Appendix to:
"An Extended Macro-Finance Model with Financial Factors"
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An Extended Macro-Finance Model with Financial Factors: Technical Appendix

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Abstract

This technical appendix provides some more technical comments concerning the EMF model, used in the paper " An Extended Macro-Finance Model with Financial Factors". We discuss consecutively the following features: (i) Model specification and more specifically transition and measurement equation; (ii) the econometric methodology including likelihood and priors and (ii) additional results including the tables containing moments of the prior and posterior distributions and impulse response functions.

1 The Model

In this section we briefly discuss the state space system estimated in the paper. This state space representation introduces the restrictions that allow us to incorporate stochastic trends into the analysis. This appendix focusses on the specification of the transition equation. The identification restrictions for money market and return generating factors and the extension of the transition equation (under no-arbitrage) to the yield curve are extensively discussed in the paper.

1.1 The transition equation

The state space incorporates a state vector combining observable macroeconomic variables - inflation (π_t), output gap (y_t) and the policy interest rate (i_t^{cb}), collected in the vector $X_t^M = [\pi_t,\ y_t,\ i_t^{cb}]'$ - with a set of latent variables. Depending on their dynamics, we distinguish two types of latent variables: three stationary latent variables $l_t = [l_{1,t},\ l_{2,t},\ l_{3,t}]'$ and two stochastic trends, $\xi_t = [\xi_{1,t},\ \xi_{2,t}]'$. The latent variables are collected in the vector $X_t^L = [\ l_t', \xi_t'\ ]'$.

We assume a dynamics for the state variables , $X_t = [\ X_t^{M\prime},\ X_t^{L\prime}]'$, which incorporates following characteristics: (i) the set of stochastic trends, ξ_t, are exogenous and independent, (ii) the stochastic trends are the main drivers of the macroeconomic variables and determine long-run expectations, i.e. $\lim_{s\to\infty} X_t^M = T^D\xi_t$ and (iii) we allow for interactions between the stationary latent factors, l_t, and the macroeconomic state. The state space dynamics, consistent with these restrictions can be summarized by a standard VAR(I) representation:

$$\begin{bmatrix} X_t^M \\ l_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} 0 \\ C^l \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{MM} & \Phi^{Ml} & 0 \\ \Phi^{lM} & \Phi^{ll} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} X_{t-1}^M \\ l_{t-1} \\ \xi_{t-1} \end{bmatrix} + \begin{bmatrix} I-\Phi^{MM} & -\Phi^{Ml} & 0 \\ -\Phi^{lM} & I-\Phi^{ll} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T^D \\ 0 \\ 0 \end{bmatrix} \xi_t$$
$$+ \begin{bmatrix} D^{MM} & 0 & 0 \\ D^{lM} & D^{ll} & 0 \\ 0 & 0 & S^{\xi\xi} \end{bmatrix} \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^l \\ \varepsilon_t^\xi \end{bmatrix}, \qquad \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^l \\ \varepsilon_t^\xi \end{bmatrix} \backsim MVN(0,I) \quad (1)$$

with $[\varepsilon_t^{M\prime},\ \varepsilon_t^{l\prime},\ \varepsilon_t^{\xi}]' = [\varepsilon_{\pi,t}, \varepsilon_{y,t}, \varepsilon_{i^{cb},t}, \varepsilon_{l_1,t}, \varepsilon_{l_2,t}, \varepsilon_{l_3,t}, \varepsilon_{\xi_\pi,t}, \varepsilon_{\xi_\rho,t}]'$, D^{MM} and D^{ll} lower triangular, $S^{\xi\xi}$ diagonal and

$$T^D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}.$$

After some elementary computations, the system can be rewritten as:

$$\begin{bmatrix} X_t^M \\ l_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} C^M \\ C^l \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{MM} & \Phi^{Ml} & \Phi^{M\xi} \\ \Phi^{lM} & \Phi^{ll} & \Phi^{l\xi} \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} X_{t-1}^M \\ l_{t-1} \\ \xi_{t-1} \end{bmatrix} + \begin{bmatrix} D^{MM} & 0 & D^{M\xi} \\ D^{lM} & D^{ll} & D^{l\xi} \\ 0 & 0 & S^{\xi\xi} \end{bmatrix} \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^l \\ \varepsilon_t^\xi \end{bmatrix} \tag{2}$$

By calling $\tilde{\Phi}$ the block of the feedback matrix that refers to the stationary variables $[X_t^{M\prime},\, l_t^\prime]^\prime$, i.e.:

$$\tilde{\Phi} = \begin{bmatrix} \Phi^{MM} & \Phi^{Ml} \\ \Phi^{lM} & \Phi^{ll} \end{bmatrix},$$

we can express the identification restrictions making sure that the stochastic endpoints are defined as the long-run expectations of the observable variables (see section 2.2.1 of the paper) as:

$$\begin{bmatrix} \Phi^{M\xi} \\ \Phi^{l\xi} \end{bmatrix} = (I - \tilde{\Phi}) \begin{bmatrix} T^D \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} C^M \\ C^l \end{bmatrix} = (I - \tilde{\Phi}) \begin{bmatrix} 0_{3\times 1} \\ \bar{C}^l_{3\times 1} \end{bmatrix},$$

$$\begin{bmatrix} D^{M\xi} \\ D^{l\xi} \end{bmatrix} = (I - \tilde{\Phi}) \begin{bmatrix} T^D \\ 0 \end{bmatrix} S^{\xi\xi}.$$

Finally, we impose restrictions on $\tilde{\Phi}$ by restricting all eigenvalues below one such that the system reverts to the long-run equilibrium implied by the stochastic trends:

$$\lim_{s\to\infty} E_t \left[X_s^M\right] = T^D \xi_t$$

$$\lim_{s\to\infty} E_t \left[l_s\right] = \bar{C}^l_{3\times 1}$$

2 Econometric methodology

We use standard Bayesian estimation techniques to estimate the model, consisting of equations (1) and (**??**) (see below). The vector containing the estimated parameters of the model is denoted by θ. The posterior of the parameters θ, $p(\theta \mid Z^T)$, is identified through Bayes rule:

$$p(\theta \mid Z^T) = \frac{L(\theta \mid Z^T)p(\theta)}{p(Z^T)}, \tag{3}$$

with Z^T the data set, $L()$ the likelihood function, $p(\theta)$ the priors and $p(Z^T)$ the marginal density of the data. In this section (i) we outline the procedure used to find the mode of the posterior distribution, (ii) we discuss in more detail the prior distributions and (iii) present the measurement equation together with the information variables.

