Regression Tables

The following table provides the coefficients, standard errors, and model fit information for the four specifications we present as our primary results. See the replication materials for the results of our robustness tests.

Table 1. Robustness Tests

Measurement strategy	EFFECTIVENESS, SUPPORT, INFLUENCE	EFFECTIVENESS, DISTANCE, INFLUENCE	EXPENDITURES, SUPPORT, INFLUENCE	EXPENDITURES, DISTANCE, INFLUENCE
Coefficients	(1)	(2)	(3)	(4)
q_0	0.173^{***}	0.0783^{***}	0.259^{***}	0.228^{***}
	(0.0264)	(0.0253)	(0.0262)	(0.0248)
b	0.259^{***}	0.276^{***}	0.268^{***}	0.292^{***}
	(0.026)	(0.0276)	(0.0265)	(0.0292)
$E[c]$	0.162^{***}	-0.116^{***}	0.112^{***}	-0.0815^{***}
	(0.0290)	(0.0281)	(0.0279)	(0.0281)
$E[c] \times b$	0.0782^{***}	-0.0960^{***}	0.0833^{***}	-0.111^{***}
	(0.0279)	(0.0298)	(0.0278)	(0.0316)
$q_0 \times E[c]$	0.151^{***}	-0.179^{***}	0.144^{***}	-0.0939^{***}
	(0.0229)	(0.0286)	(0.0261)	(0.0280)
PILOT	-0.108^{*}	-0.306^{***}	-0.166^{***}	-0.298^{***}
	(0.0591)	(0.0567)	(0.0596)	(0.0569)
LENGTH	0.348^{***}	0.422^{***}	0.360^{***}	0.432^{***}
	(0.0274)	(0.0331)	(0.0278)	(0.0335)
COMMISSION	-0.679^{***}	-0.677^{***}	-0.686^{***}	-0.664^{***}
	(0.0672)	(0.0670)	(0.0683)	(0.0680)
ADDRESSEE	-0.148	-0.155	-0.153	-0.137
	(0.186)	(0.185)	(0.187)	(0.186)
SCOPE	-0.00668^{***}	-0.119^{***}	-0.00694^{***}	-0.122^{***}
	(0.00163)	(0.0296)	(0.00169)	(0.0301)
Constant	-3.637^{***}	-0.796^{***}	-3.749^{***}	-0.808^{***}
	(0.245)	(.0370)	(0.248)	(0.0376)
Marginal Effects	(1)	(2)	(3)	(4)
q_0	0.0308^{***}	0.014^{***}	0.0456^{***}	0.0405^{***}
	(0.00492)	(0.00455)	(0.00478)	(0.00444)
b	0.0493^{***}	0.0496^{***}	0.0480^{***}	0.0518^{***}
	(0.00495)	(0.00501)	(0.00487)	(0.00520)
N	9,415	9,361	9,289	9,235
(Pseudo) R^2	0.0767	0.0738	0.0827	0.0774

Notes: Logit regressions with heteroskedasticity-robust standard errors in parentheses. Marginal effects calculated holding all other variables at their means. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Proof: Equilibrium Solution

Proof. Assume continuous, unbounded support for all k_i and c.

The Commission issues a referral to the Court when $EU_C(RF) = E[q_3(c) \mid c \leq c^\circ]b - k_1 - k_2 - k_3 \geq EU_C(\neg RF) = -k_1 - k_2$, where c° is the cost below which the government will not comply with a reasoned opinion in equilibrium. This condition simplifies to $k_3 \leq k_3^\circ = E[q_3(c) \mid c \leq c^\circ]b$. As long as the government cutpoint strategy, c°, exists, because $q_3(c)$ is monotone in c by assumption, the best response function k_3° exists. Define $\ell_3(c^\circ)$ as the belief of the government that the Commission will make a referral. By Bayes' Rule $\ell_3(c^\circ) = \Pr(RF) = \Pr(k_3 \leq E[q_3(c) \mid c \leq c^\circ]b)$.

