Supplementary Appendix for "Costly Concealment: Secret Foreign Policymaking, Transparency, and Credible Reassurance"

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We divide the appendix in two, beginning with a mapping of equilibrium strategies and then proving the remarks.

1 Equilibrium

Proposition 1. *Suppose $c_R > 1$. Then types $c_S < q+1$ defect and set $s = 0$, while types $c_S > q+1$ do not defect. R accommodates regardless of the signal.*

When $c_R > 1$, accommodating strictly dominates containing for R. That is because the worst payoff R can receive by accommodating is -1, which is still better than guaranteeing $-c_R$ through containment when the condition holds.

Given that R accommodates at both of its information sets, a generic type of S's payoff for defection is $1 - \alpha s - c_S$. This strictly decreases in s, and so the optimal s is 0. In turn, a type prefers to openly defect rather than maintain the status quo if $1 - c_S > -q$, or $c_S < q+1$. $\square$

Proposition 2. *Suppose $c_R \in \left(\frac{1}{2}, 1\right)$. Then types $c_S < q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}$ defect and set $s = \frac{1-c_R}{\alpha}$, while types $c_S > q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}$ do not defect. R accommodates following the cooperative signal and accommodates after observing defection with probability $\sigma_D = \frac{(2c_R-1)(1-t)}{(2c_R-1)(1-t)+\alpha}$.*

We begin with R's indifference condition upon observing a defection. Suppose all types of S choose the same s. Then, conditional on being at that information set, R earns $0 - c_R$ for containing and $1 - \alpha s$ for accommodating. Thus, it is indifferent if $0 - c_R = -(1 - \alpha s)$, or $s = \frac{1-c_R}{\alpha}$. All types that defect here choose that level, and thus R's indifference condition holds. Mixing between containing and accommodating is optimal.

Now consider R's decision to contain following the cooperative signal. Its posterior belief that S has defected under this circumstance is:

$$\frac{F\left(q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}\right)\frac{(1-c_R)(1-t)}{\alpha}}{F\left(q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}\right)\frac{(1-c_R)(1-t)}{\alpha} + 1 - F\left(q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}\right)}$$

This is strictly less than 1. But note that R's payoff for accommodating strictly decreases in its belief that S has defected, and R was indifferent when it knew with certainty S had defected. Thus, it has a strict preference to accommodate, as the equilibrium strategy dictates.

Meanwhile, consider S's decision to defect. Suppose that R accommodates with probability σ_D upon observing defection and never contains following the cooperative signal. Then S's objective function for defecting is:

$$(1 - \alpha s)(s(1 - t)) + (1 - s(1 - t))(\sigma_D(1 - \alpha s) + (1 - \sigma_D)(0)) - c_S$$

Taking the first order condition yields:

$$s = \frac{(1-t)(1-\sigma_D) - \alpha\sigma_D}{2\alpha(1-t)(1-\sigma_D)} \tag{1}$$

Substituting $\sigma_D = \frac{(2c_R-1)(1-t)}{(2c_R-1)(1-t)+\alpha}$ yields $s = \frac{1-c_R}{\alpha}$. This is a maximizer because the second derivative is $-2\alpha(1-t)(1-\sigma_D)$. Thus, it is optimal for all types who defect to choose that quantity, as the proposition indicates.

The final thing to verify is which types wish to defect at $s = \frac{1-c_R}{\alpha}$. Substituting that value and the equilibrium σ_D into S's objective function yields $\frac{c_R^2(1-t)}{(2c_R-1)(1-t)+\alpha} - c_S$. S earns $-q$ for not defecting. Therefore, types with $c_S < q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+\alpha}$ have a strict preference to defect, and types with $c_S > q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+\alpha}$ have a strict preference not to.[1] $\square$

Proposition 3. *Suppose* $c_R \in \left(\frac{F\left(q+\frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q+\frac{1-t}{4\alpha}\right)(1-t)+4\alpha\left(1-F(q+\frac{1-t}{4\alpha})\right)}, \frac{1}{2}\right)$. *Then types* $c_S < q + \frac{1-t}{4\alpha}$ *defect and set* $s = \frac{1}{2\alpha}$, *while types* $c_S > q + \frac{1-t}{4\alpha}$ *do not defect. R contains if it observes defection and accommodates following a cooperative signal.*

We begin by proving that S's strategies within the proposition are its best response. For any type choosing to defect, its objective function is:

$$(1-\alpha s)(s(1-t)) + 0(1-s(1-t)) - c_S$$

Thus, its first order condition is:

$$\frac{\partial}{\partial s}(1-\alpha s)(s(1-\tau)) + 0(1-s(1-t)) - c_S = 0$$

$$s = \frac{1}{2\alpha}$$

This is a maximum because the second partial derivative of its objective function with respect to s is $-2\alpha(1-t)$.

