ONLINE SUPPLEMENT TO "WHEN PARTIES MOVE TO THE MIDDLE: THE ROLE OF UNCERTAINTY"

JOHANNES LINDVALL, DAVID RUEDA, AND HAOYU ZHAI

Contents

Appendix A. Measuring Party and Voter Attributes

Formally, with L representing the lower boundary of the party-constituency interval containing the median voter, N the total vote share among all parties, CF the total vote share up to the median voter interval, and H and F the width (difference between (a) the midpoints of this and the two neighbouring classes, if present; (b) the party's own interval bounds, if not) and vote share of the median voter interval, then our measures for the position and clarity of the median voter, M and D, are defined as follows:

$$M = L + \left[\frac{\left(\frac{1}{2}N - CF\right)}{F} \right] \times H \qquad\qquad D = \frac{F}{H \times N}$$

Intuitively, the greater the vote share in the median voter class, the tighter the class is bounded, the greater the relative density of this class and hence the more clarity from the median voter's preference signal. Further, the median voter is always located around the party with the middling position on the political spectrum, in line with the (modified) spatial model of competition that we adopt.

For the median voter certainty measure specifically, we also conduct a separate analysis that compares our chosen relative frequency density (RFD) measure against three main alternatives: the aforementioned weighted standard deviation (WSD), weighted median absolute deviation (WMAD) and weighted interquartile range (WIQR), in capturing median certainty change under changing party position or vote share. We focus on two idealised party-voter distributions: a skewed distribution, where one main party occupies a substantial share of votes to either side of the political center, and a bimodal distribution, in which both main parties located off-center have comparably large proportions of votes. We also look at two party systems: a bipartisan system with two parties, one on the left and the other on the right, and a multiparty system with five parties, two of which mainstream, one centrist, and the remaining two on the far left and right. Finally, we consider two types of changes: one in which the main party[1] moves gradually away from the centre, with all parties keeping their vote shares unchanged; and the other where this party increases its vote share at the cost of others, whilst all remain fixed in their position. The expectation is that in the first case,

[1]We focus on the main left party as the moving agent here, in both position and vote based changes, for analytical tractability. Results using the main right instead would be naturally analogous.

Table 1. Simulation test setup, skewed party-constituency distribution.

	Two-party (L, R)	Five-party (FL, L, C, R, FR)
Position Change	• P_0 = {45, 55}, V_0 = {55, 45} • P_1 = {25, 55}, V_1 = {55, 45}	• P_0 = {25, 45, 50, 55, 75}, V_0 = {10, 40, 10, 30, 10} • P_1 = {25, 25, 50, 55, 75}, V_1 = {10, 40, 10, 30, 10}
Vote Change	• P_0 = {45, 55}, V_0 = {55, 45} • P_1 = {45, 55}, V_1 = {80, 20}	• P_0 = {25, 45, 50, 55, 75}, V_0 = {10, 40, 10, 30, 10} • P_1 = {25, 45, 50, 55, 75}, V_1 = {3.75, 65, 3.75, 23.75, 3.75}

Note: FL, L, C, R, FR = far-left, left, centre, right, far-right parties; P_0/V_0, P_1/V_1 = initial position/vote, final position/vote; total left-right scale range = 0-100, total vote = 100; positional change = -4/step, vote change = +5/step, total steps = 5.

Table 2. Simulation test setup, bimodal party-constituency distribution.

	Two-party (L, R)	Five-party (FL, L, C, R, FR)
Position Change	• P_0 = {45, 55}, V_0 = {50, 50} • P_1 = {25, 55}, V_1 = {50, 50}	• P_0 = {25, 45, 50, 55, 75}, V_0 = {10, 35, 10, 35, 10} • P_1 = {25, 25, 50, 55, 75}, V_1 = {10, 35, 10, 35, 10}
Vote Change	• P_0 = {45, 55}, V_0 = {50, 50} • P_1 = {45, 55}, V_1 = {75, 25}	• P_0 = {25, 45, 50, 55, 75}, V_0 = {10, 35, 10, 35, 10} • P_1 = {25, 45, 50, 55, 75}, V_1 = {3.75, 60, 3.75, 28.75, 3.75}

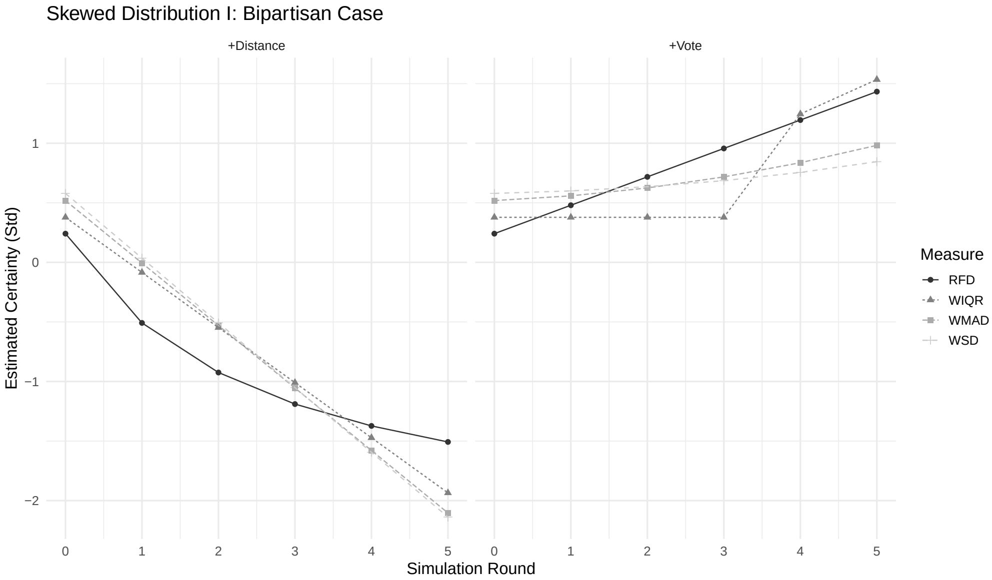
Note: FL, L, C, R, FR = far-left, left, centre, right, far-right parties; P_0/V_0, P_1/V_1 = initial position/vote, final position/vote; total left-right scale range = 0-100, total vote = 100; positional change = -4/step, vote change = +5/step, total steps = 5.

increasing the main party's distance from the center should reduce median voter certainty, while in the second increasing its vote share should increase it instead. The reason being that the former indicates weaker electoral attraction of the median region relative to the, increasingly distant, main party location, and the latter the opposite effect given growing vote-gain potentials around this region. A detailed list of key setup statistics is shown in tables 1 and 2 to save space here.

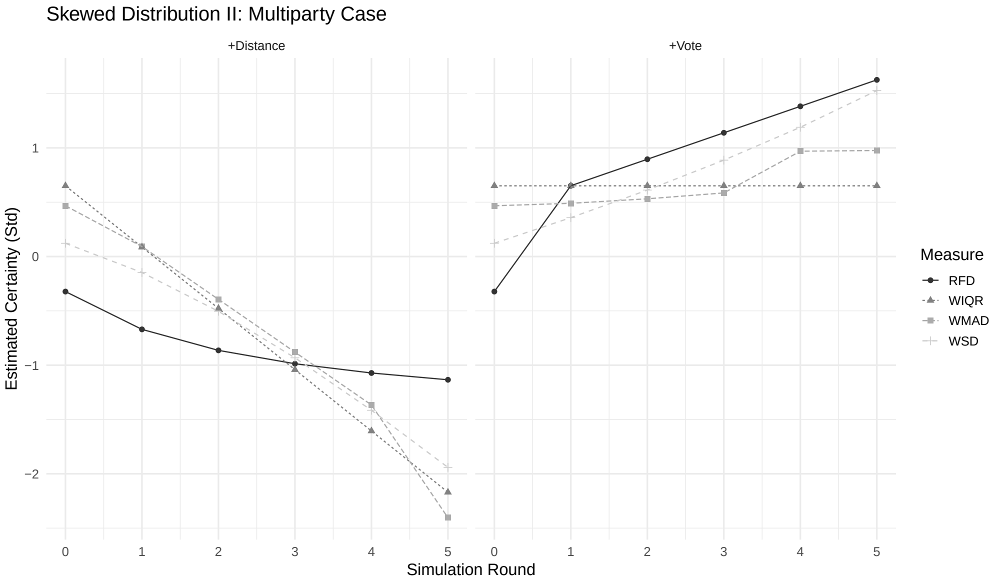
Figures 1 and 2 show results from our simulation tests, by party-voter distribution (skewed/bimodal), party system (bi/multi-party), type of change (increasing distance/vote), and step (five in total). We can see that, among the four selected measures, only our RFD measure and, to a lesser extent, the WSD measure behave in accordance to earlier expectation: both rise and fall according to the patterns just described, with a few minor deviations due to distributional specificities in some cases. The other two measures, although similar when looking at the distance from center logic, are either unresponsive or inconsistent when exploring the vote-share logic. This in addition to the earlier mentioned, better theoretical, empirical, and technical suitability of the RFD over the WSD, gives us greater confidence in the choice of this measure for our analysis in the paper.

Appendix B. Estimating Uncertainty: Some Examples

We start by exploring two scenarios with the same number of parties. Our baseline scenario has three parties, L (the main left party), C (the centrist party), and R (the main right party) with vote share equal to 40%, 20%, and 40% respectively (Ideological positions = {40, 50, 60}, estimated median position = 50, median certainty = 2). In our first illustration, we maintain the vote shares constant but change the ideological position of the parties.



