Online Appendix 2 to "The Borchardt hypothesis: a cliometric reassessment of Germany's debt and crisis during 1930-32"

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August 1, 2021

Abstract

Online Appendix 2 reports the details of the analytical model, the derivation of the equilibrium conditions, and the solution method.

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Online Appendix 2

Our analytical model is an extension of Céspedes, Chang, and Velasco (2003, 2004, 2005). For quantitative simulation, we relax the assumption that price and wage are pre-set for one period, and we use the staggered price setting a la Calvo to model price and wage rigidities. We also add two stochastic shocks to the model economy: shocks to world interest rate and to world demand for the country's exports. The model nests both fixed and flexible exchange rates, contains a clear mechanism for how foreign-currency debt affects the economy via entrepreneurs' balance sheet, and allows one to compare quantitatively the relative performance of alternative exchange rates under various scenarios.[1]

There are infinite periods denoted by $t = 1, 2, 3, \ldots$. There are two distinct agents, workers and entrepreneurs. Workers supply labor and consume an aggregate of domestic and foreign goods; while entrepreneurs supply capital and own the firms. Entrepreneurs borrow from the world capital market in order to finance investment in excess of their own net worth.

Domestic production

The production of domestic goods is monopolistic competitive and firms have a Cobb-Douglas production technology given by:

$$Y_{jt} = AK_{jt}^{\alpha}L_{jt}^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1}$$

where j denotes firm (output of variety), t denotes time period, Y_{jt} denotes output of variety j in period t, K_{jt} denotes capital input, and L_{jt} denotes labor input. Since

[1] Na et al. (2018) introduce downward nominal wage rigidity into a sovereign default model of the Eaton–Gersovitz type to explain the joint occurrence of sovereign default and devaluation of the nominal exchange rate. On the one hand, the Eaton–Gersovitz model predicts that default occurs when the endowment of tradable goods contracts. On the other hand, if there is downward nominal wage rigidity, then exchange rate devaluation lowers real wages and thus reduces involuntary unemployment. These two features allow the model to predict the joint occurrence of default and devaluation. The features also make Na et al.'s model advantageous in that it characterizes a policy that combines default and devaluation as an optimal response to external shocks, yet the tradable goods sector is an "endowment" economy and is not affected by exchange rate devaluation. Moreover, the model does not consider the potential harmful effects of exchange rate devaluation via the economy's balance sheet – a problem that concerned the Weimar German policymakers when considering an exchange rate policy.

workers' labor services are heterogeneous, L_{jt} is an CES aggregate of the services (labor) of the different workers in the economy:

$$L_{jt} = \left[\int_0^1 L_{ijt}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where workers are indexed by $i \in [0, 1]$, L_{ijt} denotes the services purchased from worker i by firm j, and $\sigma > 1$ is the elasticity of substitution among different labor types. The aggregate nominal wage, or the minimum cost of a unit of labor L_{jt}, is given by:

$$W_t = \left[\int_0^1 W_{it}^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \tag{3}$$

The j-th firm maximizes its profits, which are given by:

$$\Pi_{jt} = P_{jt} Y_{jt} - \int_0^1 W_{it} L_{ijt} di - R_t K_{jt}, \tag{4}$$

where R_t denotes return to capital, and like the the profits both are expressed in terms of the domestic currency. Cost minimization of the firms implies that:

$$\frac{R_t K_{jt}}{W_t L_{jt}} = \frac{\alpha}{1-\alpha}, \tag{5}$$

so that all firms have the same capital to labor ratio K_t/L_t. The marginal cost of production is expressed as:

$$MC_t = \frac{1}{1-\alpha} W_t \frac{1}{A} \left(\frac{\alpha}{1-\alpha} \frac{W_t}{R_t} \right)^{-\alpha} \tag{6}$$

Following the staggered price setting a la Calvo (1983), the probability that the price of a given home good can be changed in any particular period is $(1 - \theta_p)$. The problem of the home good producers is to choose price P_{jt} that maximizes discounted real profits:

$$\max_{P_{jt}} \nabla_t \sum_{\tau=0}^{\infty} (\theta_p)^{\tau} \xi_{t,t+\tau} \left\{ \left(\frac{P_{jt} - MC_{t+\tau}}{Q_{t+\tau}} \right) \left(\frac{P_{jt}}{P_{t+\tau}} \right)^{-\vartheta} C_{t+\tau}^H \right\} \tag{7}$$

