Internet Appendix to "Interest rate risk and the cross section of stock returns"

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This version: April 2014

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Abstract

This appendix to "Interest rate risk and the cross section of stock returns" presents supplementary results not included in the paper and technical appendices. Section 1 presents the comparison with nested models, while Section 2 conducts a comparison with alternative linear factor models. Section 3 presents some additional results for the benchmark linear model estimated in the paper. Section 4 presents the results for the non-linear version of our model, while Section 5 shows the results for alternative monetary asset pricing models. Sections A to F represent technical appendices.

1 Comparison with nested models

1.1 Nested models

The model stated in equation (9) in the paper represents a rich specification, since it nests other well known models from the asset pricing literature as special cases. The first special case is the Epstein and Zin (1991) model with one consumption good, which is obtained by setting $\varepsilon = 0$ in the benchmark model (that is, there is no role for real balances in the utility function),

$$\begin{aligned} \mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) &= \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right), \qquad (1) \\ \gamma_c &\equiv \frac{\theta}{\psi} = \frac{1-\gamma}{\psi - 1}, \\ \gamma_w &\equiv -(\theta - 1) = \frac{\gamma\psi - 1}{\psi - 1}, \end{aligned}$$

with both γ and ψ having the same expressions (as a function of the factor risk prices) as in equation (11) in the paper.

The second special case of CI-CAPM is the Power utility C-CAPM from Lucas (1978) and Breeden (1979), which arises as a special case of the benchmark model by imposing $\theta = 1$ (which means that $\gamma = 1/\psi$) and $\varepsilon = 0$:

$$\begin{aligned} \mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) &= \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right), \qquad (2) \\ \gamma_c &\equiv \gamma. \end{aligned}$$

The third nested case is the CAPM from Sharpe (1964) and Lintner (1965), which is obtained by imposing that $\psi \to +\infty$ and $\varepsilon = 0$ in the benchmark model:

$$\begin{aligned} \mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) &= \gamma_0 + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right), \qquad (3) \\ \gamma_w &\equiv \gamma. \end{aligned}$$

To obtain the CAPM pricing equation, notice that $\psi \to +\infty$ implies that $\gamma_c \to 0$ and also that $\theta \to 1 - \gamma$, which in turn leads to $\gamma_w \to \gamma$.

In all the above expected return-covariance equations an intercept is included, although strictly speaking this intercept should be equal to zero if there exists a real risk-free rate, as is assumed in the derivation of the Euler equation for real returns.

1.2 Empirical results

The derivations above show that the CI-CAPM can be perceived as a generalization of the Epstein and Zin (1991) model, and consequently, a generalization of either the standard C-CAPM or the CAPM. Therefore, it should be relevant to compare the pricing performance of each of these three models against the CI-CAPM. More specifically, we want to assess further whether monetary/interest risk is crucial to drive the explanatory power of the benchmark model, as suggested by the results presented in the paper. This comparison is also relevant due to the extensive previous evidence showing that both the standard C-CAPM and the CAPM do not perform well in explaining the cross-section of equity portfolio returns, and more specifically, the returns of the 25 size/BM portfolios, giving rise to the so-called size and value anomalies (Fama and French (1992)). On the other hand, it is well known that the C-CAPM with power utility is not able to price the excess market return (equity premium puzzle) at reasonable preference parameters. Furthermore, not surprisingly, there is evidence that the Epstein–Zin model cannot explain the cross-section of stock returns (e.g., Gomes, Kogan, and Yogo (2009)).

The estimation and evaluation results from first-stage GMM, for the Epstein–Zin, C-CAPM and CAPM are provided in Table 1, when the equity portfolios are SBM25 (Panel A) and SLTR25 (Panel B). Rows 1 to 3 present the results for the benchmark models, estimated without an intercept, while rows 4 to 6 present the results for the unrestricted models. To save space, in this subsection we only report the asymptotic t-statistics for the point estimates and asymptotic p-values for the χ^2 statistic. In the case of the benchmark models, the results clearly show that none of the three models performs well in explaining the excess returns of the 25 size-BM portfolios, producing negative estimates for the OLS R^2, which means that the three models perform worse than a model with only a constant factor. When one includes the intercept, the estimates for the coefficient of determination become positive, but the fit is still modest (below 24%). Furthermore, in the estimation of the unrestricted models the intercepts are economically large (between 1.5% and 2.7% per quarter), thus showing that all three models are misspecified, that is, there are relevant missing risk factors. Moreover, the intercept estimates are significant at the 5% and 1% levels for the C-CAPM and CAPM, respectively, while in the case of the Epstein–Zin model the point estimate for the intercept is significant at the 10% level.

Regarding the test with SLTR25, as in the test over SBM25, the R^2 estimates are either negative (C-CAPM and CAPM) or around zero (Epstein–Zin model). When the intercept is included in the estimation of the three models the explanatory ratios become positive, but at quite modest levels (below 15%). Moreover, the point estimates for the zero-beta excess return are above 1% per quarter in all three models, although they are only marginally significant (10% level).

Overall, the results from Table 1 show that the baseline CI-CAPM clearly outperforms the Epstein–Zin, C-CAPM, and CAPM models in pricing stock returns, and therefore, the restrictions associated with these last three models (as special cases of the benchmark model) are not confirmed by the data. These results are not surprising given the previous extensive evidence that both the CAPM or the baseline C-CAPM are not successful in explaining either the equity premium or the cross-section of stock returns (Mehra and Prescott (1985), Mankiw and Shapiro (1986), Breeden, Gibbons, and Litzenberger (1989), Fama and French (1992, 1993), Lettau and Ludvigson (2001), Parker and Julliard (2005), Yogo (2006), Jagannathan and Wang (2007), among others). Second, the interest risk factor is a crucial factor in driving the explanatory power of the consumption-interest CAPM for the excess returns of the SBM25 and SLTR25 portfolios in addition to the equity premium.

2 Comparison to alternative factor models

2.1 Alternative macro models

The findings in Section 4 in the paper show that the CI-CAPM performs relatively well in pricing the 25 size/BM portfolios in addition to the excess market return. Other macroeconomic asset pricing models, which represent extensions of the standard C-CAPM, have also been able to explain the returns of the 25 size/BM portfolios. In this subsection, we provide a brief comparison against these models. We focus the analysis on consumption-based models.[1] The criteria for choosing alternative models to compare with is that these models have been tested in the cross-section of stock returns with an expected return-covariance (beta) representation.

Among the alternative models, the conditional C-CAPM from Lettau and Ludvigson (2001) was one of the first consumption-based models to price the SBM25 portfolios. In their paper, the conditioning variable that accounts for the time-varying consumption risk price is a cointegration function of log consumption, log financial wealth, and log labor income, denoted as cay. The model's expected return-covariance representation can be defined as

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_0 + \gamma_{cay}\,\mathrm{Cov}\left(R_{j,t+1}, cay_t\right) + \gamma_{cay,c}\,\mathrm{Cov}\left(R_{j,t+1}, \Delta c_{t+1} cay_t\right) + \gamma_c\,\mathrm{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right), \tag{4}$$

where $\Delta c_{t+1} cay_t$ denotes the scaled factor resulting from the interaction between current consumption growth and the lagged conditioning variable, cay_t.

A more recent conditional version of the C-CAPM, also tested with the SBM25 portfolio returns, is the model proposed by Lustig and Van Nieuwerburgh (2005). In this model, the conditioning variable is the housing collateral ratio, my, defined as the ratio of collateralizable housing wealth (home mortgages) to non-collateralizable human wealth. The corresponding pricing equation can be represented as

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_0 + \gamma_{my}\,\mathrm{Cov}\left(R_{j,t+1}, my_t\right) + \gamma_{my,c}\,\mathrm{Cov}\left(R_{j,t+1}, \Delta c_{t+1} my_t\right) + \gamma_c\,\mathrm{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right). \tag{5}$$

The third macroeconomic model for which we compare the performance of the CI-CAPM is the three-factor model presented in Yogo (2006). Similarly to the CI-CAPM, the model from Yogo is based on Epstein–Zin intertemporal preferences. The risk factors are log non-durable consumption growth, the log market return, and log durable consumption

[1] There is a growing literature that takes a production-based approach to equilibrium asset pricing. For example, Balvers and Huang (2007) and Belo (2010) derive and test production-based asset pricing models in the cross-section of stock returns.

growth (Δd):

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_d \operatorname{Cov}\left(R_{j,t+1}, \Delta d_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right). \quad (6)$$

The fourth model analyzed is the linear version of the model with lead consumption growth from Parker and Julliard (2005),

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t,t+s}\right), \quad (7)$$

where $\Delta c_{t,t+s} = c_{t+s} - c_t = \sum_{j=1}^{s} \Delta c_{t+j}$ denotes the cumulative log consumption growth over s periods in the future.[2]

In Table 2, we present the evaluation measures for the alternative macroeconomic factor models tested on the SBM25 portfolios reported in the original studies. The R^2 estimates vary between 66% (Parker and Julliard (2005)) and 94% (Yogo (2006)), implying that these models underperform relative to the CI-CAPM, with the exception of the durable consumption model. On the other hand, both Lettau and Ludvigson (2001) and Yogo (2006) pass the χ^2 test, while the model from Parker and Julliard (2005) is rejected. Overall, these results suggest that the CI-CAPM compares favorably to these four alternative macro models in explaining the risk premia of the 25 size/BM portfolios.

2.2 Comparison to the Fama and French (1993) model

For completeness, we also compare the CI-CAPM against the Fama and French (1993) three-factor model, which represents a benchmark in the empirical asset pricing literature, with the results presented in Table 3. To save space, we only report the global fit measures since we are mainly concerned about comparing the overall explanatory power of the Fama-French model against the CI-CAPM, and not in conducting a detailed analysis of the respective risk price estimates. Thus, we compute R^2_{OLS}, MAE, and the χ^2 statistic, all associated with first stage GMM; and R^2_{WLS} from second-stage GMM. The point estimate

[2] Aït-Sahalia, Parker, and Yogo (2004), Jagannathan and Wang (2007), and Savov (2011) also test extensions of the baseline C-CAPM with the SBM25 portfolios. We do not make a comparison against these models since their tests rely on annual data, whereas the tests of the CI-CAPM are based on quarterly data.

for the intercept (γ_0) in the first-stage estimation is also presented. In the test with the SBM25 portfolios, the model's fit is quite large with a R^2_{OLS} estimate around 74% and a R^2_{WLS} estimate around 56%. In the case of the unrestricted model, the explanatory ratio increases only marginally to 79%. However, the intercept is both economically (3.7% per quarter) and statistically (1% level) significant, suggesting misspecification of the model.

When the equity portfolios are SLTR25, the OLS explanatory ratio is the same as in the test with SBM25 (74%), while the R^2_{WLS} estimate is slightly higher (69%). Thus, in the tests with either set of equity portfolios the model's explanatory power is very close to the fit obtained for the CI-CAPM. The large explanatory power of the Fama-French model in the test with SBM25 is not totally surprising given that this three-factor model was specifically designed to price the size/BM portfolios, that is, the size (SMB) and value (HML) factors are mechanic transformations of the original size and BM test portfolios.[3] What is more remarkable is that the CI-CAPM has an explanatory power that is very similar, and in some cases exceeds, to that of the Fama-French model.