Substituting $s = \frac{1}{2\alpha}$ into S's objective function generates:

$$\left(1-\alpha\left(\frac{1}{2\alpha}\right)\right)\left(\frac{1}{2\alpha}(1-t)\right) + 0(1-s(1-t)) - c_S$$

$$\frac{1-t}{4\alpha} - c_S \tag{2}$$

[1]Note that $\frac{c_R^2(1-t)}{(2c_R-1)(1-t)+\alpha}$ is constrained between 0 and 1. It is trivially greater than 0 because $c_R > \frac{1}{2}$ for this parameter space. It is less than 1 because $\frac{c_R^2(1-t)}{(2c_R-1)(1-t)+\alpha} < 1$ reduces to $(1-c_R)^2 < \frac{\alpha}{1-t}$. The left hand side maximizes in this region at $c_R = \frac{1}{2}$, and the left hand side minimizes at $\alpha = \frac{1}{2}$ (the assumption for the interior solution) and $t = 0$. Using those as the toughest case, the inequality reduces to $\frac{1}{4} < \frac{1}{2}$, which is true.

In turn, a given type prefers defecting under this optimal level of secrecy to not defecting if $\frac{1-t}{4\alpha} - c_S > -q$, or $c_S < q + \frac{1-t}{4\alpha}$. Analogously, all types with $c_S > q + \frac{1-t}{4\alpha}$ cooperate. This is the decision rule given in the proposition.

What remains is to show that R's prescribed action forms the mutual best response. The case where R observes defection is straightforward. Because each type that has defected has chosen $s = \frac{1}{2\alpha}$, R's payoff for accommodating is $-\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right)$. Meanwhile, it earns $0 - c_R$ by containing. Thus, it prefers to contain if $c_R < \frac{1}{2}$, which is given by the proposition's parameter space.

The case where R does not observe defection is more complicated. Let p represent R's posterior belief that S has defected. Given that each defecting type chose $s = \frac{1}{2\alpha}$, R prefers to accommodate if:

$$-p\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right) + (1-p)q > p(0) + (1-p)q - c_R$$

$$p < 2c_R$$

We can derive the equilibrium posterior belief through Bayes' rule. Recall that all types $c_S < q + \frac{1-t}{4\alpha}$ defect with $\frac{1}{2\alpha}$ secrecy. The probability Nature does not reveal them is therefore $\frac{1-t}{2\alpha}$. Meanwhile, all types $c_S > q + \frac{1-t}{4\alpha}$ do not defect. Therefore, the probability that S has defected conditional on not observing it is:

$$\frac{F\left(q + \frac{1-t}{4\alpha}\right)\left(\frac{1-t}{2\alpha}\right)}{F\left(q + \frac{1-t}{4\alpha}\right)\left(\frac{1-t}{2\alpha}\right) + 1 - F\left(q + \frac{1-t}{4\alpha}\right)} = \frac{F\left(q + \frac{1-t}{4\alpha}\right)(1-t)}{F\left(q + \frac{1-t}{4\alpha}\right)(1-t) + 2\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)}$$

Thus, R has a strict preference to accommodate if:

$$2c_R > \frac{F\left(q + \frac{1-t}{4\alpha}\right)(1-t)}{F\left(q + \frac{1-t}{4\alpha}\right)(1-t) + 2\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)}$$

$$c_R > \frac{F\left(q + \frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q + \frac{1-t}{4\alpha}\right)(1-t) + 4\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)}$$

This is given by the proposition's parameter space. Moreover, note that because $4\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right) > 0$, the right hand side is strictly less than $\frac{1}{2}$ and thus the parameter space is guaranteed to exist. $\square$

