# **Supplementary Figures**

## **Figure S1**

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**Figure S1.** The frequency distribution of coefficients *a* (top panel), *b* (middle panel) and *c* (bottom panel) of the asymptotic function (equation 5) for maize yield response to nitrogen, phosphorus and potassium in the database used in Tesfahunegn & Wortmann (2017).

Important note

Negative and 0 values of *a* and *b* are implausible because some yield is always obtained where no fertilizer nutrient has been applied. Acceptable values of *c* fall between 0.90 and 1.0. **When *c* < 0.90, the resultant EOR will be significantly biased downwards.**

## **Figure S2**

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**Figure S2.** Predictions (smooth lines) of yield using the asymptotic model and uncertainty around predictions (red lines) of maize yield response to N on farmers’ fields in Uganda in the 2010a season. Blue circles represent measured yields, while the smooth blue line represents predicted yield. The red lines represent the 95% confidence limits of predictions.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Sites | *a* | *b* | *c* | pR2 | AICc\* |
| Abongomola | 5.6 (3.8; 7.5) | 2.7 (-0.3; 5.7) | 0.95 (0.68; 1.22) | 0.993 | Can’t be estimated |
| Bukanga | 4.6 (2.6; 6.3) | 2.3 (-0.7; 5.3) | 0.59 (0.32; 0.86) | 0.979 | Can’t be estimated |
| Kwera | 5.5 (2.3; 8.6) | 3.2 (-2.0; 8.3) | 0.95 (0.56; 1.34) | 0.986 | Can’t be estimated |
| Kiziramfumbi | 3.8 (0.7; 7.0) | 2.1 (-3.1; 7.2) | 0.28 (-0.11; 0.67) | 0.999 | Can’t be estimated |

Values in red colour represent parameters estimated with large uncertainty. These estimates have inflated standard errors and 95% confidence limits that cover both negative and positive values.

\*AICc cannot be estimated because the four N rates do not leave enough degrees of freedom to do so.