Furthermore, the comparison with the Fama–French model is not subject to the criticisms pointed out in Lewellen, Nagel, and Shanken (2010), in the sense that obtaining a good fit for the SBM25 portfolios might represent a spurious result. First, we add to the menu of test assets the excess market return, imposing a bigger hurdle to the model than just pricing the 25 portfolios alone. Second, the model's explanatory power remains large in the second-stage GMM, with WLS coefficient of determination estimates above 50%. Third, we conduct a bootstrap simulation in order to compute more robust empirical p-values for both the t-stats on the risk prices and J-test, which basically reinforce the evidence from the asymptotic statistics. Fourth, the high explanatory power of the model does not come at the cost of very implausible estimates for the structural preference parameters in the model, that is, the implied preference parameters are economically reasonable in most cases (except the high estimates of risk aversion that are a consequence of the consumption data, as discussed in the paper). Fifth, we use an alternative set of equity portfolios (SLTR25), and the explanatory power of the model remains relatively high.

[3]Cochrane (2005) justifies the three-factor model as an APT model.

3 Additional results

In this section, we provide some additional results associated with the test of the CI-CAPM. Specifically, we present a bootstrap-based inference for the explanatory ratios in the model; we estimate an augmented version of the CI-CAPM; estimate the model by using forward consumption growth rates; estimate the CI-CAPM on alternative sets of equity portfolios; and estimate the model in expected return-beta representation.

3.1 Empirical distribution for R^2

Following Lewellen, Nagel, and Shanken (2010), we estimate an empirical distribution of the OLS/WLS coefficients of determination. We employ the bootstrap algorithm used in the paper, and described in detail in Appendix D below (to produce empirical p-values for the factor risk prices) in order to obtain 95% confidence intervals for both R^2_{OLS} and R^2_{WLS}. In this simulation, portfolio risk premia and risk factor realizations are simulated independently, without imposing the model's restrictions. The objective is to answer the following question: Under the assumption that the CI-CAPM does not hold and that the factors are useless, how likely is that one will obtain the kind of large fit found in the data?

Untabulated results show that the 95% confidence interval associated with R^2_{OLS} in the test with SBM25 is $[-0.97, 0.20]$ in both versions of the model. In the test with SLTR25, the corresponding estimates are $[-1.26, 0.16]$ and $[-1.27, 0.16]$ in the versions with TB and FED, respectively. Thus, it turns out that the R^2_{OLS} point estimates from the CI-CAPM reported in the paper are well above the upper bounds in these confidence intervals in all four cases, suggesting that the fit of the model in driving equity risk premia is not spurious, or in other words, is statistically significant.

Regarding the WLS coefficient of determination, the confidence intervals in the test with SBM25 are $[-0.89, 0.22]$ and $[-0.86, 0.22]$ in the versions with TB and FED, respectively. The corresponding estimates in the test with SLTR25 are $[-1.05, 0.20]$ and $[-1.02, 0.21]$, respectively. As in the case of R^2_{OLS}, the point estimates for R^2_{WLS} reported in the paper are well above the upper bound in these intervals. Thus, the large explanatory ratios of the CI-CAPM over a mean-variance efficient combination of the original portfolios reported in Section 4 in the paper, do not seem to be spurious.

3.2 Alternative comparison to Yogo (2006)

We conduct an alternative comparison to the Yogo (2006) model. As noted in Section 2 above, both the CI-CAPM and Yogo's model are three-factor linear models based on Epstein–Zin intertemporal preferences, and two of the risk factors are common in both models—the log non-durable consumption growth and the log equity market return. The third factor in Yogo is the log durable consumption growth, whereas in the CI-CAPM the third factor is the log interest growth (log growth in the opportunity cost of money). Both factors are key to the explanatory power of either model. We estimate the following augmented four-factor model for the 1963:III–2007:IV period:[4]

$$\begin{aligned} \mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) &= \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_d \operatorname{Cov}\left(R_{j,t+1}, \Delta d_{t+1}\right) \\ &\quad + \gamma_f \operatorname{Cov}\left(R_{j,t+1}, \Delta f_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right). \end{aligned} \tag{8}$$

The goal in estimating (8) is two-fold. First, we want to assess whether the key factor in the CI-CAPM (Δf_{t+1}) is still a priced factor in the presence of the durable consumption growth factor, Δd_{t+1}. Second, we want to assess whether Δd_{t+1} adds explanatory power to the CI-CAPM by producing a significant increase in the R^2_{OLS} estimates.

The estimation results for the augmented model are displayed in Table 4. We can see that for both sets of portfolios (SBM25 and SLTR25), the interest factor risk price is consistently negative, while the risk price estimates for durable consumption alternate in sign. Moreover, the interest risk factor is priced in the cross-section of stock returns (5% or 1% levels), while the risk price estimates for the durable consumption factor are highly insignificant. In terms of global fit, the OLS R^2 estimates (around 60-70%) are slightly below the corresponding estimates for the CI-CAPM presented in the paper. This decline in fit should be related with the slightly shorter sample used, but it also suggests that the durable consumption factor does not add relevant explanatory power to our three-factor model in terms of pricing the cross-section of average returns jointly with the market equity premium. It looks like the type of risks measured by Δd_{t+1} are captured by the interest

[4] The data on the growth rate of the stock of consumption of durable goods are obtained from Motohiro Yogo's webpage, and corresponds to the measure used in Gomes, Kogan, and Yogo (2009). The quarterly stock of durables is related with the real expenditure on durable goods by an accumulation equation (see equation (1) in Gomes, Kogan, and Yogo (2009)), and using a constant depreciation rate within each year. Due to data availability, the model is estimated for the 1963:III–2007:IV period.

rate factor.

3.3 Using forward consumption growth

Following Parker and Julliard (2005), we estimate the CI-CAPM by using the forward growth rate in the consumption of non-durables and services. Specifically, stock returns are aligned with the log consumption growth measured over the following four quarters. The estimation results are displayed in Table 5. The fit of the model decreases in comparison to the benchmark test, however, the explanatory power remains significant: the R^2_{OLS} estimates are about 66% for both versions of the interest rate factor in the test with SBM25. When the portfolios are SLTR25 the explanatory ratios vary between 46% (TB) and 57% (FED).

The estimates for the interest factor risk price are similar to the corresponding estimates in the benchmark test, and are statistically significant in all cases. On the other hand, the estimates for the consumption risk price are significantly smaller than in the test using the usual definition of consumption growth, and are statistically significant in all cases. Consequently, the implied estimates for RRA are significantly smaller than in the benchmark test, varying between 108 (TB) and 110 (FED) in the estimation with SBM25, while in the test with SLTR25 the corresponding range is between 77 (TB) and 79 (FED). The estimates for ψ vary between 3% and 5%, while the estimates for ε are now in the 8%–10% interval. Thus, by using forward consumption growth of non-durables and services, we obtain more plausible implied preference parameter estimates, especially for the risk aversion coefficient.

3.4 Alternative equity portfolios

We estimate the CI-CAPM for 10 portfolios sorted on size plus 10 portfolios sorted on BM (S10+BM10). The use of the 20 portfolios is a response to the fact that some of the portfolios within SBM25 (especially, the extreme small-growth portfolio) are difficult to price for most asset pricing models in the literature, and tend to influence the overall fit of a given model when tested on these portfolios.

Table 6 (Panel A) presents the estimation results when the equity portfolios are S10+BM10. We can see that the model's overall fit, as measured by R^2_{OLS}, is slightly higher in comparison to the test with SBM25, when the model is estimated with FED (86%). In the case of TB (77%), the fit is only marginally smaller than in the test with SBM25. These results show that in the version with FED it is slightly easier to explain the returns of the 20 portfolios in comparison to SBM25, which might be related with the difficulty of pricing the extreme small-growth portfolio (SBM_{11}) that tends to be the most problematic return to be priced within that class of portfolios. Nevertheless, the discrepancy is low, since the two sets of portfolios are, obviously, strongly correlated. As a comparison, the fit of the Fama and French (1993) model when estimated on the 20 portfolios is only marginally higher than the CI-CAPM (version with FED) with a cross-sectional R^2 of 92%.

The point estimates for the risk price associated with the growth in the opportunity cost of money have lower magnitudes (almost half) than in the test with SBM25. Nevertheless, these point estimates are statistically significant at the 5% and 10% levels when the interest rate proxies are TB and FED, respectively. The point estimates for the consumption risk price are somewhat lower than the corresponding estimates in the test with SBM25, but only in the version based on TB there is statistical significance (10% level). On the other hand, the estimates for γ_w are largely insignificant as in the test with SBM25.

The implied (raw) point estimates for ψ, γ, and ε are 0.00, 181, and 0.04, respectively, in the version with TB, while in the version with FED these estimates are 0.01, 192, and 0.03, respectively. Thus, these estimates are not very different from the corresponding estimates in the test with SBM25, in the cases of ψ and ε, while there is a decline in the magnitude of γ. In sum, the estimation results for the test with S10+BM10 are qualitatively similar to the results obtained in the test with SBM25.

We use additional portfolios as test assets in order to address the criticism from Lewellen, Nagel, and Shanken (2010) that the size/BM portfolios have a strong embedded factor structure. We use decile portfolios sorted on asset growth (AG10); total accruals (TA10); and on the Ohlson (1980) O-score measure of financial distress (OS10). These three portfolio sorts are used by Chen, Novy-Marx, and Zhang (2011) to test alternative multifactor models. The portfolio return data are available from Long Chen. We test the CI-CAPM on the additional portfolio sorts jointly with the size deciles and the aggregate equity premium, that is, each empirical test has a total of 21 test assets. Due to data availability on the portfolio returns, the sample used in these three additional empirical tests is 1972:II to 2008.III.

When the test portfolios are S10+OS10 (Panel B in Table 6) the explanatory ratio is 51% in the version with TB, while in the version with FED the fit is significantly lower (15%). Moreover, the CI-CAPM is not rejected by the χ^2 statistic when the interest rate proxy is TB (p-value of 77%). The point estimates for γ_f vary between -7.25 (FED) and -11.76 (TB), and both estimates are significant at the 5% level. In the test with S10+TA10 (Panel C), the fit is similar in the two versions of the CI-CAPM (around 55%), and the model is not rejected in the version with TB (p-value of 16%). The point estimates for the interest factor risk price are around -5, and are significant at the 5% level. When the test portfolios are S10+AG10 (Panel D), the explanatory ratio is slightly above 30% in both versions of the model, and the χ^2 statistic points to non-rejection of the model in both cases (p-value of 14%). The point estimates for γ_f are around -6, being significant at the 1% (version with TB) or 5% level (FED).

As a comparison, the R^2_{OLS} estimates associated with the Fama and French (1993) model are 5%, 10%, and 63% in the tests with S10+OS10, S10+TA10, and S10+AG10, respectively. Thus, the CI-CAPM clearly outperforms the Fama-French model in explaining the accruals and financial distress anomalies, while underperforming in pricing the asset growth anomaly. Overall, these results show that our three-factor model works relatively well in pricing these alternative equity portfolios, and in all cases, the interest rate factor is priced.

Our last empirical test uses five industry portfolios (IND5) employed by Gomes, Kogan, and Yogo (2009) to test a non-linear version of the macro model with durable consumption from Yogo (2006). The quarterly return data on IND5 are available from Motohiro Yogo's webpage. As above, we combine the industry portfolios with the size deciles and the market return, for a total of 16 test assets. The results, presented in Table 6, Panel E, show that the CI-CAPM performs well in pricing the dispersion in returns among the 16 portfolios, with explanatory ratios varying between 63% (version based on FED) and 67% (TB), while the average pricing error is 0.20% per quarter in both versions of the model. In comparison to the benchmark tests based on SBM25 and SLTR25, the risk price estimates associated with the interest rate factor have smaller magnitudes, and are statistically significant only in the version based on TB (10% level), which should be related with some degree of multicollinearity in the estimation.