As will become clear later, we use Q_t to denote the general consumption price level, P_t to denote the price of domestically produced goods and S_t to denote the price of imported goods. C_t^H denotes domestic consumption bundle to be further explained below. To avoid confusion with the real exchange rate, we use ∇ as the notation for expectation. The pricing kernel $\xi_{t,t+\tau}$ is assumed to be equal to the entrepreneurs' inter-temporal marginal rate of substitution in consumption:[2]

$$\xi_{t,t+\tau} = \beta^{\tau} \frac{V_{C,t+\tau}}{V_{C,t}} \tag{8}$$

Define $\Xi_{t,t+\tau} \equiv \xi_{t,t+\tau} C_{t+\tau}^H \frac{(P_{t+\tau})^{\vartheta}}{Q_{t+\tau}}$. The first-order condition for optimal price setting is given by:

$$P_{jt}^* = \frac{\vartheta}{\vartheta - 1} \frac{\nabla_t \sum_{\tau=0}^{\infty} (\theta_p)^{\tau} \Xi_{t,t+\tau} MC_{t+\tau}}{\nabla_t \sum_{\tau=0}^{\infty} (\theta_p)^{\tau} \Xi_{t,t+\tau}} \tag{9}$$

The equation implies that the price set at period t is equal to a weighted average of current and expected future marginal costs, multiplied by the markup factor $\frac{\vartheta}{\vartheta-1}$.[3] Assume symmetric solution for the firms so that $P_{jt}^* = P_t^*$, the price index for domestically produced goods evolves according to:

$$P_t = \left[\theta_p \left(P_{t-1}\right)^{1-\vartheta} + (1-\theta_p)\left(P_t^*\right)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{10}$$

Workers

The representative worker has preferences over consumption C_t, labor supply L_t, and real money balances $\frac{M_t}{Q_t}$ given by:

$$\log C_t - \left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\upsilon} L_t^{\upsilon} + \frac{1}{1-\varepsilon}\left(\frac{M_t}{Q_t}\right)^{1-\varepsilon}, \tag{11}$$

where $\upsilon > 1$, $\varepsilon > 0$, and Q_t is the consumer price index.

Consumption C_t is an aggregate of domestic and foreign goods:

[2] Note that we assume that the entrepreneurs and the workers have the same discount factor β. As long as the entrepreneurs' utility function is additively separable, this assumption affects only the slope of the log-linearized optimal pricing formula shown in equation (56).

[3] This equation is analogous to Kollmann (2001, equation 9).

$$C_t = \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \left(C_t^H\right)^{\gamma} \left(C_t^F\right)^{1-\gamma}, \tag{12}$$

where C_t^H is a basket of the varieties of domestically produced goods and C_t^F is a basket of imported goods. The basket of domestically produced goods, C_t^H, is aggregated through the CES function:

$$C_t^H = \left[\int_0^1 C_{jt}^{\frac{\vartheta-1}{\vartheta}} dj\right]^{\frac{\vartheta}{\vartheta-1}}, \qquad \vartheta > 1 \tag{13}$$

Using P_t to denote the domestic price of one unit of basket of domestically produced goods, P_t is expressed as:

$$P_t = \left[\int_0^1 P_{jt}^{1-\vartheta} dj\right]^{\frac{1}{1-\vartheta}} \tag{14}$$

The prices of imported goods are assumed to be fixed and are normalized to one in terms of foreign currency. The exchange rate is expressed as the price of a unit of foreign currency in terms of the domestic currency. Imports are freely traded and the Law of One Price holds, so that the domestic price of imported goods is equal to the nominal exchange rate S_t.[4]

The cost of one unit of aggregate consumption Q_t (or CPI) is given by:

$$Q_t = P_t^{\gamma} S_t^{1-\gamma} \tag{15}$$

The i-th workers' budget constraint in period t is:[5]

$$P_t C_{it}^H + S_t C_{it}^F = W_{it} L_{it} + T_t - M_{it} + M_{it-1} \tag{16}$$

Use E_t to denote the real exchange rate; that is, $E_t \equiv S_t/P_t$. We define real exchange rate as the price of foreign goods in terms of domestic goods. This definition is not unconventional, because the same definition has been used in Mussa (1986) and the

[4]Purchasing power parity (PPP) does not hold in our model. For PPP to hold, it must be that $S_t P_t^*/Q_t = 1$. Since P_t^* is normalized to 1, the above expression also implies $Q_t = S_t$ if PPP holds. However, this is not the case in our model, because we have, $Q_t = P_t^{\gamma} S_t^{1-\gamma}$.

