Online Appendix A: Supplementary Results

Electoral Probabilities and Incumbency Advantage

This result shows that the delegation region for an insulated program expands with the probability of re-election for a first-term politician. For each group i, let $\overline{\pi}_i \geq \pi_i$ be the probability of re-election, where $\overline{\pi}_i > \pi_i$ implies an incumbency advantage. Let $\mathcal{D}(\overline{\pi}_i)$ represent the corresponding delegation region. Note that with an incumbency advantage or disadvantage, it is possible that $\overline{\pi}_1 + \overline{\pi}_2 \neq 1$.

Proposition A.1. *Election Probabilities and Delegation.* For any $\overline{\pi}_i'' > \overline{\pi}_i'$, $\mathcal{D}(\overline{\pi}_i') \subseteq \mathcal{D}(\overline{\pi}_i'')$.

Proof of Proposition A.1. As derived in Proposition 2, the delegation region $\mathcal{D}$ for group i is characterized in terms of the group i incumbent politician's election probability π_i and the bureaucrat's probability of remaining in office $\pi_b = \pi_i$ under politicization. The delegation region $\mathcal{D}(\overline{\pi}_i)$ is therefore simply $\mathcal{D}$ rewritten using election probability $\overline{\pi}_i$ in place of π_i. I show that the lower bound of $\mathcal{D}(\overline{\pi}_i)$ is decreasing in $\overline{\pi}_i$ and the upper bound is non-decreasing in $\overline{\pi}_i$.

Under insulation, the lower bound of $\mathcal{D}(\overline{\pi}_i)$ is:

$$\mu_b^0(1) + \frac{\overline{\pi}_i}{\kappa_p}\left(2c\lambda_i\sqrt{\frac{\kappa_b}{\delta m_b}} - \frac{\lambda_i^2 - k\Lambda^2}{\Lambda}\right).$$

The derivative of this expression with respect to $\overline{\pi}_i$ is:

$$\frac{2c\lambda_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta m_b}} - \frac{\lambda_i^2 - k\Lambda^2}{\kappa_p\Lambda}.$$

This expression is easily verified to be negative given assumption (6).

The upper bound of $\mathcal{D}(\overline{\pi}_i)$ is the minimum of $\frac{\Lambda}{\delta} + \frac{2\overline{\pi}_i c\lambda_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta m_b}}$ and $\mu_b^0(1)$. The former expression is clearly increasing in $\overline{\pi}_i$, and the latter expression is clearly constant in $\overline{\pi}_i$, establishing the result. ∎

Politicized and Insulated Delegation Regions

To illustrate the effect of politicization on the delegation region $\mathcal{D}$, the following figure uses the same parameters as Figure 1 to compare a group 1 politician's $\mathcal{D}$ under both insulation and politicization. In this example, politicization expands the delegation region somewhat.

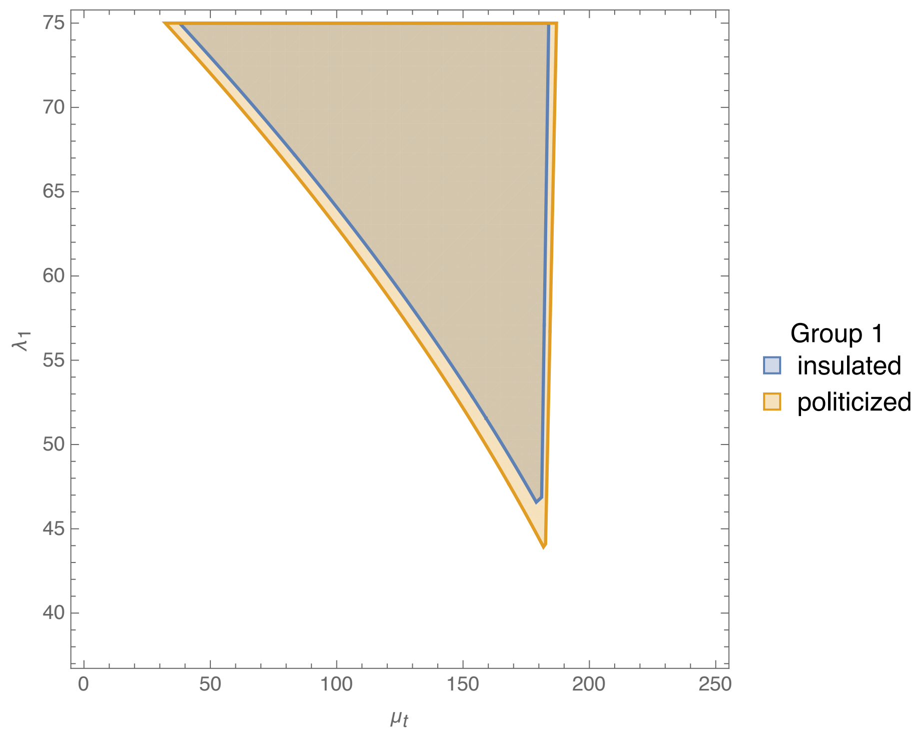

Figure A.1: Delegation Regions for Insulated and Politicized Programs. Here $\Lambda = \lambda_1 + \lambda_2 = 150$, $m_b = 75$, $c = 0.2$, $\kappa_b = 0.1$, $\kappa_p = 0.08$, $k = 0.0625$, $\pi_1 = 0.5$, and $\delta = 0.85$. Plots are of delegation regions by a newly elected group 1 politician under both insulation and politicization as functions of capacity μ_t and service demand rate λ_i.