To put these results in perspective, we estimate the linear three-factor model from Yogo (2006) on the same set of portfolios. Untabulated results show that the model with durable consumption has a similar fit to the CI-CAPM, with an OLS R^2 estimate of 69% and an average pricing error of 0.18% per quarter. However, the point estimate for the risk price associated with non-durable consumption is implausibly negative. Thus, our model compares favorably with the three-factor model from Yogo (2006) in pricing the industry portfolios jointly with the size portfolios and the aggregate equity premium.

3.5 Beta representation

We estimate the CI-CAPM in expected return-beta form by using the time-series/cross-sectional regression approach used in Brennan, Wang, and Xia (2004) and Cochrane (2005), among others. In the first step, we conduct time-series regressions to estimate the factor loadings for each portfolio,

$$R_{j,t+1} - R_{f,t+1} = \delta_j + \beta_{j,c}\Delta c_{t+1} + \beta_{j,f}\Delta f_{t+1} + \beta_{j,w} r_{w,t+1} + \varepsilon_{j,t}, \tag{9}$$

where the coefficients β_j stand for the factor loadings. In the second step, we conduct an OLS cross-sectional regression of average excess returns on the factor loadings to obtain estimates for the factor (beta) risk prices (λ):

$$\overline{R_j - R_f} = \lambda_c \beta_{j,c} + \lambda_f \beta_{j,f} + \lambda_w \beta_{j,w} + \alpha_j. \tag{10}$$

To compute the t-statistics associated with the risk prices we use the Shanken (1992) standard errors, which correct for the estimation error in the betas. The beta representation produces the same fit as the covariance representation, the only difference being the fact that the multiple regression betas account for the correlation among the different factors.

The estimation results are presented in Table 7. As expected, the coefficient of determination and MAE estimates are the same as in the test of the covariance representation shown in the paper. The estimates for λ_f are negative in all four cases, and these estimates are significant at the 5% or 1% levels. On the other hand, the estimates for λ_c are not statistically significant, while the estimates for the market risk price are positive in all cases, being significant (at the 5% level) in the test with SLTR25. Therefore, the results from the beta representation of the CI-CAPM are similar to the results from the benchmark test and show once more the importance of the interest rate factor in driving the fit of the model.

4 Euler equations

In this section, we estimate the Euler equations for excess stock returns associated with our macroeconomic asset pricing model:[5]

$$\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{\frac{R_{f,t+2}-1}{R_{f,t+2}}}{\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{\varepsilon(\gamma-1)} R_{w,t+1}^{\theta-1}(R_{j,t+1}-R_{r,t+1})\right]=0. \quad (11)$$

The objective of this exercise is to assess whether the estimates of the preference parameters, $\gamma, \psi, \varepsilon$, are significantly different from the implied estimates associated with the linear model estimated in Section 4 in the paper. Since the linear version of our model relies on a first-order Taylor approximation of the original model, in principle, the two versions can deliver different estimates of the structural preference parameters.

4.1 Econometric framework

We estimate the monetary model by a two-step GMM procedure, where the weighting matrix in the first-stage estimation is associated with the Hansen and Jagannathan (1997) distance,[6]

$$\mathbf{W}_{HJ}=\left(\frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t^e\mathbf{R}_t^{e\prime}\right)^{-1}, \quad (12)$$

where $\mathbf{R}_t^e$ is the vector of portfolio excess returns at time t. Therefore, portfolios with a larger second moment in returns are given less weight in the estimation. This method allows us to compare the results across different models (as in the GMM estimation with the identity matrix) since they do not rely on the estimation of the spectral density matrix,

[5] We exclude the time-discount factor term, δ^θ, from the SDF since it is not identifiable when the Euler equation is estimated only with excess returns. From an econometric point of view, this may also lead to a more efficient estimation of the remaining preference parameters in the model.

[6] The Hansen and Jagannathan (1997) metric has been employed in asset pricing tests by Jagannathan and Wang (1996), Hodrick and Zhang (2001), and Jacobs and Wang (2004), among others.

$\mathbf{S}$ (as it is the case with efficient GMM). Moreover, the standard errors of the parameter estimates are lower than those in the estimation with equal-weighted moment conditions. However, as in the second-stage estimation, the results associated with $\mathbf{W}_{HJ}$ are more difficult to interpret than the results with equal-weighted moments, since they often involve large negative and positive weights on the different moment conditions (portfolio pricing errors). Furthermore, often the second moments matrix of returns is near singular with the resulting difficulties in its inversion, similarly to the second-stage GMM estimation (see Cochrane (2005) for further discussion on this issue).

The Hansen-Jagannathan (HJ) distance is equal to

$$HJ = \left(\hat{\boldsymbol{\alpha}}'\mathbf{W}_{HJ}\hat{\boldsymbol{\alpha}}\right)^{\frac{1}{2}}, \tag{13}$$

where $\hat{\boldsymbol{\alpha}}$ is the vector of Euler equation errors. This metric can be interpreted as the minimum distance between a given candidate SDF and the set of all true SDFs. HJ also measures the magnitude of mispricing of a given model, and thus, can be used to compare the explanatory power of alternative asset pricing models.

The set of moment conditions correspond to the Euler equations associated with excess returns. By exploiting the linearity of conditional expectations and by using the law of total expectations, the Euler equations for excess returns can be rewritten as

$$\mathbf{0} = \mathrm{E}\left(Q_{t+1}\mathbf{R}^{e}_{t+1} \otimes \mathbf{z}_{t}\right), \tag{14}$$

where $\mathbf{z}_t$ is a vector of state variables or instruments known at time t; $\mathbf{0}$ is a vector of zeros; and $\otimes$ denotes the Kronecker product. Given the large number of test assets (26), and the inherent problems of applying non-linear GMM to a large number of moment conditions, the only instrument is a constant, $\mathbf{z}_t = 1$. Thus, in each estimation we have a total of 26 orthogonality conditions and three parameters to estimate, leading to 23 overidentifying conditions.

Following Brav, Constantinidies, and Geczy (2002) and Constantinides and Ghosh (2011), in order to estimate our macro model we use a three-dimensional parameter grid search approach. In this method, for each specified vector of values for the parameters, $(\gamma, \psi, \varepsilon)$, we evaluate the GMM objective function, $\mathbf{g}_T'\mathbf{W}\mathbf{g}_T$ (with $\mathbf{g}_T$ representing the vector of stacked moments, and **W** the weighting matrix), and choose the parameter values that minimize the objective function. The range of values for γ is $(2, 2.5, ..., 400)$; for ψ it is $(0.05, 0.1, ..., 5)$, and for ε the interval is $(0.05, 0.1, ..., 1)$. Notice that with this approach we are constraining the signs of the parameters, but we are not constraining in any way the respective magnitudes. For example, we can have large estimates of γ (as high as 400, the well-known equity premium puzzle from Mehra and Prescott (1985)), or we can also have implausible high estimates for the share of money in the utility function, ε (as high as one).

In the second-stage estimation, we compute the asymptotic test of overidentifying restrictions (J-test) to test the null hypothesis that the orthogonality conditions associated with the Euler equations are satisfied, which is distributed as $\chi^2(23)$ (see equation (15) in the paper). As in the estimation of the linear model in Section 4 in the paper, we use asymptotic heteroskedasticity-robust standard errors to compute the individual t-statistics associated with the structural parameters and the covariance matrix of the residuals from the moment conditions.[7]

4.2 Empirical results

The estimation results for the model in equation (11) when the test portfolios are SBM25, are displayed in Table 8. In the first-stage estimation (Panel A), the estimates for γ vary between 70 (version with TB) and 78.50 (FED), while both ψ and ε are estimated at 0.05 in both versions of the model. All these estimates are significant at the 5% or 1% levels. The estimates for the HJ distance are 0.45 and 0.43 in the versions with TB and FED, respectively, thus, showing that the version based on FED has marginally better fit. In the second-stage estimation (Panel B), the point estimates for γ are somewhat higher than in the estimation with first-stage GMM: 97.50 and 104.50 in the versions with TB and FED, respectively. On the other hand, both ψ and ε are estimated at 0.05, and all six point estimates are significant at the 1% level. Both versions of the model are rejected by the J-test, which might be related with a problematic inversion of the covariance matrix of the pricing errors.

The estimation results for the test with the SLTR25 portfolios are shown in Table 9,

[7] Since we are constraining the signs of the structural parameters, we use one-sided t-statistics.

which is identical to Table 8. The first-stage estimates for γ (around 60) are slightly lower than the corresponding estimates in the test with SBM25. Moreover, the estimates for both ψ and ε are the same as in the test with SBM25 (0.05), and all these estimates are significant at the 5% or 1% levels. The HJ distance is estimated at 0.48 and 0.46 in the versions with TB and FED, respectively, which shows a slightly lower fit than in the test with SBM25. In the second-stage estimation, the estimates for γ increase to 85.50 (TB) and 89.50 (FED), while the estimates for both ψ and ε remain at 0.05. As in the test with SBM25, all the second-stage estimates are significant at the 1% level.

When we compare the parameter estimates above with the implied estimates presented in Table 4 in the paper, we can see that the estimates for both ψ and ε are relatively similar in both the linear and non-linear specifications of the model. On the other hand, the estimates for γ from the non-linear model are significantly smaller (and hence, more plausible) than the corresponding implied estimates from the linear model estimated in Section 4 in the paper.

In recent related work, by estimating the Euler equations associated with a monetary model based on money real balances with 30 portfolios sorted on book-to-market, short-term past returns, and long-term past returns, Gu and Huang (2012) obtain larger magnitudes for the estimates of the risk aversion parameter (121 and 277) and the share of money in the utility function (0.27 and 0.80). These larger (and less economically plausible) estimates should be related with the fact that real balances growth (the key risk factor in their SDF) has greater measurement error and is less correlated with equity returns than the interest rate factor in our model. It is not surprising that a specification based on a price (interest rate) yields more plausible parameter estimates than a specification based on a quantity (money). However, we should note that the estimates in both models are not totally comparable since our first-stage GMM estimation is based on the (inverse of the) second-moment matrix of returns, while Gu and Huang (2012) use the identity matrix. Moreover, they use instruments in one of the alternative estimations of the Euler equations, while in our case the only instrument is the constant.

5 Alternative monetary models

This section presents two alternative monetary models, which rely on different mechanisms of introducing money in the economy.

5.1 The models

As an alternative to the money-in-the-utility Function (MIUF) approach we can use the cash-in-advance (CIA) approach, as in Clower (1967), in which the consumer needs to hold cash to be able to consume. In this case, the CIA constraint will bind at equilibrium since money has an opportunity cost: the consumer should not hold any useless money and incur an opportunity cost, unless such cash is strictly required for consumption. Since the CIA constraint is binding, consumption is equal to real balances and therefore we can substitute one for another. Therefore, with a simple CIA constraint it follows that the marginal utility of consumption (the pricing kernel) will be driven either by consumption or real balances, in addition to another factor, linked to the opportunity cost of money holdings. However, in the CIA specification we do not have any additional parameter relative to the standard real consumption model without money frictions.

We can reformulate our monetary model to have money through a CIA constraint instead of the MIUF specification. More precisely, the representative consumer program is now given by

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^{N}} U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{15}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right),$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1},$$

$$C_t \leq M_t,$$

where the last equation corresponds to the CIA constraint.