[5]$Q_t C_{it} \equiv P_t C_{it}^H + S_t C_{it}^F$.

textbook of Cecchetti and Schoenholtz (2017). This definition for real exchange rate also makes clear that other things being unchanged, real exchange rate depreciation increases the risk premium.

To minimize cost, the consumer will purchase domestic and foreign goods under the requirement that a proportion γ of consumption will be spent on domestic goods and a proportion $(1-\gamma)$ will be spent on foreign goods:

$$\frac{C_t^H P_t}{Q_t C_t} = \gamma \tag{17}$$

$$\frac{C_t^F S_t}{Q_t C_t} = 1 - \gamma \tag{18}$$

The government follows a simple policy: it is assumed that revenues from an inflation tax are rebated to workers though lump-sum transfers.

$$M_t - M_{t-1} = T_t, \qquad M_t = \int_0^1 M_{it} di \tag{19}$$

The above fiscal policy setting and worker's budget constraint mean that in equilibrium, workers consume all their nominal income:

$$Q_t C_t = W_t L_t \tag{20}$$

We assume each household specializes in one type of labor, which it supplies monopolistically. Use θ_w to denote the degree of wage stickiness, assume a symmetric solution, and omit the household's index. Following Galí (2015), the household's wage setting problem is to find W_t^* which maximizes:

$$\nabla_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} U\left(C_{t+\tau}, L_{t+\tau|t}\right) \tag{21}$$

$$st. \qquad L_{t+\tau|t} = \left(\frac{W_t^*}{W_{t+\tau}}\right)^{-\sigma} L_{t+\tau} \tag{22}$$

$$st. \qquad Q_{t+\tau}C_{t+\tau} = W_t^* L_{t+\tau|t} + T_{t+\tau} - M_{t+\tau} + M_{t+\tau-1} \tag{23}$$

$L_{t+\tau|t}$ is the quantity of labor services provided in period $t+\tau$ by a household that last reset its wage in period t. Define $\Omega_{t+\tau} \equiv \frac{1-\alpha}{\alpha} K_{t+\tau} R_{t+\tau} \left(W_{t+\tau}\right)^{\sigma-1}$. The first-order condition, or the wage setting equation, is given by:[6]

$$W_t^* = -\frac{\sigma}{\sigma-1} \frac{\nabla_t \sum_{\tau=0}^{\infty} \left(\beta\theta_w\right)^{\tau} U_{L,t+\tau}\Omega_{t+\tau}}{\nabla_t \sum_{\tau=0}^{\infty} \left(\beta\theta_w\right)^{\tau} \frac{U_{C,t+\tau}}{Q_{t+\tau}}\Omega_{t+\tau}} \tag{24}$$

The wage index evolves according to:

$$W_t = \left[\theta_w \left(W_{t-1}\right)^{1-\sigma} + \left(1-\theta_w\right)\left(W_t^*\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{25}$$

In addition, the money demand is given by:

$$\nabla_t \left(\beta \frac{1}{C_{t+1}} \frac{Q_t}{Q_{t+1}}\right) = \frac{1}{C_t} - \left(\frac{M_t}{Q_t}\right)^{-\varepsilon} \tag{26}$$

Entrepreneurs

Entrepreneurs borrow from the world capital market in order to finance investment in excess of their net worth. Entrepreneurs own a quantity K_t of capital, which is used to produce domestic goods. Assume that all debt contracts are denominated in foreign currency. Entrepreneurs' budget constraint is given by:

$$P_t N_t + S_t D_{t+1} = Q_t I_t \tag{27}$$

$$I_t = K_{t+1}, \tag{28}$$

where N_t is net worth, D_{t+1} is the amount borrowed abroad in period t and to be repaid in period $t+1$, and $I_t = K_{t+1}$ is investment in period $t+1$ capital. Here, it is assumed that capital is produced in the same fashion as consumption, so that the cost of producing

[6] This optimal condition is similar to Kollmann (2001, equation 22).

one unit of capital is also Q_t. Capital depreciates completely in production, and so there is no capital accumulation.

