

## Role Based Toughness

First consider an extension to the model where the actors can have different preferences over conflict based on whether they are in the proposer or responder role. That is, the type of a player is now a double  $(\beta^r, \beta^p)$ . and the conflict payoff is  $v - k + \beta^p$  when in the proposer role and  $v - k + \beta^r$  when in the responder role.

### Static Analysis

The analysis of the SP-SPE of the model is the same with some minor modifications to notation. If player 1 has a toughness of  $\beta_1^p$  in the proposer role and player 2 has a toughness of  $\beta_2^r$  in the responder role, then a deal is struck at the responders reservation point if:

$$\beta_1^p + \beta_2^r \leq 2k. \tag{1}$$

If this inequality does not hold, the proposer makes a low offer which is rejected.

Next, we compute the expected fitness payoff for a player with type  $\beta_j$  when matched with a player of type  $\beta_{-j}$ . When player  $j$  is in the proposer role, his objective payoff is  $v - \beta_{-j}^r + k$  if  $\beta_j^p \leq 2k - \beta_{-j}^r$  and  $v - k$  otherwise. When player  $j$  is in the responder role, their objective payoff is  $v + \beta_j^r - k$  if  $\beta_j^r \leq 2k - \beta_{-j}^p$  and  $v - k$  otherwise. So the expected

payoff as a function of  $j$ 's type is:

$$\Pi(\beta_j^p, \beta_j^r; \beta_{-j}) = \begin{cases} v + \frac{\beta_j^r - \beta_{-j}^r}{2} & \beta_j^p + \beta_{-j}^r \leq 2k \text{ and } \beta_{-j}^p + \beta_j^r \leq 2k \\ v + \frac{k - \beta_{-j}^r}{2} - \frac{k}{2} & \beta_j^p + \beta_{-j}^r \leq 2k \text{ and } \beta_{-j}^p + \beta_j^r > 2k \\ v + \frac{\beta_j^r - k}{2} - \frac{k}{2} & \beta_j^p + \beta_{-j}^r > 2k \text{ and } \beta_{-j}^p + \beta_j^r \leq 2k \\ v - k & \beta_j^p + \beta_{-j}^r > 2k \text{ and } \beta_{-j}^p + \beta_j^r > 2k \end{cases}$$

First, note that for any pair  $(\beta_h, p_h)$  that meets the equilibrium condition in the baseline model, there is an analogous equilibrium where types can differ on toughness where  $\beta^r = \beta^p = \beta_h$  with probability  $p_h$  and  $\beta^r = \beta^p = \beta_l$  with probability  $1 - p_h$ . In such an equilibrium, increasing the toughness in any role for either type leads to more conflict and hence a lower objective payoff, and decreasing the toughness either leads to a strictly lower objective payoff (when  $\beta_j^r$  decreases) or no change in the payoff (when  $\beta_j^p$  increases). So, building on Theorem 1 in the main text:

**Proposition 1.** *For any  $p_c \in [0, 1]$ , the model with asymmetric toughness based on role has a SP-SPE where the probability of conflict is  $p_c$ .*

Second, note that the objective payoff is always weakly decreasing in  $\beta_j^p$  for any  $\beta_j^p > 0$ , as becoming tough in the proposer role can only lead to more inefficient conflict. Further, the objective payoff is weakly increasing in  $\beta_j^p$  for  $\beta_j^p < 0$  as being “irrationally weak” can only lead to proposing accepted offers that give a worse objective payoff than fighting. So, for any  $\beta_{-j}$  and  $\beta_j^r$ , any type where  $\beta_j^p \neq 0$  gets a weakly lower payoff than they would with  $\beta_j^p = 0$ . Formally:

**Definition** A type  $\beta_j$  is *weakly dominated* if there exists a  $\beta_j' \neq \beta_j$  such that  $\Pi(\beta_j; \beta_{-j}) \leq \Pi(\beta_j'; \beta_{-j})$  for all  $\beta_{-j}$ .

Under this definition, any type with  $\beta_j^p \neq 0$  is weakly dominated. So, if we restrict attention to equilibria where weakly dominated types do not exist in equilibrium, we only consider types where  $\beta_j^p = 0$ . It immediately follows that for  $\beta_j^r$  to give the highest payoff possible, all types are  $\beta_j^r = 2k$ :

**Proposition 2.** *The model with asymmetry of toughness based on role has a unique SP-SPE with no weakly dominated types where all actors have  $\beta^p = 0$  and  $\beta^r = 2k$*

In such an equilibrium, the proposer always offers  $v + k$  which is accepted, i.e., there is no conflict.

## Noisy Evolution

However, the lack of conflict is still not possible with a noisy evolutionary process. Using the uniform noise, suppose that if a type  $\beta_{\max} = (\beta_{\max}^p, \beta_{\max}^r)$  gets the highest payoff, then the toughness parameters for each actor in the subsequent round are given by  $(\beta_{\max}^p + \nu_i^p, \beta_{\max}^r + \nu_i^r)$  where  $\nu_i^p$  and  $\nu_i^r$  are independent and uniformly distributed on  $[-\epsilon^p, \epsilon^p]$  and  $[-\epsilon^r, \epsilon^r]$ , respectively, where  $\epsilon^p, \epsilon^r > 0$ .