As shown in Appendix E below, by assuming that the CIA constraint is binding the

Euler equation for excess returns is given by

$$0 = \mathrm{E}_t \left[\delta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} \left(\frac{1 + \frac{R_{f,t+2}-1}{R_{f,t+2}}}{1 + \frac{R_{f,t+1}-1}{R_{f,t+1}}} \right)^{-\theta} R_{w,t+1}^{\theta-1} \left(R_{j,t+1} - R_{r,t+1} \right) \right]. \tag{16}$$

This model is denoted as model 1. We can see that the SDF in this economy is qualitatively equivalent to the SDF in our benchmark model: the presence of money brings a new factor, which is the growth rate in the real cost of one unit of the consumption good. When the investor consumes one unit of the consumption good, it costs him one (since it is the numéraire), but due to the CIA constraint he has to hold a corresponding amount of money with opportunity cost given by $\frac{R_{f,t+1}-1}{R_{f,t+1}}$. Hence the total cost of one unit of the consumption good is $1 + \frac{R_{f,t+1}-1}{R_{f,t+1}}$. However, as noted above, despite the presence of the interest rate factor, the parameters in the model are the same as in the standard Epstein-Zin model without money frictions, the RRA parameter (γ) and the elasticity of intertemporal substitution (ψ).

The representation (16) is based on the standing assumption in the literature that the CIA constraint is binding, meaning that $C = M$. In particular, because there is no additional parameter in the model, we cannot have as a nested case the standard Epstein-Zin model, since it is obtained by setting $M = 0$ (and thus $C = 0$), which does not make sense.

An alternative, and perhaps less restrictive, way to introduce money based on a transaction story is the approach adopted by Feenstra (1986), Marshall (1992), and Balvers and Huang (2009). The representative agent problem is now given by

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^{N}} U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{17}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - T(C_t, M_t) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right),$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}.$$

The function $T(\cdot)$ is the real transaction cost associated with real consumption. In this framework, money saves transaction costs while allowing the agent to trade off these transaction costs relative to the opportunity cost of money.

The usual assumptions for $T(\cdot)$ are as follows (the index denotes the variable relative to which the derivative is taken): $T > 0, T_C > 0, T_M < 0, T_{CC} \leq 0$ and $T_{MM} \geq 0$. While these conditions are necessary from an economic standpoint (the first two) and for achieving an optimal solution (the last two), there is no consensus so far as to the sign of the cross derivative. However, Feenstra (1986) suggests that this cross derivative should be negative.

The above conditions on the properties of T are enough in Balvers and Huang (2009) to obtain a factor representation of risky asset excess returns. However, if one wants to derive the Euler equation for asset returns and relate it to the properties of T, one has to add an assumption that will guaranty that the solution to the consumer's problem will be scale invariant at the optimum. For this purpose, we write the transaction cost as a function of the consumption to real balances ratio. Thus, the dynamic problem becomes,

$$\begin{aligned} \max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} U_t &= \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ \text{s.t. } W_{t+1} &= R_{w,t+1} \left[W_t - C_t \left(1 + \tau \left(\frac{C_t}{M_t} \right) \right) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right] \\ R_{w,t+1} &= \sum_{j=1}^N \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}, \end{aligned} \tag{18}$$

where we assume that $\tau > 0, \tau_C > 0, \tau_M < 0, \tau_{CC} \leq 0, \tau_{MM} \geq 0$ and $\tau_{CM} < 0$. τ has now the dimension of a percentage.

As shown in Appendix F below, the Euler equation for excess returns in such a setting is as follows,

$$0 = \mathrm{E}_t \left[\delta^\theta \left(\frac{1 + MC_{t+1}}{1 + MC_t} \right)^{-\theta} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} R_{w,t+1}^{\theta-1} \left(R_{j,t+1} - R_{r,t+1} \right) \right], \tag{19}$$

where

$$MC_t \equiv \frac{\partial}{\partial C_t} \left(C_t \tau \left(\frac{C_t}{M_t} \right) \right) = \tau \left(\frac{C_t}{M_t} \right) + \frac{C_t}{M_t} \tau' \left(\frac{C_t}{M_t} \right). \tag{20}$$

Once again, the presence of money brings an additional factor related to the opportunity cost of money holdings.

The previous results are amenable for an empirical investigation to the extent that one puts some structure on the transaction cost function. We assume the following specification (see Baumol (1952), Tobin (1956), and Feenstra (1986)):

$$\tau\left(\frac{C_t}{M_t}\right) = a\left(\frac{C_t}{M_t}\right)^b, a > 0, 0 < b < 1. \tag{21}$$

The condition on the parameter b guaranties that the second derivative of the transaction cost function relative to consumption is negative. As in Balvers and Huang (2009), the cross derivative is negative. By using the transaction function above, and substituting the money demand equation, the Euler equations becomes:

$$0 = \mathrm{E}_t\left[\delta^\theta\left(\frac{1 + a^{\frac{1}{1+b}} b^{\frac{-b}{1+b}}(1+b)\left[\frac{R_{f,t+2}-1}{R_{f,t+2}}\right]^{\frac{b}{1+b}}}{1 + a^{\frac{1}{1+b}} b^{\frac{-b}{1+b}}(1+b)\left[\frac{R_{f,t+1}-1}{R_{f,t+1}}\right]^{\frac{b}{1+b}}}\right)^{-\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta} R_{w,t+1}^{\theta-1}\left(R_{j,t+1} - R_{r,t+1}\right)\right]. \tag{22}$$

This model is labeled model 2.

5.2 Empirical results

Similarly to the case of our (non-linear) benchmark model in the last section, we estimate the two models above by a two-step GMM procedure, where the weighting matrix in the first-stage estimation is associated with the Hansen and Jagannathan (1997) distance. We use the same grid search intervals for γ and ψ as in the last section. In the case of parameter b in model 2, the range of values is $(0.05, 0.1, ..., 1)$. The parameter a is calibrated at 0.5.[8] We estimate both models 1 and 2 by using both proxies for the short-term interest rate (TB and FED) in the construction of the interest growth factor, as in our benchmark model. To save space, we estimate models 1 and 2 only for the 25 size-BM portfolios.

The results for model 1 are presented in Table 10 below. We can see that the first-stage estimates for γ are only marginally below the corresponding estimates from the benchmark model in the previous section. On the other hand, ψ is estimated at 0.05, as in the benchmark model. The HJ distance is around 0.48 in both versions of model 1, which is slightly above the magnitude of mispricing in the benchmark model. In the second-stage estimation, the point estimates for the risk aversion parameter are slightly below 100, while

[8]The estimation results of model 2 are very similar if we fix a at 1 or 1.5.

ψ is estimated at 0.05. As in the benchmark model, model 1 is rejected by the J-test.

The results for model 2 are displayed in Table 11. The estimates for γ are very close to those in model 1, while ψ continues to be estimated at 0.05. On the other hand, the estimates for b are 0.05 in both versions of the model, but these estimates are strongly insignificant as indicated by the very low t-statistics. The magnitude of mispricing, as indicated by the HJ distance, is very similar to the estimates obtained for model 1. Thus, these results show that model 2 does not improve model 1 in terms of pricing the size-BM portfolios. In the second-stage estimation, the point estimates for γ increase to values around 100, while the estimates for the other two parameters are the same as in the first-stage estimation. In particular, the estimates for b continue to be highly non-significant. Thus, the highly imprecise estimates for b do not provide support for model 2.

Overall, these results show that these two alternative monetary models perform worse than our benchmark model estimated in the last section, as indicated by the higher values for the HJ distance. Moreover, some of the parameter estimates are not statistically significant.

A Derivation of the Euler equation

The investor's intertemporal problem is given by

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} U_t = \left\{ (1-\delta) \left(C_t^{1-\varepsilon} M_t^{\varepsilon} \right)^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{A.1}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right), \tag{A.2}$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}, \tag{A.3}$$

$$\theta \equiv \frac{(1-\gamma)\,\psi}{\psi - 1}.$$

This problem can be represented in a dynamic programming setting as

$$J(W_t) \equiv \max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} \left\{ (1-\delta) \left(C_t^{1-\varepsilon} M_t^{\varepsilon} \right)^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(J(W_{t+1})^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{A.4}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right), \tag{A.5}$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}. \tag{A.6}$$

The first-order conditions relative to C_t and M_t are given respectively by

$$(1-\varepsilon)\, C_t^{\frac{1-\gamma}{\theta}(1-\varepsilon)-1} M_t^{\varepsilon \frac{1-\gamma}{\theta}} = \frac{\delta}{1-\delta} \left[\mathrm{E}_t \left(J(W_{t+1})^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} J_W(W_{t+1}) R_{w,t+1} \right], \tag{A.7}$$

and

$$\varepsilon C_t^{\frac{1-\gamma}{\theta}(1-\varepsilon)} M_t^{\varepsilon \frac{1-\gamma}{\theta}-1} \frac{R_{f,t+1}}{R_{f,t+1}-1} = \frac{\delta}{1-\delta} \left[\mathrm{E}_t \left(J(W_{t+1})^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} J_W(W_{t+1}) R_{w,t+1} \right]. \tag{A.8}$$

By combining the two first-order conditions we obtain the standard portfolio balance relationship:

$$C_t = \frac{1-\varepsilon}{\varepsilon} \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t. \tag{A.9}$$

This equation postulates that the flow of services of real balances holdings—as measured by the total opportunity cost of money holdings—is equal to the consumption flow. Similarly to Epstein and Zin (1991), let's assume that the value function is proportional to wealth in the following way:

$$J(W_t) = \phi_t W_t. \tag{A.10}$$

By substituting (A.10) in (A.7), using both the law of iterated expectations and the intertemporal budget constraint (A.5), and rearranging, we have:

$$C_t^{\frac{1-\gamma}{\theta}(1-\varepsilon)-1} M_t^{\varepsilon\frac{1-\gamma}{\theta}} = \frac{\delta}{1-\delta}\frac{1}{1-\varepsilon}\left[\mathrm{E}_t\left(J(W_{t+1})^{1-\gamma}\right)\right]^{\frac{1}{\theta}}\left(W_t - C_t - \frac{R_{f,t+1}-1}{R_{f,t+1}}M_t\right)^{-1}. \tag{A.11}$$

Moreover, the value function in (A.4) can be rearranged, leading to

$$J(W_t)^{\frac{1-\gamma}{\theta}} - (1-\delta)\left(C_t^{1-\varepsilon}M_t^{\varepsilon}\right)^{\frac{1-\gamma}{\theta}} = \delta\left[\mathrm{E}_t\left(J(W_{t+1})^{1-\gamma}\right)\right]^{\frac{1}{\theta}}. \tag{A.12}$$

By substituting (A.12) in (A.11), and after some tedious algebra, we obtain the explicit functional form for the value function,

$$J(W_t) = (1-\delta)^{\frac{\theta}{1-\gamma}}(1-\varepsilon)^{\frac{\theta}{1-\gamma}}\left(\frac{C_t}{W_t}\right)^{1-\frac{\theta}{1-\gamma}}\left(\frac{M_t}{C_t}\right)^{\varepsilon} W_t = \phi_t W_t, \tag{A.13}$$

$$\phi_t \equiv (1-\delta)^{\frac{\theta}{1-\gamma}}(1-\varepsilon)^{\frac{\theta}{1-\gamma}}\left(\frac{C_t}{W_t}\right)^{1-\frac{\theta}{1-\gamma}}\left(\frac{M_t}{C_t}\right)^{\varepsilon},$$

thus confirming the previous guess. By substituting (A.13) in (A.11), we derive the Euler equation for the return on the market portfolio:

$$1 = \mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}\left(\frac{W_{t+1}}{W_t - C_t - \frac{R_{f,t+1}-1}{R_{f,t+1}}M_t}\right)^{\theta}\right] \Leftrightarrow$$

$$1 = \mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta}\right]. \tag{A.14}$$