2.1 Mode of the posterior

We found the mode of the posterior distribution by selecting the optimal posterior density value out of a set of 1000 optimizations. More specifically, we ran for 1000 times the following algorithm:

1. Select an initial the initial value, θ_0 as:

   $$\theta_0 = x_p + \varepsilon_t, \qquad \varepsilon_t \backsim N(0, V)$$

   with θ_p the priors' mean of the parameters and $V = diag(0.1 * \max(|\theta_m|, 0.001))$

2. set $\theta_{opt}^0 = \theta_0$

3. for j = 1,...,4

   (a) starting from θ_{opt}^{j-1} run the enhanced simulating annealing procedure of Siarry et al. (1997) to find $\theta_{sa}^j = \underset{\theta}{argmax}\ f(\theta)$

   (b) starting from θ_{sa}^j run a simplex algorithm in order to find $\theta_{simp}^j = \underset{\theta}{argmax}\ f(\theta)$

   (c) starting from θ_{simp}^j run a Newton-Raphson optimization $\theta_{NR}^j = \underset{\theta}{argmax}\ f(\theta)$

   (d) starting from θ_{NR}^j run the enhanced simulating annealing procedure of Siarry et al. (1997) to find $\theta_{opt}^j = \underset{\theta}{argmax}\ f(\theta)$

This procedure was repeated 1000 times. We selected the θ^j_{NR} with the highest posterior density out of these 1000 simulations as the posterior mode.

2.2 Priors densities

Table 1 lists the specific prior distributions used for the parameters contained in θ (see section 3 of the paper).We impose normal priors on all the feedback parameters Φ. The priors related to observable macroeconomic variables, Φ^{MM}, are based on preliminary regression analysis. In particular, we introduce significant inertia in the macroeconomic variables (mean auto-regressive parameters between 0.5 for inflation and 0.95 for the output gap). Univariate analysis of the Libor, T-bill and TED spreads suggests that the spreads contain significant inertia. This inertia is taken into account in the prior for the feedback of financial factors by assuming mean autoregressive parameters in Φ^{LL} of 0.6. Loose, zero-mean priors are used for most of the off-diagonal elements of Φ. This is the case for the feedback of macroeconomic variables on liquidity and return-forecasting factors, Φ^{lM}, and for the interaction across financial factors. In line with theory, a negative feedback from spread and return-forecasting factors to macroeconomic variables (i.e. Φ^{ML}) is imposed $\mathcal{N}(-0.25, 0.5)$, modeling an a priori deflationary impact of liquidity and risk-aversion shocks. Finally, we assume the standard negative feedback from the policy rate to inflation and output gap $\mathcal{N}(-0.25, 0.25)$, a positive impact of output gap on inflation $\mathcal{N}(0.1, 0.5)$, and, in line with the Taylor rule, a positive impact of inflation $\mathcal{N}(0.25, 0.25)$ and output gap $\mathcal{N}(0.1, 0.5)$ on the policy interest rate.

The priors with respect to the impact matrix $D = \Gamma S$ in equation (1) are standard. In particular, we assume an inverted gamma distribution for the diagonal components. Depending on the type of variable, we opt for different parameterizations. An important modeling assumption in this respect is the relatively tight prior for the standard deviations of the stochastic trends, $S^{\xi\xi}$, $\mathcal{IG}(0.002, 0.2)$.[1] This choice reflects the belief that the stochastic trends, in line with long-run expectations, move smoothly over time. The priors for the off-diagonal elements of D, contained in D^{lM}, D^{MM} and D^{ll}, are assumed to be $\mathcal{N}(0, 0.02)$. This choice leaves substantial freedom in modeling the covariance between the respective shocks.

A crucial set of uniform priors is imposed on the measurement errors. This set of priors aims at facilitating the identification of the latent factors by imposing small measurement errors for certain variables in the

[1] The first parameter of the Inverse Gamma distribution refers to the mean of the distribution while the second is the standard deviation.

measurement equation. In particular, we assume zero measurement error for observable macroeconomic variables Σ_M and for the convenience yield, implying $\Sigma_{cy} = 0$. The latter assumption implies that the T-bill spread factor, $l_{1,t}$, becomes observable. Second, we impose an upper bound of 20 basis points on the standard deviation of the measurement errors of the Eurodollar - T-Bill spread. These distributional assumptions allow for the identification of the credit-related spread factor, $l_{2,t}$, which otherwise becomes excessively volatile.

2.3 Likelihood and Measurement equation

As stated in the paper, we obtain the likelihood function by means of the prediction error decomposition. Obviously, given some of the variables are latent, we use standard Kalman filter procedures to extract the latent factors. The prediction error decomposition is conditioned on the measurement equation, containing the information variables, collected in the vector Z_t, and in linearly related to the state vector X_t.

$$Z_t = A_m(\theta) + B_m(\theta)X_t + \varepsilon_t^m,\ \varepsilon_t^m \sim N(0, \Sigma_m \Sigma_m') \tag{4}$$

$$A_m(\theta) = \left[A_M', A_\rho', A_{Tb}', A_y', A_s', A_{cc}', A_{cy}'\right]', \quad B_m(\theta) = \left[B_M', B_\rho', B_{Tb}', B_y', B_s', B_{cc}', B_{cy}'\right]',$$
$$\Sigma_m = diag\left[\Sigma_M, \Sigma_\rho, \Sigma_{Tb}, \Sigma_y, \Sigma_s, \Sigma_{cc}, \Sigma_{cy}\right],$$

where the observable vector, $Z_t = [\pi_t, y_t, i_t^{cb}, \Delta g_t, y_t(1/4), y_t(1/2), ..., y_t(10), s_t(1), s_t(10), i_t^{Libor} - y_t(1/2), i_t^{Eurodollar} - y_t(1/2),\ i_t^{GC-repo} - y_t(1/2)]$, is divided in four blocks:

1. **Macroeconomic block**

   The macroeconomic contains inflation, π_t, output gap, y_t, policy interest rate, i_t^{cb}, and trend growth rate of potential output, Δg_t as information variables. We assume perfect updating, implying zero measurement errors.

2. **Yield curve block**

   We include government bond yields spanning maturities between one quarter and ten years. The measurement equation loadings for the yield curve, A_y and B_y, are obtained by imposing the no-arbitrage conditions listed in the paper. The derivation of these loadings is analogous to Ang and Piazzesi (2003).

3. **Inflation survey block**

   We use survey data of the expected average one- and ten-year ahead inflation rates in the observation vector, i.e. $s_t(1)$ and $s_t(10)$. The respective measurement equation loadings, A_s and B_s, can be derived given the transition equation. In particular, solving equation (1) for the inflation expectations, we obtain the respective loadings (see section 3 of the paper) as (see Dewachter (2008)):

$$A_{s(m)} = \tfrac{1}{m} e_\pi \sum_{j=0}^{m-1} A_\pi(j), \; B_{s(m)} = \tfrac{1}{m} e_\pi \sum_{j=0}^{m-1} B_\pi(j), \; \Sigma_s \geq 0 \tag{5}$$

   and

$$\begin{aligned} A_\pi(j) &= \Phi A_\pi(j-1) + C, \\ B_\pi(i) &= \Phi B_\pi(i-1), \end{aligned} \tag{6}$$

   with $e_\pi = [1, 0_{1\times 7}]$ and initial conditions $A_\pi(0) = 0$ and $B_\pi(0) = I_8$.

