Feudalism, Collaboration and Path Dependence in England's Political Development

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November 15 2017

A Supporting Information

A.1 Comparative Statics

So far I have focused on how the size of the elite determines whether a society faces a contracting path in which the elite size shrinks over time, or an expanding path in which the elite can expand as a response to baronial revolts. The size of the elite is the key state variable and the one that generates the feedback that gives rise to path dependence, with exogenous changes in the size of the elite potentially causing a society to switch between paths. But societies can switch between paths for other reasons; in particular, the thresholds identified in proposition 1 can shift as the parameters of the model change, and in doing so might cause a shift in the path followed by a society. Furthermore, differences in these parameters can help explain

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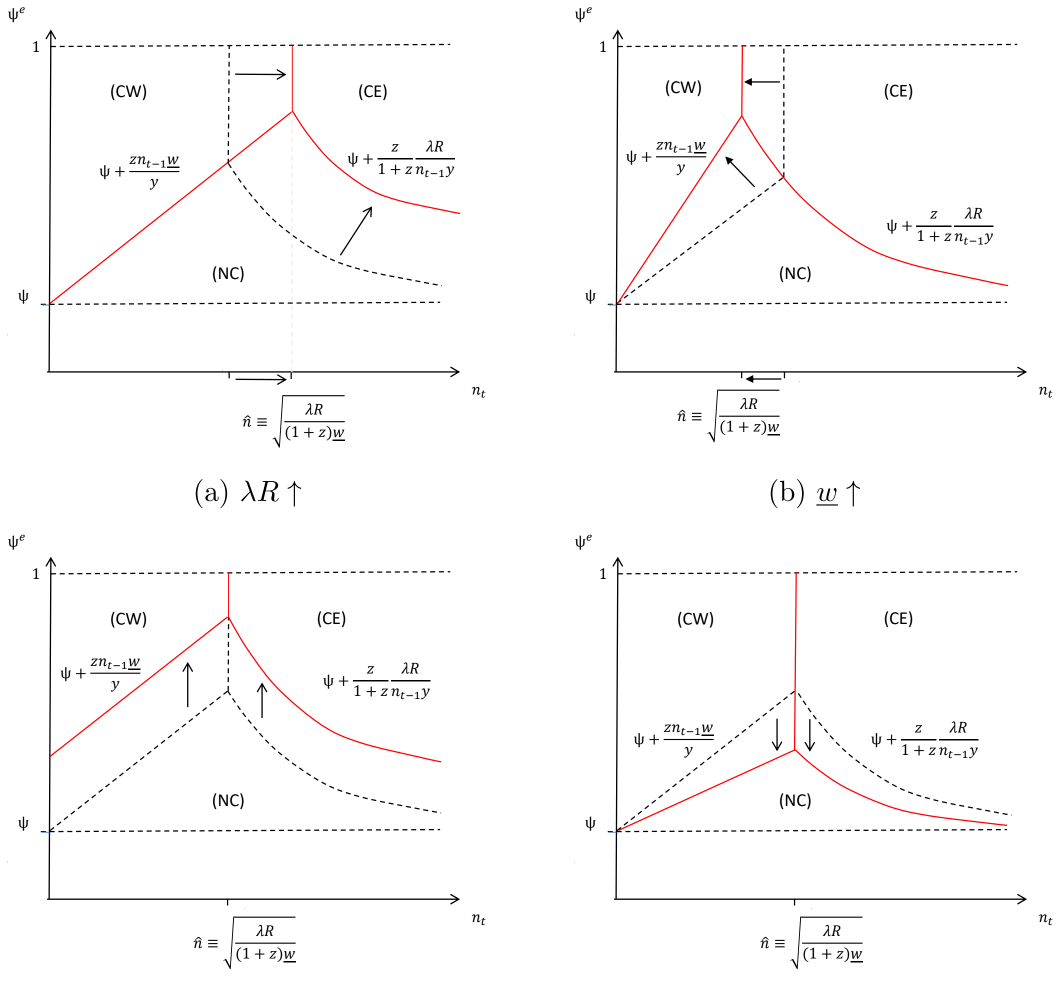