Following a similar analysis as the main uniform model, the fitness payoff for being type  $\beta_j$  when matched with a population with proposer fitness uniform on  $[\beta_m^p - \epsilon^p, \beta_m^p + \epsilon^p]$  and responder fitness uniform on  $[\beta_m^r - \epsilon^r, \beta_m^r + \epsilon^r]$  is:

$$\Pi(\beta_i^p, \beta_j^r; F; \sigma) = \frac{1}{2}\Pi^p(\beta_j^p; F; \sigma) + \frac{1}{2}\Pi^r(\beta_j^r; F; \sigma)$$

where  $\Pi^j$  is the expected fitness payoff in role  $j$ . When in the proposer role the only relevant part of the distribution is the responder toughness and when in the responder role the only relevant part of the distribution is the proposer toughness. So, finding the optimal type can be separated into finding the optimal type in each role.

For the proposer role, it is more straightforward to first consider the payoff for fixed types (when using SPNE strategies):

$$\pi^p(\beta_j^p, \beta_{-j}^r; \sigma^*) = \begin{cases} v + k - \beta_{-j}^r & \beta_j^p + \beta_{-j}^r \leq 2k \\ v - k & \beta_j^p + \beta_{-j}^r > 2k \end{cases}$$

The fitness payoff for making a deal (the first segment) is higher if  $v + k - \beta_{-j}^r < v - k$ , or  $\beta_{-j}^r < 2k$ . So if  $\beta_{-j}^r < 2k$  the optimal proposer toughness is any  $\beta_j^p$  such that  $\beta_j^p < 2k - \beta_{-j}^r$ , and if  $\beta_{-j}^r > 2k$  then the optimal proposer toughness is any  $\beta_j^p$  such that  $\beta_j^p > 2k - \beta_{-j}^r$ . So, if all responder types are greater than  $2k$  any type such that  $\beta_j^p > 0$  gets the highest possible payoff, and hence there can be no stable distribution using a definition analogous to [REFERENCE]. If all responders have toughness less than  $2k$  then any type such that  $\beta_j^p < 0$  gets the highest possible expected fitness, so there can be no stable type distribution of this form either.

So in any stable distribution, there must be some responders with  $\beta_{-j}^r > 2k$  and some with  $\beta_{-j}^r < 2k$ . The only proposer type that gets the highest fitness when matched with an individual with this distribution is  $\beta_j^p = 0$ . This is because types with  $\beta_j^p < 0$  will strike a deal with some responders with  $\beta_{-j}^r > 2k$ , which gives a lower fitness than fighting, and types with  $\beta_j^p > 0$  fight against some types with  $\beta_{-j}^r < 2k$ , which gives lower fitness than striking a deal.

So, this can only have a unique maximum at  $\beta_j^p = 0$ , and hence in any stable preference distribution  $\beta^{p,*} = 0$ . That is, there is never a benefit to having preferences that deviate from fitness payoffs *when in the proposer role*.

This also implies that the expected payoff for having toughness  $\beta_j^r$  in the responder role

in any stable preference distribution is:

$$\begin{aligned}\Pi^r(\beta_j^r; F^p; \sigma^*) &= Pr(\beta_{-j}^p \leq 2k - \beta_j^r)(v - k + \beta_j^r) + Pr(\beta_{-j}^p > 2k - \beta_j^r)(v - k) \\ &= \begin{cases} v - k + \beta_j^r & \beta_j^r \leq 2k - \epsilon^p \\ \frac{2k + \epsilon^p - \beta_j^r}{2\epsilon^p}(v - k + \beta_j^r) + \frac{\beta_j^r - 2k + \epsilon^p}{2\epsilon^p}(v - k) & \beta_j^r \in (2k - \epsilon^p, 2k + \epsilon^p] \\ v - k & \beta_j^r > 2k + \epsilon^p \end{cases}\end{aligned}$$

Again, the first segment is linear and increasing, the second segment is quadratic, and the last segment is constant, though always at a lower level than the peak of the first segment. The quadratic is maximized at  $\beta^r = k + \epsilon^p/2$ , which is above  $2k - \epsilon^p$  if and only if  $k > 3\epsilon^p/2$ , so the optimal toughness level in the stable preference distribution is:

$$\beta^{r,*} = \begin{cases} k + \epsilon^p/2 & k < 3\epsilon^p/2 \\ 2k - \epsilon^p & k > 3\epsilon^p/2 \end{cases}$$

which is double the toughness of the equilibrium average toughness is the baseline model (if  $\epsilon^p = \epsilon$ ). So, by allowing the toughness to be conditional on whether a player is the proposer or not, the *aggregate* toughness remains unchanged, though only responders are irrationally tough.