Proposition 4. *Suppose* $c_R \in \left(\frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))}, \frac{F\left(q+\frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q+\frac{1-t}{4\alpha}\right)(1-t)+4\alpha\left(1-F(q+\frac{1-t}{4\alpha})\right)}\right)$. *Then types* $c_S < c_S^*$ *(as defined below) defect and set* $s = \frac{1}{2\alpha}$. *R contains if it observes defection and accommodates with probability* $\sigma_C = \frac{4\alpha(c_S^*-q)}{1-t}$ *following a cooperative signal.*

The simplest part of this is to verify that R wishes to contain observed defection, so we begin there. The proof of Proposition 3 showed that R prefers to contain there under $s = \frac{1}{2\alpha}$ given that $c_R < \frac{1}{2}$. Because $\frac{F\left(q+\frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q+\frac{1-t}{4\alpha}\right)(1-t)+4\alpha\left(1-F(q+\frac{1-t}{4\alpha})\right)} < \frac{1}{2}$, R has an even stronger preference to contain in this parameter space.

Now consider R's indifference condition. Suppose all types below some c_S^* defect with secrecy $s = \frac{1}{2\alpha}$. Let p represent R's posterior belief that S has defected. Given $s = \frac{1}{2\alpha}$, R is indifferent between containing and not if:

$$-p\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right) + (1-p)q = p(0) + (1-p)q - c_R$$

$$p = 2c_R \tag{3}$$

R can calculate its posterior belief through Bayes' rule. Recall that all types c_S less than some c_S^* defect with secrecy $\frac{1}{2\alpha}$. The probability that Nature does not reveal them is therefore $\frac{1-t}{2\alpha}$. Meanwhile, all types $c_S > c_S^*$ do not defect. Therefore, the probability that S has defected conditional on not observing it is:

$$\frac{F(c_S^*)\left(\frac{1-t}{2\alpha}\right)}{F(c_S^*)\left(\frac{1-t}{2\alpha}\right) + 1 - F(c_S^*)} = \frac{F(c_S^*)(1-t)}{F(c_S^*)(1-t) + 2\alpha(1 - F(c_S^*))}$$

Indifference therefore requires:

$$2c_R = \frac{F\left(c_S^*\right)(1-t)}{F\left(c_S^*\right)(1-t) + 2\alpha\left(1 - F(c_S^*)\right)}$$

$$c_R = \frac{F\left(c_S^*\right)(1-t)}{2F\left(c_S^*\right)(1-t) + 4\alpha\left(1 - F(c_S^*)\right)}$$

We therefore define c_S^* as the solution to $c_R = \frac{F(c_S)(1-t)}{2F(c_S)(1-t)+4\alpha(1-F(c_S))}$. Such a solution exists and is strictly between q and $q + \frac{1-t}{4\alpha}$ due to a few points. First, the right hand side is continuous in c_S. Second, $c_R < \frac{F\left(q+\frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q+\frac{1-t}{4\alpha}\right)(1-t)+4\alpha\left(1-F(q+\frac{1-t}{4\alpha})\right)}$ for this parameter space. Finally, $c_R > \frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))}$ for this parameter space. Moreover, the solution is unique because the right hand side strictly increases in c_S.

Thus, we have shown that R must contain when it observes defection and that it is optimal for R to mix when it observes the cooperative signal given S's strategy. Now consider S's strategy. Let σ_C be the probability that R accommodates following the cooperative signal. Then any type that defects has an objective function of:

$$(s(1-t))(\sigma_C(1-\alpha s) + (1-\sigma_C)0) + (1 - s(1-t))(0) - c_S$$

Thus, its first order condition is:

$$\frac{\partial}{\partial s}((s(1-t))(\sigma_C(1-\alpha s) + (1-\sigma_C)0) + (1 - s(1-t))(0) - c_S) = 0 \tag{4}$$

$$s = \frac{1}{2\alpha}$$

This is a maximum because the second partial derivative of its objective function with respect to s is $-2\alpha\sigma_C(1-t)$. For Proposition 4's strategies to be optimal, the c_S^* type must be indifferent between defecting at $s = \frac{1}{2\alpha}$ and not. All types with $c_S < c_S^*$ would therefore have a strict preference to defect, and all types with $c_S > c_S^*$ would have a strict preference to cooperate. Drawing from the above objective function and substituting the optimal secrecy, that the c_S^* type is indifferent if:

$$\sigma_C \left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right)\left(\frac{1}{2\alpha}(1-t)\right) - c_S^* = -q$$

$$\sigma_C = \frac{4\alpha(c_S^* - q)}{1-t}$$

Note that this is a valid mixed strategy so long as $c_S^* \in \left(q, q + \frac{1-t}{4\alpha}\right)$, which the derivation of c_S^* already guaranteed to be true.