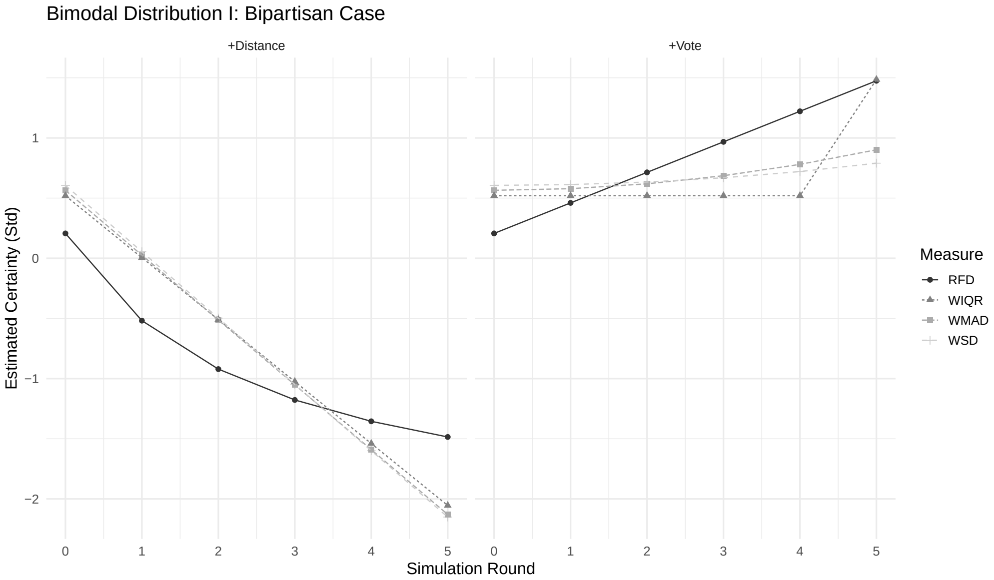
(A) Bipartisan system



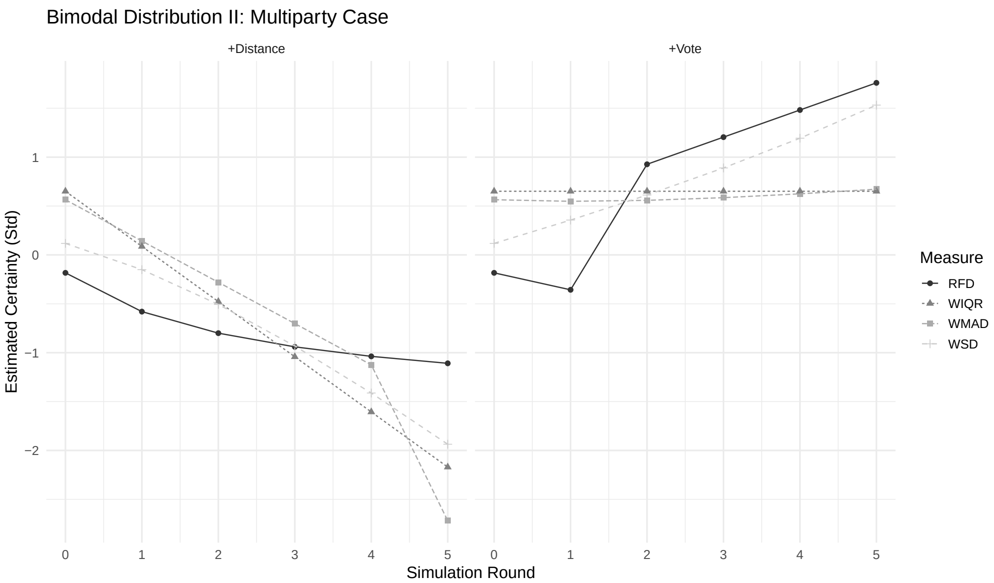
(B) Multiparty system

FIGURE 1. Simulation test results for median certainty measures, skewed party-constituency distribution. Standardised median certainty estimates by round ($0 \rightarrow 5$), scenario (vote/distance change), and party system (bi/multiparty). Top panel: bipartisan system; bottom panel: multi(5)-party system. Left panel: increasing distance from the centre; right panel: increasing vote share. Using the main left party as the moving agent.

In Figure 3 the top panels represent divergence (A) and convergence (B). In the divergent scenario, each main party (L/R) moves from the baseline position to the far left/right (20/80). The dashed outlines in the top panel show the baseline setup. In the convergent scenario, each main party moves near the middle (49/51). In the bottom panel, we show the simulated levels of median certainty (the dashed line is the benchmark median certainty estimate). As suggested by our argument, the estimates capture an exponential increase in certainty as



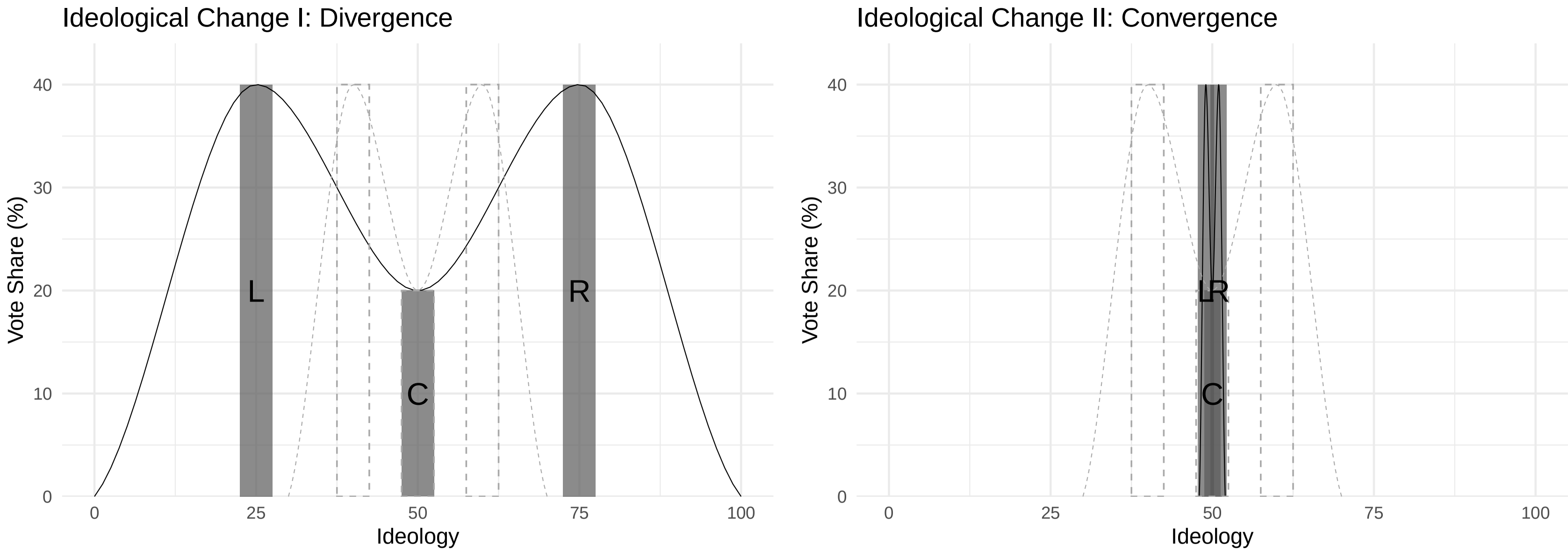
(A) Bipartisan system



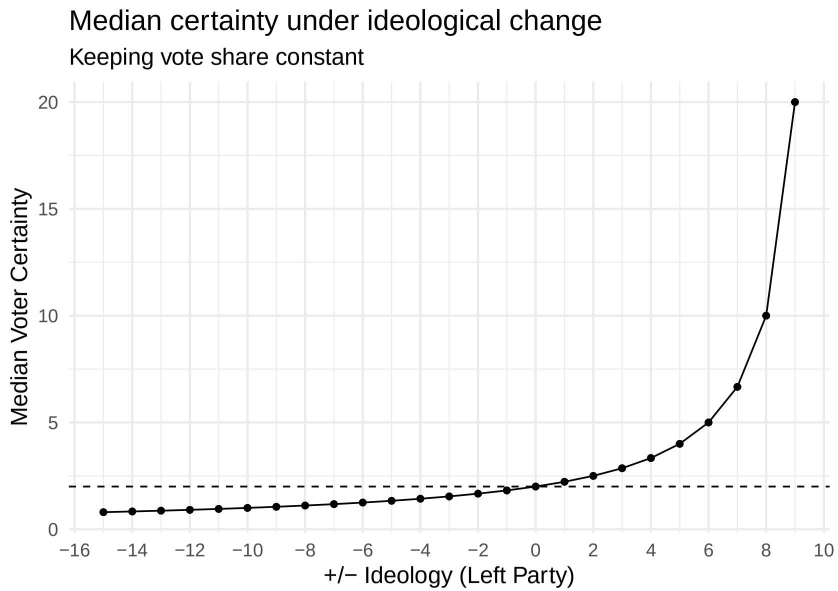
(B) Multiparty system

FIGURE 2. Simulation test results for median certainty measures, bimodal party-constituency distribution. Standardised median certainty estimates by round (0 → 5), scenario (vote/distance change), and party system (bi/multiparty). Top panel: bipartisan system; bottom panel: multi(5)-party system. Left panel: increasing distance from the centre; right panel: increasing vote share. Using the main left party as the moving agent.

the main parties converge. The reason for this increase in certainty is that, even though the party containing the median voter continues to have the same share of the vote, the party-constituency interval (bounded by the midpoints between the two neighboring parties) has become narrower. The estimation of the median voter position becomes therefore more precise as the two main parties converge.



(A) Setup: diverging and converging main party positions



(B) Result: estimated median voter certainty

FIGURE 3. Ideological change without vote share change. Simulated median voter certainty under diverging/converging main party positions.

In our second illustration, we return to our baseline scenario and this time we maintain the ideological position of the parties but change their vote shares. In Figure 4 the top panels represent bimodality (A) and unimodality (B). In the bimodal scenario, each main party increases its vote share from 40% to 47.5%. In the unimodal scenario, the center party increases its vote share from 20% to 40%, each main party decreases to 30% as a result. As expected, as unimodality increases, the certainty around the median voter estimate increases. The intuition here is again straightforward. Since we are keeping the ideological positions of the parties constant, the relative frequency density of the party-constituency interval containing the median voter increases as the vote share of the party increases.