Plots of Long Run Capacity

Figures A.2 and A.3 plot long-run average capacity levels, varying different exogenous parameters of interest. Each point is the mean of terminal capacity level μ_{1000} over 5,000 simulation runs. To ease comparisons, parameters across plots have been held constant where possible.

Figure A.2 plots capacity as a function of group 1 demand (λ_1) at different values of public service motivation (m_b), holding total demand Λ constant so that higher values of λ_1

correspond to lower levels of inequality. It shows that higher vales of m_b result in higher average capacity in the long run.

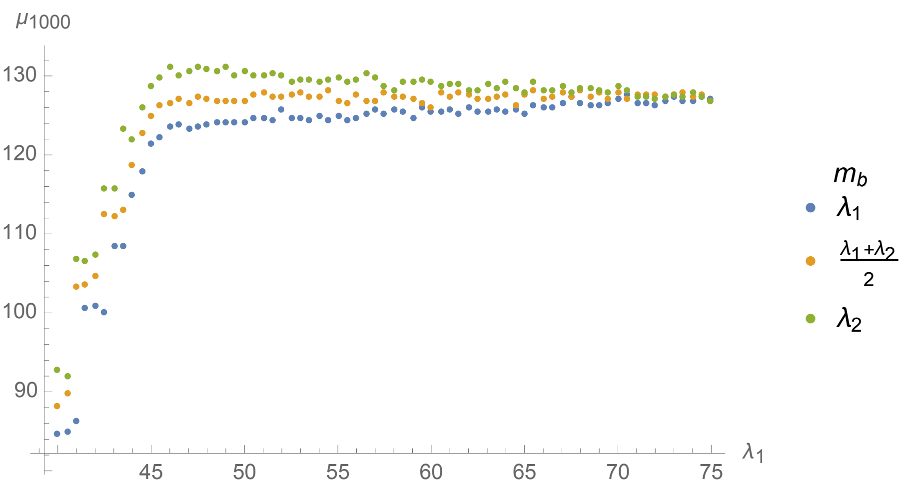

Figure A.2: Long Run Capacity (insulated program). Here $\Lambda = \lambda_1 + \lambda_2 = 150$, $c = 0.2$, $\kappa_b = 0.1$, $\kappa_p = 0.01$, $k = 0.0625$, $\pi_1 = 0.5$, $\mu_1 = 105$, and $\delta = 0.85$. Plot depicts average μ_{1000} across 5,000 simulations as a function of λ_1. Each series varies the bureaucrat's public service motivation m_b.

Figure A.3 plots capacity for an insulated program as a function of the group 1 election probability (π_1) at different values of politician marginal cost (k_p). Higher values of π_1 imply lower values of π_2 and hence a disadvantage for the high-demand group. This figure is discussed in the "Long Run Survival and Quality" section.

Online Appendix B: Proofs of Main Results

Proof of Proposition 3. (i) First observe that $\underline{\lambda}_i(1)$ (20) is the value of λ_i that solves:

$$\mu_b^0(1) + \frac{\pi_i}{\kappa_p}\left(2c\lambda_i\sqrt{\frac{\kappa_b}{\delta m_b}} - \frac{\lambda_i^2 - k\Lambda^2}{\Lambda}\right) = 0, \tag{32}$$

where the left-hand side of (32) is the infimum of $\mathcal{D}_i$, the group i delegation region (19), as defined in expression (30) in the proof of Proposition 2. Thus for $\lambda_i > \underline{\lambda}_i(1)$, a group i politician delegates for any arbitrarily low value of μ_t.