# **Supplementary Tables**

Table S1. Examples of problematic EOR estimates for nitrogen (N) and phosphorus (P) for trials covering maize in the literature from SSA (summarized from Wortmann, C.S. & Sones K. (ed) (2016). *Fertilizer Use Optimization in Sub-Saharan Africa*. CAB International, Nairobi, Kenya).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Target region and/or agroecological zones | Estimated parameters | | | | EOR  (kg ha-1) | Existing recommended  rate (kg ha-1) |
| Nutrient | Country | Asymptote  (*a*) | Amplitude  (*b*) | Curvature  (*c*) |  |
| Nitrogen | Kenya | Rift Valley upper | 3.7 | 0.2 | 0.886 |  | 6 | 75 |
|  |  | Eastern Upper | 2.4 | 0.8 | 0.875 |  | 9 | 75 |
|  |  | Rift Valley upper | 7.5 | 2.6 | 0.903 |  | 16 | 75 |
|  |  | Coastal lowlands | 2.1 | 0.4 | 0.950 |  | 11 | 75 |
|  | Mali | Sahel zone | 1.3 | 0.7 | 0.951 |  | 31 | 84 |
|  | Tanzania | Lake zone | 2.3 | 0.8 | 0.947 |  | 16 | 50-80 |
|  |  | Lake zone | 2.1 | 1.0 | 0.960 |  | 21 | 50 |
|  |  | Lake zone | 4.5 | 1.4 | 0.966 |  | 27 | 80 |
|  |  | Central zone | 4.7 | 1.2 | 0.862 |  | 27 | NA |
|  | Uganda | Eastern Uganda | 2.6 | 0.8 | 0.930 |  | 28 | 60 |
| Phosphorus | Ethiopia | Hot-warm lowland | 2.8 | 0.6 | 0.897 |  | 4 | 20 |
|  | Ghana | Derived savanna trans. | 0.9 | 0.2 | 0.780 |  | 0 | 17 |
|  | Kenya | Eastern lower | 2.0 | 0.1 | 0.885 |  | 0 | NA |
|  |  | Rift Valley upper | 4.7 | 0.8 | 0.990 |  | 0 | NA |
|  | Mali | Sahel zone | 1.3 | 0.7 | 0.951 |  | 0 | 7 |
|  |  | North Sudan savanna | 2.9 | 0.3 | 0.928 |  | 3 | 7 |
|  |  | South Sudan savanna | 2.9 | 0.3 | 0.928 |  | 2 | 7 |
|  | Niger | Sahel AEZ | 2.9 | 0.03 | 0.938 |  | 0 | 0 |
|  | Nigeria | Derived savanna | 0.9 | 0.2 | 0.780 |  | 0 | 26 |
|  |  | Mid-altitude zone | 2.6 | 0.2 | 0.860 |  | 0 | 26 |
|  |  | Sudan savanna | 2.9 | 0.3 | 0.928 |  | 0 | 26 |
|  |  | Sahel savanna | 1.3 | 0.7 | 0.951 |  | 0 | 26 |
|  |  | South Guinea savanna | 3.2 | 0.3 | 0.880 |  | 5 | 26 |
|  | Rwanda | North-eastern AEZ | 4.7 | 0.4 | 0.899 |  | 1 | 20 |
|  | Tanzania | Lake zone | 2.0 | 0.1 | 0.858 |  | 0 | 8 |
|  |  | Central zone | 3.4 | 0.1 | 0.917 |  | 0 | NA |
|  |  | Southern zone | 3.7 | 0.3 | 0.958 |  | 3 | 10 |
|  |  | Southern highland | 3.8 | 0.5 | 0.600 |  | 5 | 20-40 |
|  | Uganda | Central region | 3.9 | 0.2 | 0.840 |  | 5 | 25 |
|  |  | Easter Uganda | 3.9 | 0.2 | 0.840 |  | 5 | 25 |
|  |  | Northern & western | 3.9 | 0.2 | 0.840 |  | 5 | 25 |
|  |  | S. Western highlands | 3.9 | 0.2 | 0.840 |  | 5 | 25 |
| Potassium | Ghana | Derived savanna | 3.8 | 0.04 | 0.550 |  | 0 | 50 |
|  |  |  | 2.6 | 0.4 | 0.855 |  | 0 | 33 |
|  | Malawi | Mid-elevation plateau | 4.1 | 0.1 | 0.900 |  | 2 | 6-8 |
|  |  | Lakeshore | 4.1 | 0.1 | 0.900 |  | 2 | 6-8 |
|  | Nigeria | Derived savanna | 3.8 | 0.04 | 0.550 |  | 2 | 62 |
|  | Tanzania | Southern highlands | 2.8 | 0.1 | 0.800 |  | 7 | NA |
|  | Zambia | Region II | 4.1 | 0.1 | 0.900 |  | 0 | 17 |

Values in red colour represent significantly biased parameter estimates resulting in a significant downward bias in EORs. Acceptable values of *c* fall between 0.90 and 1.0. **When *c* < 0.90, the resultant EOR will be significantly biased downwards.**

Table S2. Comparison of different models for maize yield response to nitrogen on contrasting sites where (1) yields are still increasing and (2) apparent maximum yields have been reached