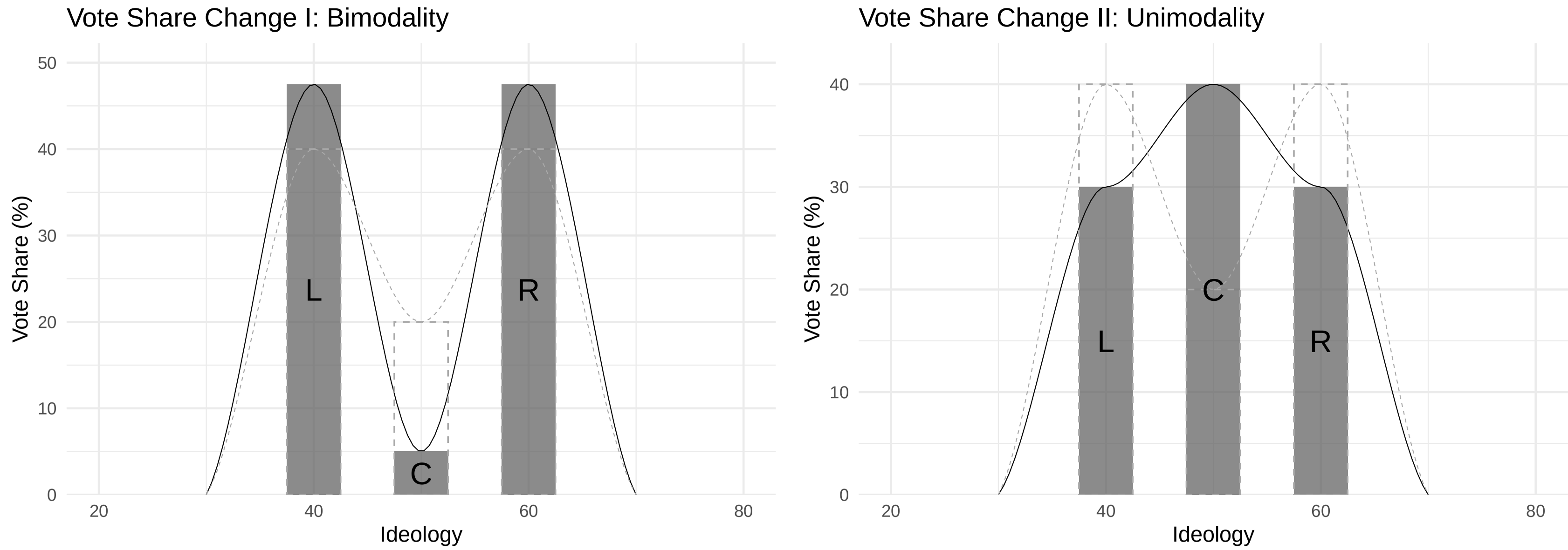
The third and final illustration of the implications of our model concerns the entry of new parties into electoral competition. In this case, however, whether the entry of a new party has any effect over the certainty of the median voter estimate depends on where, in the party system, the new party appears. Figure 5 illustrates two scenarios that have different implications for what parties can learn from the results of an election. In both scenario (A) (on the left-hand side of the figure) and scenario (B) (on the right-hand side), we start with two parties, L and R, with L being larger and therefore having the support of the median voter. In scenario (A), a new party, $L1$, enters to the left of the median-voter party, winning some of the vote that previously went to L. This means that the uncertainty around the estimate of the median voter's position decreases: the width of the interval that L represents decreases, so the density increases. When a new party, $R1$, enters to the right of R, however, there is no new information about the location of the median voter, and the level of uncertainty remains the same: since the median voter is known to have voted for L, the distribution of votes between R and $R1$ tells us nothing new about the median voter's location.[2]

Appendix C. Centrist Parties and Median Uncertainty

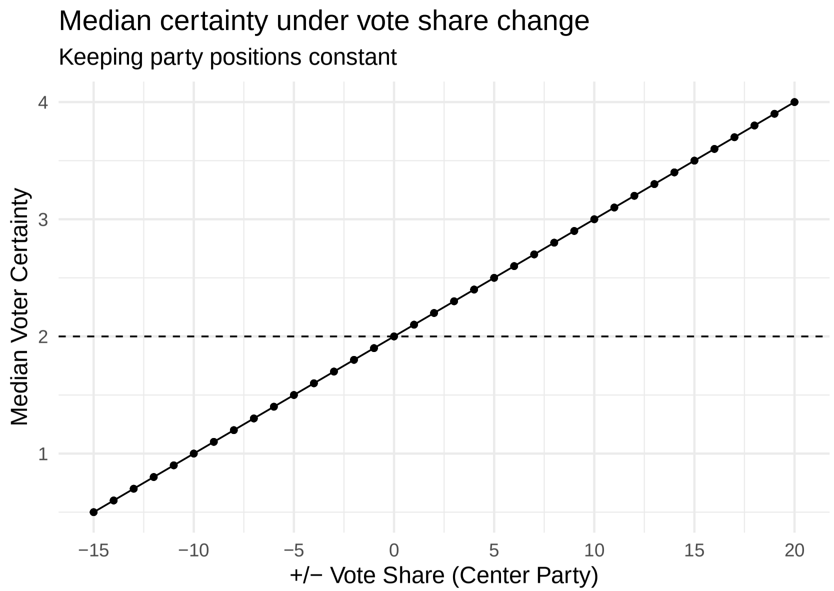
We check the correlation between median voter certainty and centrist party vote share in our sample. We define a centrist party as one whose left-right ideological position lies between the positions of the mainstream left and right parties in a national election. On average, 72.5% of elections in our sample have at least one centrist party. But the overall correlation coefficient between certainty and centrist vote share is a very low 2.9% ($t = 0.431$, $p = 0.667$).

Moreover, Table 3 presents country-specific estimates for the correlation between uncertainty and centrist party vote share. In most countries, this relationship is insignificant. In Italy and Norway is significant and positive, while in the Netherlands is significant and negative.

[2]Total voter mass and density distribution are held as constant. Dotted line in the background is the unobserved voter preference distribution (normalised to unity mass), rectangles with/without fills are observed current/previous party-constituency intervals, the vertical dashed line is inferred median voter position from party-constituency distribution.

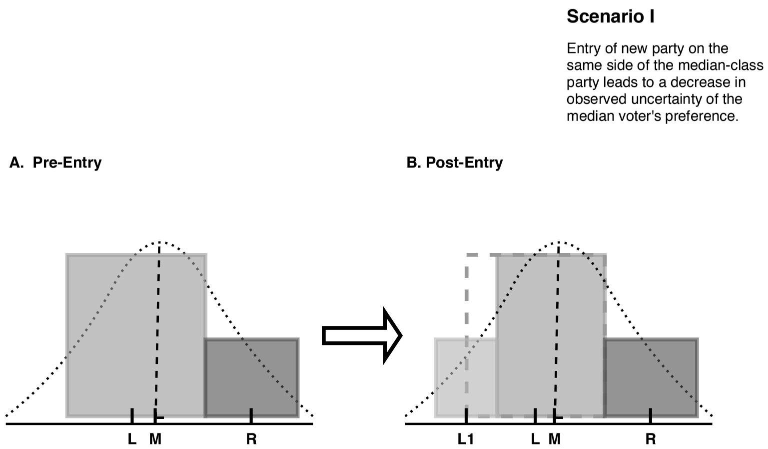


(A) Setup: decreasing and increasing main party vote shares

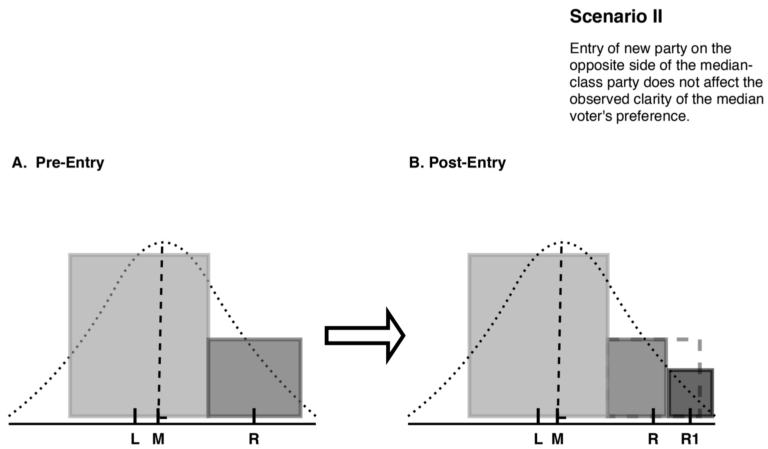


(B) Result: estimated median voter certainty

FIGURE 4. Vote share change without ideological change. Simulated median voter certainty under increasing/decreasing main party vote shares.



(A) Scenario I: Entry on the same side



(B) Scenario II: Entry on the opposite side

FIGURE 5. Illustration of the impact of party system change (new party entry) on estimated median voter certainty.

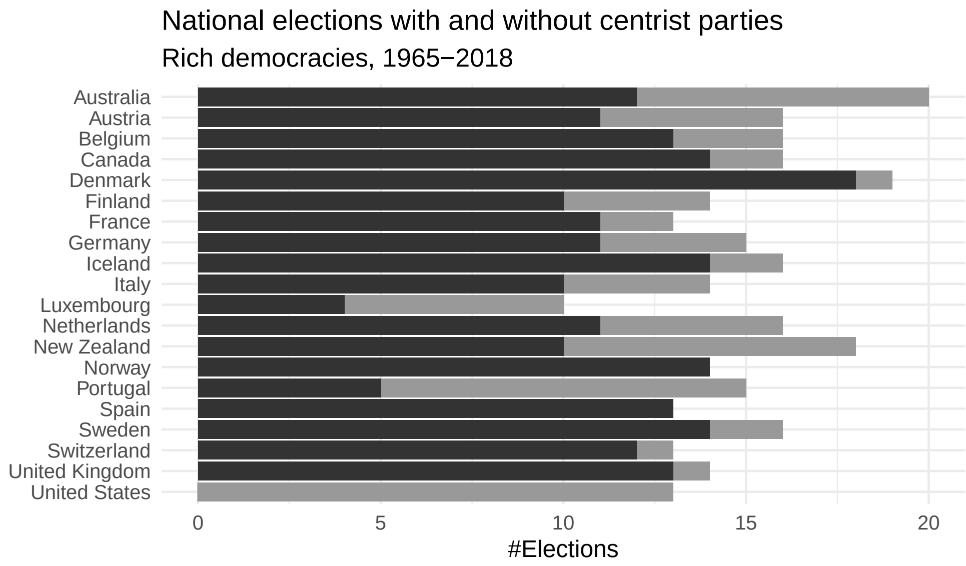


FIGURE 6. Tally of national elections with and without at least one centrist party. Grey bars show the total number of national elections in our core sample. Black bars show the number of national elections with one or more centrist parties. Centrist parties are defined as those parties whose left-right positions lie between those of the main left and right parties. Note that the United States does not have a centrist party in our sample.