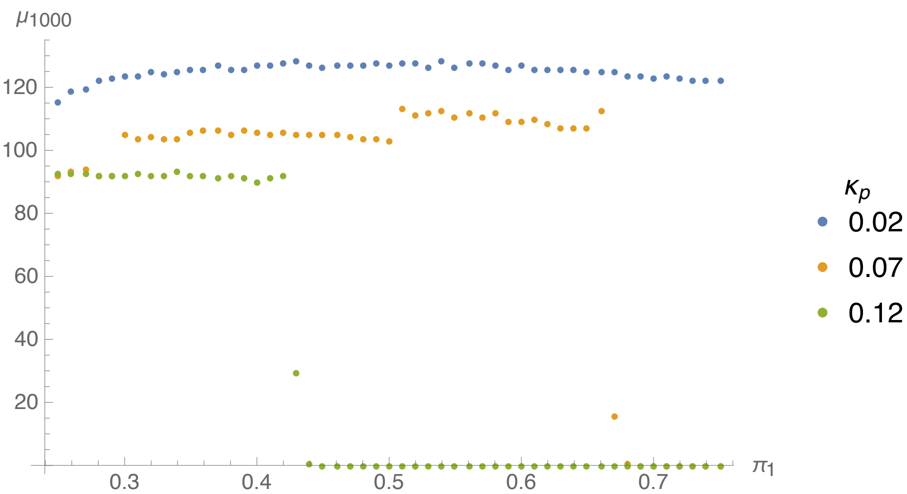

Figure A.3: Long Run Capacity and Electoral Advantage (insulated program). Here $\Lambda = 150$, $\lambda_1 = 60$, $m_b = 75$, $c = 0.2$, $\kappa_b = 0.1$, $k = 0.0625$, $\mu_1 = 105$, and $\delta = 0.85$. Plot depicts average μ_{1000} across 5,000 simulations as a function of π_1 for $\kappa_p = 0.02$, 0.07, and 0.12.

To show sufficiency, suppose that $\lambda_i > \underline{\lambda}_i(1)$ for group i. Thus for any μ_t and even-numbered period t, there is an age 2 bureaucrat and with probability $\pi_i > 0$ either (i) μ_t is higher than the supremum of $\mathcal{D}_i$, or (ii) delegation and investment will occur with certainty. This clearly ensures program survival.

To show necessity, suppose to the contrary that $\lambda_i < \underline{\lambda}_i(1)$ for both groups. Recall that under the political Markov process $\mathcal{P}_t$, delegation and investment occur only in states (1, 1, 1) and (2, 1, 1). I construct a sequence of elections that begins in any state of the form (i, 1, 2) and any initial capacity μ_t that results in a limit of zero capacity.

For politicians of each group i, $\lambda_i < \underline{\lambda}_i(1)$ implies that the left-hand side of (32) is strictly positive. I define the following as the minimum of the lower bounds on $\mathcal{D}_1$ and $\mathcal{D}_2$:

$$\mu_D = \min\left\{\mu_b^0(1) + \frac{\pi_1}{\kappa_p}\left(2c\lambda_1\sqrt{\frac{\kappa_b}{\delta m_b}} - \frac{\lambda_1^2 - k\Lambda^2}{\Lambda}\right), \mu_b^0(1) + \frac{\pi_2}{\kappa_p}\left(2c\lambda_2\sqrt{\frac{\kappa_b}{\delta m_b}} - \frac{\lambda_2^2 - k\Lambda^2}{\Lambda}\right)\right\}$$

Starting from a state (i, 1, 2) and capacity μ_t, let the incumbent politician be re-elected in period $t + 1$. Then let the incumbent politician (of either group) be re-elected in every period $t+j$, for $j = 3, 5, \ldots, \bar{j}$, where j is odd and $\bar{j}$ is the lowest odd integer satisfying:

$$\bar{j} > \left\lceil \frac{\log \mu_D - \log \mu_t}{\log \delta} \right\rceil,$$

if such an integer exists, and 0 otherwise. By construction, $\delta^{\bar{j}} \mu_t < \mu_D$, and thus after $\bar{j}$ periods of the specified sequence of electoral outcomes, no politician delegates. As capacity declines exponentially in each period, we have that $\lim_{t\to\infty} \mu_t = 0$.

For $\bar{j} = 0$, μ_t is sufficiently low at period t to ensure no delegation. For $\bar{j} \geq 1$, the probability of this sequence is:

$$\pi_i(\pi_1^2 + \pi_2^2)^{\frac{\bar{j}-1}{2}}. \tag{33}$$

Finally, since the states (i, 1, 2) are positive recurrent with stationary probability $\pi_i/(1+\pi_i)$ and the probability in (33) is clearly bounded away from zero, capacity drops below μ_D with probability one: contradiction.

(ii) The result on deference is derived by using $\pi_b = \pi_i$ in the sufficiency part of the proof of part (i). For the result on $\underline{\lambda}_i^p$, I derive conditions for the delegation region to be large enough to contain two periods of non-investment.

Observe that under politicization, the combination of newly-elected politicians and age-1 bureaucrats appears at least every other period. Thus, conditional upon investments by politicians of either group that brings capacity to some $\mu_b^0(\pi_i)$, investment by both groups is guaranteed at least every other period if after two periods capacity depreciates to a level within $\mathcal{D}_1 \cap \mathcal{D}_2$. Equivalently, for each group i:

$$\delta^2 \mu_b^0(\pi_1) \in \mathcal{D}_i \text{ and } \delta^2 \mu_b^0(\pi_2) \in \mathcal{D}_i. \tag{34}$$

Observe that the period 1, group i incumbent brings capacity to $\mu_b^0(\pi_i)$ by the assumption that $\mu_1 \in \mathcal{D}_i$ in period 1.