To compute the probability of conflict, write the type a actor  $j$  is in role  $i$  as  $\beta^{i,*} + \nu_j$ . So, the probability of conflict in the stable preference distribution is:

$$Pr(\beta^r + \beta^p > 2k) = Pr(\beta^{r,*} + \nu^r + \nu^p > 2k) = \begin{cases} Pr(\nu^r + \nu^p > k - \epsilon^p/2) & k < 3\epsilon^p/2 \\ Pr(\nu^r + \nu^p > \epsilon^p) & k > 3\epsilon^p/2 \end{cases} \quad (2)$$

Determining the probability of conflict for either case requires computing the distribution of  $\nu^p + \nu^r$ . It is useful to first state a general result about the sum of uniform random variables

centered at zero but with different range:

**Lemma 1.** *Let  $\nu_h \sim U[-\epsilon_h, \epsilon_h]$  and  $\nu_l \sim U[-\epsilon_l, \epsilon_l]$ , where  $\epsilon_l \leq \epsilon_h$ . Then:*

*i. the cumulative density function of  $\nu_h + \nu_l$  is given by:*

$$F^{\nu_h + \nu_l}(x) = \begin{cases} 0 & x < -\epsilon_h - \epsilon_l \\ \frac{(x + \epsilon_l + \epsilon_h)^2}{8\epsilon_l\epsilon_h} & x \in (-\epsilon_h - \epsilon_l, \epsilon_l - \epsilon_h) \\ x/(2\epsilon_h) + 1/2 & x \in (\epsilon_l - \epsilon_h, \epsilon_h - \epsilon_l) \\ 1 - \frac{(-x + \epsilon_l + \epsilon_h)^2}{8\epsilon_l\epsilon_h} & x \in (\epsilon_h - \epsilon_l, \epsilon_l + \epsilon_h) \\ 1 & x > \epsilon_l + \epsilon_h \end{cases}$$

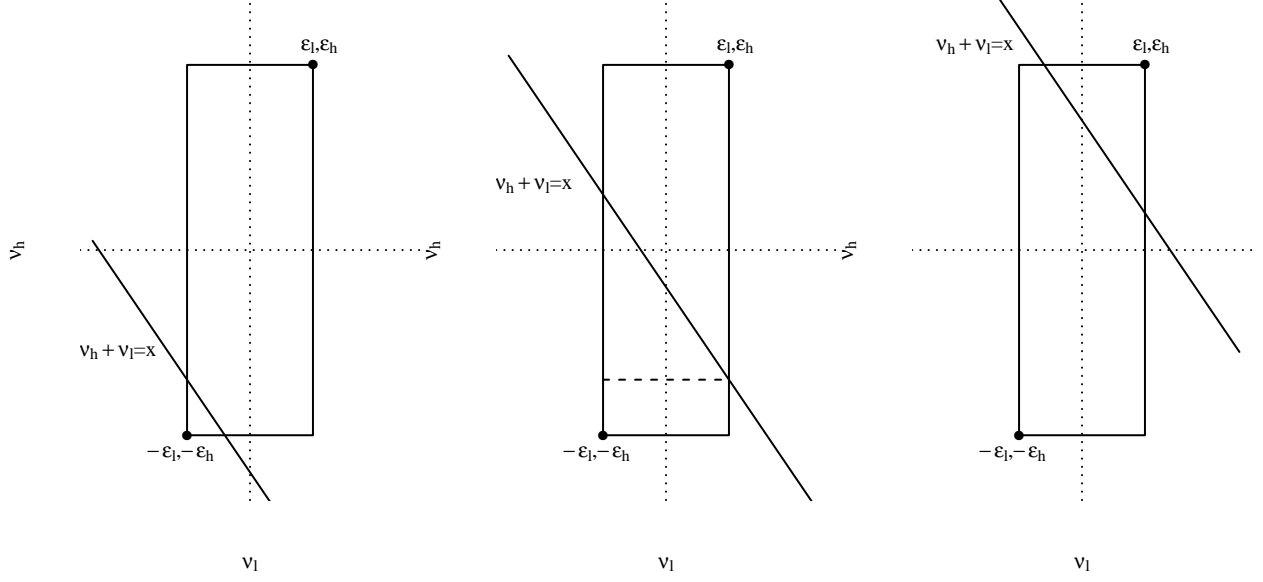
*ii.  $F^{\nu_h + \nu_l}(-x) = 1 - F^{\nu_h + \nu_l}(x)$*

**Proof** For part i, figure plots the joint density of  $\nu_l$  and  $\nu_h$ . For any  $x$ , the distribution function is the relative area of the rectangle drawn by the bounds of the distribution below the line  $\nu_l + \nu_h = x$  times the density over the rectangle, which is  $\frac{1}{4\epsilon_l\epsilon_h}$  (i.e., the product of the individual densities). Clearly for  $x < -\epsilon_l - \epsilon_h$  none of the rectangle is under the diagonal, so the distribution function is 0, and when  $x > \epsilon_l + \epsilon_h$  the entire rectangle is below and the distribution function is 1.