4. **Money market information**.

   Next to the federal funds rate and the yield curve, three money market interest rates are included, i.e. the 3-month Libor, the 3-month Eurodollar and the 3-month GC repo rates. The actual loadings are discussed in the paper.

The logarithm function that we maximize, $f(x)$, has the following expression:

$$f(x) = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' \log\left|F_t^{-1}\right| v_t$$

with:

N = number of measured variables

T = total number data points for observable variable

$v_t = Z_t - [A_m + B_m(C + \Phi X_{t-1|t-1})]$

$F_t = B_m(\Xi_{t-1|t-1} + DD')B_m' + \Sigma_m \Sigma_m'$

with $X_{t-1|t-1}$ and $\Xi_{t-1|t-1}$ obtained via the standard Kalman filter recursions:

$$X_{t|t} = C + \Phi X_{t-1|t-1} + K_t([Z_t - [A_m + B_m(C + \Phi X_{t-1|t-1})]]) \tag{7}$$

$$\Xi_{t|t} = (\Xi_{t-1|t-1} + DD') - K_t B_t(\Xi_{t-1|t-1} + DD') \tag{8}$$

$$K_t = (\Xi_{t-1|t-1} + DD')B_t' F_t^{-1} \tag{9}$$

where C, Φ and D refer to equation (2) of the paper and $X_{0|0}$ has been estimated and $\Xi_{0|0}$ has been obtained by iterating forward the Riccati equations (7), (8) and (9) .

3 Estimation results

This section collects the tables containing (i) the description of the posterior densities and (ii) the variance decomposition of the macroeconomic, money market and return forecasting factors. Given the reduced form nature of the transition equation, it is not very informative to discuss the posterior densities in detail. For completeness, however, we include Table 3, containing the posterior for the parameters in Φ, Table 4 containing the posterior for the impact matrix D and Table 5 summarizing the posterior densities for the remaining parameters, including prices of risk and measurement errors.

Finally, Table 2 and Figures 1 and 2 allows us to identify the impact of the respective shocks. In particular, the IRFs allow us to verify the identification (labeling) of the shocks (see section 4.3 note 19). We differentiate between three types of shocks: macroeconomic, money market and risk premium shocks. Within the class of macroeconomic shocks, we distinguish five types: three transitory shocks - supply (ε_π), demand (ε_y) and monetary policy (ε_i) shocks- and two permanent shocks - long run inflation/inflation target (ε_{π^*}) and an equilibrium growth rate (ε_ρ) shock. The responses of the three transitory shocks (Figure 1 panels (a) to (c)) are in line with a structural interpretation of supply, demand and policy rate shocks. Two qualifications should be kept in mind, however: the inflation response to the demand shocks is imprecisely estimated and we observe a price puzzle in the response to a policy shock. The two permanent shocks (Figure 2, panels (c) and (d)) generate the required permanent effects. An increase in the inflation target triggers a permanent increase in inflation and interest rate and generates substantial transitory expansionary effects. Increases in the equilibrium growth rate lead to transitory expansionary effects in inflation and output and a permanent increase in the interest rate (due to the higher natural real interest rate).

The IRFs identify two types of money market shocks, which we label respectively as 'flight to quality' and 'credit crunch' shocks. The flight to quality shocks primarily impact on the convenience yield (T-bill spread) while credit crunch shocks affect the money market spread (Libor spread). The flight to quality shocks typically generate a decrease in government bond yields, a decrease in inflation and a monetary policy easing. Typical credit crunch shocks, increasing the money market spread, have stagflationary effects on the economy. Somewhat controversial is the response of the policy rate, which (countering higher inflation) increases. Finally, by construction, return-generating factor shocks are neutral with respect to the macroeconomy and the money market.

References

Dewachter, H., "Imperfect information, macroeconomic dynamics and the yield curve: an encompassing macro-finance model", *NBB Working Paper,* 144 *(*2008).

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Table 1: PRIOR DISTRIBUTIONS OF THE PARAMETERS

		Distr.	*Mean*	*Std. Dev.*			*Distr.*	*Mean*	*Std. Dev.*
$\Phi^{MM}(1,1)$		$\mathcal{N}$	0.500	0.250	$D^{ll}(j,i)$	$j>i$	$\mathcal{N}$	0.000	0.020
$\Phi^{MM}(2,1)$		$\mathcal{N}$	0.000	0.500	$D^{ll}(j,j)$	$j=1,2$	$\mathcal{IG}$	0.010	2.000
$\Phi^{MM}(3,1)$		$\mathcal{N}$	0.250	0.250	$D^{ll}(j,j)$	$j=3$	$\mathcal{IG}$	0.020	2.000
$\Phi^{MM}(1,2)$		$\mathcal{N}$	0.100	0.500	$S^{\xi,\xi}(j,j)$	$j=1,2$	$\mathcal{IG}$	0.002	0.200
$\Phi^{MM}(2,2)$		$\mathcal{N}$	0.950	0.250	$\Lambda_0(j)$	$j=1,...,8$	$\mathcal{N}$	0.000	20.000
$\Phi^{MM}(3,2)$		$\mathcal{N}$	0.100	0.500	$\Lambda_1(j,6)$	$j=1,...,8$	$\mathcal{N}$	0.000	50.000
$\Phi^{MM}(1,3)$		$\mathcal{N}$	-0.250	0.250	$\Sigma_m(j,j)$	$j=4,...,12$	$\mathcal{U}^*$	0.000	0.005
$\Phi^{MM}(2,3)$		$\mathcal{N}$	-0.250	0.250	$\Sigma_m(j,j)$	$j=14$	$\mathcal{U}^*$	0.000	0.002
$\Phi^{MM}(3,3)$		$\mathcal{N}$	0.800	0.250	$A(j)$	$j=4$	$\mathcal{N}$	0.000	0.010
$\Phi^{lM}(j,i)$	$j,i=1,2,3$	$\mathcal{N}$	0.000	0.500	$A(j)$	$j=14$	$\mathcal{N}$	0.000	0.002
$\Phi^{Ml}(j,i)$	$j=1;i=1,2$	$\mathcal{N}$	-0.250	0.250	$B(j)$	$j=4$	$\mathcal{N}$	1.000	0.500
$\Phi^{Ml}(j,i)$	$j=2,3;i=1,2$	$\mathcal{N}$	-0.250	0.500	$X_0(j)$	$j=4,5$	$\mathcal{U}^*$	-0.050	0.050
$\Phi^{ll}(j,j)$	$j=1,2,3$	$\mathcal{N}$	0.600	0.500	$X_0(j)$	$j=6$	$\mathcal{U}^*$	-0.100	0.300
$\Phi^{ll}(j,i)$	$j\neq i$	$\mathcal{N}$	0.000	0.500	$X_0(j)$	$j=7,8$	$\mathcal{U}^*$	-0.010	0.050
$D^{MM}(j,j)$	$j=1,2,3$	$\mathcal{IG}$	0.010	2.000	$\bar{C}(j)$	$j=4,5$	$\mathcal{U}^*$	0.000	0.015
$D^{MM}(j,i)$	$j>i$	$\mathcal{N}$	0.000	0.020	$\bar{C}(j)$	$j=6$	$\mathcal{U}^*$	0.000	0.200
$D^{lM}(j,i)$	$j,i=1,2,3$	$\mathcal{N}$	0.000	0.020					