If the return on total wealth is rewritten as $R_{w,t+1} = \sum_{j=1}^{N}\omega_{j,t}\left(R_{j,t+1} - R_{r,t+1}\right) + R_{r,t+1}$, the first-order condition with respect to $\omega_{j,t}$ is given by

$$\mathrm{E}_t\left[J(W_{t+1})^{-\gamma} J_W(W_{t+1})\left(R_{j,t+1} - R_{r,t+1}\right)\right] = 0, \tag{A.15}$$

and by using (A.5), (A.13), and the definition of θ, leads to

$$\begin{aligned} &\mathrm{E}_t\left[C_{t+1}^{(1-\gamma)(1-\varepsilon)-\theta} M_{t+1}^{\varepsilon(1-\gamma)} W_{t+1}^{\theta-1}\left(R_{j,t+1}-R_{r,t+1}\right)\right]=0 \Leftrightarrow \\ &\mathrm{E}_t\left[C_{t+1}^{(1-\gamma)(1-\varepsilon)-\theta} M_{t+1}^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta-1}\left(R_{j,t+1}-R_{r,t+1}\right)\right]=0. \end{aligned} \tag{A.16}$$

By multiplying both terms of Equation (A.16) by $\delta^{\theta} C_t^{-(1-\gamma)(1-\varepsilon)+\theta} M_t^{-\varepsilon(1-\gamma)} \omega_{j,t}$ and summing over N returns, leads to

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta-1} \sum_{j=1}^{N} \omega_{j,t}\left(R_{j,t+1}-R_{r,t+1}\right)\right]=0. \tag{A.17}$$

If we notice that $\sum_{j=1}^{N} \omega_{j,t}\left(R_{j,t+1}-R_{r,t+1}\right)=R_{w,t+1}-R_{r,t+1}$, it follows that

$$\begin{aligned} &\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta}\right]= \\ &=\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta-1} R_{r,t+1}\right] \Leftrightarrow \\ &\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta}\right]= \\ &=\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta-1}\right] R_{r,t+1}, \end{aligned} \tag{A.18}$$

where the last equality comes from the assumption that $R_{r,t+1}$ is risk-free. By substituting (A.14) into (A.18), we have the following Euler equation for the real interest rate:

$$1=\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)} R_{w,t+1}^{\theta-1}\right] R_{r,t+1}. \tag{A.19}$$

Moreover, by rearranging Equation (A.17), and using Equation (A.19), we have the

Euler equation for an arbitrary risky return, $R_{j,t+1}$:

$$\sum_{j=1}^{N}\omega_{j,t}\,\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}R_{w,t+1}^{\theta-1}\left(R_{j,t+1}-R_{r,t+1}\right)\right]=0\Leftrightarrow$$

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}R_{w,t+1}^{\theta-1}\left(R_{j,t+1}-R_{r,t+1}\right)\right]=0\Leftrightarrow$$

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}R_{w,t+1}^{\theta-1}R_{j,t+1}\right]=$$

$$=\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}R_{w,t+1}^{\theta-1}R_{r,t+1}\right]\Leftrightarrow$$

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{(1-\gamma)(1-\varepsilon)-\theta}\left(\frac{M_{t+1}}{M_t}\right)^{\varepsilon(1-\gamma)}R_{w,t+1}^{\theta-1}R_{j,t+1}\right]=1. \tag{A.20}$$

Finally, we can use the portfolio balance relationship (A.9) to obtain the representations of the SDF with the nominal interest rate growth as a factor:

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{\frac{R_{f,t+2}-1}{R_{f,t+2}}}{\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{\varepsilon(\gamma-1)}R_{w,t+1}^{\theta-1}R_{j,t+1}\right]=1, \tag{A.21}$$

$$\mathrm{E}_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{\frac{R_{f,t+2}-1}{R_{f,t+2}}}{\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{\varepsilon(\gamma-1)}R_{w,t+1}^{\theta-1}\right]R_{r,t+1}=1. \tag{A.22}$$

B Intertemporal budget constraint

The intertemporal budget constraint (A.5) can be rewritten in a more intuitive way. By combining Equations (A.5) and (A.6), we have,

$$W_{t+1}=\left(W_t-C_t-\frac{R_{f,t+1}-1}{R_{f,t+1}}M_t\right)\left[\sum_{j=1}^{N}\omega_{j,t}\left(R_{j,t+1}-R_{r,t+1}\right)+R_{r,t+1}\right], \tag{B.23}$$

which can be rearranged as follows:

$$W_{t+1}=\sum_{j=1}^{N}\omega_{j,t}\left(W_t-C_t-\frac{R_{f,t+1}-1}{R_{f,t+1}}M_t\right)\left(R_{j,t+1}-R_{r,t+1}\right)+\left(W_t-C_t\right)R_{r,t+1}-M_t\frac{R_{f,t+1}-1}{R_{f,t+1}}R_{r,t+1}. \tag{B.24}$$

By noting that $R_{r,t+1} = \frac{R_{f,t+1}}{1+\pi_{t+1}}$, where π_{t+1} is the inflation rate, leads to

$$W_{t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(W_t - C_t - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right) (R_{j,t+1} - R_{r,t+1}) + (W_t - C_t - M_t)\, R_{r,t+1} + \frac{M_t}{1+\pi_{t+1}}. \tag{B.25}$$

Finally, by defining the absolute demands, $a_{jt} \equiv \omega_{j,t} \left(W_t - C_t - \frac{R_{f,t+1}-1}{R_{f,t+1}} M_t \right)$, it follows:

$$W_{t+1} = \sum_{j=1}^{N} a_{jt} R_{j,t+1} + \left(W_t - C_t - M_t - \sum_{j=1}^{N} a_{jt} \right) R_{r,t+1} + \frac{M_t}{1+\pi_{t+1}}. \tag{B.26}$$

C GMM formulas

The GMM system contains $N+3$ moment conditions, with the first N sample moments corresponding to the pricing errors for each of the N test returns:

$$g_T(\mathbf{b}) \equiv$$

$$\frac{1}{T}\sum_{t=0}^{T} \left\{ \begin{array}{c} (R_{j,t+1} - R_{r,t+1}) - \gamma_0 - \gamma_c R_{j,t+1} (\Delta c_{t+1} - \mu_c) - \gamma_f R_{j,t+1} (\Delta f_{t+1} - \mu_f) - \\ \gamma_w R_{j,t+1} (r_{w,t+1} - \mu_w) \\ \Delta c_{t+1} - \mu_c \\ \Delta f_{t+1} - \mu_f \\ r_{w,t+1} - \mu_w \end{array} \right. = \mathbf{0},$$

$$i = 1, ..., N, \tag{C.27}$$

where (μ_c, μ_f, μ_w) denote the means of $(\Delta c_{t+1}, \Delta f_{t+1}, r_{w,t+1})$. The last three moment conditions in system (C.27) allow us to estimate the factor means. Thus, the estimated covariance risk prices from the first N moments account for the estimation error in the factors' means, as in Cochrane (2005) (Chapter 13), Yogo (2006), and Maio (2012).[9] We use one-sided p-values for the tests of individual significance of the risk prices since the respective signs are constrained by theory, as stressed in Section 2 in the paper.

Following Cochrane (2005) and Maio and Santa-Clara (2012), the weighting matrix

[9] In the case of the benchmark specification that does not include a constant term, the GMM system is obtained by setting $\gamma_0 = 0$.

associated with the GMM system (C.27) is given by

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_K \end{bmatrix}, \tag{C.28}$$

where $\mathbf{W}^*$ $(N \times N)$ is the weighting matrix associated with the first N moments; $\mathbf{0}$ denotes a conformable matrix of zeros; and $\mathbf{I}_K$ denotes a K-dimensional identity matrix. In this specification, $\mathbf{W}^*$ is the weighting matrix for the first N moment conditions (corresponding to the N pricing errors), while $\mathbf{I}_K$ is the weighting matrix associated with the last K orthogonality conditions that identify the factor means.

In the first-step GMM (OLS cross-sectional regression) $\mathbf{W}^*$ corresponds to the identity matrix, $\mathbf{W}^* = \mathbf{I}_N$, and in the second-step GMM (GLS cross-sectional regression), $\mathbf{W}^*$ is the inverse of the first $(N \times N)$ block of the spectral density matrix, $\mathbf{W}^* = \mathbf{S}_N^{-1}$.[10]

The parameter estimates, $\hat{\mathbf{b}}$, have variance formulas given by

$$\text{Var}(\hat{\mathbf{b}}) = \frac{1}{T}(\mathbf{d}'\mathbf{W}\mathbf{d})^{-1}\mathbf{d}'\mathbf{W}\hat{\mathbf{S}}\mathbf{W}\mathbf{d}(\mathbf{d}'\mathbf{W}\mathbf{d})^{-1}, \tag{C.29}$$

where $\mathbf{d} \equiv \frac{\partial g_T(\mathbf{b})}{\partial \mathbf{b}'}$ represents the matrix of moments' sensitivities to the parameters, and $\hat{\mathbf{S}}$ is an estimator for the spectral density matrix, $\mathbf{S}$, derived under the heteroskedasticity-robust standard errors (White (1980)), that is, no lags of the moment functions are considered in the computation of $\hat{\mathbf{S}}$.

The variance–covariance matrix for the moments from first-stage GMM is given by

$$\text{Var}\left(g_T(\hat{\mathbf{b}})\right) = \frac{1}{T}\left(\mathbf{I}_{N+K} - \mathbf{d}(\mathbf{d}'\mathbf{W}\mathbf{d})^{-1}\mathbf{d}'\mathbf{W}\right)\hat{\mathbf{S}}\left(\mathbf{I}_{N+K} - \mathbf{W}\mathbf{d}(\mathbf{d}'\mathbf{W}\mathbf{d})^{-1}\mathbf{d}'\right), \tag{C.30}$$

where both $\mathbf{I}_{N+K}$ and $\mathbf{W}$ denote an identity matrix of $N+K$ dimension. The first (N, N) block of (C.30) denotes the covariance matrix of the pricing errors.