To characterize the minimum value of λ_i satisfying (34) and provide closed form solutions, there are two cases. First, using expression (27), if $\lambda_i \geq \pi_b m_b \kappa_p/(2\pi_i c \kappa_b)$, then the supremum of $\mathcal{D}_i$ for a given λ_i is $\hat{\mu}_i^- = \mu_b^0(\pi_b)$. Obviously $\delta^2 \mu_b^0(\pi_j) < \mu_b^0(\pi_j)$, and so to satisfy $\delta^2 \mu_b^0(\pi_j) \in$

$\mathcal{D}_i$ for each group j, it is sufficient to verify that:

$$\delta^2 \mu_b^0(\pi_j) \geq \tilde{\mu}_i, \tag{35}$$

where $\tilde{\mu}_i$ is the infimum of $\mathcal{D}_i$ for a given λ_i, as provided by expression (30). Using the fact that $\pi_b = \pi_i$ under politicization, solving for λ_i satisfying (35) produces the unique non-negative lower bound:

$$\lambda_i \geq \underline{\lambda}_{i,j}^p \equiv \Lambda \left(c\sqrt{\frac{\kappa_b}{\delta \pi_i m_b}} + \sqrt{\frac{c^2 \kappa_b}{\delta \pi_i m_b} + (1-\delta^2)\frac{\kappa_p}{\delta \pi_i} + \frac{\kappa_p}{\Lambda}\sqrt{\frac{m_b}{\delta \pi_i \kappa_b}}\left(1 - \delta^2 \sqrt{\frac{\pi_j}{\pi_i}}\right) + k} \right).$$

Second, if $\lambda_i < \pi_b m_b \kappa_p / (2\pi_i c \kappa_b)$, then the supremum of $\mathcal{D}_i$ for a given λ_i is $\hat{\mu}_i^- = \frac{\Lambda}{\delta} + \frac{2\pi_i c \lambda_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta \pi_b m_b}}$, which is less than $\mu_b^0(\pi_b)$. In addition to satisfying (35), $\delta^2 \mu_b^0(\pi_j) \in \mathcal{D}_i$ additionally requires that $\delta^2 \mu_b^0(\pi_j) \leq \hat{\mu}_i^-$. Solving for λ_i meeting this condition produces:

$$\lambda_i \geq \underline{\lambda}_{i,j}^{p'} \equiv \frac{\kappa_p}{2c}\sqrt{\frac{m_b}{\delta \pi_i \kappa_b}}\left(\delta^3 \sqrt{\frac{\pi_j m_b}{\delta \kappa_b}} - \Lambda(1-\delta^2)\right).$$

Combining results, for each group i, the minimum value of λ_i satisfying (34) is then:

$$\underline{\lambda}_i^p = \begin{cases} \max_j \{\underline{\lambda}_{i,j}^p\} & \text{if } \max_j \{\underline{\lambda}_{i,j}^p\} \geq \pi_b m_b \kappa_p / (2\pi_i c \kappa_b) \\ \max_j \{\underline{\lambda}_{i,j}^p, \underline{\lambda}_{i,j}^{p'}\} & \text{otherwise.} \quad \blacksquare \end{cases}$$

The following definitions and two lemmas are used in the proof of Proposition 4. Full deference extends the notion of deference to capture situations where politicians are willing to delegate not only for arbitrarily low capacity, but also after one period of depreciation. For the subsequent discussion, it will be convenient to define a modified version of $\mathcal{Q}_t$ to describe the evolution of quality. Let $\mathcal{Q}_t'$ have states denoted by the 4-tuple $(i,\ \theta_i,\ \theta_b,\ j)$, where i is the group of the incumbent politician, and θ_i and θ_b are the politician's term and the bureaucrat's age from the immediately preceding period, respectively. The integer $j = 1$, 2, ... summarizes capacity in the subsequent period, where after j periods of non-investment $\mu_t = \delta^j \mu_b^0(\pi_b)$.

Definition 4. *A group i politician satisfies* full deference *if she satisfies deference and:*

$$\sqrt{\frac{\delta \pi_b m_b}{\kappa_b}} - \frac{2c\lambda_i \pi_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta \pi_b m_b}} < \Lambda\left(\frac{1}{\delta} - 1\right). \tag{36}$$

Lemma 2. Delegation Under Full Deference. *If group i politicians satisfy full deference, then they delegate whenever the political state is $(i, 1, 1, j)$ for any $j \geq 1$.*

Proof of Lemma 2. The result holds if deference and expression (36) imply that $\delta^j \mu_b^0(\pi_b) \in (\tilde{\mu}_i, \hat{\mu}_i^-)$ for any $j \geq 1$, where $\tilde{\mu}_i$ and $\hat{\mu}_i^-$ are the limit points of the group i delegation region $\mathcal{D}_i$, as defined in equation (31) in the proof of Proposition 2.