TABLE 3. Correlation between median voter certainty and centrist party vote share by country

Country	Pearson's ρ	p-value	95% CI
Australia	-0.441	0.151	[-0.81, 0.178]
Austria	0.059	0.863	[-0.561, 0.636]
Belgium	-0.207	0.498	[-0.68, 0.388]
Canada	0.313	0.276	[-0.261, 0.723]
Denmark	-0.185	0.461	[-0.6, 0.308]
Finland	-0.033	0.927	[-0.649, 0.609]
France	-0.080	0.815	[-0.649, 0.546]
Germany	-0.195	0.565	[-0.712, 0.458]
Iceland	0.141	0.631	[-0.421, 0.625]
Italy	0.664	0.036	[0.059, 0.912]
Luxembourg	-0.699	0.301	[-0.993, 0.799]
Netherlands	-0.674	0.023	[-0.907, -0.125]
New Zealand	0.219	0.544	[-0.476, 0.746]
Norway	0.568	0.034	[0.053, 0.844]
Portugal	-0.660	0.225	[-0.975, 0.532]
Spain	-0.196	0.521	[-0.674, 0.398]
Sweden	-0.371	0.191	[-0.753, 0.198]
Switzerland	0.263	0.409	[-0.366, 0.727]
United Kingdom	-0.428	0.145	[-0.792, 0.161]

Table 4. Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Median Position	299	46.94	7.22	25.11	76.55
Median Certainty	299	40.33	14.64	4.44	100.00
Average Position	300	47.91	7.43	29.34	71.14
Distance	300	15.84	9.43	0.43	52.40
Left Competitor	301	0.62	0.49	0	1
Right Competitor	301	0.54	0.50	0	1
Polarization	288	4.23	3.67	0.01	19.62
Voter Turnout	297	78.98	14.21	35.00	95.80
Effective N. Parties	297	4.27	1.55	2.00	10.29
Govt. Partisanship	293	97.52	1.31	95.00	99.00
Union Density	258	43.32	20.53	7.69	93.69
Trade Openness	294	67.62	39.54	9.56	325.34
GDP Growth (Real)	287	2.81	1.55	−1.55	7.84
Unemployment Rate	294	5.53	3.74	0.00	23.78

Appendix D. Summary Statistics

For summary statistics, see Table 4.

Appendix E. Robustness Checks

As mentioned in the main text, we have conducted a comprehensive set of tests to check the robustness of our main analysis results. In this section we go through each test in more detail. In total, we have two types of tests: a technical one where we focus on issues of measurement and model specification, and a mechanism-based one where we instead look at possible confounds and competing explanations for our results. We progress in the same order (technical first, mechanism next) below. *Unless otherwise stated, all our checks are conducted on a larger, full sample of 34 OECD countries over the same period of 1965-2018.*[3] This ensures the robustness of our main findings to the scope of sample coverage.

First, we check our results with a different scaling method for party and voter positions, using the aforementioned log-odds ratio scale developed by Lowe et al. (2011). We re-estimate the four main party and median voter variables with this method and re-run our models using the new scales. Results are shown in tables 5 and 6, for the less easily interpreted interaction models, conditional marginal effect plots in figure 7 as well.[4] We detect little change in terms of effect size or significance relative to the main analysis results. Our results are therefore robust against scaling method choice.

[3]The 14 additional countries are (in alphabetical order): the Czech Republic, Estonia, Greece, Hungary, Ireland, Israel, Japan, Latvia, Mexico, Poland, Slovakia, Slovenia, South Korea, and Turkey.

[4]All models in these robustness tests are run with a corresponding set of control variables, models (2) and (3) in the main text. To save space, we do not report the estimates for the controls which are however available upon request.

Table 5. Robustness check I: using log-ratio scales. DV = Main party average position.

	FE		LDV	
	(2)	**(3)**	**(2)**	**(3)**
Median Position × Certainty	0.01**	0.01***	0.01***	0.01***
	(0.003)	(0.003)	(0.003)	(0.003)
Median Position	−0.29*	−0.31*	−0.29	−0.32*
	(0.14)	(0.14)	(0.16)	(0.16)
Median Certainty	−0.49**	−0.50**	−0.52**	−0.56***
	(0.15)	(0.15)	(0.16)	(0.17)
Average Position (Lag)			0.16*	0.15*
			(0.06)	(0.07)
Constant	155.22***	151.83***	103.38***	103.18***
	(24.82)	(25.11)	(22.25)	(23.11)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	315	315	314	314
R-squared	0.60	0.60	0.45	0.45
Adj. R-squared	0.44	0.44	0.32	0.32
Residual Std. Error	4.37	4.37	4.84	4.83
F Statistic	3.86***	3.78***	3.44***	3.38***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

Table 6. Robustness check I: using log-ratio scales. DV = Main party distance.

	FE		LDV	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty	−0.11***	−0.11***	−0.11***	−0.11***
	(0.03)	(0.03)	(0.03)	(0.03)
Distance (Lag)			0.11*	0.11*
			(0.05)	(0.05)
Constant	71.87*	72.98*	56.16	62.18
	(34.02)	(34.11)	(32.02)	(32.03)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	315	315	314	314
R-squared	0.53	0.53	0.40	0.45
Adj. R-squared	0.35	0.35	0.27	0.31
Residual Std. Error	6.85	6.83	7.28	7.04
F Statistic	2.97***	2.95***	2.96***	3.38***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

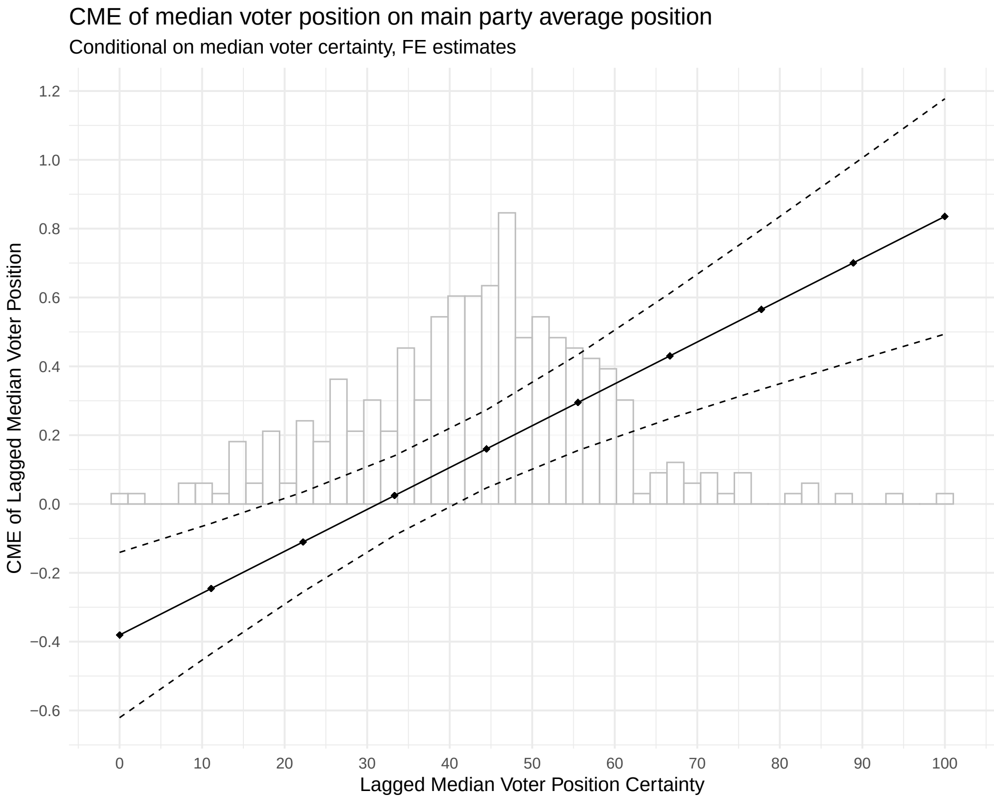


FIGURE 7. Robustness check I: using log-ratio scales. DV = Main party average position.

Next, we examine the use of linear interaction model for our first hypothesis. Hainmueller, Mummolo, and Xu (2018) have diagnosed two main problems in using such models: the linearity assumption of marginal effects, and the condition of common support in the moderator.[5] To check whether our models have met these conditions, we use a flexible kernel estimator developed by the same authors to re-estimate our interaction models. This is essentially a semiparametric smooth varying-coefficient model where one flexibly estimates a series of local effects with a kernel reweighting scheme.[6] We display the model results graphically in figure 8 and, as can be seen from the strongly linear, upward sloping effect curve in the middle, and the normally spreadout histogram showing the moderator's sample distribution at the bottom, both conditions appear to be largely met in our case.

We then look at the issue of sample composition, in particular, the sensitivity of our results to country coverage and/or time period. We do three tests for these checks: one in which we use a jackknife approach, where we drop one country at a time from our core sample of 20 advanced democracies and re-run our models; another where we split our core sample into two periods, one before and one after 1990 (1965-1990 and 1991-2018, respectively); and the third where we use a rolling window approach in which we apply a 50-year time window starting from the early postwar years (1945) to the full sample, and run the models on this subset of

[5]See Hainmueller, Mummolo, and Xu (2018), section "Multiplicative Interaction Models" for details.
[6]See the same paper, section "4.2 Kernel estimator" for details.