In computing the asymptotic standard errors of the preference parameter estimates, $\widehat{\boldsymbol{\theta}} \equiv \left(\widehat{\psi}, \widehat{\gamma}, \widehat{\varepsilon}\right)'$, we use the delta method:

$$\text{Var}(\widehat{\boldsymbol{\theta}}) = \frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}'} \text{Var}(\widehat{\mathbf{b}}) \frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}}, \tag{C.31}$$

[10] Notice that the GLS cross-sectional regression does not correspond to a fully efficient GMM estimation, since in this case, the weighting matrix would be the inverse of the full spectral density matrix, $\mathbf{S}^{-1}$.

where $\widehat{\mathbf{b}} \equiv (\widehat{\gamma}_c, \widehat{\gamma}_f, \widehat{\gamma}_w)'$ denotes the vector of risk price estimates. The matrix of derivatives, $\frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}'}$, is given by

$$\frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}'} \equiv \begin{bmatrix} \frac{\partial \widehat{\psi}}{\partial \widehat{\gamma}_c} & \frac{\partial \widehat{\psi}}{\partial \widehat{\gamma}_f} & \frac{\partial \widehat{\psi}}{\partial \widehat{\gamma}_w} \\ \frac{\partial \widehat{\gamma}}{\partial \widehat{\gamma}_c} & \frac{\partial \widehat{\gamma}}{\partial \widehat{\gamma}_f} & \frac{\partial \widehat{\gamma}}{\partial \widehat{\gamma}_w} \\ \frac{\partial \widehat{\varepsilon}}{\partial \widehat{\gamma}_c} & \frac{\partial \widehat{\varepsilon}}{\partial \widehat{\gamma}_f} & \frac{\partial \widehat{\varepsilon}}{\partial \widehat{\gamma}_w} \end{bmatrix} = \begin{bmatrix} \frac{\widehat{\gamma}_w - 1}{\widehat{\gamma}_c^2} & 0 & -\frac{1}{\widehat{\gamma}_c} \\ 1 & 0 & 1 \\ \frac{\widehat{\gamma}_f}{(1-\widehat{\gamma}_w-\widehat{\gamma}_c)^2} & \frac{1}{1-\widehat{\gamma}_w-\widehat{\gamma}_c} & \frac{\widehat{\gamma}_f}{(1-\widehat{\gamma}_w-\widehat{\gamma}_c)^2} \end{bmatrix}. \quad \text{(C.32)}$$

D Bootstrap simulation

The asymptotic theory embedded in the GMM robust standard errors might suffer from several problems in the current application. More specifically, the small sample size in the time-series (181 observations) can imply that the asymptotic approximation is not valid. This might be especially relevant in the second stage GMM, since the inverse of the spectral density matrix is poorly behaved when the number of moments is high relative to the number of observations. On the other hand, the moment functions might not be distributed as a martingale difference sequence (MDS) as it is assumed in the standard GMM theory used in this paper. In both cases, the asymptotic p-vales associated with the t-statistics and J-test statistics will not be close to the true p-values. To account for this problem, we conduct a bootstrap simulation to produce more robust (empirical) p-values for the tests of individual significance of the parameters and also for the J-test. The bootstrap simulation consists of 10,000 replications, and in each replication, the portfolio return data and the factors are simulated independently, without imposing the asset pricing model's restrictions, that is, the data are simulated under the hypothesis that the model is not true.

The bootstrap algorithm used in this paper consists of the following steps:

1. The model is estimated by first-stage GMM, and we save the consumption, interest growth and market return risk prices, $(\widehat{\gamma}_c, \widehat{\gamma}_f, \widehat{\gamma}_w)$; the χ^2-statistic; and the implied estimates for the preference parameters, $(\widehat{\psi}, \widehat{\gamma}, \widehat{\varepsilon})$. We repeat the same procedure for second-stage GMM.

2. In each replication $m = 1, ..., 10,000$, we construct a pseudo-sample of excess returns

and real returns for each asset (of size T) by drawing with replacement,

$$\{(R_{j,t+1} - R_{r,t+1})^m, t = s_{j1}^m, s_{j2}^m, ..., s_{jT}^m\}, j = 1, ..., N,$$
$$\{R_{j,t+1}^m\}, t = s_{j1}^m, s_{j2}^m, ..., s_{jT}^m\}, j = 1, ..., N,$$

where the time indices $s_{j1}^m, s_{j2}^m, ..., s_{jT}^m$ are created randomly from the original time sequence $1, ..., T$. Notice that both the excess return and real return of a given asset have the same time index.

3. For each replication $m = 1, ..., 10,000$, we also construct an independent pseudo-sample of the factors,

$$\{\Delta c_{t+1}^m, t = r_1^m, r_2^m, ..., r_T^m\},$$
$$\{\Delta f_{t+1}^m, t = p_1^m, p_2^m, ..., p_T^m\},$$
$$\{r_{w,t+1}^m, t = q_1^m, q_2^m, ..., q_T^m\},$$

where the time sequences $(r_1^m, r_2^m, ..., r_T^m)$, $(p_1^m, p_2^m, ..., p_T^m)$, and $(q_1^m, q_2^m, ..., q_T^m)$, are mutually independent and also independent from $s_{j1}^m, s_{j2}^m, ..., s_{jT}^m$.

4. In each replication, we estimate the CI-CAPM by first-stage GMM, but using the artificial data rather than the original data. The moment conditions are given by

$$g_T(\mathbf{b}) \equiv$$
$$\frac{1}{T}\sum_{t=0}^{T} \left\{ \begin{array}{c} (R_{j,t+1} - R_{r,t+1})^m - \gamma_c^m R_{j,t+1}^m \left(\Delta c_{t+1}^m - \mu_c\right) - \gamma_f^m R_{j,t+1}^m \left(\Delta f_{t+1}^m - \mu_f\right) - \\ \gamma_w^m R_{j,t+1}^m \left(r_{w,t+1}^m - \mu_w\right) \\ \Delta c_{t+1}^m - \mu_c^m \\ \Delta f_{t+1}^m - \mu_f^m \\ r_{w,t+1}^m - \mu_w^m \end{array} \right. = \mathbf{0}.$$

We estimate the individual factor risk prices, $(\widehat{\gamma}_c^m, \widehat{\gamma}_f^m, \widehat{\gamma}_w^m)$; the χ^2-statistic, χ^{2m}; and the implied preference parameter estimates, $(\widehat{\psi}^m, \widehat{\gamma}^m, \widehat{\varepsilon}^m)$. In result, we have empirical distributions of the factor risk prices; χ^2 statistic; and preference parameter estimates. We repeat the procedure for second-stage GMM.

5. The empirical p-values associated with the risk prices (for a one-sided test) are computed as

$$p(\gamma_c) = [\#\{\widehat{\gamma}_c^m \geq \widehat{\gamma}_c\}]/10,000,$$
$$p(\gamma_f) = [\#\{\widehat{\gamma}_f^m \leq \widehat{\gamma}_f\}]/10,000,$$
$$p(\gamma_w) = [\#\{\widehat{\gamma}_w^m \geq \widehat{\gamma}_w\}]/10,000,$$

and the empirical p-value associated with the structural estimates are given by

$$p(\gamma) = [\#\{\widehat{\gamma}^m \geq \widehat{\gamma}\}]/10,000,$$
$$p(\psi) = [\#\{\widehat{\psi}^m \geq \widehat{\psi}\}]/10,000,$$
$$p(\varepsilon) = [\#\{\widehat{\varepsilon}^m \geq \widehat{\varepsilon}\}]/10,000.$$

The empirical p-value for the χ^2-statistic is computed as

$$p(\chi^2) = [\#\{\chi^{2m} \geq \chi^2\}]/10,000.$$

We accept $\mathbf{H}_0$ if $p > \alpha$, and reject otherwise, where α is the nominal significance level. We repeat the procedure for second-stage GMM.

E Derivation of model 1

The representative consumer intertemporal problem is given by

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} U_t = \left\{ (1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{E.33}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right),$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1},$$

$$C_t \leq M_t.$$

It is understood that one of the risky assets is an asset which pays as dividend the lump sum transfer of the exogenous change in nominal money supply from the monetary authority to the consumers. This approach has been used by others (see Tristani (2009)), and it eases considerably the derivations. As is standard in the literature, we assume that the cash in advance constraint is binding. It is not rational for a utility maximizer to hold money and incur an opportunity cost if this cash is not for acquiring consumption goods. Relaxing this assumption and introducing a Lagrange multiplier for the cash in advance constraint will simply not be tractable, unless one puts more structure on the economy (dynamics of exogenous processes) and uses approximations. In fact, the enormous literature on DSGE (Dynamic Stochastic General Equilibrium) models used by central banks for conducting monetary policy rely on the MIUF setting (see Fernández-Villaverde (2010) for a review on this literature).

This problem can be represented in a dynamic programming setting as

$$J\left(W_t\right) \equiv \max_{C_t,\{\omega_{j,t}\}_{j=1}^{N}} \left\{(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta\left[\mathrm{E}_t\left(J\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \tag{E.34}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1}\left[W_t - C_t\left(1+\frac{R_{f,t+1}-1}{R_{f,t+1}}\right)\right],$$

$$R_{w,t+1} = \sum_{j=1}^{N}\omega_{j,t}\left(R_{j,t+1}-R_{r,t+1}\right)+R_{r,t+1}.$$

The first order conditions relative to C_t is given by

$$(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}-1} = \delta\left[\mathrm{E}_t\left(J\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{\frac{1}{\theta}-1}\mathrm{E}_t\left[J\left(W_{t+1}\right)^{-\gamma}J_W\left(W_{t+1}\right)R_{w,t+1}\left(1+\frac{R_{f,t+1}-1}{R_{f,t+1}}\right)\right]. \tag{E.35}$$

Similarly to Epstein and Zin (1991), let's assume that the value function is proportional to wealth:

$$J\left(W_t\right) = \phi_t W_t. \tag{E.36}$$

By substituting (E.36) in (E.35), using the intertemporal budget constraint and the linearity of conditional expectations, and rearranging, we have:

$$0 = (1-\delta)\, C_t^{\frac{1-\gamma}{\theta}-1} - \delta\left[\mathrm{E}_t\left(\phi_{t+1}^{1-\gamma}W_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\theta}}\frac{1+\frac{R_{f,t+1}-1}{R_{f,t+1}}}{W_t - C_t\left(1+\frac{R_{f,t+1}-1}{R_{f,t+1}}\right)}. \tag{E.37}$$

Moreover, the value function in (E.34) can be rearranged, leading to

$$\phi_t^{\frac{1-\gamma}{\theta}} W_t^{\frac{1-\gamma}{\theta}} - (1-\delta) C_t^{\frac{1-\gamma}{\theta}} = \delta \left[\mathrm{E}_t \left(\phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} . \tag{E.38}$$

By substituting (E.38) in (E.37), and after some tedious algebra, we obtain the explicit functional form for the value function,

$$\phi_t = \left(1 + \frac{R_{f,t+1} - 1}{R_{f,t+1}} \right)^{-\frac{\theta}{1-\gamma}} (1-\delta)^{\frac{\theta}{1-\gamma}} \left(\frac{C_t}{W_t} \right)^{1-\frac{\theta}{1-\gamma}} , \tag{E.39}$$

thus confirming the previous guess. By substituting (E.39) in (E.37), we derive the Euler equation for the return on the market portfolio:

$$1 = \mathrm{E}_t \left[\delta^\theta \left(\frac{1 + \frac{R_{f,t+2}-1}{R_{f,t+2}}}{1 + \frac{R_{f,t+1}-1}{R_{f,t+1}}} \right)^{-\theta} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} R_{w,t+1}^{\theta} \right] . \tag{E.40}$$

By performing similar steps as in Appendix A above, the Euler equation for an arbitrary risky return, $R_{j,t+1}$, is given by

$$1 = \mathrm{E}_t \left[\delta^\theta \left(\frac{1 + \frac{R_{f,t+2}-1}{R_{f,t+2}}}{1 + \frac{R_{f,t+1}-1}{R_{f,t+1}}} \right)^{-\theta} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} R_{w,t+1}^{\theta-1} R_{j,t+1} \right] . \tag{E.41}$$

F Derivation of model 2

The representative consumer intertemporal problem is given by

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{F.42}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left(W_t - C_t - T(C_t, M_t) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right),$$

$$R_{w,t+1} = \sum_{j=1}^{N} \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}.$$

It is again understood that one of the risky assets is an asset which pays as dividend the lump sum transfer of the exogenous change in nominal money supply from the monetary authority to the consumers.