**Mean=Lower bound, Std. Dev.=Upper bound*

Notes: These two panels report the priors density for the parameters estimated in the extended Macro-Finance model. $\mathcal{N}$ stands for Normal, $\mathcal{IG}$ for Inverse Gamma and $\mathcal{U}$ for Uniform. The paremeters contained in the table refer to the following state space system:

$$Z_t = A_m + B_m X_t + \varepsilon_t^m, \ \varepsilon_t^m \sim N(0, \Sigma_m \Sigma_m') \quad \text{(Meas. Eq.)}$$
$$X_t = C + \Phi X_{t-1} + \Gamma S \varepsilon_t, \quad \varepsilon_t \sim N(0, I) \quad \text{(Trans. Eq.)}$$

Where the observable and state vectors are

$$Z_t = [\pi_t, y_t, i_t^{cb}, \Delta g_t, y_t(1/4), ..., y_t(10), s_t(1), s_t(10), i_t^{Libor} - i_t^{cb}, i_t^{Eurodollar} - i_t^{cb}]'$$
$$X_t = [\pi_t,\ y_t,\ i_t^{cb},\ l_{1,t},\ l_{2,t},\ l_{3,t},\ \xi_{1,t},\ \xi_{2,t}]$$

and the parameters of the state equation are given by:

$$C = \begin{bmatrix} C^M \\ C^l \\ 0 \end{bmatrix}, \Phi = \begin{bmatrix} \Phi^{MM} & \Phi^{Ml} & \Phi^{M\xi} \\ \Phi^{lM} & \Phi^{ll} & \Phi^{l\xi} \\ 0 & 0 & I \end{bmatrix}, D = \Gamma S = \begin{bmatrix} D^{MM} & 0 & D^{M\xi} \\ D^{lM} & D^{ll} & D^{M\xi} \\ 0 & 0 & S^{\xi\xi} \end{bmatrix}$$

with

$$\begin{bmatrix} C^M \\ C^l \end{bmatrix} = \left(I - \begin{bmatrix} \Phi^{MM} & \Phi^{Ml} \\ \Phi^{lM} & \Phi^{ll} \end{bmatrix} \right) \begin{bmatrix} 0_{3\times 1} \\ \bar{C}^l_{3\times 1} \end{bmatrix}$$

Finally the parameters Λ_0 and Λ_1 are related to the stochastic discount factor used for pricing the government bonds:

$$M_{t+1} = \exp(-i_t - \tfrac{1}{2}\Lambda_t S S' \Lambda_t' - \Lambda_t S \varepsilon_{t+1}).$$

with $i_t = y_t(1/4)$ and $\Lambda_t = \Lambda_0 + \Lambda_1 X_t$

Table 2: VARIANCE DECOMPOSITION STATE VARIABLES

Inflation

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	99.90%	0.00%	0.00%	0.00%	0.00%	0.00%	0.10%	0.00%
2Q	98.42%	0.00%	0.00%	0.39%	0.93%	0.00%	0.25%	0.00%
4Q	95.12%	0.00%	0.05%	1.12%	2.92%	0.00%	0.77%	0.01%
10Q	88.05%	0.02%	1.02%	2.13%	5.29%	0.00%	3.46%	0.03%
40Q	68.13%	0.02%	1.76%	2.23%	5.03%	0.00%	22.79%	0.04%
100Q	46.27%	0.01%	1.20%	1.52%	3.42%	0.00%	47.56%	0.02%

Output gap

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	1.56%	98.39%	0.00%	0.00%	0.00%	0.00%	0.04%	0.00%
2Q	2.46%	93.99%	2.78%	0.54%	0.07%	0.00%	0.15%	0.02%
4Q	5.07%	82.04%	11.12%	0.52%	0.63%	0.00%	0.56%	0.06%
10Q	13.84%	50.91%	26.30%	1.71%	5.12%	0.00%	1.93%	0.19%
40Q	23.05%	28.23%	29.05%	5.60%	10.87%	0.00%	2.95%	0.25%
100Q	23.10%	28.15%	29.02%	5.62%	10.90%	0.00%	2.95%	0.25%

Fed rate

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	2.49%	0.67%	96.83%	0.00%	0.00%	0.00%	0.02%	0.00%
2Q	3.73%	1.54%	89.50%	3.39%	1.82%	0.00%	0.01%	0.00%
4Q	6.12%	2.81%	75.67%	9.06%	6.29%	0.00%	0.02%	0.03%
10Q	10.63%	4.43%	60.24%	12.86%	10.89%	0.00%	0.67%	0.28%
40Q	10.08%	4.49%	48.95%	11.27%	9.76%	0.00%	12.76%	2.70%
100Q	7.67%	3.41%	37.20%	8.57%	7.42%	0.00%	29.79%	5.94%

T-bill spread

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	3.10%	0.93%	53.07%	42.47%	0.00%	0.00%	0.40%	0.04%
2Q	5.37%	0.75%	58.36%	34.54%	0.10%	0.00%	0.80%	0.08%
4Q	8.58%	1.25%	57.92%	29.22%	1.55%	0.00%	1.37%	0.13%
10Q	12.23%	2.42%	51.30%	27.15%	4.93%	0.00%	1.83%	0.15%
40Q	13.01%	2.95%	49.72%	26.80%	5.49%	0.00%	1.87%	0.15%
100Q	13.02%	2.95%	49.72%	26.80%	5.49%	0.00%	1.87%	0.15%

Libor spread

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	0.02%	0.39%	22.22%	15.91%	61.46%	0.00%	0.00%	0.00%
2Q	0.34%	1.24%	22.35%	14.13%	61.93%	0.00%	0.00%	0.00%
4Q	1.48%	2.93%	21.68%	13.40%	60.50%	0.00%	0.01%	0.00%
10Q	3.57%	4.99%	21.25%	12.69%	57.35%	0.00%	0.14%	0.01%
40Q	6.18%	5.06%	22.08%	12.33%	53.86%	0.00%	0.46%	0.04%
100Q	6.20%	5.06%	22.07%	12.34%	53.83%	0.00%	0.46%	0.04%