The left panel shows that for  $x \in (-\epsilon_l - \epsilon_h, \epsilon_l - \epsilon_h)$  the region below the diagonal is a right triangle with equal base and height. The diagonal intersects  $\nu_h = -\epsilon_h$  at  $\nu_h = x + \epsilon_h$ , so the sides are length  $x + \epsilon_h - (-\epsilon_l)$ , and hence the area is  $\frac{(x + \epsilon_l + \epsilon_h)^2}{2}$ , and multiplying by the density gives  $\frac{(x + \epsilon_l + \epsilon_h)^2}{8\epsilon_l\epsilon_h}$ .

The middle panel shows that for  $x \in (\epsilon_l - \epsilon_h, \epsilon_h - \epsilon_l)$ , the area is a right triangle with area  $\frac{(2\epsilon_l)^2}{2}$  plus a rectangle with area  $2\epsilon_l(x - (\epsilon_l - \epsilon_h))$ . Adding these and multiplying by the

Figure 1: Illustration of CDF of Sum of Uniform Random Variables



density gives:

$$\frac{2\epsilon_l(x - (\epsilon_l - \epsilon_h)) + 2\epsilon_l^2}{4\epsilon_l\epsilon_h} = \frac{x}{\epsilon_h} + \frac{1}{2}$$

The right panel shows that for  $x \in (\epsilon_h - \epsilon_l, \epsilon_l + \epsilon_h)$ , the area under the diagonal is the area of the entire rectangle  $4\epsilon_l\epsilon_h$  minus the upper triangle with area  $\frac{(\epsilon_l + \epsilon_h - x)^2}{2}$ . Combining these pieces gives part i.

Part ii follows from part i (or the symmetry of the densities of  $\nu_l$  and  $\nu_h$  around 0). ■

So, there are many cases to consider for the probability of conflict, depending on the signs of  $\epsilon^r - \epsilon^p$  and  $k - \epsilon^p/2$ , and then where  $k - \epsilon^p/2$  and  $\epsilon^p$  lie in the five segments of the CDF of  $\nu^r + \nu^p$ . Rather than enumerating all possible cases, we focus on comparative statics analogous to those in the main text:

**Proposition 3.** *In the unique stable preference distribution to the model with role-based toughness, the probability of conflict is:*

- i. equal to the probability of conflict in the baseline if  $\epsilon^p = \epsilon^r$ ,
- ii. weakly decreasing in  $k$ , and
- iii. for any  $k > 3\epsilon^p/2$ , equal to

$$\underline{p} = \begin{cases} \frac{\epsilon^r}{8\epsilon^p} & \epsilon^r < \epsilon^p \\ \frac{\epsilon^p}{8\epsilon^r} & \epsilon^r \in (\epsilon^p, 2\epsilon^p) \\ \frac{1}{2} - \frac{\epsilon^p}{2\epsilon^r} & \epsilon^r > 2\epsilon^p \end{cases}$$

**Proof** Part i follows from evaluating the fact that  $\beta^{*,r} + \beta^{*,p} = 2\beta^*$  (where  $\beta^*$  is the average toughness for the main model) and if  $\epsilon^r = \epsilon^p = \epsilon$  the distribution of  $\epsilon^r + \epsilon^p$  is the triangle distribution with CDF given by equation [REFERENCE].

Part ii follows immediately from equation 2.

For part iii, for any  $k > 3\epsilon^p/2$ , the probability of conflict is  $1 - F(\epsilon^p) = F(-\epsilon^p)$ . When  $\epsilon^p > \epsilon^r$ ,  $-\epsilon^p$  must lie on the second segment of the CDF and is hence the probability of conflict is:

$$\frac{(-\epsilon^p + \epsilon^p + \epsilon^r)^2}{8\epsilon^p\epsilon^r} = \frac{\epsilon^r}{8\epsilon^p}.$$

When  $\epsilon^p < \epsilon^r$ ,  $-\epsilon^p$  lies on the second segment if  $-\epsilon^p < \epsilon^p - \epsilon^r \implies \epsilon^r < 2\epsilon^p$  and on the third segment otherwise. Plugging  $-\epsilon^p$  into the relevant segment of the PDF gives the desired result. ■

Part i implies that allowing role-based toughness changes the equilibrium *preferences* but not the equilibrium probability of conflict if the amount of noise in the evolutionary process is the same. Parts ii-iii examines what happens if the evolution of the toughness in different roles is more or less noisy. The only case where conflict approaches 0 is if  $k$  is large and  $\epsilon^r \rightarrow 0$ . This is because in this case  $\beta^{r,*} \rightarrow 2k - \epsilon^p$ . So, the probability of conflict approaches



$$Pr(\nu^p > 2k - \epsilon_p) = 0.$$

On the other hand, as  $\epsilon^p \rightarrow 0$ , the probability of conflict can become much larger than in the baseline model. This is because  $\beta^{r,*} \rightarrow 2k$ , and hence any responder with a positive draw of  $\nu^r$  will fight every proposer.

## Sacred Values

Next, consider a class of utility functions which depart more starkly from standard bargaining models to capture the idea that players care about “sacred land” or values. In particular, in this formulation players do not vary on the *conflict* payoff but on their payoff for making agreements. The intuition is that there is some share of what players are bargaining over  $x$  such that they are always willing to take any agreement where they get at least  $x$  but are unwilling to accept any agreement less than  $x$ . For example, an actor that assigns a “sacred value” to getting  $v$  is one who demands at least half of the prize, perhaps due to strong feelings about a norm of fairness.