The government complies with a reasoned opinion when $EU_G(C_{RO}) = c \geq EU_G(\neg C_{RO}) = \ell_3(c^\circ)(q_3(c)(c-j) + (1-q_3(c))(0)) + (1-\ell_3(c))(0)$. This condition simplifies to $c \geq c^\circ = -\frac{\ell_3(c^\circ)q_3(c^\circ)j}{1-\ell_3(c^\circ)q_3(c^\circ)}$. Because $\frac{dq_3(c)}{dc} > 0$, $\lim_{c \to -\infty} q_3(c) \to 0$, and $\lim_{c \to 0} q_3(c) \to 1$ by assumption, $\frac{d\ell_3(c^\circ)}{dc^\circ} > 0$, and $j > 0$ by assumption, $\lim_{c^\circ \to -\infty} -\frac{\ell_3(c^\circ)q_3(c^\circ)j}{1-\ell_3(c^\circ)q_3(c^\circ)} \to 0$. Since $-\frac{\ell_3(c^\circ)q_3(c^\circ)j}{1-\ell_3(c^\circ)q_3(c^\circ)}$ is decreasing as $\lim_{c^\circ \to 0}$, for $\ell_3(c^\circ)$ sufficiently large (i.e., as long as the distribution of $k_3 \leq E[q_3(c) \mid c \leq c^\circ]b$ has sufficient density) a $c^\circ < 0$ exists. Otherwise $c^\circ = 0$. Define p_2 as the belief of the Commission that the government will comply with a reasoned opinion. By Bayes' Rule $p_2(c^\circ) = \Pr(C_{RO}) = \Pr(c \geq c^\circ \mid c < 0)$. We demonstrate below that the government complies with letters of formal notice whenever $c \geq 0$, thus the conditional probability.

The Commission issues a reasoned opinion when $EU_C(RO) = p_2(c^\circ)(b - k_1 - k_2) + (1-p_2(c^\circ))(\ell_3(c^\circ)(E[q_3(c) \mid c \leq c^\circ]b - k_1 - k_2 - E[k_3 \mid k_3 \leq k_3^\circ]) + (1-\ell_3(c^\circ))(-k_1 - k_2)) \geq EU_C(\neg RO) = -k_1$. Solving for k_2 yields the Commission's best reply function $k_2 \leq k_2^\circ = p_2(c^\circ)b + (1-p_2(c^\circ))\ell_3(c^\circ)(E[q_3(c) \mid c \leq c^\circ]b - E[k_3 \mid k_3 \leq k_3^\circ])$. Define $\ell_2(c^\circ)$ as the belief of the government that the Commission will bring a reasoned opinion. By Bayes' Rule $\ell_2(c^\circ) = pr(RO) = pr(k_2 \leq b(p_2(c^\circ) + \ell_3(c^\circ)E[q_3(c) \mid c \leq c^\circ] - p_2(c^\circ)\ell_3(c^\circ)E[q_3(c) \mid c \leq c^\circ]) - \ell_3(c^\circ)(1-p_2(c^\circ))E[k_3 \mid k_3 \leq k_3^\circ])$.

The government complies with a letter of formal notice when $EU_G(C_{LFN}) \geq EU_G(\neg C_{LFN})$. Note that $EU_G(C_{LFN}) = c$ and $EU_G(\neg C_{LFN}) = \ell_2(c^\circ)(\max\{EU_G(C_{RO}), EU_G(\neg C_{RO})\}) + (1-\ell_2(c^\circ))(0)$. If $c \geq 0$ it is a weakly dominant strategy for the government to comply, because $c \geq \max\{EU_G(C_{RO}), EU_G(\neg C_{RO})\}$ when $c \geq 0$. $EU_G(C_{RO}) = c$, and $EU_G(\neg C_{RO}) < c$ ($EU_G(\neg C_{RO})$ is a convex combination of payoffs of 0 and $c-j$). If $c < 0$, it is a strictly dominant strategy for the government not to comply. Because $EU_G(C_{RO}) = c$, the government can assure itself at least a convex combination of payoffs of c and 0 by playing $EU_G(\neg C_{LFN})$. Define p_1 as the belief of the Commission that the government will comply with a letter of formal notice. By Bayes' Rule, $p_1(c^\circ) = \Pr(C_{LFN})$, where $\Pr(C_{LFN}) = \Pr(c > 0 \mid \neg C_0) = \frac{q_0 \Pr(c \geq 0)}{1-p_0}$. We demonstrate below that the government plays C_0 whenever $c \geq 0$, and thus $c \geq 0$ only occurs here if there is an accidental instance of noncompliance.