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Country | Site | Model | *Ymax*  (t ha-1) | *Xmax*  (kg ha1) | R2 or  pR2 | Converged |
| Niger | 1. Bengou (Luvisols) | Quadratic | ND† | ND | 0.663 | NANA |
|  |  | Asymptotic | 1.2 | 130 | 0.353 | No |
|  |  | Mitscherlich | 1.0 | ND | 0.316 | No |
|  |  | Linear-plateau | 1.2 | 70 | 0.825 | Yes |
|  | 2. Maradi (Fluvisols) | Quadratic | 1.8 | 67 | 0.907 | NA |
|  |  | Asymptotic | 2.0 | 160 | 0.939 | Yes |
|  |  | Mitscherlich | 2.0 | 210 | 0.939 | Yes |
|  |  | Linear-plateau | 1.9 | 40 | 0.991 | Yes |
| Tanzania | 1. Selian (Chernozems) | Quadratic | ND† | ND | 0.667 | NA |
|  |  | Asymptotic | 6.3 | 200 | -0.718 | No |
|  |  | Mitscherlich | ND† | ND | None | No |
|  |  | Linear-plateau | 6.4 | 143 | 0.833 | No |
|  | 2. Ilonga (Fluvisols) | Quadratic | 5.5 | 105 | 0.987 | NA |
|  |  | Asymptotic | 5.7 | 200 | 0.978 | Yes |
|  |  | Mitscherlich | 5.7 | 200 | 0.987 | Yes |
|  |  | Linear-plateau | 5.5 | 73.9 | 0.981 | Yes |
| Uganda | 1. Abongomola (Soil: unknown) | Quadratic | 5.6 | 113 | 0.790 | NA |
|  |  | Asymptotic | 5.6 | 130 | 0.993 | Yes |
|  |  | Mitscherlich | 5.6 | 130 | 0.993 | Yes |
|  |  | Linear-plateau | 5.6 | 54 | 0.993 | Yes |
|  | 2. Bukanga (Soil: unknown) | Quadratic | 4.9 | 98 | 0.790 | NA |
|  |  | Asymptotic | 4.4 | 90 | 0.979 | Yes |
|  |  | Mitscherlich | 4.5 | 90 | 0.979 | Yes |
|  |  | Linear-plateau | 4.5 | 48 | 0.978 | Yes |
| Rwanda | 1. Burera (Regosols) | Quadratic | ND† | ND | 0.933 | NA |
|  |  | Asymptotic | 6.7 | 200 | -0.625 | Yes |
|  |  | Mitscherlich | ND† | ND† | 0.967 | No |
|  |  | Linear-plateau | 6.7 | 108 | 0.974 | Yes |
|  | 2. Bugesera (Ferralsols) | Quadratic | 5.5 | 148 | 0.927 | NA |
|  |  | Asymptotic | 6.7 | 400 | 0.957 | Yes |
|  |  | Mitscherlich | 6.7 | 450 | 0.957 | Yes |
|  |  | Linear-plateau | 5.3 | 90 | 0.996 | Yes |
| Malawi | 1. Bunda (Luvisols) | Quadratic | ND† | ND | 0.983 | NA |
|  |  | Asymptotic | 5.3 | 170 | -1.995 | No |
|  |  | Mitscherlich | 4.9 | ND | None | No |
|  |  | Linear-plateau | 5.5 | 141 | 0.942 | Yes |
|  | 2. Salima (Cambisols) | Quadratic | ND | ND | 0.997 | NA |
|  |  | Asymptotic | 6.1 | 190 | -0.844 | No |
|  |  | Mitscherlich | 4.0 | ND | 0.308 | No |
|  |  | Linear-plateau | 8.2 | 119 | 0.998 | Yes |
| Kenya | 1. Eldoret (Nitisols) | Quadratic | ND | ND | 0.968 | NA |
|  |  | Asymptotic | 8.0 | ND | -0.807 | Yes |
|  |  | Mitscherlich | 6.0 | ND | None | No |
|  |  | Linear-plateau | 8.1 | 130 | 0.967 | Yes |
|  | 2. Migori (Ferralsols) | Quadratic | 3.0 | 94 | 0.943 | NA |
|  |  | Asymptotic | 3.0 | 210 | 0.897 | Yes |
|  |  | Mitscherlich | 3.0 | 210 | 0.897 | Yes |
|  |  | Linear-plateau | 2.9 | 58.8 | 0.890 | Yes |

†ND = The maximum yield could not be determined, i.e., yields were still increasing but N application was limited to segment A; NA = Not applicable

Values in red represent problematic models

Estimates of the asymptotic or plateau yield (*Ymax*), the N rate (*Xmax*) that gives *Ymax* estimated using the different models. The R2 is given for linear models and pseudo R2 (pR2) for nonlinear models. The last column shows convergence of the algorithm (yes or no) for non-linear models.