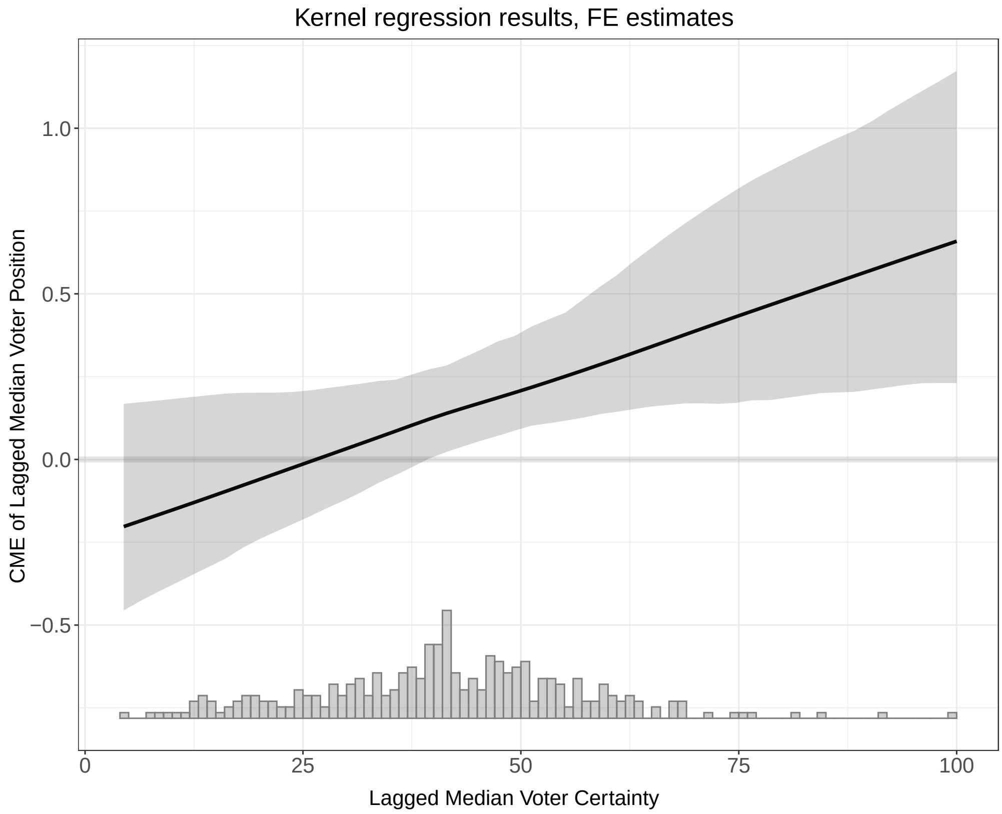


Figure 8. Robustness check II: kernel regression estimates of the conditional marginal effect of median voter position, depending on median voter certainty. Adaptive bandwidth selected via 10-fold least-squares cross-validation.

cases only. We report model results graphically in figures 11, 12, and 13, this time for both interaction (left panel) and linear (right panel) models. We see from all three figures that the key parameter estimates, the interaction effect of median voter position and certainty and the singular effect of median certainty respectively, are barely affected by changing sample coverage, whether spatially or temporally, in any substantial way. (The singular effect of median voter certainty on main party distance temporarily lost its significance among the exclusively pre-2000 samples in panel 13b but this is more likely to be a power issue than anything substantively meaningful, since fewer countries enter the sample in these earlier time intervals.)[7] We may thus also rule out any major influence of case selection on our analysis.

To address common issues in observational studies, we also adopt a statistical machine learning approach (generalized random forest, GRF, or causal forest) that is designed to provide efficient and consistent estimation of common and heterogeneous effects against selection bias and clustering (Athey, Tibshirani, and Wager 2019; Athey and Wager 2019). Briefly, the GRF uses a generalized version of the random forest algorithm with local weighting which maximizes detected effect heterogeneity through recursively splitting the observed data across

[7]For the two cases where the fixed-effect estimates also temporarily lose their statistical significance in figures 12a and 12b at the 5% level, for the party position and party distance models respectively, we note that this is likely due to the fact that the split samples have much shorter time lengths thus inflated dominance by the unit fixed effects. This means that these models are likely underestimating the true effects in these two cases.

Table 7. Robustness check III: Estimated average treatment effects of median voter preferences and main party platforms

	Left-Right Position	Left-Right Distance
Lagged Median Voter Position	0.29 (0.04)	
Lagged Median Voter Certainty (log)	−0.01 (0.25)	−0.21 (0.04)

Note: cluster-robust standard errors in parentheses.

the covariates, with clustered sampling and fitting if needed. The method has been shown to work well in identifying heterogeneous treatment effects in both simulated and real data (Athey and Wager 2019). Importantly for our purpose, it allows for a very flexible identification of the moderating effect of one covariate upon the effect of a main predictor on the outcome, which lends itself readily to our present task of testing the interaction between past median voter position and certainty in driving main party positions.

Table 7 displays our out-of-sample estimates of the key "average treatment effect" of our two median voter measures. The effect estimates are aggregated across the conditional average effects depending on other covariates' sample values. The cluster-robust standard error for each effect estimate is displayed in brackets. We note that in line with our main results, (1) the revealed position of the median voter in the earlier election does have a positive and precisely estimated effect on the average position of the main parties in the subsequent election, and that (2) the certainty around this revealed position also has a precisely estimated negative effect on the distance between the main parties afterwards. The overall effect of median voter certainty on main party position is not significant at the 5% level again revealing its dependent, conditioning role in this relationship.

Figure 9 presents estimates of the conditional average treatment effect (CATE) of the main "treatment" variable, lagged median voter position, at different (in fact, every observed) levels of the main "moderator" variable, lagged median voter certainty, on the outcome, main parties' average position. All covariate values are taken from their sample realizations. We use an 80/20% split of the full data for training and test samples respectively, to avoid overfitting and to generate more reliable estimates of the conditional effects. The forest is first fitted on the 80% training sample then deployed on the held-out 20% to make predictions for the latter. We incorporate the full set of covariates in the main model as well as the country-based clustering structure to best align the current and previous modelling exercises. Each diamond dot in the graph represents an estimated conditional effect at a specific level of the moderator, and the error bars stretching into either side of a dot give the 95% (asymptotic) confidence interval for each point estimate. A line of best linear fit is also added with its own confidence bands. The marginal distribution of the moderating variable in the test sample is given in short bars along the bottom.

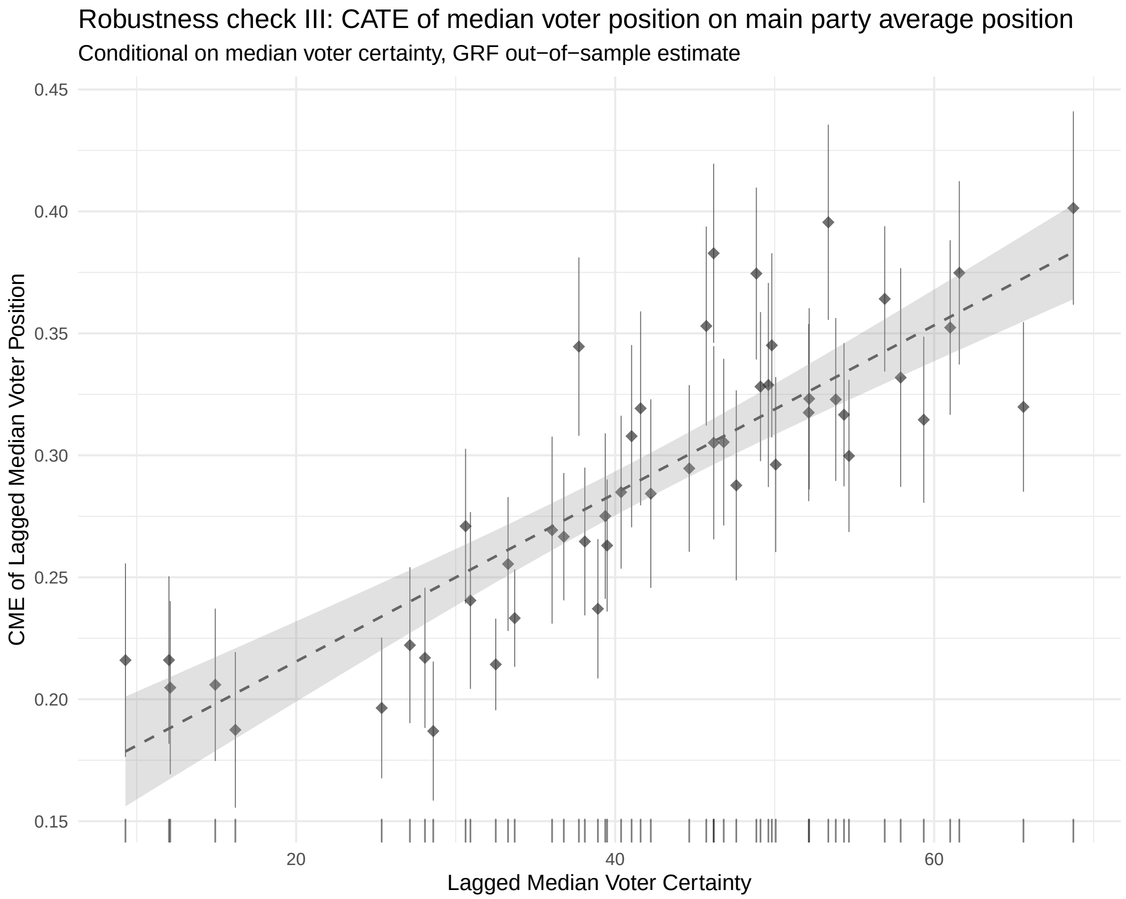


Figure 9. Generalized random forest estimates of the conditional average treatment effect (CATE) of lagged median voter position on main parties' average position, at all values of lagged median voter certainty, based on an 80/20 training-test data split. Fitted with 10,000 trees and country clustered splitting scheme.

Figure 9 makes clear that even when using a non-parametric and versatile approach that minimizes assumptions, the hypothesized interactive relationship between median voter's position and clarity in affecting the average position of the main parties remains very strong. The upwardly trending movement of the estimated CATEs, albeit in a more curvilinear manner, confirms the positive conditioning effect of median voter certainty. In addition, variable importance estimates from the same forest, displayed in figure 10, also single out median certainty as the most influential factor in inducing heterogeneity in the main effect of median voter position.