(a) $\lambda R \uparrow$ (b) $\underline{w} \uparrow$

(c) $\psi \uparrow$ (d) $y \uparrow$

FIGURE 1: Parameter changes and how they affect the king's actions. In panels (a) and (b) the value of $\widehat{n}$ changes: an increase in rents or their rivalry can shift a society from the expanding to the contracting path. An increase in the reservation wage has the opposite effect: it can shift a society from the contracting to the expanding path. In panels (c) and (d) an increase in the crown's strength or income affects the range of values of ψ^e for which collaboration happens, but these changes do not affect $\widehat{n}$ and therefore cause no shift in a society's path.

differences across societies; even if two societies have elites of the same size, they may follow different paths if their parameters – and consequently their thresholds – are different.

It is useful to think about the parameter space as being partitioned by two axes: (i) the value $\widehat{n} = \sqrt{\frac{\lambda R}{(1+z)\underline{w}}}$ partitions the elite size space into two, corresponding graphically to the left and right of the cutoff value $\widehat{n}$ in the x-axis, and (ii) the collaboration threshold on the vertical axis, which depends on which side of the elite cutoff $\widehat{n}$ a society is in. If to the left ($n < \widehat{n}$), then the relevant cutoff is given by $\psi + \frac{zn_{t-1}\underline{w}}{y}$; if $\psi^e \leq \psi + \frac{zn_{t-1}\underline{w}}{y}$ there is no collaboration, while if $\psi^e > \psi + \frac{zn_{t-1}\underline{w}}{y}$ there is collaboration and compensation is in wages. If to the right ($n \geq \widehat{n}$), then the relevant cutoff is given by $\psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}$; if $\psi^e \leq \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}$ there is no collaboration, while if $\psi^e > \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}$ there is collaboration and compensation is in elite rights.

A.1.1 The size of rents or their rivalry

Suppose that λR increases either because of an increase in the size of rents R or their rivalry λ; this has an impact on both the elite size threshold $\widehat{n}$ and the shock threshold to the right of the elite threshold, with both shifting to the right. The effect is captured graphically in panel (a) of figure 1. As a result, the range of elite sizes for which the decision is between NC and CW is increased. Furthermore, for the range of elite sizes still facing the choice between NC and CE, the values of ψ^e for which CE is optimal is reduced to include only large realizations of ψ^e. Therefore, an increase in λR makes it more likely that societies will not expand their elites unless their elites are already large or the shock is large. In terms of the dynamics, the set of elite sizes for which contractions happens is larger, and the transition probabilities for all other cases are reduced. As a result, countries with large λR are less likely to have expanding elites, and when they do, their expansion will be slower.

A.1.2 The size of the outside wage

An increase in the outside wage $\underline{w}$ decreases $\widehat{n}$, which increases the region of elite sizes for which the extension of rights is possible. It also increases the threshold for collaboration to the left of $\widehat{n}$. The effect is shown graphically in panel (b) of figure 1.

This has two implications: first, the range of elite sizes for which no extensions take place is reduced, but the minimum value of ψ^e for which collaboration happens goes up. In other words, for small elite sizes it is now less likely that collaboration will happen, because the cost of collaborators is higher. Second, the threshold $\widehat{n}$ for elite expansion is lowered, and so the elite will start expanding in some cases where before it would not have done so. This is because in these cases it becomes cost-effective to expand the elite instead of paying wages. For larger elite sizes nothing changes: the line marking the threshold does not change. In term of the dynamics, states might change from the contracting to the expanding path. The transition probabilities are unaffected, because they do not depend on wages. As a result, societies with larger $\underline{w}$ are more likely to find themselves in the expanding path; if they are not, then collaboration happens less often than it would otherwise.