Proposition 5. *Suppose $c_R < \frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))}$. Then all types $c_S < q$ defect with secrecy $s = \frac{1}{2\alpha}$. R contains regardless of the signal.*

We begin with S's defection strategy. Any type that defects earns $0 - c_S$ regardless of s because R contains at both its information sets. Thus, types with $0 - c_S < -q$, or $c_S > q$, must cooperate. Likewise, types with $c_S < q$ must defect. Moreover, all types that defect are indifferent among their s choices, so choosing $s = \frac{1}{2\alpha}$ is optimal. Note further that this is the *unique* maximizer if S anticipates any trembles in R's response to a cooperative signal, as Line 4 showed that $s = \frac{1}{2\alpha}$ is the solution to the first order condition for any σ_C.

Now consider R's decision to contain following observed defection. It earns $-\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right)$ for accommodating and $0 - c_R$ for containing. Therefore, it contains if $c_R < \frac{1}{2}$, which is true because $\frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))} < \frac{1}{2}$.

R's decision to contain following the cooperative signal is more complicated. Let p be its posterior belief that S has defected, all at $s = \frac{1}{2\alpha}$. Then its payoff for containment is $p(0) + (1-p)q - c_R$ and its payoff for accommodating is $-p\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right) + (1-p)q$. Thus, it contains if $2c_R < p$.

Note from before that $\frac{F(q)(1-t)}{F(q)(1-t)+2\alpha(1-F(q))}$ is R's posterior given that all types $c_S < q$ defect with secrecy $\frac{1}{2\alpha}$. Substituting that value for p yields $c_R < \frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))}$, which is the bound for this parameter space.[2] $\square$

[2]Because the types are indifferent among every s, multiple equilibria exist. Indeed, any strategy mapping from c_S to s sustains the equilibrium as long as R's posterior belief p is greater than $\frac{c_R}{1-s}$. In fact, the strategies can be more complex, with each type choosing a different s value. If the strategy mapping $s(c_S)$

2 Uniqueness Proof

We now show that the equilibrium is unique given the refinement on trembles. This is a two-step process. First, we show that the five types of strategies R can undertake in equilibrium are the five types of strategies covered in Propositions 1–5. These are (1) accommodate at both information sets, (2) accommodate following the cooperative signal and mix following observed defection, (3) accommodate following the cooperative signal and contain following observed defection, (4) mix following the cooperative signal and contain following observed defection, and (5) contain at both information sets. Second, we show that the equilibrium conditions described in those propositions are necessary as well as sufficient.

2.1 Restrictions on R's Equilibrium Strategies

Broadly, the equilibria we have seen so far all feature R accommodating with weakly greater probability following a cooperative signal than after observing defection. That is, $\sigma_C \geq \sigma_D$. We now show that this is true for *any* equilibrium.

Suppose in an equilibrium, R was weakly more likely to accommodate an observed defection than a cooperative signal. That is, $\sigma_D \geq \sigma_C$. In this case, S's optimal secrecy is $s = 0$; any secrecy made results in a weakly higher probability that R contains it, and S's defection is weaker whenever R accommodates instead. In turn, R must contain the observed defection as a pure strategy if $c_R < 1$. This exhausts one case where $\sigma_D = \sigma_C$. The other pure strategy way for $\sigma_D = \sigma_C$ is if the receiver contains as a pure strategy at both information sets, so any s value is optimal.

The last possibility is that R mixes at both information sets. However, this is not possible. Line 4 showed that, given that R will accommodate with positive probability following a cooperative signal, any type that defects has a unique optimal secrecy level $s = \frac{1}{2\alpha}$. Note

is differentiable, the condition for this is $\frac{\int_0^q s(c_S)(1-t)f(s)ds}{\int_0^q s(c_S)(1-t)f(s)ds+1-F(q)}$. Going further, it can grow even more complex and involve non-differentiable mappings. The key is that R's expected benefit for containing is greater than its cost to do so.

that this is independent of type and strictly less than 1. Note further that indifference after observed defection requires that $c_R = 1 - \alpha s$. Meanwhile, let p be R's posterior belief that R has defected following a cooperative signal. Then R is indifferent between containing and accommodating if if $p(-c_R) + (1-p)(q - c_R) = p(-(1-\alpha s)) + (1-p)(q)$, or $c_R = p(1-\alpha s)$. It is therefore impossible for R to be indifferent at both information sets. It would require $1-\alpha s = p(1-\alpha s)$, but $p < 1$ because S's mixing implies positive probability on both actions.