We now look at more theoretically pertinent, mechanism-related issues. Several objections may be launched against our empirical findings, based on insights from existing studies. One is the partly conflicting results from Adams et al. (2004), as mentioned in the main text, which find no evidence that parties adjust their ideological positions in response to past election results. This is despite that fact that parties do respond to shifts in public opinion (albeit, only when voters are moving away from a party's policy position), and that dispersion in the voter distribution is still correlated with that in the electoral positions of parties. We note

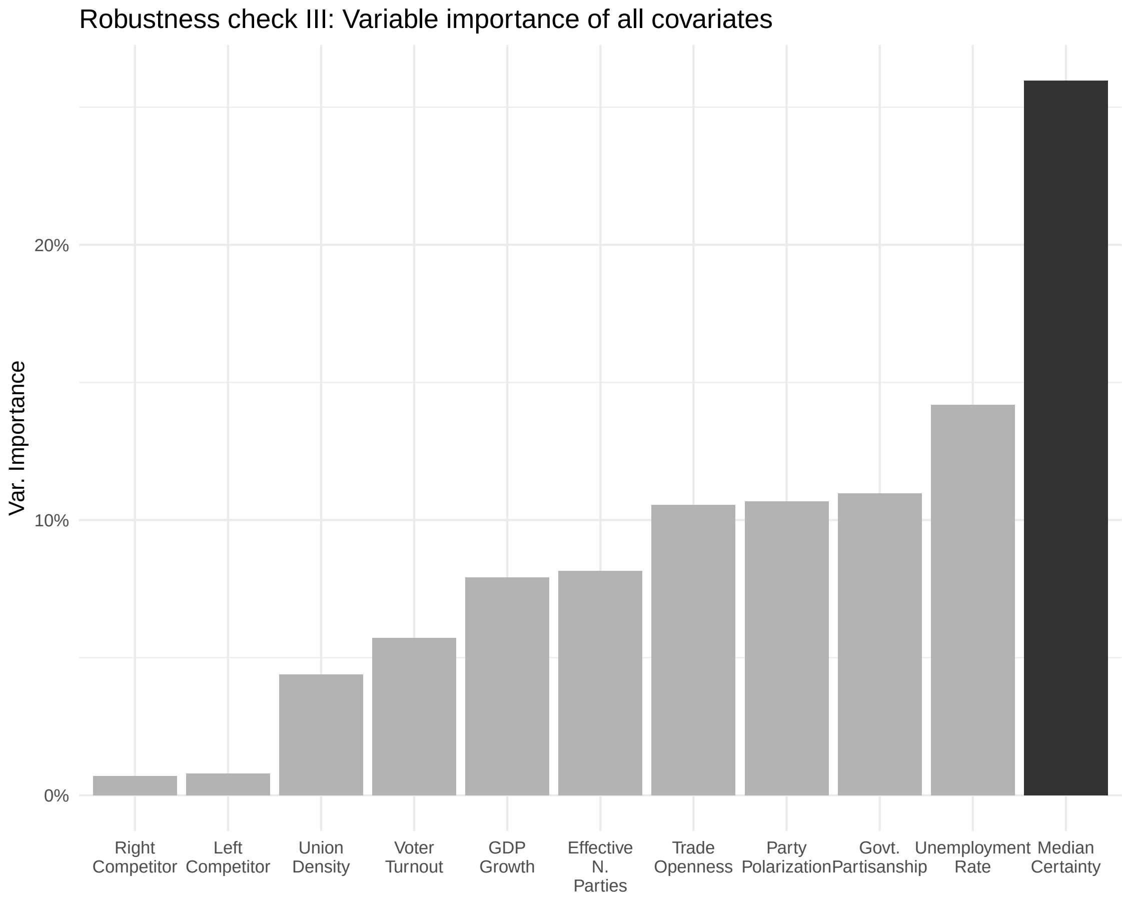
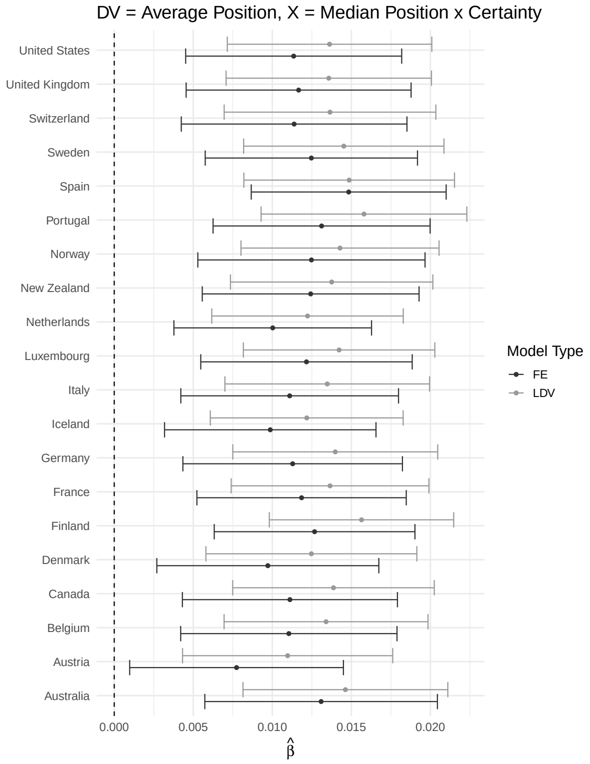


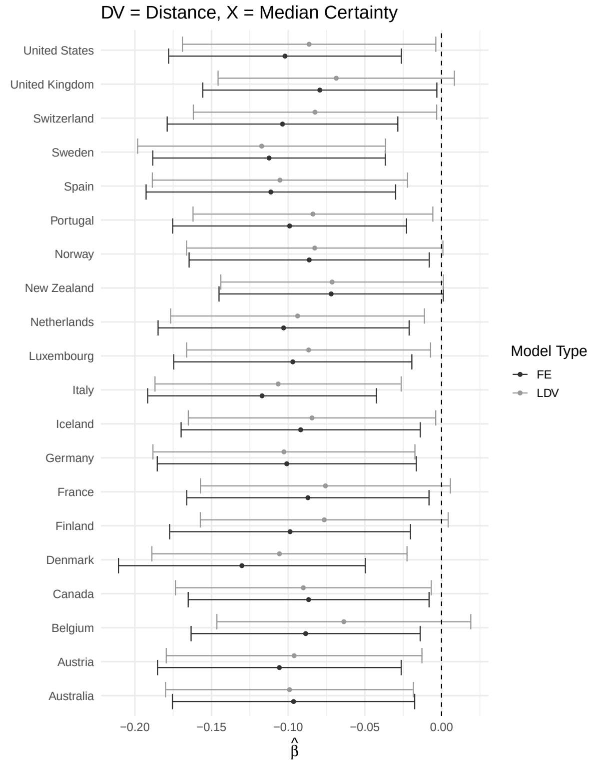
Figure 10. Variable importance of all covariates. Calculated as the weighted sum of the total number of instances where one covariate was split on at each depth (maximum depth = 4) in the forest. Based on 10,000 trees and country clustered splitting scheme.

that the authors take a different measurement approach to ours and that our measures differ in important ways as a result. Most importantly, whilst Adams et al. look at the general dispersion in the electorate (defined in terms of standard deviation in voter self-placement), we look instead at the specific uncertainty — local dispersion as it might be termed — around the median voter position more precisely (as relative frequency density of the median-containing interval). The two measures should be moderately correlated, since they both fundamentally concern voter uncertainty, yet differ significantly in terms of the relative weight assigned to the median area, computing method irrespective.

Indeed we find empirical support that our relative density measure is indeed negatively correlated with Adams et al.'s standard deviation measure, at a magnitude of about 20% under either additive or log-odds scales. Note that we use the CMP data for voter position estimation for the author's measure too, to ensure its comparability with our own KM measure, instead of their originally used Eurobarometer data with a much smaller coverage. To further check if our results are vulnerable to the omission of this partly correlated voter uncertainty measure, we enter this term as an additional control into our main models and, as results in tables 8 and 9 reveal, none of our main results are in any way affected if at all. This shows not only



(A) DV = Average Position, X = Median Position × Certainty.

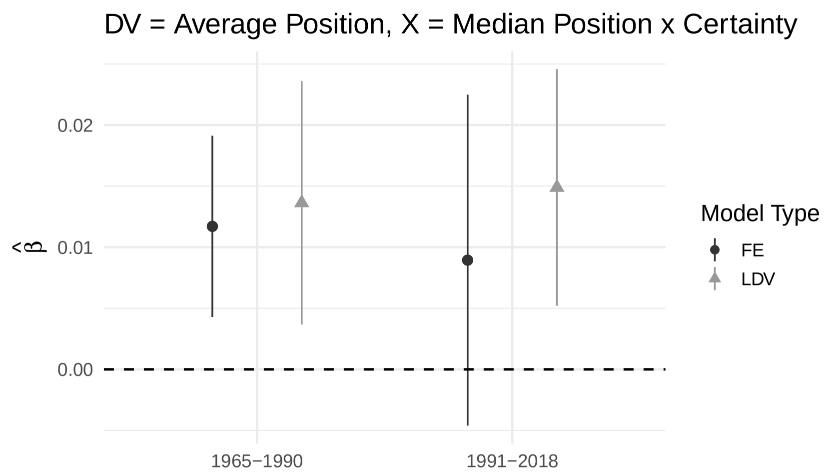


(B) DV = Distance, X = Median Certainty.

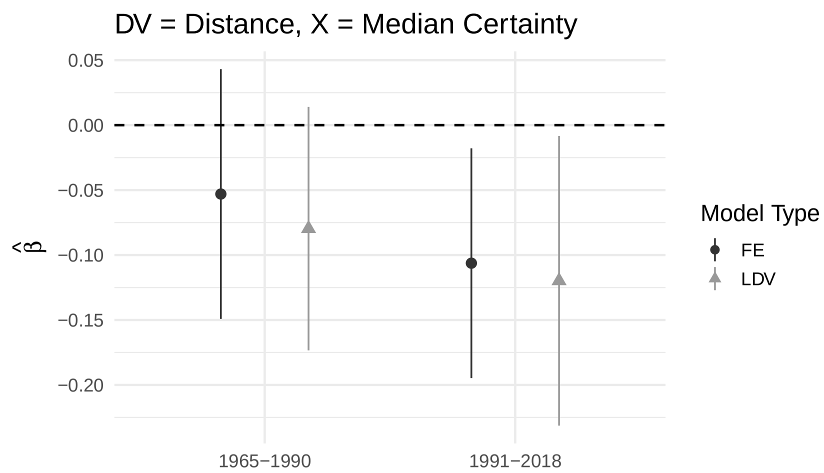
FIGURE 11. Robustness check IV: country jackknifing results. Jackknife regression estimates of key parameters with 95% confidence intervals. Dropping one country at a time. Left panel shows estimates for the interaction effect between median voter position and certainty on main party average position. Right panel shows estimates for the single effect of median voter certainty on main party distance. Dashed lines give the 0 effect boundary. CIs estimated with heteroscedasticity-consistent standard errors clustered by country.

the distinctiveness of our measure and results from Adams et al., and the latter's robustness an validity in this respect, but more importantly our theoretical novelty and contribution as well, in terms of understanding the genuine relationship between median voter and main party preferences. We can safely conclude that our results are not affected by the omission of some "general dispersion" effects in the electorate a lá Adams et al. (2004).