A.1.3 The crown's strength and income

An increase in the crown's strength (ψ) causes the minimum value of ψ^e for which there is collaboration to increase for all elite sizes, but it does not shift $\widehat{n}$. Therefore collaboration happens less often, but the society does not shift between the contracting and expanding paths. Similarly, an increase in crown income (y) reduces the threshold for collaboration for all elite sizes (with the exception of the limiting cases of $n_{t-1} \to 0$ and $n_{t-1} \to \infty$), but does not shift $\widehat{n}$. Therefore it results in collaboration happening more often, but does not induce shifts between the contracting and expanding paths.[1]

1. It is also possible to do the comparative statics with respect to z, but this is a parameter for which it is difficult to find an empirical counterpart in the historical literature.

A.2 Proofs

Lemma 1

Proof. If the king offered wages and does not renege, his payoff is

$$y + \frac{n_{t-1} - \lambda(n_{t-1} - 1)}{n_{t-1}} R - z n_{t-1} w,$$

which equals crown income plus the rents he receives, minus the total wages he must pay. If he tries to renege, his payoff is

$$\psi\left(y + \frac{n_{t-1} - \lambda(n_{t-1} - 1)}{n_{t-1}} R\right) + (1-\psi)\left(\frac{n_{t-1} - \lambda(n_{t-1} - 1)}{n_{t-1}} R - z n_{t-1} w\right),$$

since with probability ψ he succeeds and avoids paying the wages, but with probability $1-\psi$ he is defeated and loses the crown income while still having to pay the wages. The king tries to renege if the latter is greater than the former. This condition is equivalent to $n_{t-1} > \frac{(1-\psi)y}{\psi z w}$.

If the king granted elite rights and does not try to renege, his payoff is

$$y + \left(1 - \lambda + \frac{\lambda}{(1+z)n_{t-1}}\right) R$$

and if he tries to renege his payoff is

$$\psi\left[y + (1 - \lambda + \frac{\lambda}{n_{t-1}})R\right] + (1-\psi)\left(1 - \lambda + \frac{\lambda}{(1+z)n_{t-1}}\right) R,$$

which is equal to the probability of winning ψ times the payoff (crown income plus undiluted rents), plus the probability of losing $(1-\psi)$ times the diluted rents. The king tries to renege if the latter is greater than the former. This condition is equivalent to $n_{t-1} < \frac{z}{1+z} \frac{\psi \lambda R}{(1-\psi)y}$. ■

Lemma 2

Proof. If the king does not attack the barons, he receives

$$y + \frac{n_{t-1} - \lambda(n_{t-1} - 1)}{n_{t-1}} R - z n_{t-1} w,$$

which is always less than what he receives when he attacks them, succeeds with certainty, and expropriates all their rents for that period:

$$y + R.$$

■

Lemma 3

Proof. (i) Suppose that the king is rewarding a collaborator with both a wage and elite rights. Then this must be sub-optimal; from assumption 1 it follows that the king could ensure collaboration without paying a wage, as long as elite rights are granted. This would be enough to satisfy the collaborator's reservation wage, saving the king the amount paid as wage.

(ii) This result follows from the fact that the cost of extending the elite goes down as the elite expands, while the wage a collaborator receives does not vary with the number of collaborators. Suppose that πe collaborators are given rights, while $(1-\pi)e$ are paid in wages. For this to be an equilibrium then it must be that (i) it is not worthwhile to change one collaborator's compensation from wages to elite rights, which is equivalent to the condition:

$$y+\left(1-\lambda+\frac{\lambda}{n_{t-1}+\pi e}\right)R-(1-\pi)e\underline{w} > y+\left(1-\lambda+\frac{\lambda}{n_{t-1}+\pi e+1}\right)R-(1-\pi)(e-1)\underline{w}$$

where the left-hand side is the payoff from not deviating, while the right-hand side is the payoff from deviating by paying one fewer wage and instead offering that collaborator rights. This is equivalent to

$$\underline{w} < \left[\frac{1}{n_{t-1} + \pi e} - \frac{1}{n_{t-1} + \pi e + 1}\right] \frac{\lambda R}{1 - \pi} \equiv \underline{w}^h.$$