To be general, we allow the actors to have a different minimal acceptable value depending on their role in the bargaining game. In particular:

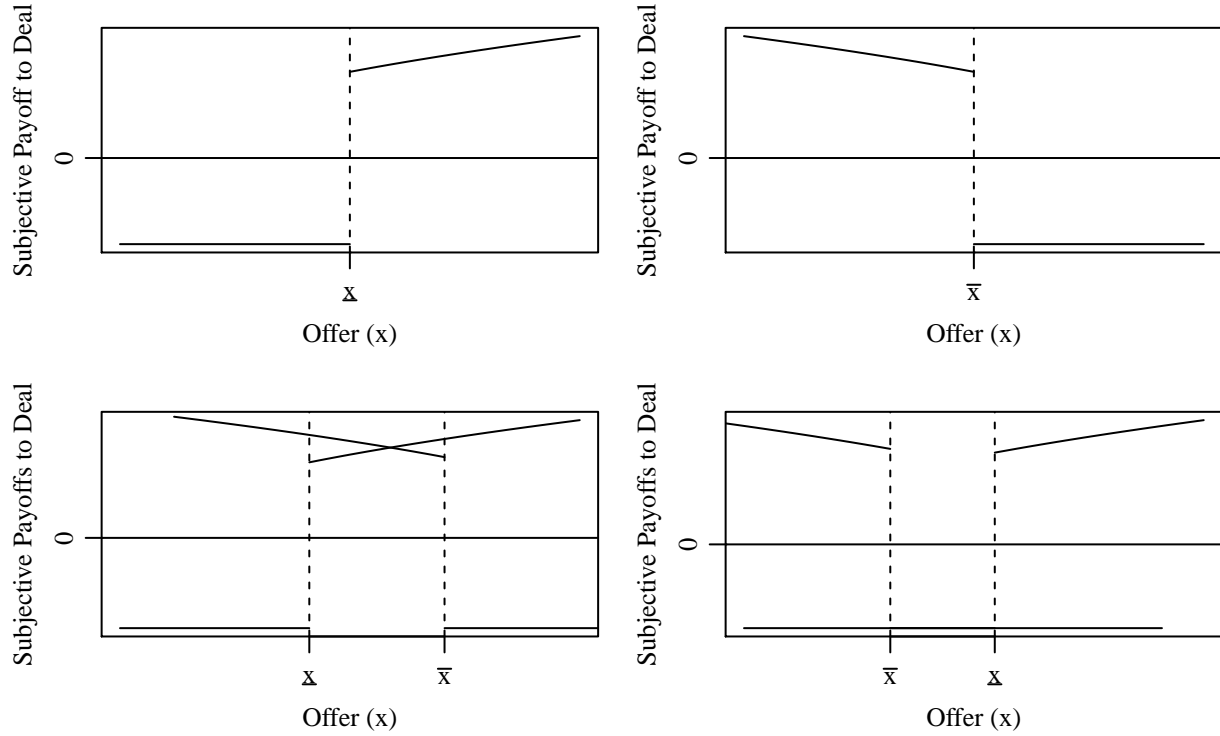
**Definition** A player has  $(\bar{x}, \underline{x})$ -*sacred value preferences* if her subjective utility function in role  $i$  is:

$$u(i, x, a) = \begin{cases} s & a = 1, i = p, x > \bar{x} \text{ or } a = 1, i = r, x < \underline{x} \\ 0 & a = 0 \\ g_i(x) & a = 1, i = p, x \leq \bar{x} \text{ or } a = 1, i = r, x \geq \underline{x} \end{cases}$$

for any  $s < 0$ , strictly positive and increasing  $g_r(x)$ , and strictly positive and decreasing  $g_p(x)$ .

That is, when in the responder role any deal which give less than  $\underline{x}$  is strictly (and

Figure 2: Sacred Values Preferences and the Bargaining Range



discontinuously) worse than fighting. Similarly, when in the proposer role giving anything more than  $\bar{x}$  is discontinuously worse than fighting.

The top two panels in figure 2 show what these payoffs look like for a responder (top right) and proposer (top left). For the responder, any deal below  $\underline{x}$  is below zero and hence “unacceptable”, while deals above  $\underline{x}$  are better than fighting, get better as the offer increases. Conversely, for a proposer, anything above  $\bar{x}$  is unacceptable, and lower accepted deals are better.

The bottom two panels illustrate the two important cases for determining the equilibrium behavior. In the bottom left panel, the responders minimal acceptable offer is below the proposer maximal acceptable offer, so any offer between  $(\underline{x}, \bar{x})$  is preferred to fighting for both actors. In the bottom right panel,  $\underline{x}$  for the responder is above  $\bar{x}$  for the proposer, so

there is no mutually acceptable deal.