Deference implies that $\tilde{\mu}_i = 0$, and thus $\delta^j \mu_b^0(\pi_b) > \tilde{\mu}_i$. To show that $\delta^j \mu_b^0(\pi_b) < \hat{\mu}_i^-$, note that as defined in (27), $\hat{\mu}_i^-$ takes the value of either $\mu_b^0(\pi_b)$ or $\frac{\Lambda}{\delta} + \frac{2\pi_i c \lambda_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta \pi_b m_b}}$. If the former, then the desired condition holds trivially. If the latter, then the condition holds if:

$$\delta\left(\frac{\Lambda}{\delta} + \sqrt{\frac{\pi_b m_b}{\delta \kappa_b}}\right) \quad < \quad \frac{\Lambda}{\delta} + \frac{2\pi_i c \lambda_i}{\kappa_p}\sqrt{\frac{\kappa_b}{\delta \pi_b m_b}}.$$

Further simplification produces expression (36). ∎

Lemma 3. Irreducibility. *For both insulated and politicized agencies, $\mathcal{Q}_t'$ is irreducible.*

Proof of Lemma 3. First note that under both politicization and insulation, the only states for which $j = 1$ are of the form $(i,\ 1,\ 1,\ 1)$. Furthermore, by Lemma 2, full deference implies that $j = 1$ whenever $\theta_i = \theta_b = 1$.

Under politicization, $\theta_i = \theta_b$ in all states. By full deference, non-investment can occur if and only if a politician is re-elected. Thus, the transition matrix can be written as follows:

	$(1,1,1,1)$	$(1,2,2,2)$	$(2,1,1,1)$	$(2,2,2,2)$
$(1,1,1,1)$	0	π_1	π_2	0
$(1,2,2,2)$	π_1	0	π_2	0
$(2,1,1,1)$	π_1	0	0	π_2
$(2,2,2,2)$	π_1	0	π_2	0

These states clearly form a communicating class, and because investment under any other possible state must result in a state of the form $(i,\ 1,\ 1,\ 1)$, the class is unique. Thus $\mathcal{Q}_t'$ is irreducible.

For an insulated agency, full deference implies that non-investment occurs if and only if a politician is re-elected or $\theta_b = 2$. The communicating states for each j are as follows.

For $j = 2$, states of the form (i, 1, 1, 2) are clearly impossible. States of the form (i, 2, 1, 2) are also impossible because they imply state (i, 1, 2, 1) in the preceding period. Thus the only possible states are of the forms (i, 1, 2, 2) and (i, 2, 2, 2), which are accessible from ($-i$, 1, 1, 1) and (i, 1, 1, 1), respectively.

For $j = 3$, note that whenever $\theta_i = \theta_b = 2$ and $j = 2$, the subsequent state is of the form (i, 1, 1, 1) for some i. Thus the only states for which $j = 3$ follow states where $\theta_i = 1$ and $\theta_b = 2$, and are therefore of the form (i, 2, 1, 3).

For $j = 4$, the only possible successors to (i, 2, 1, 3) are (1, 1, 2, 4) or (2, 1, 2, 4). The successor to (i, 1, 2, 4) is ($-i$,1,1,1) with probability π_{-i}.

Following this logic, generally for any odd $j \geq 3$, only states of the form (i, 2, 1, j) exist. For any even $j \geq 4$, only states of the form (i, 1, 2, j) exist. The states (i, 1, 1, 1) are reached with probability π_i from any state of the form ($-i$, 1, 2, j), where $j \geq 4$ is even. Therefore, all states communicate.

Combining the results, states of the form (i, 1, 1, 1), (i, 1, 2, 2), (i, 2, 2, 2), (i, 2, 1, j), and (i, 1, 2, $j+1$) for $i \in \{1, 2\}$ and $j \geq 3$ odd form a communicating class. This class is unique because any optimal investment decision results in some state (i,1,1,1). Thus $\mathcal{Q}'_t$ is irreducible. ∎

Proof of Proposition 4. By Lemma 3, the Markov chains $\mathcal{Q}'_t$ induced by both insulated and politicized agencies are irreducible. Therefore a unique stationary distribution q exists that solves $q = q\mathbf{Q}'$ if and only if $\mathcal{Q}'_t$ is positive recurrent, where $\mathbf{Q}'$ is the probability transition matrix associated with $\mathcal{Q}'_t$. Existence is demonstrated through direct computation of q. (For the politicized case, positive recurrence is also guaranteed by the finiteness of $\mathcal{Q}'_t$.)