We use ρ_t to denote the world interest rate and η_{t+1} to denote risk premium. Here, ρ_t is a mean-reversion process with a mean value of ρ (the world safe interest rate). Assume that the risk premium is increasing in the ratio of the value of investment to net worth.

$$1+\eta_{t+1}=\left(\frac{Q_t I_t}{P_t N_t}\right)^{\mu}=\left(1+\frac{E_t D_{t+1}}{N_t}\right)^{\mu}, \qquad \mu \geq 0 \tag{29}$$

The above endogenous risk premium follows Bernanke and Gertler (1989). Entrepreneurs will borrow to finance investment so that the expected return is equal to the cost of borrowing, namely, expected yield on capital in dollars must equal the cost of foreign borrowing. This condition holds, because entrepreneurs borrow from abroad to finance their investment, and thus entrepreneurs must balance the cost and benefit of borrowing in terms of foreign currency.

$$\nabla_t \frac{R_{t+1}}{Q_t}=\left(1+\rho_t\right)\left(1+\eta_{t+1}\right)\nabla_t\left(\frac{S_{t+1}}{S_t}\right) \tag{30}$$

The equation presents the interest parity (arbitrage relation), which is analogous to the uncovered interest rate parity. The equation, which is analogous to the form of uncovered interest rate parity that Flood and Marion (2000) adopt, states that the domestic interest rate deviates from the foreign interest rate by the expected rate of change of the exchange rate plus a risk premium.

The interest parity implies that we assume the German interest rate (expected return on capital) then is equal to the world interest rate plus risk premium. As explained above, the model's interest rate is determined by the interest parity. Remember that Germany was on a gold standard, which is a kind of strict fixed exchange rate. This means that as long as Germany remained on the gold standard, its interest rate was equal to the foreign interest rate plus the risk premium. This is so, because entrepreneurs were borrowing from abroad to finance investment and because of the fixed exchange rate. Policy autonomy of the Reichsbank was limited under the gold standard arrangement established in August 1924. The Reichsbank was placed under international supervision and had to follow a set of strict rules, including a tight limit on lending to the government (Ritschl, 2013). The gold parity of the Reichsmark, being guaranteed nationally by the Reichsbank Act itself and internationally by the treaty of the Young Plan, left Germany's currency policy only a small amount of autonomous scope for action (Schiemann, 1980, Chapter 1). To maintain the gold parity, notes in circulation were required to be covered by gold or foreign exchange by at least 40 percent. The monetary policy instrument of the Reichsbank was essentially limited to the discount policy. In contrast to the U.S., open market policy was of no relevance in Germany. With its hands tied by the gold standard, the Reichsbank relied on indirect means such as moral suasion to make its influence felt. In sum, the institutional arrangement of Reichsbank indicate that as long as the Reichsmark retained its gold parity, the German interest rate followed closely the gold-standard center's interest rate plus a risk premium. This is true, because of international arbitrage and the gold standard arrangement and has little to do with Germany's economic scale.

Entrepreneurs receive the profits of the firms as well as the rent on capital. Assume that capitalists consume a portion $(1-\delta)$ of their net worth, and they only consume imported goods. Entrepreneurs' net worth therefore is:

$$\begin{aligned} P_t N_t &= \delta \left[R_t K_t + \Pi_t - \left(1+\rho_{t-1}\right)\left(1+\eta_t\right) S_t D_t \right] \\ &= \delta \left[P_t Y_t - W_t L_t - \left(1+\rho_{t-1}\right)\left(1+\eta_t\right) S_t D_t \right], \end{aligned} \tag{31}$$

where Π_t is firm profits in domestic currency, and D_t is dollar debt repayment in period t.

Market clearing conditions

The market clearing condition for home goods is given by:

$$Y_t = \gamma \left(\frac{Q_t}{P_t} \right) (I_t + C_t) + (E_t)^{\chi} X_t, \tag{32}$$

where E_t is the real exchange rate already mentioned above, $\chi > 0$, $(E_t)^{\chi} X_t$ denotes the domestic goods demanded by the rest of the world, and X_t is exogenous world demand for domestic goods.