So, when two actors  $i$  and  $j$  are matched with  $i$  in the proposer role and  $j$  in the responder role, by standard logic  $i$  will offer  $\underline{x}_j$  and it will be accepted if  $\underline{x}_j \leq \bar{x}_i$  and will make an offer which is rejected if  $\underline{x}_j > \bar{x}_i$ .

**Proposition 4.** *An actor with  $(\bar{x}, \underline{x})$ -sacred value preferences uses the same SPNE strategy and hence gets the same objective payoff as an actor with toughness  $(\beta^r, \beta^p) = (v + k - \bar{x}^p, v - k + \underline{x}^r)$*

Now consider a noisy evolutionary process where in each generation the type that gets the highest fitness payoff (call these  $(\bar{x}_{\max}, \underline{x}_{\max})$ ) reproduces, and the next generation has sacred value preferences where  $\bar{x}_i$  is uniformly distributed on  $[\bar{x}_i - \epsilon^p, \bar{x}_i + \epsilon^p]$  and  $\underline{x}_i$  is uniformly distributed on  $[\underline{x}_i - \epsilon^r, \underline{x}_i + \epsilon^r]$ . Then by an identical analysis, there is a unique stable distribution of preferences centered around  $\bar{x}^* = v + k - \beta^{p,*} = v + k$  and:

$$\underline{x}^* = v - k + \beta^{r,*} = \begin{cases} v + \epsilon^p/2 & k < 3\epsilon^p/2 \\ v + k - \epsilon^p & k \geq 3\epsilon^p/2 \end{cases}$$

and the probability of conflict is the same as in the role-based toughness case.

## More General Preferences

This equivalence suggests that the same equilibrium behavior and chance of conflict can occur for a much wider class of preferences differing from the objective payoffs. A complete description of a players preferences is a 4-tuple  $u = (w^p, w^r, a^p(x), a^r(x))$ , where  $w^r \in \mathbb{R}$  and  $w^p \in \mathbb{R}$  are the preferences over conflict when in the responder and proposer role, respectively, and  $a^p(x)$  and  $a^r(x)$  are the subjective utility when offer  $x$  is accepted in the respective roles. The only restrictions we place on the preferences are the following:

**Assumption 1.** *The preferences for the actors  $u$  are such that: i)  $a^p$  is weakly decreasing in  $x$  and  $a^r$  is weakly increasing in  $x$*   
*ii) there exists an  $\bar{x} \in \mathbb{R}$  such that  $\bar{x} = \min\{x : a^p(x) \geq w^p\}$  and an  $\underline{x} \in \mathbb{R}$  such that  $\underline{x} = \max\{x : a^r(x) \leq w^r\}$ .*

In words, i implies the proposer always prefers smaller offers and the responder always prefers higher offers. Part ii implies that there is a well defined “highest acceptable offer” for the proposer and a “lowest acceptable offer” for the responder. A somewhat more intuitive assumption which implies this property is if  $a^p(x)$  is right-continuous and  $w^p \in \text{Range}(a^p)$ , and similarly  $a^r(x)$  is left-continuous and  $w^r \in \text{Range}(a^r)$ . That is, the cases that need to be ruled out are when either the fighting payoffs lie outside the range of possible payoffs for acceptance or there is a discontinuity in the acceptance payoffs which renders the minimum or maximum expressions undefined.

Suppose two players are matched to play the bargaining game, and call the player in the proposer role  $i$  and in the responder role  $j$ . Then the proposer role either offers  $\underline{x}_j$  or an offer which is rejected, and prefers to offer  $\underline{x}_j^r$  if and only if  $\underline{x}_j^r \leq \bar{x}_i$ . That is, if there is a division weakly preferred to war for both players, they strike a bargain at the minimal offer accepted by the responder. Otherwise, they fight. So, a more general statement of lemma ?? is:

**Lemma 2.** *Suppose players  $i$  and  $j$  have preferences meeting assumption 1, and  $i$  is placed in the proposer role with  $j$  in the responder role. Then in any SPNE:*

- i. If  $\underline{x}_j^r \leq \bar{x}_i^p$ , then the proposer offers  $\underline{x}_j^r$  and it is accepted*
- ii. If  $\underline{x}_j^r > \bar{x}_i^p$ , then the proposer makes an offer less than  $\underline{x}_j^r$  which is rejected.*

So, any preferences meeting equation 1 induce identical behavior as the  $(\underline{x}, \bar{x})$ -sacred value preferences. While explicitly modeling the evolution of preferences is more complex, as it requires specifying not just how a real-valued parameter changes but how the entire

preference function evolves. However, as long as the resulting  $\underline{x}$  and  $\bar{x}$  behave in a similar manner defined above, identical results arise in this more general setting.

## Partially Observed Preferences

So far all of the analysis has assumed that the preferences of the players are known. In the final extension, we relax this assumption. In particular, suppose that when two players are matched, their preferences are observed with probability  $q$ , and with probability  $1 - q$  the actors only know the distribution of preferences.