(i) Under an insulated bureaucracy and full deference, $\mu_1 \in \mathcal{D}_i$ and Lemma 2 imply that the states (1, 1, 1, 1) and (2, 1, 1, 1) coincide with the states (1, 1, 1) and (2, 1, 1) in the political process. Thus Table 2 implies the same long-run probabilities for states of the form $(i,\ 1,\ 1,\ 1)$:

$$q_{i,1,1,1} = \frac{\pi_i}{2(1+\pi_i)}$$

Since investments take place under under political states (1, 1, 1) and (2, 1, 1), $q_{i,\theta_i,\theta_b,1} = 0$ for all other states where $j = 1$. Observe also that any state where $\theta_b = 1$ (2) must be preceded by one where $\theta_b = 2$ (1). Finally, any state such that $j > 1$ can be accessed only through states of the form $(i, \theta_i, \theta_b, j-1)$. Thus for any $j \geq 2$, the stationary probability for each group i, where it exists, is given by:

$$q_{i,1,1,j} = 0 \tag{37}$$

$$q_{i,1,2,j} = \pi_i \left(q_{1,2,1,j-1} + q_{2,2,1,j-1} + q_{-i,1,1,j-1}\right) \tag{38}$$

$$q_{i,2,1,j} = \pi_i q_{1,1,2,j-1} \tag{39}$$

$$q_{i,2,2,j} = \pi_i q_{1,1,1,j-1} \tag{40}$$

I establish the probabilities for j up to 5 iteratively. Applying the $j = 1$ results, simplifying (37)-(40) for $j = 2$ produces the following probabilities:

$$q_{i,1,2,2} = \pi_i q_{-i,1,1,1} = \frac{\pi_1\pi_2}{2(1+\pi_{-i})}$$
$$q_{i,2,2,2} = \pi_i q_{i,1,1,1} = \frac{\pi_i^2}{2(1+\pi_i)}$$

Note that $q_{i,1,1,2} = q_{i,2,1,2} = 0$ in equilibrium.

Performing the same exercise for $j = 3$ produces:

$$q_{i,2,1,3} = \pi_i q_{i,1,2,2} = \pi_i^2 q_{-i,1,1,1} = \frac{\pi_i^2\pi_{-i}}{2(1+\pi_{-i})}$$

Note that $q_{i,1,1,3} = q_{i,1,2,3} = q_{i,2,2,3} = 0$ in equilibrium.

Repeating this exercise for $j = 4$ produces the following positive stationary probabilities:

$$\begin{aligned} q_{i,1,2,4} &= \pi_i \left(q_{1,2,1,3} + q_{2,2,1,3}\right) = \pi_i \left(\pi_1^2 q_{2,1,1,1} + \pi_2^2 q_{1,1,1,1}\right) \\ &= \frac{\pi_i^2\pi_{-i}}{2}\left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right). \end{aligned}$$

Finally, for $j = 5$ the positive stationary probabilities probabilities are:

$$
\begin{aligned}
q_{i,2,1,5} &= \pi_i q_{i,1,2,4} = \pi_i^2 \left(\pi_1^2 q_{2,1,1,1} + \pi_2^2 q_{1,1,1,1}\right) \\
&= \frac{\pi_i^3 \pi_{-i}}{2} \left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right).
\end{aligned}
$$

I show by induction that for any even integer $j' > 4$,

$$
q_{i,1,2,j'} = \left(\pi_1^2 + \pi_2^2\right)^{\frac{j'}{2}-2} \frac{\pi_i^2 \pi_{-i}}{2} \left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right).
$$

And for $j'+1$ (i.e., odd),

$$
q_{i,2,1,j'+1} = \left(\pi_1^2 + \pi_2^2\right)^{\frac{j'}{2}-2} \frac{\pi_i^3 \pi_{-i}}{2} \left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right).
$$

These expressions are clearly true for $j' = 4$.

For the induction step, apply the transition probabilities (37)-(40), which produces for $j'+2$ (even):

$$
\begin{aligned}
q_{i,1,2,j'+2} &= \pi_i \left(q_{1,2,1,j'+1} + q_{2,2,1,j'+1}\right) \\
&= \left(\pi_1^2 + \pi_2^2\right)^{\frac{j'}{2}-1} \frac{\pi_i^2 \pi_{-i}}{2} \left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right).
\end{aligned}
$$

Correspondingly, for $j'+3$ (odd):

$$
\begin{aligned}
q_{i,2,1,j'+3} &= \pi_i q_{i,1,2,j'+2} \\
&= \left(\pi_1^2 + \pi_2^2\right)^{\frac{j'}{2}-1} \frac{\pi_i^3 \pi_{-i}}{2} \left(\frac{\pi_1}{1+\pi_2} + \frac{\pi_2}{1+\pi_1}\right).
\end{aligned}
$$