2.2 The Equilibrium Parameter Spaces Are Comprehensive

Now we turn to showing that the strategies listed in the propositions can each only occur in the parameter spaces listed. All the earlier proofs started by considering equilibrium strategies and then derived constraints on model parameters. This part shows that the parameters imply the strategies.

First, consider the set of strategies where R accommodates following an observed defection. When this happens, S must set $s = 0$ when it defects. Because R knows S has defected upon observing the non-cooperative signal, R earns $0 - c_R$ for containing and -1 for accommodating. Thus, for this strategy to occur in equilibrium, it must be that $c_R \geq 1$.

Meanwhile, R's payoff for containing following a cooperative signal is $(1-p)q - c_R$, where p is its posterior belief that S has defected. Its payoff for accommodating depends on its belief about which types defected and what their level of secrecy is. However, for each type that defected, R's payoff must be strictly greater than -1. Using $-1+\epsilon$ as a placeholder, R still prefers not resisting if $p(-1+\epsilon) + (1-p)q > (1-p)q - c_R$. This reduces to $c_R > 1 - \epsilon$. This is covered by the requirement that $c_R \geq 1$. Therefore, equilibria where R accommodates after observing defection require that R accommodates following a cooperative signal. The only parameters where this can happen are found in Proposition 1.

Second, consider the set of strategies where R mixes between containing and accommodating after observing defection. Mixing here requires indifference. The first section of the uniqueness proof showed that this implies that R must accommodate as a pure strategy upon observing a cooperative signal. Line 1 showed that types that wish to defect under those circumstances have a solution to the first order condition that is not a function of c_S. They therefore must select the same s value. The proof of Proposition 2 then showed that $s = \frac{1-c_R}{\alpha}$ is the unique strategy that generates R's necessary indifference condition, that $\sigma_D = \frac{(2c_R-1)(1-t)}{(2c_R-1)(1-t)+\alpha}$ is the unique strategy that makes $\frac{1-c_R}{\alpha}$ the solution to the first order condition, and $c_S = q + \frac{c_R^2(1-t)}{(2c_R-1)(1-t)+a}$ is the cutpoint the cutpoint for which S wants to defect given that secrecy. Thus, the only parameters where R mixes only after observing defection are found in Proposition 2.

Finally, consider the set of strategies where R contains after observing defection. There are three subcases to consider: R contains as a pure strategy following a cooperative signal, R accommodates following a cooperative signal, and R mixes between its two options following a cooperative signal. Propositions 3 and 5 derived the bounds on when S's response to those strategies would self-reinforce R's strategies. Thus, the remaining work needs to verify that the mixing subcase is unique to Proposition 4's parameters.

To make progress there, begin by noting that Line 4 showed that any type that defects has a unique solution to the first order condition for secrecy, setting it at $\frac{1}{2\alpha}$. Given that, all types $c_S < c_S^*$ must defect to obtain R's indifference condition in any cutpoint equilibrium[3], and R must contain in response to a cooperative signal with probability $\frac{4\alpha(c_S^*-q)}{1-t}$ to maintain the c_S^* type's defection. Thus, the only parameters where R mixes after a cooperative signal are found in Proposition 4.

3 Proof of the Remarks

The proof of Remark 1 is straightforward. Within the parameters of Proposition 3, the probability that a type engages in secret defection is $F\left(q + \frac{1-t}{4\alpha}\right)$. The CDF is strictly increasing, so demonstrating that the probability decreases is equivalent to showing that

[3] The equilibrium must be in cutpoint strategies because the types have utilities monotonic in type.