Equilibrium conditions

Define $\Pi_{p,t} \equiv P_t/P_{t-1}$, $\tilde{P}_t \equiv P_t^*/P_t$, $\Pi_{w,t} \equiv W_t/W_{t-1}$, $\tilde{W}_t \equiv W_t^*/W_t$, $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{jt}}{P_t} \right)^{-\vartheta} dj$, and $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_{it}}{W_t} \right)^{-\sigma} di$. We summarize the equilibrium conditions of the model as follows:

$$Y_t = \frac{1}{\Delta_{p,t}} \frac{1}{\Delta_{w,t}} A K_t^{\alpha} L_t^{1-\alpha} \tag{33}$$

$$\Delta_{p,t} = (1 - \theta_p) \left(\tilde{P}_t \right)^{-\vartheta} + \theta_p \left(\Pi_{p,t} \right)^{\vartheta} \Delta_{p,t-1} \tag{34}$$

$$\Delta_{w,t} = (1 - \theta_w) \left(\tilde{W}_t \right)^{-\sigma} + \theta_w \left(\Pi_{w,t} \right)^{\sigma} \Delta_{w,t-1} \tag{35}$$

$$\frac{R_t K_t}{W_t L_t} = \frac{\alpha}{1 - \alpha} \tag{36}$$

$$\frac{MC_t}{P_t} = \frac{1}{1-\alpha} \frac{W_t}{P_t} \frac{1}{A} \left(\frac{\alpha}{1-\alpha} \frac{W_t}{R_t} \right)^{-\alpha} \tag{37}$$

$$P_t^* = \frac{\vartheta}{\vartheta - 1} \frac{\nabla_t \sum_{\tau=0}^{\infty} (\theta_p)^{\tau} \Xi_{t,t+\tau} MC_{t+\tau}}{\nabla_t \sum_{\tau=0}^{\infty} (\theta_p)^{\tau} \Xi_{t,t+\tau}} \tag{38}$$

$$1 = \left[\theta_p \left(\Pi_{p,t} \right)^{\vartheta - 1} + (1 - \theta_p) \left(\tilde{P}_t \right)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}} \tag{39}$$

$$\frac{Q_t}{P_t} = \frac{P_t^{\gamma} S_t^{1-\gamma}}{P_t} = \left(\frac{S_t}{P_t}\right)^{1-\gamma} \tag{40}$$

$$\frac{S_t}{P_t} \equiv E_t \tag{41}$$

$$C_t = \frac{W_t}{Q_t} L_t \tag{42}$$

$$W_t^* = -\frac{\sigma}{\sigma - 1} \frac{\nabla_t \sum_{\tau=0}^{\infty} \left(\beta\theta_w\right)^{\tau} U_{L,t+\tau} \Omega_{t+\tau}}{\nabla_t \sum_{\tau=0}^{\infty} \left(\beta\theta_w\right)^{\tau} \frac{U_{C,t+\tau}}{Q_{t+\tau}} \Omega_{t+\tau}} \tag{43}$$

$$1 = \left[\theta_w \left(\Pi_{w,t}\right)^{\sigma-1} + (1-\theta_w) \left(\tilde{W}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{44}$$

$$\nabla_t \left(\beta \frac{1}{C_{t+1}} \frac{Q_t}{Q_{t+1}}\right) = \frac{1}{C_t} - \left(\frac{M_t}{Q_t}\right)^{-\varepsilon} \tag{45}$$

$$N_t + \frac{S_t}{P_t} D_{t+1} = \frac{Q_t}{P_t} I_t \tag{46}$$

$$I_t = K_{t+1} \tag{47}$$

$$1 + \eta_{t+1} = \left(\frac{Q_t}{P_t} \frac{I_t}{N_t}\right)^{\mu} \tag{48}$$

$$\nabla_t \frac{R_{t+1}}{Q_t} = (1+\rho_t) \left(1+\eta_{t+1}\right) \nabla_t \left(\frac{S_{t+1}}{S_t}\right) \tag{49}$$

$$N_t = \delta \left[Y_t - \frac{W_t}{P_t} L_t - \left(1+\rho_{t-1}\right) (1+\eta_t) \frac{S_t}{P_t} D_t\right] \tag{50}$$

$$Y_t = \gamma \left(\frac{Q_t}{P_t}\right) (I_t + C_t) + (E_t)^{\chi} X_t \tag{51}$$

Steady state

The steady state of the model is the same regardless of the assumption about monetary policy (fixed or floating exchange rate regime), and regardless of the assumption about price and wage rigidity. We have imposed $P \equiv 1$. It follows that:

$$\Pi_p = \Pi_w = \Delta_p = \Delta_w = \tilde{P} = \tilde{W} = L = 1$$

The solution of other variables starts from solving η. Equation (52) solves for η.