Return-forecasting factor

Shocks	*Supply*	*Demand*	*Mon. pol.*	*Flight to qual.*	*Credit cr.*	*Risk premia*	*Infl. target*	*Eq. gr. rate*
1Q	6.54%	0.35%	18.42%	8.33%	0.07%	66.29%	0.00%	0.00%
2Q	5.83%	0.33%	17.12%	8.93%	0.06%	67.72%	0.00%	0.00%
4Q	4.96%	0.34%	15.30%	9.40%	0.05%	69.94%	0.00%	0.00%
10Q	4.07%	0.46%	12.81%	9.24%	0.04%	73.38%	0.00%	0.00%
40Q	3.84%	0.74%	11.65%	8.74%	0.06%	74.94%	0.00%	0.01%
100Q	3.84%	0.75%	11.66%	8.74%	0.07%	74.93%	0.00%	0.01%

Notes: This table reports the forecasting error variance decomposition of the state variables, computed at the mode of the posterior distribution of the parameters. *Mon. pol.* stands for Monetary policy, *Flight to qual.* for Flight to quality, *Credit cr.* for Credit crunch, *Infl. target* for Inflation target and *Eq. gr. rate* for Equilibrium growth rate.

Table 3: PRIOR AND POSTERIOR DISTRIBUTION OF THE PHI MATRIX

	Prior			Posterior						
	Distr.	Mean	Std.Dev.	0.5 %	5 %	50 %	95 %	99.5 %	Mode	Mean
$\Phi(1,1)$	$\mathcal{N}$	0.500	0.250	0.411	0.426	0.491	0.548	0.558	0.490	0.489
$\Phi(2,1)$	$\mathcal{N}$	0.000	0.500	-0.085	-0.073	-0.008	0.054	0.068	-0.013	-0.007
'$\Phi(3,1)$	$\mathcal{N}$	0.250	0.250	0.121	0.125	0.146	0.167	0.172	0.150	0.148
$\Phi(4,1)$	$\mathcal{N}$	0.000	0.500	0.061	0.067	0.096	0.118	0.122	0.101	0.098
$\Phi(5,1)$	$\mathcal{N}$	0.000	0.500	-0.016	-0.011	0.020	0.052	0.059	0.015	0.022
$\Phi(6,1)$	$\mathcal{N}$	0.000	0.500	-0.228	-0.221	-0.180	-0.146	-0.138	-0.184	-0.181
$\Phi(1,2)$	$\mathcal{N}$	0.100	0.500	0.032	0.039	0.065	0.093	0.099	0.068	0.064
$\Phi(2,2)$	$\mathcal{N}$	0.950	0.250	0.871	0.878	0.916	0.957	0.964	0.937	0.916
'$\Phi(3,2)$	$\mathcal{N}$	0.100	0.500	0.113	0.114	0.121	0.127	0.128	0.123	0.121
$\Phi(4,2)$	$\mathcal{N}$	0.000	0.500	0.016	0.017	0.026	0.035	0.038	0.024	0.027
$\Phi(5,2)$	$\mathcal{N}$	0.000	0.500	-0.095	-0.092	-0.072	-0.050	-0.046	-0.075	-0.070
$\Phi(6,2)$	$\mathcal{N}$	0.000	0.500	-0.237	-0.234	-0.223	-0.214	-0.213	-0.223	-0.224
$\Phi(1,3)$	$\mathcal{N}$	-0.250	0.250	0.015	0.022	0.063	0.101	0.110	0.067	0.062
$\Phi(2,3)$	$\mathcal{N}$	-0.250	0.250	-0.112	-0.101	-0.049	0.012	0.026	-0.056	-0.043
'$\Phi(3,3)$	$\mathcal{N}$	0.800	0.250	1.058	1.061	1.078	1.099	1.101	1.072	1.077
$\Phi(4,3)$	$\mathcal{N}$	0.000	0.500	0.076	0.079	0.104	0.128	0.133	0.102	0.098
$\Phi(5,3)$	$\mathcal{N}$	0.000	0.500	-0.060	-0.053	-0.023	0.004	0.009	-0.018	-0.025
$\Phi(6,3)$	$\mathcal{N}$	0.000	0.500	-0.168	-0.161	-0.126	-0.093	-0.086	-0.110	-0.128
$\Phi(1,4)$	$\mathcal{N}$	-0.250	0.250	-0.159	-0.129	0.023	0.185	0.216	0.030	0.018
$\Phi(2,4)$	$\mathcal{N}$	-0.250	0.500	-0.594	-0.558	-0.368	-0.207	-0.170	-0.358	-0.387
'$\Phi(3,4)$	$\mathcal{N}$	-0.250	0.500	-0.614	-0.600	-0.532	-0.481	-0.471	-0.547	-0.529
$\Phi(4,4)$	$\mathcal{N}$	0.600	0.500	0.259	0.278	0.383	0.483	0.503	0.379	0.393
$\Phi(5,4)$	$\mathcal{N}$	0.000	0.500	0.009	0.031	0.135	0.231	0.248	0.141	0.135
$\Phi(6,4)$	$\mathcal{N}$	0.000	0.500	0.521	0.538	0.638	0.783	0.811	0.674	0.641
$\Phi(1,5)$	$\mathcal{N}$	-0.250	0.250	0.268	0.300	0.442	0.585	0.610	0.406	0.446
$\Phi(2,5)$	$\mathcal{N}$	-0.250	0.500	-0.175	-0.149	0.006	0.184	0.219	-0.006	-0.004
'$\Phi(3,5)$	$\mathcal{N}$	-0.250	0.500	0.201	0.207	0.236	0.264	0.268	0.241	0.236
$\Phi(4,5)$	$\mathcal{N}$	0.000	0.500	-0.175	-0.165	-0.112	-0.060	-0.053	-0.100	-0.112
$\Phi(5,5)$	$\mathcal{N}$	0.600	0.500	0.475	0.486	0.563	0.646	0.659	0.602	0.565
$\Phi(6,5)$	$\mathcal{N}$	0.000	0.500	-0.686	-0.676	-0.629	-0.574	-0.548	-0.618	-0.630
$\Phi(6,6)$	$\mathcal{N}$	0.600	0.500	0.773	0.785	0.837	0.891	0.902	0.843	0.837

Notes: This table reports the priors and the posterior density for the parameters of Φ matrix in Eq. 1. The first three columns report the distributions, means and standard deviations of the prior distributions. The fourth to the eight columns report the .5-th, 5-th, the 50-th and the 95-th 99.5-th percentile of the posterior distributions, respectively. The last two columns report the modes and the means of the posterior distributions. All results were obtained using the Metropolis Hastings algorithm.