$q + \frac{1-t}{4\alpha}$ decreases in t, which is obviously true. $\square$

The proof of Remark 2 requires more work. From Proposition 3, R's posterior belief that S has not defected upon observing cooperative signals is:

$$\frac{F\left(q + \frac{1-t}{4\alpha}\right)(1-t)}{F\left(q + \frac{1-t}{4\alpha}\right)(1-t) + 2\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)}$$

Thus, we need to show:

$$\frac{\partial}{\partial t}\left(\frac{F\left(q + \frac{1-t}{4\alpha}\right)(1-t)}{F\left(q + \frac{1-t}{4\alpha}\right)(1-t) + 2\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)}\right) < 0 \tag{5}$$

It will be helpful to instead define $g(t) = F\left(q + \frac{1-t}{4\alpha}\right)(1-t)$ and $h(t) = 2\alpha\left(1 - F(q + \frac{1-t}{4\alpha})\right)$. Using the quotient rule, Line 5 holds if:

$$\frac{g'(t)(g(t) + h(t)) - (g'(t) + h'(t))g(t)}{(g(t) + h(t))^2} < 0$$

$$g'(t)h(t) < h'(t)g(t)$$

Both $g(t)$ and $h(t)$ are strictly positive. Note that $g'(t) = -F\left(q + \frac{1-t}{4\alpha}\right) - (1-t)\left(\frac{f\left(q + \frac{1-t}{4\alpha}\right)}{4\alpha}\right)$ and is therefore negative. Meanwhile, $h'(t) = \frac{f\left(q + \frac{1-t}{4\alpha}\right)}{4\alpha}$ and is therefore positive. Consequently, the left hand side is negative and the right hand side is positive, so Line 5 holds. $\square$

Proving Remark 3 is more complicated than the other two. First, consider how changes to t affect S's payoff under Proposition 5. Types with $c_S < q$ always defect. Because R contains regardless of the signal, such a type earns $0 - c_S$. This is unchanging in t.

Note that the cutpoint on Proposition 5 is $\frac{F(q)(1-t)}{2F(q)(1-t) + 4\alpha(1-F(q))}$. This decreases in t and thus can transition the parameters into Proposition 4.[4] Recall that R contains upon observing defection and accommodates upon observing the cooperative signal with probability $\frac{4\alpha(c_S^* - q)}{1-t}$. Also, the type in question chooses $s = \frac{1}{2\alpha}$. Thus, within Proposition 4's parameter

[4]That is, the derivative of $\frac{F(q)(1-t)}{2F(q)(1-t)+4\alpha(1-F(q))}$ with respect to t is $-\frac{\alpha F(q)(1-F(q))}{(2\alpha(1-F(q))+F(q)(1-t))^2}$.

space, such a type earns:

$$\left(\frac{4\alpha(c_S^* - q)}{1-t}\right)\left(1 - \alpha\left(\frac{1}{2\alpha}\right)\right)\left(\frac{1-t}{2\alpha}\right) - c_S$$

$$c_S^* - q - c_S$$

In turn, showing that the utility increases requires demonstrating that c_S^* increases in t. Recall that c_S^* is the unique solution to $c_R = \frac{F(c_S)(1-t)}{2F(c_S)(1-t)+4\alpha(1-F(c_S))}$. Increasing t decreases the right hand side. Thus, c_S must change in a manner to exactly offset the change to maintain the equality. The value increases in c_S, and so c_S^* increases in t.[5] Note that because $c_S^* > q$, this amount is larger than what it earns under Proposition 5.

Note that the upper cutpoint on Proposition 4 is $\frac{F\left(q+\frac{1-t}{4\alpha}\right)(1-t)}{2F\left(q+\frac{1-t}{4\alpha}\right)(1-t)+4\alpha\left(1-F(q+\frac{1-t}{4\alpha})\right)}$. This decreases in t and thus can transition the parameters into Proposition 3.[6] Within that parameter space, Line 2 calculated the given type's payoff as $\frac{1-t}{4\alpha} - c_S$. This decreases in t and goes to $-c_S$ as t goes to 1. We therefore have a nonmonotonicity as claimed. $\square$

[5]More formally, let $G(c_S, t) = \frac{F(c_S)(1-t)}{2F(c_S)(1-t)+4\alpha(1-F(c_S))} - c_R$. Then the implicit function theorem says that the derivative of c_S^* with respect to t is $-\frac{\partial G}{\partial t} / \frac{\partial G}{\partial c_S}$. Because G increases in c_S but decreases in t, the sign of that derivative is positive.

[6]The cutpoint is the value from Line 5 multiplied by the scalar $\frac{1}{2}$. We have previously shown that this value decreases in t, and so does the same value multiplied by a positive scalar.