$$1 - \delta \left[1 - \frac{\vartheta - 1}{\vartheta} (1 - \alpha) \right] \frac{1}{\alpha} \frac{\vartheta}{\vartheta - 1} (1 + \rho)(1 + \eta)(1 + \eta)^{\frac{1}{\mu}} + \delta (1 + \rho)(1 + \eta) \left((1 + \eta)^{\frac{1}{\mu}} - 1 \right) = 0 \tag{52}$$

In the special case that $\vartheta = \infty$ (perfect competition):

$$\eta = \frac{1}{\delta (1 + \rho)} - 1$$

For a compact notational expression, we define coefficients λ_1 and λ_2 as:

$$\lambda_1 \equiv \left[1 - \frac{\vartheta - 1}{\vartheta} \gamma (1 - \alpha) - \frac{\vartheta - 1}{\vartheta} \frac{\alpha \gamma}{(1 + \rho)(1 + \eta)} \right]$$

$$\lambda_2 \equiv \frac{\vartheta - 1}{\vartheta} \alpha \frac{A^{\frac{1}{\alpha}}}{(1 + \rho)(1 + \eta)}$$

The steady states of other variables are solved in the following sequence. Note that the prices of imported goods are normalized to one in terms of foreign currency. The model also normalizes the steady-state prices of home goods in home currency to one. Therefore, in the steady state, nominal exchange rate is equal to real exchange rate.

$$S = \left[\lambda_2 \left(\frac{X}{\lambda_1} \right)^{\frac{\alpha - 1}{\alpha}} \right]^{\frac{1}{1 - \gamma + \chi \frac{1 - \alpha}{\alpha}}}$$

$$Y = \frac{(S)^{\chi} X}{\lambda_1}$$

$$K=\left(\frac{Y}{A}\right)^{\frac{1}{\alpha}}$$

$$Q=S^{1-\gamma}$$

$$C=\frac{\vartheta-1}{\vartheta}\left(1-\alpha\right)\frac{Y}{Q}$$

$$N=QK\left(1+\eta\right)^{-\frac{1}{\mu}}$$

$$D=\frac{QK-N}{S}$$

$$M=Q\left(\frac{1-\beta}{C}\right)^{\frac{-1}{\varepsilon}}$$

$$E=S$$

$$I=K$$

$$R=\alpha\frac{\vartheta-1}{\vartheta}\frac{Y}{K}$$

$$W=\left(1-\alpha\right)\frac{\vartheta-1}{\vartheta}\frac{Y}{L}$$

Log-linearization

We use a linear method to solve the model. The log-linearized equilibrium conditions are expressed below (when i_t is replaced by k_{t+1}). With the exception of η_t $(\hat{\eta}_t)$ and ρ_t $(\hat{\rho}_t)$, we use lower case to denote the linearized variables. All variables are expressed as log-deviation from their steady states, and only ρ_t is expressed as deviation from the steady state.

$$y_t = \alpha k_t + (1-\alpha)\, l_t \tag{53}$$

$$r_t + k_t = w_t + l_t \tag{54}$$

$$(mc_t - p_t) = (w_t - p_t) - \alpha\,(w_t - r_t) \tag{55}$$

$$\pi_{p,t} = \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}\,(mc_t - p_t) + \beta\pi_{p,t+1} \tag{56}$$

$$(q_t - p_t) = (1-\gamma)\,(s_t - p_t) \tag{57}$$

$$c_t + q_t = w_t + l_t \tag{58}$$

$$\pi_{w,t} = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\,[(\upsilon - 1)\, l_t + c_t + q_t - w_t] + \beta\pi_{w,t+1} \tag{59}$$

$$\left(\beta\frac{1}{C}\right)(-c_{t+1} + q_t - q_{t+1}) = \frac{-1}{C}c_t + \varepsilon\left(\frac{M}{Q}\right)^{-\varepsilon}(m_t - q_t) \tag{60}$$

$$n_t + \frac{SD}{NP}\,(s_t - p_t + d_{t+1}) = \frac{QI}{NP}\,(q_t - p_t + k_{t+1}) \tag{61}$$

