This document offers a complete description of the Molinari bounds presented in the *Sensitivity Analysis* section of the paper "Does Issue Linkage Work?" and how they are applied to that study.

Molinari (2010) offers an approach for estimating average treatment effects in observational studies. Beginning with the same basic setup as Manski (1997), Molinari assumes that each member j of a population of interest, J, is (1) characterized by some covariates, $x_j \in \mathbf{X}$, (2) is exposed to a set of mutually exclusive and exhaustive treatments, T, and (3) has a specific response function, $y_j(\cdot) : T \to Y$, mapping treatments $t \in T$ into outcomes $y_j(t) \in Y$. If $z_j \in T$ is the treatment that individual j actually receives, then $y_j \equiv y_j(z_j)$ is the realized outcome, while $y_j(t)$ is a latent outcome for $t \neq z_j$. In the context of my study, the assumption is that each negotiation is (1) characterized by distance, economic size, and military size, (2) witnesses or does not witness the offer of economic issue linkage, and (3) the linkage (or lack thereof), leads to either the successful or unsuccessful conclusion of an alliance agreement.

Denote by d_j a binary variable which takes on a value of 1 if the treatment received by individual j is observed, 0 otherwise. Here, d_j indicates if it is observed and known that a negotiation did/did not witness a linkage offer. Finally, it is assumed there is perfect observability of realized outcomes as well as covariates and that all variables are correctly measured. These assumptions allow one to focus on the problem of missing treatments.

The researcher learns the distribution of $P[y, x, d]$ (realized outcomes, covariates, and observability of realized treatments) and of $P[z|x, y, d = 1]$ (realized treatments given covariates, realized outcomes, and the treatment is observed). The researcher's problem is to learn the distribution of the response functions, $P[y(\cdot)|x]$, in order to infer the effect of a treatment. This leads Molinari to address two questions. First, what can be learned about $\Pr[y(t) = 1], t \in T$, i.e. the distribution of outcomes under a mandatory policy? Second, what can be learned about $\Pr[y(1) = 1] - \Pr[y(0) = 1]$, i.e. the classical treatment effect (ATE)? For my purposes, I am interested in the second of these two questions.

Decomposing the ATE using the law of total probability,[1] Molinari (citing Manski (1995) Chapter 2) identifies the sharp lower and upper bounds of the ATE

$$LB_{ATE} \leq \Pr[y(1) = 1] - \Pr[y(0) = 1] \leq UB_{ATE} \tag{1}$$

[1]The *Law of Total Probability* is used to break down the computation of a probability into distinct cases. Suppose we are interested in the probability of event E: $\Pr(E)$. Suppose also that the random experiment can evolve in two different ways; that is, two different cases X and $\overline{X}$ are possible. Suppose also that it is easy to find the probability of each case ($\Pr(X)$ and $\Pr(\overline{X})$) and it is easy to find the probability of the event in each case ($\Pr(E|X)$ and $\Pr(E|\overline{X})$).

where

$$LB_{ATE} = -P[d=0] - \left(\begin{array}{c} P[z=1|d=1] + P[y=1|d=1,\ z=0]P[z=0|d=1] \\ -P[y=1|d=1,\ z=1]P[z=1|d=1] \end{array} \right) P[d=1]$$

and

$$UB_{ATE} = P[d=0] - \left(\begin{array}{c} P[z=0|d=1] + P[y=1|d=1,\ z=1]P[z=1|d=1] \\ -P[y=1|d=1,\ z=0]P[z=0|d=1] \end{array} \right) P[d=1]$$

A key probability for computing these bounds is $\Pr[z=1|d=0]$, which is the probability of the treatment being received given that it is unobserved (i.e., the probability that the alliance negotiation witnessed an offer of issue linkage, given that the negotiation failed). This probability cannot be derived from the data. Molinari calls this probability p and in the absence of knowledge of p, the sharp bounds on $\Pr[y(1)=1|d=0] - \Pr[y(1)=1|d=0]$ are

$$-1 \leq \Pr[y(1)=1|d=0] - \Pr[y(0)=1|d=0] \leq 1 \tag{2}$$

which are not informative. Together with equation (2), the sharp bounds on the ATE in the absence of knowledge of p is

$$LB_{ATE} \leq \Pr[y(1)=1] - \Pr[y(0)=1] \leq UB_{ATE} \tag{3}$$

where

$$LB_{ATE} = -P[d=0] - \left(\begin{array}{c} P[z=1|d=1] \\ +P[y=1|d=1,\ z=0]P[z=0|d=1] \\ -P[y=1|d=1,\ z=1]P[z=1|d=1] \end{array} \right) P[d=1]$$

Then we have the following

$$\begin{array}{c} \Pr[y(1)=1] - \Pr[y(0)=1] \\ \Pr(E \cap X) + \Pr(E \cap \overline{X}) \\ \Pr(E \cap X) \cdot \frac{\Pr(X)}{\Pr(X)} + \Pr(E \cap \overline{X}) \cdot \frac{\Pr(\overline{X})}{\Pr(\overline{X})} \\ \Pr(E|X) \cdot \Pr(X) + \Pr(E|\overline{X}) \cdot \Pr(\overline{X}) \end{array}$$

where moving to the last line from the second to the last line requires using the definition of a conditional probability.

and

$$UB_{ATE} = P[d=0] - \begin{pmatrix} P[z=0|d=1] \\ +P[y=1|d=1,\ z=1]P[z=1|d=1] \\ -P[y=1|d=1,\ z=0]P[z=0|d=1] \end{pmatrix} P[d=1]$$

Applying this setup to my study, I let $y = 1$ if an alliance is formed ($y = 0$ otherwise), let $z = 1$ if economic linkage is offered ($z = 0$ otherwise) and let $d = 1$ if economic linkage (or lack thereof) is observed ($d = 0$ otherwise). I will assume that $d = 1$ (meaning we observe the treatment) only for successfully formed alliances and failures in which I identified a linkage offer. Therefore, $d = 0$ for those negotiation failures in which I did not identify an offer of a linkage provision.