The function $T(\cdot)$ is the real transaction cost of real consumption. The usual assumption for $T(\cdot)$ are that: $T > 0$, $T_C > 0$, $T_M < 0$, $T_{CC} \leq 0$, and $T_{MM} \geq 0$.

In order for the solution to be scale invariant we define the transaction cost function as a function of the consumption to real balances ratio. Hence we rewrite the problem as follows,

$$\max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{F.43}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left[W_t - C_t \left(1 + \tau \left(\frac{C_t}{M_t} \right) \right) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right],$$

$$R_{w,t+1} = \sum_{j=1}^N \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1},$$

where we assume that $\tau > 0$, $\tau'(\cdot) > 0$, and $\tau''(\cdot) < 0$. This problem can be represented in a dynamic programming setting as

$$J(W_t) \equiv \max_{C_t, M_t, \{\omega_{j,t}\}_{j=1}^N} \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[\mathrm{E}_t \left(J(W_{t+1})^{1-\gamma} \right) \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{F.44}$$

$$\text{s.t. } W_{t+1} = R_{w,t+1} \left[W_t - C_t \left(1 + \tau \left(\frac{C_t}{M_t} \right) \right) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} M_t \right]$$

$$R_{w,t+1} = \sum_{j=1}^N \omega_{j,t} \left(R_{j,t+1} - R_{r,t+1} \right) + R_{r,t+1}.$$

The first-order conditions relative to consumption, C_t, and real balances holdings, M_t, are given by

$$\begin{aligned} 0 &= (1-\delta) C_t^{\frac{1-\gamma}{\theta}-1} \\ &\quad -\delta \left[\mathrm{E}_t \left(J(W_{t+1})^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} J_W(W_{t+1}) R_{w,t+1} \left(1 + \tau \left(\frac{C_t}{M_t} \right) + \frac{C_t}{M_t} \tau' \left(\frac{C_t}{M_t} \right) \right) \right] \end{aligned} \tag{F.45}$$

$$0 = \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} J_W(W_{t+1}) R_{w,t+1} \left(\frac{C_t^2}{M_t^2} \tau' \left(\frac{C_t}{M_t} \right) - \frac{R_{f,t+1} - 1}{R_{f,t+1}} \right) \right] \tag{F.46}$$

From (F.45) and (F.46) we have,

$$\frac{C_t^2}{M_t^2} \tau' \left(\frac{C_t}{M_t} \right) = \frac{R_{f,t+1} - 1}{R_{f,t+1}}, \tag{F.47}$$

which simply states that the marginal transaction cost savings from money holdings has to be equal to the respective opportunity cost.

As in Appendix E above, we assume that the value function is proportional to wealth,

$$J\left(W_t\right) = \phi_t W_t, \tag{F.48}$$

and by substituting (F.48) in (F.45) and rearranging, we have:

$$\begin{aligned} 0 &= (1-\delta)\, C_t^{\frac{1-\gamma}{\theta}-1} \\ & -\delta \left[\mathrm{E}_t\left(\phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\theta}} \frac{1+\tau\left(\frac{C_t}{M_t}\right)+\frac{C_t}{M_t}\tau'\left(\frac{C_t}{M_t}\right)}{W_t - C_t\left(1+\tau\left(\frac{C_t}{M_t}\right)\right)-\frac{R_{f,t+1}-1}{R_{f,t+1}}M_t}. \end{aligned} \tag{F.49}$$

Moreover, the value function in (F.44) can be rearranged, leading to

$$J(W_t) - (1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} = \delta\left[\mathrm{E}_t\left(\phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\theta}}. \tag{F.50}$$

By substituting (F.50) in (F.49), and after some tedious algebra, we obtain the explicit functional form for the value function,

$$\phi_t = \left[\frac{C_t^{-1}W_t - \left(1+\tau\left(\frac{C_t}{M_t}\right)\right)-\frac{R_{f,t+1}-1}{R_{f,t+1}}C_t^{-1}M_t}{1+\tau\left(\frac{C_t}{M_t}\right)+\frac{C_t}{M_t}\tau'\left(\frac{C_t}{M_t}\right)}+1\right]^{\frac{\theta}{1-\gamma}} (1-\delta)^{\frac{\theta}{1-\gamma}}\,\frac{C_t}{W_t}, \tag{F.51}$$

thus confirming the previous guess. By substituting (F.51) in (F.49), we derive the Euler equation for the return on the market portfolio:

$$1 = \mathrm{E}_t\left[\delta^\theta\left(\frac{1+\tau\left(\frac{C_t}{M_t}\right)+\frac{C_t}{M_t}\tau'\left(\frac{C_t}{M_t}\right)}{1+\tau\left(\frac{C_{t+1}}{M_{t+1}}\right)+\frac{C_{t+1}}{M_{t+1}}\tau'\left(\frac{C_{t+1}}{M_{t+1}}\right)}\right)^\theta \frac{C_{t+1}^{1-\gamma-\theta}}{C_t^{1-\gamma-\theta}} R_{w,t+1}^\theta\right]. \tag{F.52}$$

Following the same approach as in the previous appendix, the Euler equation for an arbitrary risky return, $R_{j,t+1}$, is given by

$$1 = \mathrm{E}_t\left[\delta^\theta\left(\frac{1+\tau\left(\frac{C_t}{M_t}\right)+\frac{C_t}{M_t}\tau'\left(\frac{C_t}{M_t}\right)}{1+\tau\left(\frac{C_{t+1}}{M_{t+1}}\right)+\frac{C_{t+1}}{M_{t+1}}\tau'\left(\frac{C_{t+1}}{M_{t+1}}\right)}\right)^\theta \frac{C_{t+1}^{1-\gamma-\theta}}{C_t^{1-\gamma-\theta}} R_{w,t+1}^{\theta-1} R_{j,t+1}\right]. \tag{F.53}$$

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Table 1: Epstein–Zin model

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_0 + \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right).$$

The test assets consist of the value-weighted market return and equity portfolios. The equity portfolios are the 25 size/book-to-market portfolios (Panel A) and 25 size/long term return reversal portfolios (Panel B). The estimation procedure is first-stage GMM with equally weighted errors. In everything else this table is identical to Table 2 in the paper.

Row	γ_0	γ_c	γ_w	χ^2	R^2_{OLS}	MAE(%)
			Panel A (SBM25)			
1		240.51		34.40	−0.18	0.66
		(1.71)		(0.10)		
2			2.60	60.86	−0.42	0.72
			(**2.69**)	(0.00)		
3		258.50	−0.20	31.52	−0.18	0.66
		(1.85)	(−0.09)	(0.14)		
4	0.015	71.61		52.95	0.04	0.61
	(1.77)	(0.73)		(0.00)		
5	0.027		−0.71	57.25	0.02	0.65
	(**2.99**)		(−0.46)	(0.00)		
6	0.026	243.88	−3.25	32.84	0.23	0.55
	(*1.55*)	(2.21)	(*−1.42*)	(0.08)		
			Panel B (SLTR25)			
1		273.14		25.08	−0.12	0.56
		(1.83)		(0.46)		
2			2.86	57.64	−0.01	0.53
			(**2.96**)	(0.00)		
3		63.93	2.20	48.48	0.00	0.52
		(0.60)	(1.25)	(0.00)		
4	0.014	113.55		43.69	0.13	0.49
	(*1.37*)	(1.28)		(0.01)		
5	0.012		1.38	56.31	0.06	0.52
	(*1.44*)		(0.89)	(0.00)		
6	0.019	185.39	−1.45	33.74	0.14	0.48
	(*1.44*)	(1.78)	(−0.61)	(0.07)		

Table 2: Alternative macroeconomic factor models

This table reports the evaluation results for alternative macroeconomic factor models. The factor models are the conditional C-CAPM from Lettau and Ludvigson (2001) (LL2001); the conditional C-CAPM from Lustig and Van Nieuwerburgh (2005) (LVN2005); the model with durable consumption from Yogo (2006) (Y2006); and the consumption-CAPM with lead consumption growth from Parker and Julliard (2005) (PJ2005). The test assets consist of the 25 size/book-to-market portfolios. The column χ^2 indicates whether the model passes the χ^2 test. The column R^2_{OLS} denotes the OLS cross-sectional R^2. All the values are obtained from the original studies.

Model	Sample	R^2_{OLS}	Pass χ^2 test
LL2001	1963.III–1998.III	0.70	yes
LVN2005	1952.I–2002.IV	0.68	not reported
Y2006	1951.I–2001.IV	0.94	yes
PJ2005	1963.III–1999.IV	0.66	no

Table 3: Fama–French three factor model

This table reports the estimation and evaluation results for the Fama and French (1993) model. The test assets consist of the value-weighted market return and equity portfolios. The portfolios are the 25 size/book-to-market portfolios (Panel A) and the 25 size/long term return reversal portfolios (Panel B). In row 1 (2) the models are estimated without (with) an intercept. The column labeled γ_0 show the point estimates for the intercept in the estimation with first-stage GMM. Below the intercept estimates are reported the asymptotic GMM robust t-statistics (in parenthesis). The column J presents the levels (first line) and associated asymptotic p-values (second line) for the χ^2 statistic. The column R^2_{OLS} denotes the OLS cross-sectional R^2. The column R^2_{WLS} represents the WLS cross-sectional R^2. MAE(%) denotes the average absolute pricing error from first-stage GMM. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	γ_0	R^2_{OLS}	MAE(%)	J	R^2_{WLS}
		Panel A (SBM25)			
1		0.74	0.28	49.14	0.56
				(0.00)	
2	0.037	0.79	0.25	42.35	
	(**3.18**)			(0.01)	
		Panel B (SLTR25)			
1		0.74	0.23	61.43	0.69
				(0.00)	
2	0.011	0.76	0.24	56.23	
	(0.95)			(0.00)	

Table 4: Augmented factor model

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_d \operatorname{Cov}\left(R_{j,t+1}, \Delta d_{t+1}\right) + \gamma_f \operatorname{Cov}\left(R_{j,t+1}, \Delta f_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right).$$

The test assets consist of the value-weighted market return and equity portfolio returns. The portfolios are the 25 size/book-to-market portfolios (Panel A) and the 25 size/long term return reversal portfolios (Panel B). The estimation procedure is first-stage GMM with equally weighted errors. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the covariance risk price estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the levels (first line) and associated asymptotic p-values (second line) for the χ^2 statistic. The column R^2_{OLS} denotes the OLS cross-sectional R^2. MAE(%) denotes the average absolute pricing error. The sample is 1963:III–2007:IV. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Row	γ_c	γ_d	γ_f	γ_w	χ^2	R^2_{OLS}	MAE(%)
			Panel A (SBM25)				
TB	236.06	102.00	−11.79	−1.01	19.84	0.69	0.28
	(1.65)	(0.50)	(−2.16)	(−0.39)	(0.59)		
FED	312.99	−5.04	−10.27	−3.23	22.81	0.65	0.35
	(2.25)	(−0.02)	(−**2.99**)	(−1.24)	(0.41)		
			Panel B (SLTR25)				
TB	384.30	−115.58	−8.81	−1.15	18.30	0.58	0.29
	(1.84)	(−0.57)	(−**2.79**)	(−0.41)	(0.69)		
FED	335.08	−73.17	−7.37	−2.17	23.37	0.63	0.28
	(*1.63*)	(−0.41)	(−2.31)	(−0.75)	(0.38)		