$$\hat{\eta}_{t+1} = \left(\frac{Q}{P}\frac{I}{N}\right)^{\mu}\frac{\mu}{\eta}\,(q_t + k_{t+1} - p_t - n_t) \tag{62}$$

$$(r_{t+1} - q_t) = (s_{t+1} - s_t) + (1+\rho)\,\frac{Q}{R}\,(\eta\hat{\eta}_{t+1}) + (1+\eta)\,\frac{Q}{R}\,(\hat{\rho}_t) \tag{63}$$

$$N n_t = \delta \left[Y y_t - \frac{WL}{P} (w_t - p_t + l_t) - (1+\rho)(1+\eta) \frac{SD}{P} \left(s_t - p_t + d_t + \frac{\eta}{1+\eta} \hat{\eta}_t + \frac{1}{1+\rho} \hat{\rho}_{t-1} \right) \right] \tag{64}$$

$$y_t = \gamma \frac{Q}{P} \frac{I}{Y} (k_{t+1} + q_t - p_t) + \gamma \frac{Q}{P} \frac{C}{Y} (c_t + q_t - p_t) + \frac{(E)^{\chi} X}{Y} (x_t + \chi e_t) \tag{65}$$

$$s_t - p_t = e_t \tag{66}$$

$$\pi_{p,t} = p_t - p_{t-1} \tag{67}$$

$$\pi_{w,t} = w_t - w_{t-1} \tag{68}$$

Recovering shocks

The model contains two structural shocks X_t and ρ_t. Given the model parameters and two observables, the exogenous shocks can be extracted recursively by inverting the observation equation (Guerrieri and Iacoviello, 2017; Kollmann, 2017).

More specifically, the model solution takes the form:

$$X_t^m = P \cdot X_{t-1}^m + Q \cdot u_t, \tag{69}$$

where X_t^m include both state and control variables of the model, P and Q are coefficient matrices, and u_t denotes exogenous shocks. Let H be a selection matrix and $Y_t = H \cdot X_t^m$ is the vector of observed series. Multiply both sides of equation (69) by H and we obtain:

$$Y_t = H \cdot X_t^m = H \cdot P \cdot X_{t-1}^m + H \cdot Q \cdot u_t. \tag{70}$$

Given X_{t-1}^m and the current realization of Y_t, equation (70) represents a system of non-linear equations that allow us to solve for u_t recursively. A necessary condition for the inversion filter is that matrix $H \cdot Q$ is invertible. To initiate the inversion filter, we also assume that X_0^m coincide with the model's steady state.

IS-LM-BP framework

We simplify the model to two periods and assume that prices and wages are pre-set for one period. We adopt the convention that no subscript indicates an initial period variable, while a subscript indicates a final period variable. Capital-case variables with a bar denote their no-shock steady-state values. Lower case letters denote variables presented in percentage deviation from the no-shock steady state. The model can be summed up in a system of three equations analogous to the IS-LM-BP framework:

$$y = \alpha_i i + \alpha_x x + \alpha_e e \tag{IS}$$

$$\alpha_i = \left[1 - \gamma (1-\alpha) \left(1 - \vartheta^{-1}\right)\right]^{-1} \left[1 - \gamma \left(1 - \vartheta^{-1}\right)\right]^{-1} \frac{\gamma \bar{Q} \bar{I}}{\left(\gamma \bar{Q} \bar{I} + \bar{E}^{\chi} \bar{X}\right)}$$

$$\alpha_x = \left[1 - \gamma (1-\alpha) \left(1 - \vartheta^{-1}\right)\right]^{-1} \left[1 - \gamma \left(1 - \vartheta^{-1}\right)\right]^{-1} \left[1 - \frac{\gamma \bar{Q} \bar{I}}{\left(\gamma \bar{Q} \bar{I} + \bar{E}^{\chi} \bar{X}\right)}\right]$$

$$\alpha_e = \left[1 - \gamma (1-\alpha) \left(1 - \vartheta^{-1}\right)\right]^{-1} \left[1 - \gamma \left(1 - \vartheta^{-1}\right)\right]^{-1} \left[\chi + \frac{\gamma \bar{Q} \bar{I}}{\left(\gamma \bar{Q} \bar{I} + \bar{E}^{\chi} \bar{X}\right)} (1 - \gamma - \chi)\right]$$

$$m = \beta_y y + \beta_e e - \beta_i i \tag{LM}$$

$$\beta_y = \frac{1}{\varepsilon \left[1 - \beta \frac{\bar{Q} \bar{C}}{\bar{Q}_1 \bar{C}_1}\right] (1-\alpha)}$$