It must also be true that (ii) it is not worthwhile to change one collaborator's compensation from elite rights to wages, which is equivalent to the condition

$$y + \left(1 - \lambda + \frac{\lambda}{n_{t-1} + \pi e}\right) R - (1-\pi) e \underline{w} > y + \left(1 - \lambda + \frac{\lambda}{n_{t-1} + \pi e - 1}\right) R - (1-\pi)(e+1)\underline{w}$$

where the left-hand side is the payoff from not deviating, while the right-hand side is the payoff from deviating by extending rights to one fewer collaborator and instead paying her a wage. This is equivalent to

$$\underline{w} > \left[\frac{1}{n_{t-1} + \pi e - 1} - \frac{1}{n_{t-1} + \pi e}\right] \frac{\lambda R}{1 - \pi} \equiv \underline{w}^l$$

Some additional algebra shows that $\underline{w}^l > \underline{w}^h$, and so this cannot be an equilibrium. The equilibrium must consequently be at a 'corner', where all collaborators are compensated with wages or where all are compensated with elite rights. In this case there is only one deviation condition and the contradiction shown here does not arise. ■

Lemma 4

Proof. If a peasant is approached with an offer of collaboration, she needs to decide whether to accept it. If offered a wage $\underline{w}$ and $n_{t-1} \leq \frac{1-\psi}{\psi} \frac{y}{z\underline{w}}$, then the king will not try to renege (from lemma 1). In this case the peasant accepts as long as $w \geq \underline{w}$, and so the king offers $w = \underline{w}$, since this amount is enough to get the peasant to collaborate and paying her more would result in the king receiving a lower payoff without affecting the peasant's collaboration decision.

If instead $n_{t-1} > \frac{1-\psi}{\psi}\frac{y}{z\underline{w}}$, the king will renege and so the peasant will no longer accept an offer of $\underline{w}$. The king must now offer $w = \frac{\underline{w}}{1-\psi}$, which is the lowest offer the peasant will accept. And since at the given value of n_{t-1} an offer of $\underline{w}$ led the king to renege, it follows from lemma 1 that an offer of $\frac{\underline{w}}{1-\psi}$ will lead him to renege too; this is consistent with the premium being applied to the wage. ■

Lemma 5

Proof. Suppose that the barons have revolted, and that the king decides to collaborate. He has four options: (i) offer a wage and not renege, (ii) offer a wage and try to renege, (iii) offer elite rights and not renege, and (iv) offer elite rights and try to renege.

(i) The king can only offer a wage of $\underline{w}$ and not renege if $n_{t-1} \leq \frac{1-\psi}{\psi}\frac{y}{z\underline{w}}$. In this case he collaborates only if

$$\psi^e \left[y + \left(1 - \lambda + \frac{\lambda}{n_{t-1}}\right) R - zn_{t-1}\underline{w}\right] + (1-\psi^e)\left[\left(1 - \lambda + \frac{\lambda}{n_{t-1}}\right) R - zn_{t-1}\underline{w}\right]$$
$$> \psi \left[y + \left(1 - \lambda + \frac{\lambda}{n_{t-1}}\right) R\right] + (1-\psi)\left[\left(1 - \lambda + \frac{\lambda}{n_{t-1}}\right) R\right]$$

where the first term is the probability of success when collaborating times the payoff (taking into account that the wage must be paid), the second term is the probability of defeat when collaborating times the payoff in that case (the king loses the crown income and pays the wage bill), the third term is the probability of success when not collaborating times the payoff, and the fourth term is the probability of defeat when not collaborating times the payoff in that case (the king loses the crown income). Simplying the expression yields

$$\psi^e > \psi + \frac{zn_{t-1}\underline{w}}{y}.$$

Notice that if we solve this expression for n_{t-1} we get

$$n_{t-1} < \frac{(\psi^e - \psi)y}{z\underline{w}} \leq \frac{1-\psi}{\psi}\frac{y}{z\underline{w}},$$

and so this is consistent with the king not reneging.