We then look at a second possible objection, namely that our uncertainty measure is simply taking up party/electoral system effects. In the sense that, since our measure is based on the median interval's relative density, merely by virtual of having less party-constituency intervals to begin with, bipartisan and majoritarian systems — the close connection being one of the Duvergerian (1963) type, as is well-known — will inevitably have higher median certainty than their multiparty and proportional (PR) counterparts, as the less parties to share all the votes with the more votes each party may keep. It could be that our median density measure is capturing more of a mechanical, party-system compositional effect than a substantive, voter-signal effect. Recall from Appendix B we have already touched upon the issue with discussion based on idealised party-voter scenarios; here we more formally and

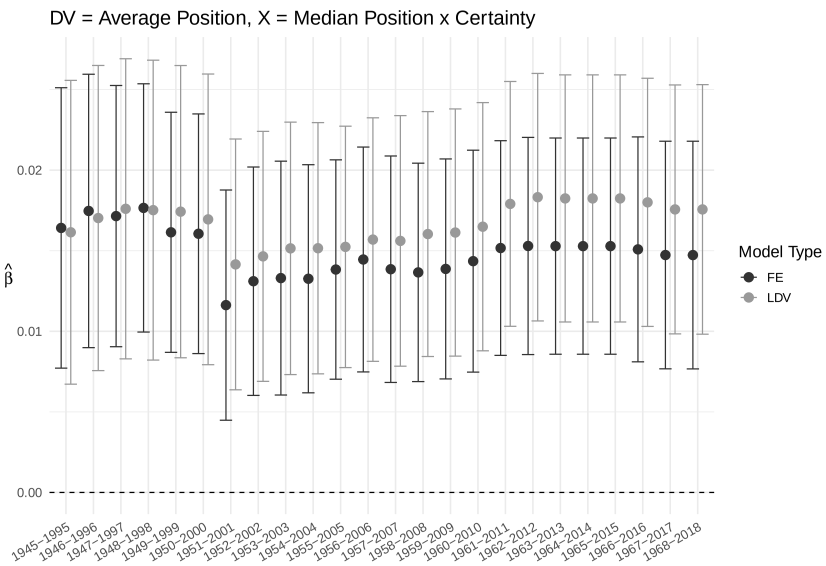


(A) DV = Average Position, X = Median Position × Certainty.

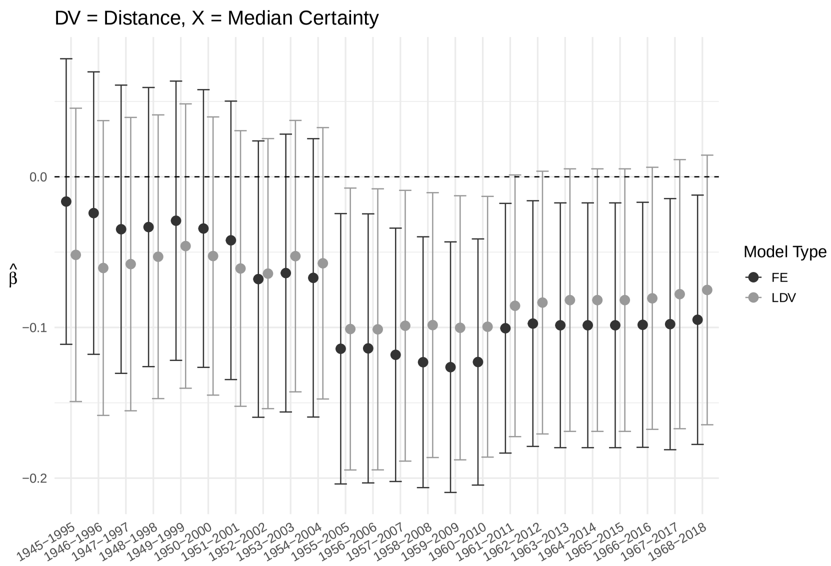


(B) DV = Distance, X = Median Certainty.

FIGURE 12. Robustness check V: split-sample results. Period-specific estimates of key parameters with 95% confidence intervals. Core sample split at the 1990 threshold (1965-1990 and 1991-2018). Left panel shows estimates for the interaction effect between median voter position and certainty on main party average position. Right panel shows estimates for the single effect of median voter certainty on main party distance. Dashed lines gives the 0 effect boundary. CIs estimated with heteroscedasticity-consistent standard errors clustered by country.



(A) DV = Average Position, X = Median Position × Certainty.



(B) DV = Distance, X = Median Certainty.

FIGURE 13. Robustness check VI: rolling window results. Rolling window regression of key parameters with 95% confidence intervals. Using a 50-year sliding window on the complete subsample (country cases with consistent observations during 1940s-2010s, N=385). Left panel shows estimates for the interaction effect between median voter position and certainty on main party average position. Right panel shows estimates for the single effect of median voter certainty on main party distance. Dashed lines gives the 0 effect boundary. CIs estimated with heteroscedasticity-consistent standard errors clustered by country.

Table 8. Robustness check VII: adding Adam et al.'s (2004) voter dispersion measure as an additional control. DV = Main party average position.

	FE		LDV	
	(2)	**(3)**	**(2)**	**(3)**
Median Position × Certainty	0.01***	0.01**	0.01***	0.01***
	(0.003)	(0.003)	(0.003)	(0.003)
Median Position	−0.29*	−0.23	−0.46***	−0.43**
	(0.13)	(0.13)	(0.14)	(0.14)
Median Certainty	−0.60***	−0.46**	−0.63***	−0.63***
	(0.14)	(0.14)	(0.14)	(0.14)
Average Position (Lag)			0.40***	0.30***
			(0.08)	(0.08)
Voter Dispersion	0.06*	0.06*	0.08*	0.06*
	(0.03)	(0.03)	(0.03)	(0.03)
Constant	185.05***	165.43***	103.80***	102.77***
	(27.77)	(27.53)	(26.14)	(27.34)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R-squared	0.60	0.66	0.46	0.50
Adj. R-squared	0.46	0.52	0.34	0.38
Residual Std. Error	5.48	5.15	6.03	5.88
F Statistic	4.22***	4.78***	3.94***	4.01***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

systematically address it, with empirical data from our CMP sample. As a first look, we compare the observed party system configuration, measured as the total number of parties, in each election in our sample against the corresponding median voter certainty estimate. We see little to none correlation between the two: indeed the linear correlation coefficient has a magnitude of 5%, which is also below the conventional level of significance ($p = 0.20$). It is therefore doubtful that the median certainty measure is in any way directly affected by party system attributes.

We then tackle the issue more formally, by controlling for the additive and multiplicative terms (with our key median voter variables) of party/electoral system in all our main models. For completeness, we use two measures of the system effects: a categorical one where countries are sorted into one of the three bins, majoritarian (base category), modified PR, and full PR; and a continuous one where we directly enter the effective number of parties into the models.[8] Results are displayed in tables 10-11 and 12-13, and as the table entries show, neither

[8]Recall from the main text that, this effective-number-of-parties measure is also a covariate in the main models (variable "Effective N. Parties"). This in itself reveals the unlikely confounding effect of the measure on our main results, since it has already been there when we first observed the latter with an acceptable level of statistical significance.

Table 9. Robustness check VII: adding Adam et al.'s (2004) voter dispersion measure as an additional control. DV = Main party distance.

	FE		**LDV**	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty	-0.23^{***}	-0.16^{***}	-0.08^{**}	-0.06^{*}
	(0.03)	(0.03)	(0.03)	(0.03)
Distance (Lag)			0.66^{***}	0.59^{***}
			(0.06)	(0.05)
Voter Dispersion	0.39^{***}	0.34^{***}	0.59^{***}	0.55^{***}
	(0.04)	(0.03)	(0.03)	(0.03)
Constant	45.05	20.31	−15.80	−0.76
	(35.43)	(34.66)	(21.64)	(22.55)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R-squared	0.58	0.68	0.68	0.75
Adj. R-squared	0.43	0.55	0.62	0.69
Residual Std. Error	7.31	6.51	5.98	5.45
F Statistic	3.92^{***}	5.35^{***}	10.40^{***}	12.21^{***}

Notes: $^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

measure seems to have affected our key results upon their inclusion. In particular, we find little evidence for its moderating effect on either the joint effect of median voter position and certainty on main party average position, or the singular effect of median certainty on main party distance. This gives us greater evidence in refuting the notion of a hidden mechanical effect of party/electoral system composition on our main relationships of interest.

Last but not least, we examine one final issue worth checking: the potential effect of inter-electoral time lapse on our main relationships under study. Simply stated, since our theory rests upon the assumption of main party detection, processing, and reaction to median voter signals in the previous election, it might be the case that the time spent between elections play a key role in amplifying/attenuating such relationship. The longer the time lapse, the older and weaker the signal received, and the less responsive the main parties may be to such signal. Conversely, the shorter the time lapse, the fresher and stronger the signal and the better the parties react to it. To check if this is the case in practice, and whether we have wrongfully omitted it from our main analysis, we enter an inter-election time term (measured as months between two consecutive elections in each country) into our models.

Tables 14 and 15 present new model results and, again for easier visual inspection, the key interaction effect estimates are graphically displayed in figure 14. We interact the time lapse term with our key median voter measures to directly examine its suspected conditioning effect. Reassuringly, we do not detect any significant effect from this variable, which gives

Table 10. Robustness check VIII (1): checking electoral system effect, using categorical measure (majoritarian, modified PR, and full PR). DV = Main party average position.

	FE		LDV	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Median Position × Modified PR	−0.01	−0.01	−0.02	−0.02
	(0.01)	(0.01)	(0.02)	(0.02)
Median Certainty × Median Position × Full PR	−0.002	−0.003	−0.01	−0.01
	(0.01)	(0.01)	(0.01)	(0.01)
Median Certainty × Median Position	0.01	0.01	0.02	0.02
	(0.01)	(0.01)	(0.01)	(0.01)
Median Position × Modified PR	0.58	0.27	1.30	1.24
	(0.64)	(0.66)	(0.72)	(0.74)
Median Position × Full PR	0.18	0.18	0.51	0.49
	(0.54)	(0.55)	(0.59)	(0.60)
Median Certainty × Modified PR	0.58	0.29	1.08	1.02
	(0.67)	(0.69)	(0.75)	(0.78)
Median Certainty × Full PR	0.11	0.15	0.41	0.35
	(0.60)	(0.61)	(0.66)	(0.69)
Median Position	−0.50	−0.43	−0.97	−0.95
	(0.52)	(0.53)	(0.57)	(0.59)
Median Certainty	−0.75	−0.62	−1.05	−1.02
	(0.56)	(0.58)	(0.63)	(0.67)
Average Position (Lag)			0.36***	0.28**
			(0.08)	(0.09)
Modified PR	−22.22	−8.14	−64.55	−60.05
	(32.31)	(32.34)	(35.51)	(36.30)
Full PR	−10.49	−5.66	−26.69	−23.94
	(27.17)	(27.76)	(28.82)	(29.67)
Constant	185.31***	167.31***	142.13***	129.24***
	(37.40)	(36.89)	(37.56)	(38.88)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R^2	0.60	0.66	0.47	0.50
Adjusted R^2	0.45	0.50	0.33	0.36
Residual Std. Error	5.54	5.26	6.08	5.95
F Statistic	3.84***	4.24***	3.48***	3.53***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. Base category = majoritarian system. Electoral system measure taken from CPDS (item *prop*). All models are run with an appropriate set of control variables (estimates available from the authors).

us additional confidence in our main results' reliability. It does seem unlikely that the main relationships of interest are in any way dependent on the temporal gap between elections.