$$\beta_e = -\left(\varepsilon^{-1} - 1\right)(1 - \gamma)$$

$$\beta_i = -\left(\left[1-\beta\frac{\bar{Q}\bar{C}}{\bar{Q}_1\bar{C}_1}\right]^{-1}-1\right)\varepsilon^{-1}\left(\gamma+\chi-1\right)\frac{\alpha}{\chi}$$

$$\begin{aligned} i \;=\; & \frac{-1}{(1-\alpha+\alpha\chi^{-1}+\mu)}\hat{\rho} \\ & +\frac{\mu\left(1+\frac{\bar{S}\bar{D}}{\bar{P}\bar{N}}\right)}{(1-\alpha+\alpha\chi^{-1}+\mu)\left[1-(1-\alpha)\left(1-\vartheta^{-1}\right)\right]\vartheta}y \\ & +\frac{\left[\gamma-\mu\left(1-\gamma+\frac{\bar{S}\bar{D}}{\bar{P}\bar{N}}\right)\right]}{(1-\alpha+\alpha\chi^{-1}+\mu)}e \end{aligned} \tag{BP}$$

The IS and LM curves are standard in the literature. The non-standard feature of the model is the BP curve, which represents equilibrium in the international loan market. We use the IS-LM-BP system to explain how exchange rate and price level are determined in the model.

We start with definitions of both fixed and flexible exchange rates. Our definitions of the two rates are identical to the ones that Céspedes, Chang, and Velasco (2004) employ. We see Germany's monetary problem in the early 1930s as making a choice between price stability (meaning neither deflation nor inflation) and exchange rate stability (as represented by the gold standard). In fact, this conflict between stable exchange rates and stable internal prices had been highlighted by John Maynard Keynes (1923) in his book *A Tract on Monetary Reform*. "Keynes noted that a country whose central bank was devoted to fixing the value of its currency in terms of gold could not also use monetary policy to ensure stable domestic prices. Alternatively, a country whose central bank had the objective of stabilizing domestic prices could not be assured of having stable exchange rates." (Irwin, 2012, p. 34). Following this logic, we define a regime of flexible exchange rates as one in which the central bank uses its policy instrument to target the price of home output P_t while letting the nominal exchange rate S_t adjust to market conditions. In contrast, a regime of fixed exchange rates is one in which the monetary authority keeps the nominal exchange rate constant. Under a fixed exchange rate, real exchange rate depreciation can only be accomplished through deflation of the home goods price.

We now illustrate how prices and exchange rates are determined in the initial and the final periods by using this simplified version of the model. Under flexible exchange rates, $p = p_1 = 0$, and IS-LM-BP jointly determines investment (i), output (y), and real exchange rate (e). Since $e = s - p = s$, this solves the nominal exchange rate (s) as well.

Under fixed exchange rates, $s = s_1 = 0$. Since prices are pre-set at the initial period, the real exchange rate at the initial period is equal to $e = s - p = 0$ and is predetermined. IS-LM-BP jointly determines investment (i), output (y), and money balances (m). At the final period, the real exchange rate is equal to $e_1 = s_1 - p_1 = -p_1$. Since the nominal exchange rate is fixed, the adjustment of the real exchange rate is accomplished through deflation of the home goods prices. It can further be shown that home goods producers' optimality condition for the employment of labor and capital[7] and the interest parity[8] jointly solve e_1 and thus p_1.[9]

[7] $r_1 - p_1 = -(1 - \alpha)\, i$.

[8] $(r_1 - p_1) - q = \hat{\rho} + \hat{\eta} + e_1 - s$.

[9] As equations (IS) and (BP) show, floating the exchange rate stimulates the economy through two channels. First, currency depreciation in the current period implies expected currency appreciation between the current period and the subsequent period, which by interest parity reduces the cost of borrowing and thus stimulates investment. This is an interest rate channel. Second, currency depreciation stimulates foreign demand for domestic goods (exports). This is an exchange rate channel. Floating the exchange rate stimulates the economy and not because the German monetary authority could lower the domestic financing costs of entrepreneurs. This is because we assume that entrepreneurs borrow only abroad. The assumption, meant to be restrictive as it appears, is to mimic the monetary situation that Germany faced at that time.

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