Your Place or Mine?

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Supplementary Mathematical Appendix

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In this supplementary appendix, we solve our game-theoretic model. The purpose of the model is to shed light on strategic bargaining under institutional capture. In the absence of institutional capture, we assume the current policy in the organization is such that neither the challenger nor the defender needs to reform so as to ensure that the other remains a member. Thus, the model is based on the assumption that institutional capture has created a bargaining situation. Our first hypothesis thus stems from the substantive logic underlying the relationship between institutional capture and bargaining. The game-theoretic model establishes the deductive validity of our remaining hypotheses.

Equilibrium

Our solution concept is the perfect Bayesian equilibrium. In equilibrium, the following must hold:

1. The third party joins, $J^* = 1$, if and only if it obtains a payoff 1 from doing so;

2. The challenger accepts all offers such that $x \geq (1 - \lambda^{join}) - c + d$ and rejects anything else.

3. The defender selects x to maximize its expected payoff based on its prior beliefs.

We need not characterize posterior beliefs because they do not influence the third party's decision and the defender moves before the challenger.

We already have a full characterization of the third party's behavior. What about the challenger? If it is resolute, so that $\underline{c}$, it accepts all x such that $x \geq x^{generous} = \lambda^{join} - \underline{c} + d$. If it is irresolute, it accepts all x such that $x \geq x^{meager} = \lambda^{join} - \overline{c} + d$. Note that $x^{generous} > x^{meager}$. To avoid trivial outcomes, we choose parameters throughout so that $1 \geq x^{generous} > x^{meager} \geq 0$.

Consider now the challenger's offer. Given that a zone of agreement exists, with $c - d > 0$, the choice is between $x^{generous}$ and x^{meager}. If $x^* = x^{generous}$, a new organization is not created. If $x^* = x^{meager}$, a new organization is created with probability $p^{low} = 1 - p^{high}$.

The payoff from $x^{generous}$ is

$$1 - \left[\lambda^{join} - \underline{c} + d\right].$$

The payoff from x^{meager} is

$$p^{high} \cdot \left(1 - \left[\lambda^{join} - \overline{c} + d\right]\right) + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join}\right).$$

This implies that $x^{generous}$ is optimal whenever

$$\begin{aligned} 1-\left[\lambda^{join}-\underline{c}+d\right] &> p^{high}\cdot\left(1-\left[\lambda^{join}-\overline{c}+d\right]\right)+\left(1-p^{high}\right)\cdot\left(1-\lambda^{join}\right)\Leftrightarrow \\ \underline{c}-d &> p^{high}\cdot(\overline{c}-d)\Leftrightarrow \\ \underline{c}-p^{high}\cdot\overline{c} &> d\cdot\left(1-p^{high}\right). \end{aligned}$$

This condition shows that as d increases, the condition for a generous offer is more difficult to meet. This proves the deductive validity of our second hypothesis. The claim also holds that the probability of success, λ^{join}, is irrelevant for the choice between the two alternate strategies as long as a zone of agreement exists. This proves the deductive validity of our third hypothesis.

What about the fourth hypothesis? The model shows that in equilibrium, a new organization is created with zero probability if the offer is $x^* = x^{generous}$. If $x^* = x^{meager}$, then a new organization is created with a positive probability, $p^{low} = 1 - p^{high}$. All else constant, in equilibrium we expect that non-attempts to create new organizations are positively associated with the size of the reform offer. Additionally, note that as d increases, the size of the generous offer $x^{generous}$ must increase. Thus, for low values of d, the generosity of this higher potential equilibrium offer decreases. This proves the deductive validity of our fourth hypothesis.

Subjective Expected Probabilities

We now augment the model such that the expected probability of success λ^{join} may differ across the two players. In this model, the two players are not rational in the sense of the game structure being common knowledge. Suppose $\lambda^{join,A}$ is the challenger's subjective probability and $\lambda^{join,B}$ the defender's. Each player is aware of $\lambda^{join,A}, \lambda^{join,B}$ but irrationally fails to learn from the discrepancy.

The challenger accepts if and only if $x \geq \lambda^{join} - \underline{c} + d$. This defines $x^{generous}$ and x^{meager}as above.

The defender's offer is again either $x^{generous}$ and x^{meager}. The payoff from $x^{generous}$ is

$$1-\left[\lambda^{join,B}-\underline{c}+d\right].$$

The payoff from x^{meager} is

$$p^{high} \cdot \left(1 - \left[\lambda^{join,B} - \overline{c} + d\right]\right) + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,A}\right).$$

The condition for $x^{generous}$ is now

$$1 - \left[\lambda^{join,B} - \underline{c} + d\right] > p^{high} \cdot \left(1 - \left[\lambda^{join,B} - \overline{c} + d\right]\right) + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,A}\right).$$

Simplifying,

$$\begin{aligned} \underline{c} - d + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,B}\right) &> p^{high} \cdot (\overline{c} - d) + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,A}\right) \Leftrightarrow \\ \underline{c} - p^{high} \cdot \overline{c} + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,B}\right) &> d \cdot \left(1 - p^{high}\right) + \left(1 - p^{high}\right) \cdot \left(1 - \lambda^{join,A}\right). \end{aligned}$$

This condition is otherwise identical except that the probability λ^{join} does not disappear. The defender has stronger incentives to offer $x^{generous}$, so that the new institution is not created, when $\lambda^{join,B}$ decreases and $\lambda^{join,A}$ remains unchanged. Conversely, a decrease in $\lambda^{join,A}$ increases the challenger's incentive to make a generous offer when $\lambda^{join,B}$ remains unchanged.