References

Adams, James, Michael Clark, Lawrence Ezrow, and Garrett Glasgow. 2004. "Understanding change and stability in party ideologies: Do parties respond to public opinion or to past election results?" *British Journal of Political Science* 34 (04):589–610.

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Table 11. Robustness check VIII (1): checking electoral system effect, using categorical measure (majoritarian, modified PR, and full PR). DV = Main party distance.

	FE		**LDV**	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Modified PR	0.02	0.03	0.02	0.07
	(0.11)	(0.10)	(0.13)	(0.11)
Median Certainty × Full PR	0.11	0.15	0.05	0.10
	(0.08)	(0.08)	(0.10)	(0.09)
Median Certainty	−0.24**	−0.21**	−0.15	−0.17
	(0.08)	(0.07)	(0.10)	(0.09)
Distance (Lag)			0.27***	0.21***
			(0.06)	(0.05)
Modified PR	32.14***	34.85***	−1.26	0.23
	(9.77)	(9.24)	(6.27)	(5.49)
Full PR	9.80	9.20	−4.05	−2.01
	(7.10)	(6.21)	(5.19)	(4.56)
Constant	1.67	−19.49	22.90	31.84
	(38.23)	(35.73)	(36.72)	(35.34)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R^2	0.43	0.58	0.33	0.48
Adjusted R^2	0.22	0.41	0.18	0.35
Residual Std. Error	8.52	7.48	8.75	7.83
F Statistic	2.07***	3.36***	2.23***	3.65***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. Base category = majoritarian system. Electoral system measure taken from CPDS (item *prop*). All models are run with an appropriate set of control variables (estimates available from the authors).

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Table 12. Robustness check VIII (2): checking electoral system effect, using continuous measure (effective number of parties). DV = Main party average position.

	FE		**LDV**	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Median Position × Effective N. Parties	0.0004	0.001	−0.001	−0.001
	(0.002)	(0.002)	(0.002)	(0.002)
Median Certainty × Median Position	0.01	0.01	0.02*	0.02*
	(0.01)	(0.01)	(0.01)	(0.01)
Median Position × Effective N. Parties	−0.02	−0.04	0.06	0.02
	(0.09)	(0.09)	(0.09)	(0.09)
Median Certainty × Effective N. Parties	−0.004	−0.01	0.09	0.04
	(0.09)	(0.10)	(0.10)	(0.10)
Median Position	−0.21	−0.07	−0.67	−0.48
	(0.34)	(0.39)	(0.36)	(0.36)
Median Certainty	−0.55	−0.39	−0.96*	−0.77*
	(0.35)	(0.39)	(0.37)	(0.37)
Effective N. Parties	0.37	1.58	−4.09	−1.72
	(4.21)	(4.63)	(4.51)	(4.56)
Average Position (Lag)			0.37***	0.29***
			(0.08)	(0.08)
Constant	180.84***	158.66***	119.28***	110.74***
	(31.30)	(31.24)	(30.91)	(31.20)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R^2	0.60	0.66	0.46	0.50
Adjusted R^2	0.45	0.51	0.33	0.36
Residual Std. Error	5.54	5.21	6.09	5.94
F Statistic	3.96***	4.53***	3.64***	3.75***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

Table 13. Robustness check VIII (2): checking electoral system effect, using continuous measure (effective number of parties). DV = Main party distance.

	FE		**LDV**	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Effective N. Parties	−0.01	0.01	−0.02	-0.04^{*}
	(0.02)	(0.02)	(0.02)	(0.02)
Median Certainty	−0.14	−0.17	−0.02	0.06
	(0.09)	(0.09)	(0.09)	(0.08)
Effective N. Parties	2.37	1.10	1.75	3.51^{***}
	(1.27)	(1.19)	(1.07)	(0.97)
Distance (Lag)			0.28^{***}	0.21^{***}
			(0.06)	(0.05)
Constant	38.30	17.22	6.51	23.34
	(38.33)	(38.14)	(34.48)	(33.37)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R^2	0.42	0.55	0.34	0.48
Adjusted R^2	0.21	0.38	0.20	0.36
Residual Std. Error	8.58	7.67	8.67	7.78
F Statistic	2.03^{***}	3.15^{***}	2.40^{***}	3.88^{***}

Notes: $^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

Table 14. Robustness check IX: testing interaction effects between the time lapsed between two consecutive elections and previous median voter attributes. DV = Main party average position.

	FE		LDV	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Median Position × Inter-Election Time	−0.0001	−0.0001	−0.0003	−0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Median Certainty × Median Position	0.01	0.01	0.02**	0.02
	(0.01)	(0.01)	(0.01)	(0.01)
Median Position × Inter-Election Time	−0.003	−0.001	0.01	0.002
	(0.01)	(0.01)	(0.01)	(0.01)
Median Certainty × Inter-Election Time	0.01	0.01	0.02	0.01
	(0.01)	(0.01)	(0.01)	(0.01)
Median Position	−0.17	−0.15	−0.77**	−0.45
	(0.31)	(0.32)	(0.29)	(0.30)
Median Certainty	−0.79*	−0.64	−1.26**	−0.88*
	(0.37)	(0.36)	(0.42)	(0.42)
Inter-Election Time	−0.03	−0.17	−0.61	−0.34
	(0.42)	(0.43)	(0.40)	(0.41)
Average Position (Lag)			0.39***	0.28***
			(0.08)	(0.08)
Constant	175.88***	157.34***	124.97***	115.50***
	(30.96)	(30.50)	(29.93)	(30.55)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R-squared	0.61	0.68	0.47	0.52
Adj. R-squared	0.47	0.54	0.35	0.39
Residual Std. Error	5.42	5.08	6.01	5.80
F Statistic	4.24***	4.87***	3.85***	4.08***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

TABLE 15. Robustness check IX: testing interaction effects between the time lapsed between two consecutive elections and previous median voter attributes. DV = Main party distance.

	FE		**LDV**	
	(2)	**(3)**	**(2)**	**(3)**
Median Certainty × Inter-Election Time	−0.0005	0.001	−0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.003)
Median Certainty	−0.14	−0.14	−0.06	−0.12
	(0.14)	(0.13)	(0.14)	(0.12)
Inter-Election Time	−0.01	−0.01	0.02	−0.04
	(0.15)	(0.12)	(0.14)	(0.11)
Distance (Lag)			0.28***	0.21***
			(0.06)	(0.05)
Constant	31.96	16.27	4.95	33.26
	(39.73)	(38.17)	(35.17)	(33.86)
Country FE	Yes	Yes	No	No
Year FE	Yes	Yes	Yes	Yes
N	328	315	326	314
R-squared	0.39	0.55	0.33	0.48
Adj. R-squared	0.18	0.37	0.18	0.35
Residual Std. Error	8.74	7.69	8.74	7.84
F Statistic	1.86***	3.09***	2.29***	3.71***

Notes: ***p < .001; **p < .01; *p < .05; Heteroscedasticity-consistent standard errors clustered by country in brackets. All models are run with an appropriate set of control variables (estimates available from the authors).

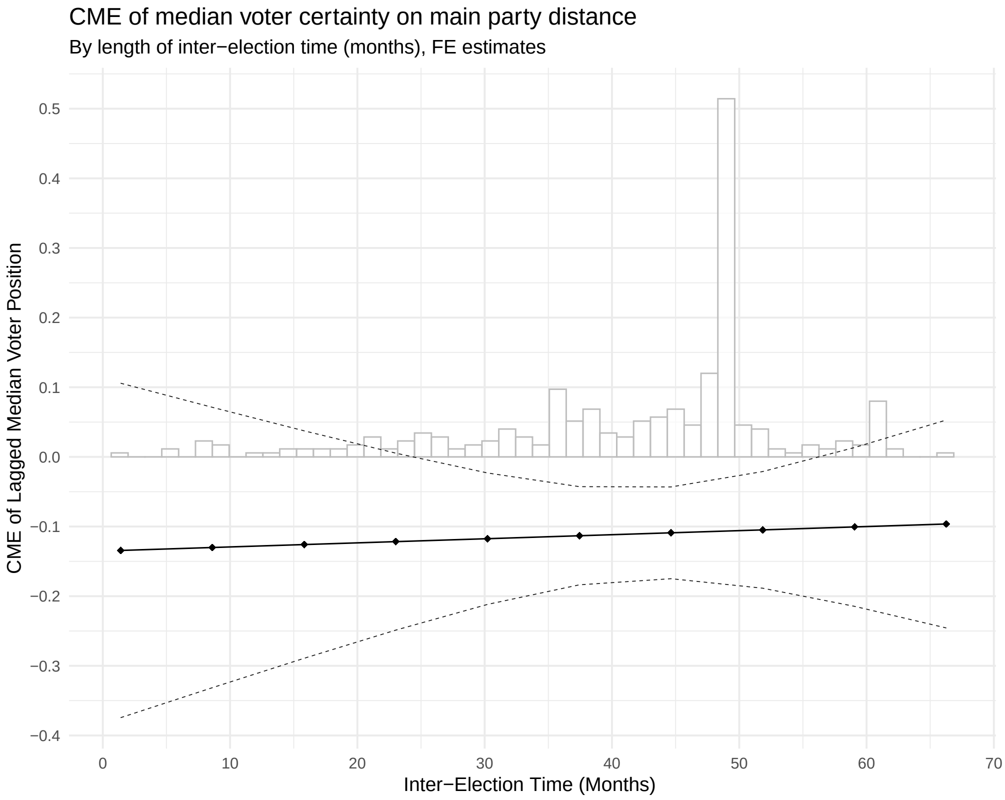

FIGURE 14. Robustness check IX: conditioning effect of inter-election time length on the relationship between main party average position and lagged median voter certainty. Time measured in months.