(ii) If the king compensates with a wage and then tries to renege (if he defeats the barons), he will have to offer a wage of $w = \frac{\underline{w}}{1-\psi}$ and it must be that $n_{t-1} > \frac{1-\psi}{\psi}\frac{y}{z\underline{w}}$. The king will collaborate if

$$\begin{aligned}\psi^e \Bigg[\psi\left[y + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right] + (1-\psi)\left[\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R - \frac{n_{t-1}z\underline{w}}{1-\psi}\right]\Bigg] \\ +(1-\psi^e)\left[\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R - \frac{n_{t-1}z\underline{w}}{1-\psi}\right] \\ > \psi\left[y + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right] + (1-\psi)\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\end{aligned}$$

where the first term is the probability of success when collaborating multiplied by the expected payoff from trying to renege (the king loses the crown income and pays the wage bill if his attempt to renege fails), the second term is the probability of defeat while collaborating times the payoff (the king loses the crown income and pays the wage), the third term is the probability of success when not collaborating times the payoff, and the fourth term is the probability of defeat when not collaborating times the payoff in that case (the king loses the crown income). This condition is never satisfied.

(iii) If the king compensates in rights and does not renege, then it must be that $n_{t-1} \geq \frac{z}{1+z}\frac{\psi\lambda R}{(1-\psi)y}$. The king will choose to collaborate if

$$\begin{aligned}\psi^e\left[y + \left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R\right] + (1-\psi^e)\left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R \\ > \psi\left[y + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right] + (1-\psi)\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\end{aligned}$$

which simplifies to

$$\psi^e > \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}.$$

Notice that if we solve this expression for n_{t-1} we get

$$n_{t-1} > \frac{z}{1+z}\frac{1}{\psi^e - \psi}\frac{\lambda R}{y} \geq \frac{z}{1+z}\frac{\psi\lambda R}{(1-\psi)y},$$

and so this is consistent with the king not reneging.

(iv) If the king compensates in rights and then tries to renege (if he defeats the barons), then it must be that $n_{t-1} < \frac{z}{1+z}\frac{\psi\lambda R}{(1-\psi)y}$. The king will collaborate if

$$\begin{aligned} \psi^e\left[\psi\left[y + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right] + (1-\psi)\left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R\right] \\ +(1-\psi^e)\left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R > \\ \psi\left[y+\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right] + (1-\psi)\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R \end{aligned}$$

which is never satisfied. ■

Proposition 1

Proof. The previous lemma establishes the king's optimal actions conditional on the type of compensation offered to collaborators. In order to establish whether he collaborates and how he compensates the collaborators, I now need to check whether collaboration is optimal, and if so, which form of compensation is cheaper.

Suppose that there is a baronial rebellion. (i) If the king does not collaborate he defeats the rebellion with probability ψ. He will choose not to collaborate when it is not worthwhile to pay wages in order to ensure collaboration, which from lemma 5 is when:

$$\psi^e \leq \psi + \frac{z n_{t-1}\underline{w}}{y},$$

and when it is not worthwhile to grant elite rights to ensure collaboration, which from lemma 5 is when:

$$\psi^e \leq \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}.$$

Both conditions are satisfied when:

$$\psi^e \leq min\left\{\psi + \frac{zn_{t-1}\underline{w}}{y}, \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}\right\}.$$

(ii) If the king collaborates and pays a wage of $\underline{w}$ he wins with probability ψ^e and does not renege (from lemma 5). Furthermore, we know that if wages are being paid, collaboration happens if

$$\psi^e > \psi + \frac{zn_{t-1}\underline{w}}{y}.$$

For wages to be paid instead of elite rights, it must be that they are a cheaper form of compensation, which requires that

$$\psi + \frac{n_{t-1}z\underline{w}}{y} < \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}$$

which simplifies to

$$n_{t-1} < \sqrt{\frac{\lambda R}{(1+z)\underline{w}}}.$$

Assumption 2 ensures that $\psi + \frac{zn_{t-1}\underline{w}}{y} < 1$ whenever $n_{t-1} < \widehat{n}$.

