \times USret_t + a_{6,i} \times PE_t + a_{7,i} \times AAA_BBB_t + a_{8,i} \times BBB_BB_t + \epsilon_t,$$

where $i = 1, 2, 3$ and t is the month index. The dependent variables $F_{i,t}$ denote the principal components, $\hat{x}_{t|t}$ is the filtered consumption forecast and $\hat{\sigma}_t$ the filtered conditional consumption volatility.

VARIABLES	(1) F1	(2) F2	(3) F1	(4) F2	(5) F1	(6) F2	(7) F1	(8) F2	(9) F1	(10) F2	(11) F1	(12) F2	(13) F1	(14) F2
$\hat{x}_t _t$	-119.89* (62.88)	246.84*** (33.84)	-19.63 (59.32)	284.68*** (36.01)	-111.46** (48.53)	246.71*** (31.76)	258.95*** (83.11)	343.26*** (46.55)	166.76*** (44.77)	333.98*** (41.68)	83.75** (39.24)	306.41*** (37.84)	247.72*** (65.07)	348.85*** (48.82)
$\hat{\sigma}_t$	474.16*** (72.85)	309.75*** (51.56)	456.88*** (60.20)	303.53*** (50.47)	503.91*** (76.97)	316.31*** (52.15)	371.01*** (51.21)	368.26*** (50.30)	277.99*** (52.44)	431.42*** (47.05)	297.86*** (45.98)	371.93*** (48.36)	371.93*** (44.31)	267.79*** (49.06)
VRP	3.09 (1.41)	1.41 (4.46)	0.66* (0.38)	0.25*** (0.09)	-0.60* (0.35)	-0.10 (0.16)	-0.10 (0.16)	-0.10 (0.16)	-0.05*** (0.01)	-0.01*** (0.00)	16.77*** (2.40)	5.15*** (1.30)	12.29*** (1.84)	3.65*** (0.95)
VIX500														
USret														
PE														
IG_AAA_BBB														
HY_BBBBB														
Constant	-1.21*** (0.17)	-0.70*** (0.12)	-1.27*** (0.15)	-0.72*** (0.12)	-0.71*** (0.18)	-0.71*** (0.12)	0.35 (0.27)	-0.30* (0.18)	-1.10*** (0.13)	-0.66*** (0.11)	-1.26*** (0.12)	-0.71*** (0.11)	3.65*** (0.95)	4.05* (2.10)
Observations	88	88	88	88	88	88	88	88	88	88	88	88	88	88
R-squared	0.75	0.60	0.80	0.64	0.77	0.60	0.65	0.91	0.68	0.90	0.67	0.90	0.94	0.71
adj R2	0.74	0.58	0.79	0.62	0.76	0.58	0.88	0.63	0.91	0.90	0.66	0.90	0.94	0.68

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