Table 5: CI-CAPM: future consumption growth

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+4}\right) + \gamma_f \operatorname{Cov}\left(R_{j,t+1}, \Delta f_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right).$$

The test assets consist of the value-weighted market return and equity portfolio returns. The portfolios are the 25 size/book-to-market portfolios (Panel A) and the 25 size/long term return reversal portfolios (Panel B). Consumption growth is measured over the next four quarters. The estimation procedure is first-stage GMM with equally weighted errors. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the covariance risk price estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the levels (first line) and associated asymptotic p-values (second line) for the χ^2 statistic. The column R^2_{OLS} denotes the OLS cross-sectional R^2. MAE(%) denotes the average absolute pricing error. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Row	γ_c	γ_f	γ_w	χ^2	R^2_{OLS}	MAE(%)
			Panel A (SBM25)			
TB	111.52	−11.18	−3.09	13.67	0.65	0.34
	(**2.69**)	(*−1.58*)	(−0.89)	(0.94)		
FED	115.26	−9.95	−5.11	15.37	0.66	0.35
	(2.25)	(−2.23)	(*−1.33*)	(0.88)		
			Panel B (SLTR25)			
TB	77.89	−6.35	−1.04	29.81	0.46	0.32
	(2.23)	(−2.18)	(−0.46)	(0.15)		
FED	81.38	−6.13	−2.40	28.09	0.57	0.28
	(2.02)	(**−2.33**)	(−0.92)	(0.21)		

Table 6: CI-CAPM: alternative portfolios

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \gamma_c \operatorname{Cov}\left(R_{j,t+1}, \Delta c_{t+1}\right) + \gamma_f \operatorname{Cov}\left(R_{j,t+1}, \Delta f_{t+1}\right) + \gamma_w \operatorname{Cov}\left(R_{j,t+1}, r_{w,t+1}\right).$$

The test assets consist of the value-weighted market return and equity portfolios. The equity portfolios are 10 size plus 10 book-to-market portfolios (Panel A); 10 size portfolios plus 10 portfolios sorted on the O-score measure of financial distress (Panel B); 10 size plus 10 portfolios sorted on total accruals (Panel C); 10 size plus 10 asset growth portfolios (Panel D); and 10 size plus 5 industry portfolios (Panel E). The estimation procedure is first-stage GMM with equally weighted errors. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the covariance risk price estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the levels (first line) and associated asymptotic p-values (second line) for the χ^2 statistic. The column R^2_{OLS} denotes the OLS cross-sectional R^2. MAE(%) denotes the average absolute pricing error. In Panel A, the sample is 1963:III–2008:III, in Panels B to D the sample is 1972:II–2008:III, and in Panel E the sample is 1963:III–2007:IV. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Row	γ_c	γ_f	γ_w	χ^2	R^2_{OLS}	MAE(%)
			Panel A (S10+BM10)			
TB	180.51	−7.23	0.73	16.52	0.77	0.17
	(*1.54*)	(−2.06)	(0.42)	(0.56)		
FED	192.77	−6.41	−0.77	18.27	0.86	0.14
	(1.24)	(*−1.51*)	(−0.42)	(0.44)		
			Panel B (S10+OS10)			
TB	202.85	−11.76	1.42	13.33	0.51	0.34
	(1.00)	(−1.71)	(0.57)	(0.77)		
FED	122.21	−7.25	−0.22	30.53	0.15	0.43
	(0.79)	(−1.65)	(−0.12)	(0.03)		
			Panel C (S10+TA10)			
TB	91.78	−5.49	1.99	23.81	0.55	0.21
	(*1.31*)	(−1.88)	(*1.34*)	(0.16)		
FED	85.69	−4.72	0.78	29.46	0.54	0.20
	(1.18)	(−1.79)	(0.59)	(0.04)		
			Panel D (S10+AG10)			
TB	154.64	−6.89	1.52	24.52	0.35	0.30
	(*1.32*)	(−**2.58**)	(0.79)	(0.14)		
FED	238.58	−6.25	−0.91	24.43	0.33	0.28
	(*1.39*)	(−1.73)	(−0.39)	(0.14)		
			Panel E (S10+IND5)			
TB	168.01	−5.16	0.48	22.09	0.67	0.20
	(0.86)	(*−1.28*)	(0.22)	(0.05)		
FED	119.07	−3.26	0.45	26.74	0.63	0.20
	(0.68)	(−0.91)	(0.18)	(0.01)		

Table 7: CI-CAPM: beta representation

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left(R_{j,t+1} - R_{r,t+1}\right) = \lambda_c \beta_{j,c} + \lambda_f \beta_{j,f} + \lambda_w \beta_{j,w}.$$

The test assets consist of the value-weighted market return and equity portfolio returns. The portfolios are the 25 size/book-to-market portfolios (Panel A) and the 25 size/long term return reversal portfolios (Panel B). The estimation procedure is the time-series/cross-sectional regressions approach. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the beta risk price estimates and the the second line reports the t-statistics based on Shanken standard errors (in parenthesis). The column χ^2 presents the levels (first line) and associated asymptotic p-values (second line) for the χ^2 statistic. The column R^2_{OLS} denotes the OLS cross-sectional R^2. MAE(%) denotes the average absolute pricing error. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Row	λ_c	λ_f	λ_w	χ^2	R^2_{OLS}	MAE(%)
			Panel A (SBM25)			
TB	0.12	−27.30	1.09	14.34	0.84	0.23
	(0.39)	(<u>−2.17</u>)	(*1.34*)	(0.92)		
FED	0.23	−22.01	0.87	16.63	0.75	0.29
	(0.77)	(<u>−2.19</u>)	(1.08)	(0.83)		
			Panel B (SLTR25)			
TB	0.18	−12.50	1.52	39.15	0.64	0.26
	(0.60)	(**−2.46**)	(<u>2.21</u>)	(0.02)		
FED	0.28	−12.02	1.32	32.39	0.67	0.26
	(0.78)	(<u>−2.29</u>)	(<u>1.92</u>)	(0.09)		

Table 8: Euler equations: SBM25

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{\frac{R_{f,t+2}-1}{R_{f,t+2}}}{\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{\varepsilon(\gamma-1)} R_{w,t+1}^{\theta-1}(R_{j,t+1}-R_{r,t+1})\right]=0.$$

The test assets consist of the value-weighted market return and the 25 size/book-to-market portfolios. The estimation procedure is by two-step GMM with the Hansen and Jagannathan (1997) weighting matrix in the first-stage estimation. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the parameter estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the level (first line) and associated p-value (in parenthesis) for the J-test. HJ denotes the Hansen-Jagannathan distance. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	γ	ψ	ε	χ^2	HJ
Panel A (First-stage)					
TB	70	0.05	0.05		0.448
	(**2.67**)	(**3.07**)	(2.27)		
FED	78.50	0.05	0.05		0.429
	(**2.58**)	(**3.06**)	(**2.76**)		
Panel B (Second-stage)					
TB	97.50	0.05	0.05	58.72	
	(**3.53**)	(**4.34**)	(**3.48**)	(0.00)	
FED	104.50	0.05	0.05	57.47	
	(**3.49**)	(**4.52**)	(**3.90**)	(0.00)	

Table 9: Euler equations: SLTR25

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{\frac{R_{f,t+2}-1}{R_{f,t+2}}}{\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{\varepsilon(\gamma-1)} R_{w,t+1}^{\theta-1}(R_{j,t+1}-R_{r,t+1})\right]=0.$$

The test assets consist of the value-weighted market return and the 25 size/long term return reversal portfolios. The estimation procedure is by two-step GMM with the Hansen and Jagannathan (1997) weighting matrix in the first-stage estimation. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the parameter estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the level (first line) and associated p-value (in parenthesis) for the J-test. HJ denotes the Hansen-Jagannathan distance. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	γ	ψ	ε	χ^2	HJ
	Panel A (First-stage)				
TB	60	0.05	0.05		0.476
	(**2.46**)	(**2.67**)	(<u>1.86</u>)		
FED	62.50	0.05	0.05		0.462
	(**2.46**)	(**2.47**)	(<u>2.07</u>)		
	Panel B (Second-stage)				
TB	85.50	0.05	0.05	62.28	
	(**3.05**)	(**3.73**)	(**3.11**)	(0.00)	
FED	89.50	0.05	0.05	54.00	
	(**3.05**)	(**3.77**)	(**3.37**)	(0.00)	

Table 10: Euler equations: Model 1

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{1+\frac{R_{f,t+2}-1}{R_{f,t+2}}}{1+\frac{R_{f,t+1}-1}{R_{f,t+1}}}\right)^{-\theta} R_{w,t+1}^{\theta-1}(R_{j,t+1}-R_{r,t+1})\right]=0.$$

The test assets consist of the value-weighted market return and the 25 size/book-to-market portfolios. The estimation procedure is by two-step GMM with the Hansen and Jagannathan (1997) weighting matrix in the first-stage estimation. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the parameter estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the level (first line) and associated p-value (in parenthesis) for the J-test. HJ denotes the Hansen-Jagannathan distance. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	γ	ψ	χ^2	HJ
	Panel A (First-stage)			
TB	69	0.05		0.476
	(**2.59**)	(**3.09**)		
FED	68	0.05		0.478
	(**2.57**)	(**3.06**)		
	Panel B (Second-stage)			
TB	99.50	0.05	61.47	
	(**4.34**)	(**4.28**)	(0.00)	
FED	98	0.05	61.63	
	(**4.31**)	(**4.23**)	(0.00)	

Table 11: Euler equations: Model 2

This table reports the estimation and evaluation results for the following model:

$$\mathrm{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma-\theta}\left(\frac{1+a^{\frac{1}{1+b}}b^{\frac{-b}{1+b}}(1+b)\left[\frac{R_{f,t+2}-1}{R_{f,t+2}}\right]^{\frac{b}{1+b}}}{1+a^{\frac{1}{1+b}}b^{\frac{-b}{1+b}}(1+b)\left[\frac{R_{f,t+1}-1}{R_{f,t+1}}\right]^{\frac{b}{1+b}}}\right)^{-\theta} R_{w,t+1}^{\theta-1}(R_{j,t+1}-R_{r,t+1})\right]=0.$$

The test assets consist of the value-weighted market return and the 25 size/book-to-market portfolios. The estimation procedure is by two-step GMM with the Hansen and Jagannathan (1997) weighting matrix in the first-stage estimation. The interest rate proxies used to compute the interest growth factor are the three-month Treasury bill rate (TB) and the Fed funds rate (FED). The first line associated with each row presents the parameter estimates and the second line reports the asymptotic GMM robust t-statistics (in parenthesis). The column χ^2 presents the level (first line) and associated p-value (in parenthesis) for the J-test. HJ denotes the Hansen-Jagannathan distance. The sample is 1963:III–2008:III. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	γ	ψ	b	χ^2	HJ
Panel A (First-stage)					
TB	69.5	0.05	0.05		0.476
	(**2.72**)	(**3.09**)	(0.05)		
FED	69	0.05	0.05		0.477
	(**2.87**)	(**3.02**)	(0.06)		
Panel B (Second-stage)					
TB	100	0.05	0.05	61.44	
	(**4.14**)	(**4.29**)	(0.09)	(0.00)	
FED	99	0.05	0.05	61.49	
	(**3.96**)	(**4.23**)	(0.10)	(0.00)	