(iii) If the king collaborates and pays by extending elite rights he wins with probability ψ^e and does not renege (from lemma 5). Furthermore, we know that for elite rights to be extended it must be that

$$\psi^e > \psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y}.$$

For elite rights to be granted it must be that they are cheaper or cost just as much as wages, which requires that

$$\psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y} \leq \psi + \frac{n_{t-1}z\underline{w}}{y}$$

which simplifies to

$$n_{t-1} \geq \sqrt{\frac{\lambda R}{(1+z)\underline{w}}}$$

Assumption 2 ensures that $\psi + \frac{z}{1+z}\frac{\lambda R}{n_{t-1}y} < 1$ whenever $n_{t-1} \geq \widehat{n}$. ∎

Proposition 2

Proof. (i) If the barons do not attack the king, the king attacks them, takes their rents, and a randomly-chosen fraction $\frac{z}{1+z}$ of them are removed from the elite (lemma 2). Consequently, they all receive a payoff of 0. If there is a baronial revolt, each baron's payoff depends on the actions taken by the king:

If the king chooses (NC), each baron receives:

$$\psi\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R + (1-\psi)\left[\frac{y}{n_{t-1}-1} + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right]$$

since with probability ψ the king defeats the barons and they just earn rents, while with probability $1-\psi$ the barons win, receive rents, and one of them becomes the new king. Therefore a baron will receive crown income y with probability $\frac{1}{n_{t-1}-1}$. If the king chooses (CW), each baron receives:

$$\psi^e\left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R + (1-\psi^e)\left[\frac{y-n_{t-1}z\underline{w}}{n_{t-1}-1} + \left(1-\lambda+\frac{\lambda}{n_{t-1}}\right)R\right]$$

since with collaboration the king defeats the barons with probability ψ^e and the barons just earn rents, while with probability $1-\psi^e$ the barons win, receive rents, and one of them becomes the new king. Therefore a baron receives the crown income

(y minus the wages paid by the king) with probability $\frac{1}{n_{t-1}-1}$. If the king chooses (CE), each baron receives:

$$\psi^e\left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R+(1-\psi^e)\left[\frac{y}{n_{t-1}-1}+\left(1-\lambda+\frac{\lambda}{(1+z)n_{t-1}}\right)R\right]$$

where with probability ψ^e the king wins and the barons receive diluted rents (since the elite has expanded to include the king's collaborators), while with probability $1-\psi^e$ the barons win and receive the diluted rents, but now one of them becomes the new king. Therefore a baron will receive crown income y with probability $\frac{1}{n_{t-1}-1}$. All of these payoffs are greater than 0, and so the barons always attempt a revolt ($a=1$).

(ii) With probability σ the barons solve their collective action problem, in which case proposition 1 describes the rest of the equilibrium.

(ii) With probability $1-\sigma$ the barons fail to solve their collective action problem, and so cannot revolt. In that case the king attacks the barons, succeeds with probability 1, and takes all rents and $\frac{z}{1+z}n_{t-1}$ barons are chosen at random and removed from the elite (lemma 2). ∎

Proposition 3

Proof. Recall that $p\left(e_i\right)=1-\frac{z}{1+z}\frac{\lambda R}{(1-\psi)e_i y}$ and $e_{i+1}=(1+z)e_i>e_i$, and so it follows that $p\left(e_{i+1}\right)>p\left(e_i\right)$. The assumption on the transition probabilities for the last state ensures the last part of the proposition holds. ∎