Table 4: Prior and Posterior Distribution of the Impact Matrix

	Prior			Posterior						
	Distr.	Mean	Std. Dev.	0.5 %	5 %	50 %	95 %	99.5 %	Mode	Mean
$D(1,1)$	$\mathcal{IG}$	0.010	2.000	1.05	1.07	1.16	1.27	1.29	1.15	1.16
$D(2,1)$	$\mathcal{N}$	0.000	0.020	-0.21	-0.18	-0.09	0.00	0.02	-0.09	-0.09
$D(3,1)$	$\mathcal{N}$	0.000	0.020	0.05	0.05	0.10	0.16	0.18	0.11	0.09
$D(4,1)$	$\mathcal{N}$	0.000	0.020	-0.01	-0.01	0.05	0.10	0.12	0.05	0.04
$D(5,1)$	$\mathcal{N}$	0.000	0.020	0.00	0.01	0.07	0.14	0.16	0.08	0.07
$D(6,1)$	$\mathcal{N}$	0.000	0.020	-0.13	-0.12	-0.05	0.03	0.05	-0.02	-0.04
$D(2,2)$	$\mathcal{IG}$	0.010	2.000	0.64	0.65	0.71	0.77	0.79	0.69	0.71
$D(3,2)$	$\mathcal{N}$	0.000	0.020	0.01	0.02	0.06	0.10	0.11	0.06	0.06
$D(4,2)$	$\mathcal{N}$	0.000	0.020	-0.09	-0.08	-0.03	0.02	0.02	-0.03	-0.02
$D(5,2)$	$\mathcal{N}$	0.000	0.020	-0.12	-0.11	-0.05	0.01	0.02	-0.05	-0.05
$D(6,2)$	$\mathcal{N}$	0.000	0.020	-0.01	-0.01	0.03	0.06	0.07	0.03	0.02
$D(3,3)$	$\mathcal{IG}$	0.010	2.000	1.19	1.21	1.32	1.44	1.46	1.30	1.32
$D(4,3)$	$\mathcal{N}$	0.000	0.020	0.38	0.39	0.45	0.52	0.54	0.44	0.46
$D(5,3)$	$\mathcal{N}$	0.000	0.020	-0.21	-0.20	-0.13	-0.07	-0.06	-0.12	-0.13
$D(6,3)$	$\mathcal{N}$	0.000	0.020	-1.74	-1.68	-1.38	-1.11	-1.05	-1.29	-1.41
$D(4,4)$	$\mathcal{IG}$	0.010	2.000	0.35	0.36	0.41	0.46	0.47	0.40	0.41
$D(5,4)$	$\mathcal{N}$	0.000	0.020	-0.15	-0.14	-0.06	0.02	0.04	-0.06	-0.06
$D(6,4)$	$\mathcal{N}$	0.000	0.020	-0.57	-0.55	-0.42	-0.26	-0.23	-0.42	-0.42
$D(5,5)$	$\mathcal{IG}$	0.010	2.000	0.37	0.38	0.43	0.48	0.49	0.41	0.43
$D(6,5)$	$\mathcal{N}$	0.000	0.020	0.03	0.07	0.30	0.51	0.55	0.37	0.31
$D(6,6)$	$\mathcal{IG}$	0.020	2.000	1.18	1.21	1.43	1.71	1.77	1.35	1.41
$D(7,7)$	$\mathcal{IG}$	0.002	0.200	0.19	0.19	0.21	0.24	0.25	0.21	0.21
$D(8,8)$	$\mathcal{IG}$	0.002	0.200	0.04	0.04	0.05	0.06	0.07	0.05	0.05

Notes: This table reports the priors and the posterior density for the parameters of ΓS matrix in Eq. 1. The first three columns report the distributions, means and standard deviations for the prior distributions. The fourth to the eight columns report the .5-th, 5-th, the 50-th and the 95-th 99.5-th percentile of the posterior distributions, respectively. The last two columns report the modes and the means of the posterior distributions. All the statistics of all the posterior distribution are multiplied by 100. The results were obtained using the Metropolis Hastings algorithm.

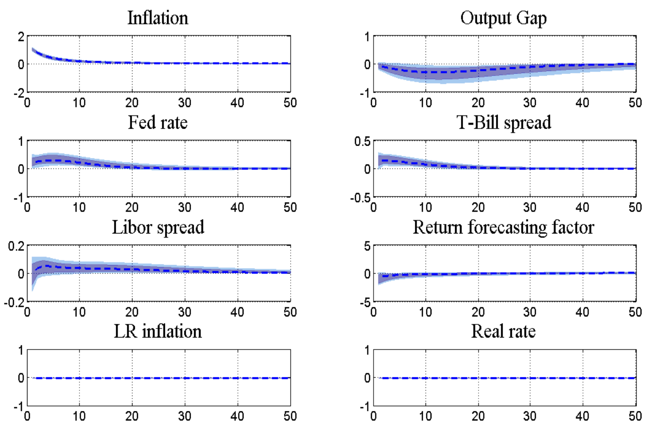
Table 5: Prior and Posterior Distribution of the Other Parameters