This completes the induction. Given these probabilities, expected equilibrium capacity is the sum of capacity levels weighted by $q_{i,\theta_i,\theta_b,j}$:

$$\begin{aligned}
&\sum_{i=1}^{2}\sum_{\theta_i=1}^{2}\sum_{\theta_b=1}^{2}\sum_{j=1}^{\infty}\delta^j q_{i,\theta_i,\theta_b,j}\mu_b^0(1)\\
=&\mu_b^0(1)\left[\delta\sum_{i=1}^{2}q_{i,1,1,1}+\delta^2\sum_{i=1}^{2}\sum_{\theta_i=1}^{2}q_{i,\theta_i,2,2}+\delta^3\sum_{i=1}^{2}q_{i,2,1,3}+\sum_{i=1}^{2}\sum_{j=4}^{\infty}\delta^j(q_{i,1,2,j}+q_{i,2,1,j})\right]\\
=&\mu_b^0(1)\left[\delta\sum_{i=1}^{2}\frac{\pi_i}{2(1+\pi_i)}+\delta^2\sum_{i=1}^{2}\left(\frac{\pi_i^2}{2(1+\pi_i)}+\frac{\pi_1\pi_2}{2(1+\pi_{-i})}\right)+\delta^3\sum_{i=1}^{2}\frac{\pi_i^2\pi_{-i}}{2(1+\pi_{-i})}+\right.\\
&\quad\sum_{i=1}^{2}\sum_{\chi=1}^{\infty}\delta^{4+2\chi}(\pi_1^2+\pi_2^2)^\chi\frac{\pi_i^2\pi_{-i}}{2}\left(\frac{\pi_1}{1+\pi_2}+\frac{\pi_2}{1+\pi_1}\right)+\\
&\quad\left.\sum_{i=1}^{2}\sum_{\chi=1}^{\infty}\delta^{5+2\chi}(\pi_1^2+\pi_2^2)^\chi\frac{\pi_i^3\pi_{-i}}{2}\left(\frac{\pi_1}{1+\pi_2}+\frac{\pi_2}{1+\pi_1}\right)\right]\\
=&\mu_b^0(1)\left[\frac{\delta\left(\delta^2\pi_1^3\pi_2+\pi_1^2(\delta+(2+\delta)\delta\pi_2)+\pi_1\left(\delta^2\pi_2^3+\delta(2+\delta)\pi_2^2+2(1+\delta)\pi_2+1\right)+\pi_2(1+\delta\pi_2)\right)}{2(1+\pi_1)(1+\pi_2)}+\right.\\
&\quad\sum_{i=1}^{2}\frac{\delta^4}{1-\delta^2(\pi_1^2+\pi_2^2)}\frac{\pi_i^2\pi_{-i}}{2}\left(\frac{\pi_1}{1+\pi_2}+\frac{\pi_2}{1+\pi_1}\right)+\\
&\quad\left.\sum_{i=1}^{2}\frac{\delta^5}{1-\delta^2(\pi_1^2+\pi_2^2)}\frac{\pi_i^3\pi_{-i}}{2}\left(\frac{\pi_1}{1+\pi_2}+\frac{\pi_2}{1+\pi_1}\right)\right].
\end{aligned}$$

Substituting $\pi_2 = 1 - \pi_1$ and simplifying produces the result.

(ii) Under politicization and full deference, $\mu_1 \in \mathcal{D}_i$ and Lemma 2 imply that states of the form (i, 1, 1, 1) and (2, 1, 1, 1) occur whenever a new politician is elected. Furthermore, the only other states occur when a new politician is re-elected, and are thus of the form (i, 2, 2, 2). Applying re-election probabilities, the long run probabilities of each state is characterized by the following system:

$$\begin{aligned}
q_{1,1,1,1} &= \pi_1\left(q_{1,2,2,2}+q_{2,1,1,1}+q_{2,2,2,2}\right)\\
q_{1,2,2,2} &= \pi_1 q_{1,1,1,1}\\
q_{2,1,1,1} &= \pi_2\left(q_{1,1,1,1}+q_{1,2,2,2}+q_{2,2,2,2}\right)\\
q_{2,2,2,2} &= \pi_2 q_{2,1,1,1}
\end{aligned}$$

Solving this system produces:

$$
\begin{aligned}
q_{i,1,1,1} &= \frac{\pi_i}{1+\pi_i} \\
q_{i,2,2,2} &= \frac{\pi_i^2}{1+\pi_i}.
\end{aligned}
$$

Noting that delegation produces investment result $\mu_b^0(\pi_i)$ for each group i, the expected capacity level is then given by:

$$
\begin{aligned}
&\delta\left(q_{1,1,1,1}+\delta q_{1,2,2,2}\right)\mu_b^0(\pi_1)+\delta\left(q_{2,1,1,1}+\delta q_{2,2,2,2}\right)\mu_b^0(\pi_2) \\
&=\sum_{i=1}^{2}\frac{\pi_i(1+\delta\pi_i)\left(\Lambda+\sqrt{\delta\pi_i m_b/\kappa_b}\right)}{1+\pi_i}. \quad \blacksquare
\end{aligned}
$$

Proof of Corollary 2. (i) Taking the first order condition of the expected quality under insulation (21) produces:

$$
\frac{(1-\delta)(1+\delta)^2(2\pi_1-1)\left(\delta^2\left(2\pi_1^4-4\pi_1^3+6\pi_1^2-4\pi_1-1\right)-3\right)\left(\Lambda+\sqrt{\delta m_b/\kappa_b}\right)}{2(\pi_1-2)^2(1+\pi_1)^2\left(\delta^2\left(2\pi_1^2-2\pi_1+1\right)-1\right)^2}=0.
$$