Motivated by the analysis of role-based toughness as well as to simplify the analysis, we assume that all actors preferences are equal to the objective payoffs when in the proposer role, and in the responder role the conflict payoff is  $v - k + \beta^r$ . We also assume uniform noise, so if the type that gets the highest payoff is  $\beta_{\max}^r$ , then the payoffs in the next period are uniformly distributed on  $[\beta_{\max} - \epsilon^r, \beta_{\max}^r = \epsilon^r]$ .

So, if the population is distributed on  $[\beta_m - \epsilon^r, \beta_m + \epsilon^r]$ , when the type is unobserved that proposer payoff for making offer  $x$  is:

$$u^p(x; \beta_m) = \begin{cases} v - k & x < v - k + \beta_m^r - \epsilon^r \\ \frac{x - (v - k + \beta_m^r - \epsilon^r)}{2\epsilon^r}(2v - x) + \frac{v - k + \beta_m^r + \epsilon^r - x}{2\epsilon^r}(v - k) & x \in (v - k + \beta_m^r - \epsilon^r, v - k + \beta_m^r + \epsilon^r) \\ 2v - x & x \geq v - k + \beta_m^r + \epsilon^r \end{cases}$$

The middle segment is a quadratic maximized at  $v + \frac{\beta_m^r - \epsilon^r}{2}$ . If this maximum lies below  $v - k + \beta_m^r - \epsilon^r$ , then the proposer maxes an offer which is always rejected. If the maximum of the quadratic is above  $v - k + \beta_m^r + \epsilon^r$ , then the proposer makes this offer as it buys off all types. If the quadratic is maximized on the relevant interval, the proposer makes that offer.

So:

$$x_u^* = \begin{cases} v - k + \beta_m^r - \epsilon^r & \beta_m^r > 2k + \epsilon^r \\ v + \frac{\beta_m^r - \epsilon^r}{2} & \beta_m^r \in [2k - 3\epsilon^r, 2k + \epsilon^r] \\ v - k + \beta_m^r + \epsilon^r & \beta_m^r < 2k - 3\epsilon^r \end{cases}$$

When the types are observed, the equilibrium behavior is the same as in the main model (with  $\beta^p = 0$ ). So, if  $\beta_m^r < 2k - 3\epsilon^r$ , then the proposer buys off all types when the type is unobserved. This inequality also implies that the highest type is  $2k - 2\epsilon$ , so a deal is reached when the type is observed, and so the highest type always gets the highest fitness payoff, and hence the distribution is not stable. Conversely, if  $\beta_m^r > 2k + \epsilon^r$ , then all types fight regardless of whether the type is observed, which also violates the stability condition. So, in any stable equilibrium  $\beta_m^r \in [2k - 3\epsilon^r, 2k + \epsilon^r]$  and an interior offer is made when the type is unobserved.

Next, we compute the fitness payoff for a responder with toughness  $\beta_j^r$  when the mean toughness is  $\beta_m^r$  (within the range of types in the distribution). When the type is observed the fitness payoff is  $v - k + \beta_j^r$  for  $\beta_j^r \leq 2k$  and  $v - k$  otherwise. When the type is unobserved, the responder accepts the offer made if and only if:

$$v - k + \beta_j^r \leq v + \frac{\beta_m^r - \epsilon^r}{2} \implies \beta_j^r \leq k + \frac{\beta_m^r - \epsilon^r}{2}$$

So, the expected fitness is:

$$\Pi(\beta_j^r; \beta_m^r) = \begin{cases} q(v - k + \beta_j^r) + (1 - q) \left( v + \frac{\beta_m^r - \epsilon^r}{2} \right) & \beta_j^r \leq k + \frac{\beta_m^r - \epsilon^r}{2} \\ q(v - k + \beta_j^r) + (1 - q)(v - k) & k \in [k + \frac{\beta_m^r - \epsilon^r}{2}, 2k] \\ v - k & \beta_j^r > 2k \end{cases}$$

which is a piecewise linear function with two (downward) discontinuities. For there to be a stable distribution at the first discontinuity, it must be the case that the center of this distribution is in fact at the first peak, and there is no type at a higher  $\beta_j^r$  that gets a higher payoff For the first condition:

$$\beta_m^r = k + \frac{\beta_m^r - \epsilon^r}{2} \implies \beta_m^r = 2k - \epsilon^r$$

Note this implies that the toughest type in the distribution lies exactly at the other peak of  $2k$ . So, for this to represent a stable preference distribution it must be case that

$$\begin{aligned} \Pi(2k - \epsilon^r; 2k - \epsilon^r) &\geq \Pi(2k; 2k - \epsilon^r) \\ q(v - k + (2k - \epsilon^r) + (1 - q) \left( v + \frac{2k - \epsilon_r - \epsilon^r}{2} \right) &\geq q(v + k) + (1 - q)(v - k) \\ (v + k - \epsilon^r) &\geq v + q2k - kq \leq 1 - \frac{\epsilon^r}{2k} \end{aligned}$$