	Prior			Posterior						
	Distr.	Mean	StdDev	0.5 %	5 %	50 %	95 %	99.5 %	Mode	Mean
$\Lambda_0\,(1)$	$\mathcal{N}$	0.000	20.000	-9.900	-9.553	-7.202	-5.199	-4.926	-7.375	-7.539
$\Lambda_0\,(2)$	$\mathcal{N}$	0.000	20.000	2.992	4.034	5.953	7.985	8.263	6.224	6.200
$\Lambda_0\,(3)$	$\mathcal{N}$	0.000	20.000	0.166	0.217	0.585	0.899	0.947	0.654	0.560
$\Lambda_0\,(4)$	$\mathcal{N}$	0.000	20.000	-0.198	-0.062	0.445	0.875	0.928	0.345	0.451
$\Lambda_0\,(5)$	$\mathcal{N}$	0.000	20.000	0.679	0.870	2.177	3.183	3.383	1.895	1.946
$\Lambda_0\,(6)$	$\mathcal{N}$	0.000	20.000	-0.603	-0.547	-0.282	0.035	0.078	-0.325	-0.244
$\Lambda_0\,(7)$	$\mathcal{N}$	0.000	20.000	0.630	0.872	2.116	3.212	3.349	1.827	2.163
$\Lambda_0\,(8)$	$\mathcal{N}$	0.000	20.000	-8.825	-7.490	0.146	6.580	7.359	-0.940	-0.457
$\Lambda_1\,(1,6)$	$\mathcal{N}$	0.000	50.000	13.623	16.762	42.610	70.767	75.660	48.226	45.790
$\Lambda_1\,(2,6)$	$\mathcal{N}$	0.000	50.000	-29.468	-25.848	-1.044	27.027	31.529	2.407	-3.660
$\Lambda_1\,(3,6)$	$\mathcal{N}$	0.000	50.000	-13.939	-12.245	-4.257	2.187	4.184	-7.003	-3.928
$\Lambda_1\,(4,6)$	$\mathcal{N}$	0.000	50.000	12.689	15.591	26.617	37.311	38.761	25.070	26.736
$\Lambda_1\,(5,6)$	$\mathcal{N}$	0.000	50.000	-53.391	-50.922	-35.579	-20.766	-18.474	-34.763	-35.419
$\Lambda_1\,(6,6)$	$\mathcal{N}$	0.000	50.000	13.305	14.426	19.152	23.343	24.137	18.394	19.621
$\Lambda_1\,(7,6)$	$\mathcal{N}$	0.000	50.000	-92.219	-87.632	-64.699	-46.256	-42.243	-62.885	-67.314
$\Lambda_1\,(8,6)$	$\mathcal{N}$	0.000	50.000	-108.143	-95.645	-29.765	40.021	55.644	-17.790	-29.369
$\Sigma_m\,(4,4)$	$\mathcal{U}^*$	0.000	0.005	0.001	0.001	0.004	0.010	0.012	0.001	0.005
$\Sigma_m\,(5,5)$	$\mathcal{U}^*$	0.000	0.005	0.231	0.237	0.267	0.305	0.313	0.268	0.268
$\Sigma_m\,(6,6)$	$\mathcal{U}^*$	0.000	0.005	0.197	0.201	0.224	0.252	0.257	0.221	0.225
$\Sigma_m\,(7,7)$	$\mathcal{U}^*$	0.000	0.005	0.237	0.241	0.265	0.294	0.300	0.266	0.266
$\Sigma_m\,(8,8)$	$\mathcal{U}^*$	0.000	0.005	0.101	0.103	0.115	0.129	0.131	0.115	0.115
$\Sigma_m\,(9,9)$	$\mathcal{U}^*$	0.000	0.005	0.002	0.002	0.010	0.026	0.031	0.001	0.012
$\Sigma_m\,(10,10)$	$\mathcal{U}^*$	0.000	0.005	0.207	0.211	0.234	0.259	0.264	0.233	0.234
$\Sigma_m\,(11,11)$	$\mathcal{U}^*$	0.000	0.005	0.360	0.369	0.420	0.476	0.484	0.404	0.422
$\Sigma_m\,(12,12)$	$\mathcal{U}^*$	0.000	0.002	0.002	0.004	0.022	0.055	0.061	0.001	0.025
$\Sigma_m\,(14,14)$	$\mathcal{U}^*$	0.000	0.002	0.189	0.191	0.198	0.200	0.200	0.200	0.197
$X_0\,(4)$	$\mathcal{U}^*$	-0.015	0.015	0.007	0.009	0.024	0.038	0.041	0.025	0.024
$X_0\,(5)$	$\mathcal{U}^*$	-0.015	0.015	0.006	0.010	0.028	0.044	0.046	0.029	0.028
$X_0\,(6)$	$\mathcal{U}^*$	-0.100	0.200	0.041	0.049	0.086	0.126	0.136	0.088	0.087
$X_0\,(7)$	$\mathcal{U}^*$	-0.010	0.050	-0.006	-0.005	0.003	0.012	0.014	0.005	0.004
$X_0\,(8)$	$\mathcal{U}^*$	-0.010	0.050	0.017	0.018	0.025	0.032	0.033	0.027	0.025
$\bar{C}\,(4)$	$\mathcal{U}^*$	0.000	0.015	0.003	0.004	0.005	0.006	0.007	0.005	0.005
$\bar{C}\,(5)$	$\mathcal{U}^*$	0.000	0.015	0.005	0.005	0.006	0.007	0.007	0.006	0.006
$\bar{C}\,(6)$	$\mathcal{U}^*$	0.000	0.120	0.044	0.047	0.066	0.087	0.090	0.066	0.066
$A(4)$	$\mathcal{N}$	0.000	0.010	-0.022	-0.020	-0.007	0.005	0.007	-0.010	-0.007
$A(14)$	$\mathcal{N}$	0.000	0.002	-0.001	-0.001	-0.001	-0.001	0.000	-0.001	-0.001
$B(4)$	$\mathcal{N}$	0.000	0.002	1.365	1.427	1.802	2.264	2.361	1.786	1.818

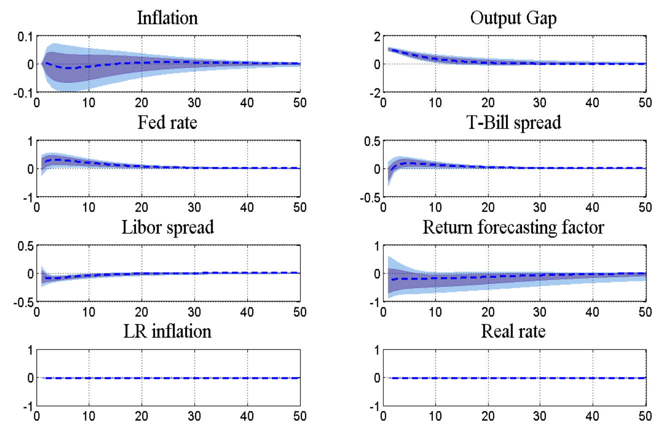
* : for the uniform distribution we report lower and upper bound of the support.

Notes: This table reports the priors and the posterior density for the parameters of Σ_m in eq.**??**, Λ_0 and Λ_1 in eq. **??**, C in eq. 1, and the initial values of the latent variables, X_0. The first three columns report the distributions, means and standard deviations of the prior distributions. The fourth to the eight columns report the .5-th, 5-th, the 50-th and the 95-th 99.5-th percentile of the posterior distributions, respectively. The last two columns report the modes and the means of the posterior distributions. The statistics of all the posterior distribution of Σ_m and A are multiplied by 100. For the uniform distribution the lower and upper bounds are reported instead of mean and standard deviation, respectively. The results were obtained using the Metropolis Hastings algorithm.

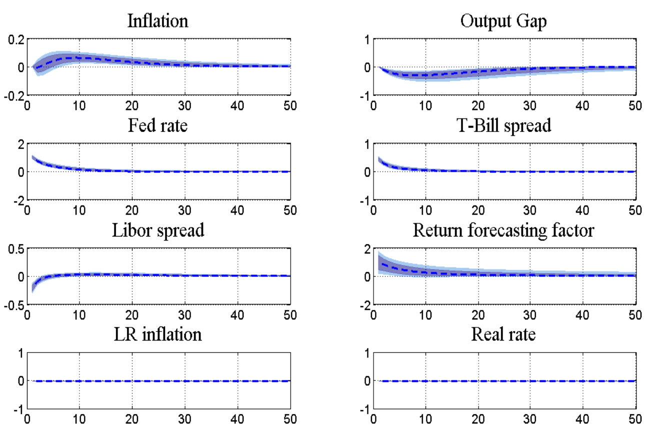
Figure 1: Implulse responses: response to demand, supply, monetary policy and flight to quality shocks.