This produces the solutions for π_1 at $1/2$, $1/2\pm\left(\sqrt{-2\sqrt{6/\delta^2+6}-3}\right)/2$, and $1/2\pm\left(\sqrt{2\sqrt{6/\delta^2+6}-3}\right)/2$. Of these, only $1/2$ is in $[0,1]$. Evaluating the second derivative of (21) at $\pi_1=1/2$ produces.

$$
\frac{8(1+\delta)^2\left(5\delta^3-5\delta^2+8\delta-8\right)\left(\Lambda+\sqrt{\delta m_b/\kappa_b}\right)}{27\left(2-\delta^2\right)^2}.
$$

This expression is clearly negative. Since the objective is continuous on $[0,1]$, (21) is maximized at $\pi_1=1/2$.

(ii) Taking the first order condition of quality under politicization (22) with respect to π_1 (keeping in mind $\pi_2=1-\pi_1$) produces:

$$
\begin{aligned}
&\frac{2\Lambda(1+2\delta\pi_1)+(3+5\delta\pi_1)\sqrt{\frac{\delta m_b\pi_1}{w}}}{2(1+\pi_1)}-\frac{\pi_1(1+\delta\pi_1)\left(\Lambda+\sqrt{\frac{\delta m_b\pi_1}{w}}\right)}{(1+\pi_1)^2}- \\
&\frac{(1-\pi_1)(\delta(1-\pi_1)+1)\left(\Lambda+\sqrt{\frac{\delta m_b(1-\pi_1)}{w}}\right)}{(2-\pi_1)^2}+\frac{2\Lambda(2\delta(1-\pi_1)+1)+(3+5\delta(1-\pi_1))\sqrt{\frac{\delta m_b(1-\pi_1)}{w}}}{2(2-\pi_1)}.
\end{aligned}
$$

Substituting in $\pi_1 = 1/2$ produces a value of 0. To check for local concavity, the second order condition at $\pi_1 = 1/2$ evaluates to:

$$-\frac{1}{54}\left(64(1-\delta)\Lambda + \sqrt{2}(2-83\delta)\sqrt{\frac{\delta m_b}{w}}\right).$$

This expression is obviously strictly positive (resp., negative) at $\delta = 1$ (resp., 0). Taking the second derivative with respect to δ produces $\frac{2+249\delta}{108}\sqrt{\frac{m_b}{2\delta^3\kappa_b}} > 0$. Thus there exists a unique $\delta_p \in (0,1)$ such that the $\pi = 1/2$ is not a local maximum for $\delta > \delta_p$.

(iii) Define $\Delta(\pi_1, \delta)$ as expression (21) minus expression (22), or the payoff advantage of insulation over politicization.

At $\delta = 1$, expected quality under insulation is higher if:

$$\Delta(\pi_1, 1) = \sqrt{\frac{m_b}{\kappa_b}}\left[\left(1-\sqrt{1-\pi_1}\right) + \pi_1\left(\sqrt{1-\pi_1} - \sqrt{\pi_1}\right)\right] > 0. \tag{41}$$

It is straightforward to verify that (41) is strictly positive, concave, and maximized at $\pi_1 = 1/2$, establishing the result for $\delta = 1$. Moreover, since $\Delta(\pi_1, \delta)$ is continuous in δ, it must be strictly positive for a neighborhood of $\delta = 1$.

At $\pi_1 = 1/2$, it is easily verified that:

$$\begin{aligned}\Delta(1/2, 0) &= -\frac{\Lambda}{3}\\ \Delta(1/2, 1) &= \left(1-\frac{\sqrt{2}}{2}\right)\sqrt{\frac{m_b}{\kappa_b}}.\end{aligned}$$

Since $\Delta(1/2, 0) < 0 < \Delta(1/2, 1)$, there is a unique $\hat{\delta} \in (0,1)$ if $\Delta(1/2, \delta)$ is concave in δ. Evaluating the second derivative of $\Delta(\cdot)$ with respect to δ at $p = 1/2$ produces:

$$-\frac{2\left(\delta^3+3\delta^2+6\delta+2\right)\Lambda}{3(\delta^2-2)^3} - \frac{\sqrt{m_b}}{24(\delta^2-2)^3\sqrt{\delta\kappa_b}}\Big[3(\sqrt{2}-1)\delta^6 + (1-2\sqrt{2})\delta^5 - 6(3\sqrt{2}-4)\delta^4 + 12(\sqrt{2}+2)\delta^3 + 4(9\sqrt{2}+17)\delta^2 + (84-24\sqrt{2})\delta - 24(\sqrt{2}-2) + \frac{16}{\delta}(\sqrt{2}-1)\Big].$$

It is straightforward to verify that this expression is negative for $\delta \in [0,1]$. ■