The second peak always lies at  $2k$ . If the center of the preference distribution is at the second peak, the first peak lies at  $k + \frac{2k - \epsilon^r}{2} = 2k - \epsilon^r/2$  which is above the lowest possible type  $2k - \epsilon^r$ . So, for there to be a stable preference distribution at  $2k$  it must be the case that:

$$\begin{aligned} \Pi(2k - \epsilon^r; 2k) &\leq \Pi(2k; 2k) \\ q(v - k + (2k - \epsilon^r/2)) + (1 - q)(v + \frac{2k - \epsilon^r/2 - \epsilon^r}{2}) &\leq q(v + k) + (1 - q)(v - k) \\ q &\geq 1 - \frac{\epsilon^r}{4k} \end{aligned}$$

Since  $1 - \frac{\epsilon^r}{2k} < 1 - \frac{\epsilon^r}{4k}$ , there may be a range of  $q$  where neither peak represents an equilibrium. While there is no stable distribution of preferences in this case, there is a stable *cycle* where generations alternate between being centered at  $2k$  and  $2k - \epsilon^r/2$ . This is

because the optimal level of toughness when  $\beta_m = 2k$  is  $2k - \epsilon^r/2$ , and when  $\beta_m = 2k - \epsilon^r/2$  the optimal level of toughness is  $2k$ .

Finally, consider the equilibrium probability of conflict in the stable (or cyclically stable) distribution. When  $q \leq 1 - \frac{\epsilon^r}{2k}$ , the mean of the preference distribution is  $2k - \epsilon^r$ , which implies there is never conflict when the type is observed. When the type is unobserved, the center of the preference distribution is indifferent between accepting the offer made or not, so the probability of conflict is  $1/2$ . So, the overall probability of conflict is  $(1 - q)/2$ .

When  $q \geq 1 - \frac{\epsilon^r}{4k}$ , the preference distribution is centered at  $2k$ . When the type is unobserved, there is always conflict, and when the type is observed there is conflict with probability  $1/2$ . So the overall probability of conflict is  $q/2 + 1 - q = 1 - q/2$ .

When  $q \in (1 - \frac{\epsilon^r}{2k})$ , for half of rounds the preference distribution is centered at  $2k$  giving probability of conflict  $1 - q/2$ . For the other half, the distribution is centered at  $2k - \epsilon^r/2$ . So, when the type is observed, the probability of conflict is  $\frac{2k + \epsilon^r - (2k - \epsilon^r/2)}{2\epsilon^r} = 1/4$ . When the type is unobserved, the offer made is accepted if  $\beta^r > k + \frac{2k - \epsilon^r/2 - \epsilon^r}{2} = 2k - \frac{3\epsilon^r}{4}$ , which occurs with probability  $\frac{2k + \epsilon^r - (2k - \frac{3\epsilon^r}{4})}{2\epsilon^r} = \frac{7}{8}$ . So, the total probability of conflict is:

$$\frac{1}{2}(1 - q/2) + \frac{1}{2}(q/4 + (1 - q)(7/8))$$

## General Single Reproducer Preferences

Suppose the toughness in the population follows density  $f(\beta_{-j})$  with support on  $[\underline{\beta}, \bar{\beta}]$  (where these bounds can be infinite). Then the expected payoff for having toughness  $\beta_j$  in this population is (suppressing the strategy argument):

$$\Pi(\beta_j) = \int_{\underline{\beta}}^{\bar{\beta}} \pi(\beta_j; \beta_{-j}) f(\beta_{-j}) d\beta_{-j} = \int_{\underline{\beta}}^{2k - \beta_j} v + \frac{\beta_j - \beta_{-j}}{2} f(\beta_{-j}) d\beta_{-j} + \int_{2k - \beta_j}^{\bar{\beta}} (v - k) f(\beta_{-j}) d\beta_{-j}$$



If  $\beta_j < 2k - \bar{\beta}$ , this is equal to  $v + \frac{\beta_j - \beta_{-j}}{2}$  and if  $\beta_j > 2k - \bar{\beta}$  this is equal to  $v - k$ . so: So on the range of  $\beta_j$  where both parts of the integral are used:

$$\frac{\partial \Pi(\beta_j)}{\partial \beta_j} = \begin{cases} 1/2 & \beta_j < 2k - \bar{\beta} \\ -\beta_j f(2k - \beta_j) + \int_{-\infty}^{2k - \beta_j} f(\beta_{-j})/2 d\beta_{-j} & \beta_j \in (2k - \bar{\beta}, 2k - \underline{\beta}) \\ 0 & \beta_j > 2k - \underline{\beta} \end{cases}$$

If the middle segment is always increasing, then the objective function is maximized at any  $\beta_j > 2k - \underline{\beta}$ . If the middle segment is always decreasing, the objective function is maximized at  $\beta_j = 2k - \bar{\beta}$ .

A sufficient condition for there to be a unique maximizer is if the middle segment is concave, which becomes:

$$\begin{aligned} \frac{\partial^2 \Pi(\beta_j)}{\partial^2 \beta_j} &= -\frac{3}{2}f(2k - \beta_j) + \beta_j f'(2k - \beta_j) < 0 \\ \frac{2}{3}\beta_j &\leq \frac{f'(2k - \beta_j)}{f(2k - \beta_j)} \end{aligned}$$