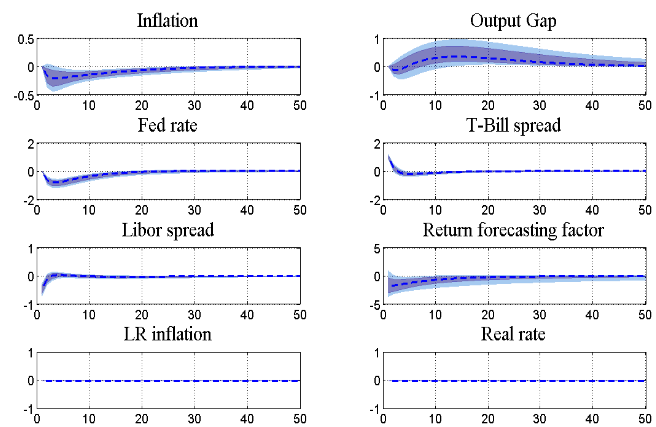
(a) Responses to supply shocks.



(b) Responses to demand shocks.



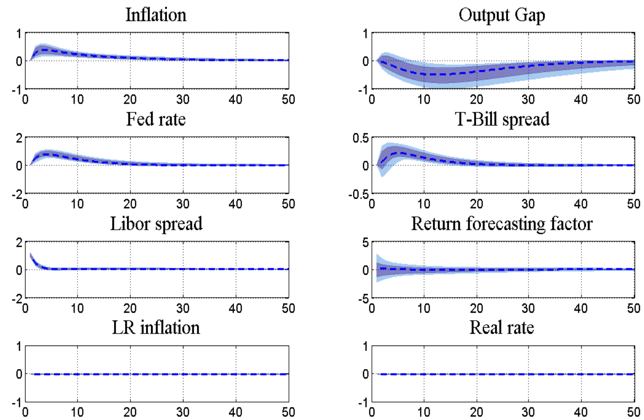
(c) Responses to policy shocks.



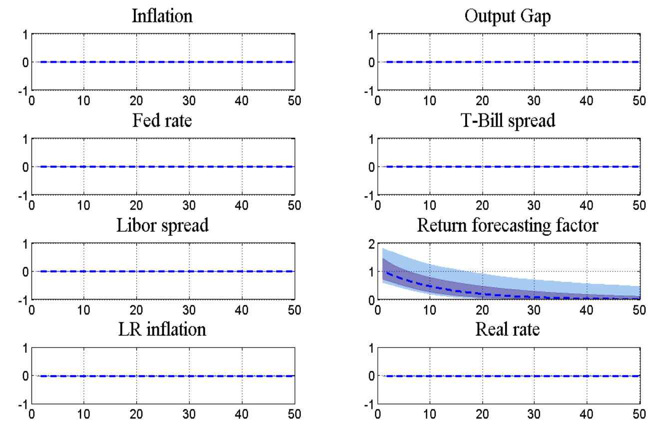
(d) Responses to flight to quality shocks.

Notes: This figure depicts the impulse response functions (IRFs) of the state variables to supply shocks (top-left panel), to demand shocks (top-right panel), to policy rate shocks (bottom-left panel) and to flight to quality shocks (bottom-right panel). The IRFs are standardized by the standard deviation the respective shocks. For example, the responses to demand shocks are divided by the standard deviation of the demand shock. The dashed (dark blue) lines are the IRFs evaluated the mode of the posterior distribution of the parameters. The shadowed area refers to the 90 % (darker shadow) and 99 % error bands.

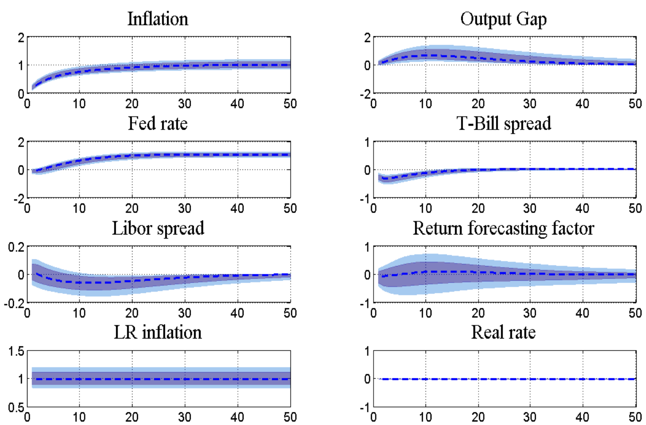
Figure 2: Implulse responses: response to credit crunch, risk premia, target inflation and equilibrium growth rate shocks.



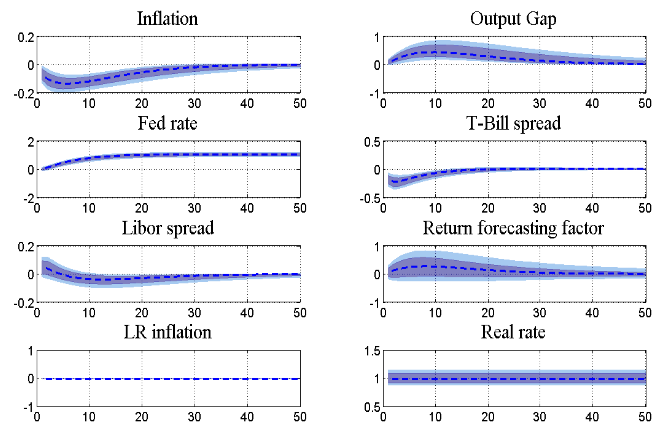
(a) Responses to credit crunch shocks.



(b) Responses to risk premia shocks.



(c) Responses to inflation target shocks.



(d) Response to equilibrium growth rate shocks.

Notes: This figure depicts the impulse response functions (IRFs) of the state variables to credit crunch shocks (top-left panel), to risk premia shocks (top-right panel), to inflation target shocks (bottom-left panel) and to equilibrium growth shocks (bottom-right panel). The IRFs are standardized by the standard deviation the respective shocks. For example, the responses to equilibrium growth rate shocks are divided by the standard deviation of the equilibrium growth rate shocks. The dashed (dark blue) lines are the IRFs evaluated the mode of the posterior distribution of the parameters. The shadowed area refers to the 90 % (darker shadow) and 99 % error bands.