The Commission issues a letter of formal notice when $EU_C(LFN) = p_1(b - k_1) + (1 - p_1)(\ell_2(c^\circ)(p_2(c^\circ)(b - k_1 - E[k_2 \mid k_2 \leq k_2^\circ]) + (1 - p_2(c^\circ))(\ell_3(c^\circ)(E[q_3(c) \mid c \leq c^\circ]b - k_1 - E[k_2 \mid k_2 \leq k_2^\circ] - E[k_3 \mid k_3 \leq k_3^\circ]) + (1 - \ell_3(c^\circ))(-k_1 - E[k_2 \mid k_2 \leq k_2^\circ]))) + (1 - \ell_2(c^\circ))(-k_1)) \geq EU_C(\neg LFN) = 0$. Solving for k_1 yields the Commission's best reply function $k_1 \leq k_1^\circ = p_1 b + (1 - p_1)(\ell_2(c^\circ)(p_2(c^\circ)b + (1 - p_2(c^\circ))(\ell_3(c^\circ)(E[q_3(c) \mid c \leq c^\circ]b - E[k_3 \mid k_3 \leq k_3^\circ])) - E[k_2 \mid k_2 \leq k_2^\circ])$. Define $\ell_1(c^\circ)$ as the belief of the government that the Commission will bring a letter of formal notice. By Bayes' Rule, $\ell_1(c^\circ) = \Pr(LFN) = \Pr(k_2 \leq k_2^\circ)$.

The government ex ante complies when $EU_G(C_0) \geq EU_G(\neg C_0)$. Note that $EU_G(C_0) = c$ and $EU_G(\neg C_0) = \ell_1(\max\{EU_G(C_{LFN}), EU_G(\neg C_{LFN})\}) + (1 - \ell_1)(0)$. If $c \geq 0$ it is a weakly dominant strategy for the government to comply. $c \geq \max\{EU_G(C_{LNF}), EU_G(\neg C_{LFN})\}$ when $c \geq 0$ since $EU_G(C_{LNF}) = c$, $EU_G(C_{RO}) = c$, and $EU_G(\neg C_{RO}) < c$. If $c < 0$, it is a strictly dominant strategy for the government not to comply. Because $EU_G(C_{LFN}) = c$, the government can assure itself at least a convex combination of payoffs of c and 0 by playing $EU_G(\neg C_0)$. Define p_0 as the belief of the Commission that the government will ex ante comply. By Bayes' Rule, $p_0 = \Pr(C_0) = \Pr(c \geq 0)(1 - q_0)$.

This system of best replies defines the Perfect Bayesian Equilibrium for the game. It is summarized below. □

$$C_0^* = \begin{cases} C_0 & \textit{if } c \geq 0 \\ \neg C_0 & \textit{otherwise} \end{cases}$$

$$LFN^* = \begin{cases} LFN & \textit{if } k_1 \leq k_1^* \\ \neg LFN & \textit{otherwise} \end{cases}$$

$$C_{LFN}^* = \begin{cases} C_{LFN} & \textit{if } c \geq 0 \\ \neg C_{LFN} & \textit{otherwise} \end{cases}$$

$$RO^* = \begin{cases} RO & \textit{if } k_2 \leq k_2^* \\ \neg RO & \textit{otherwise} \end{cases}$$

$$C_{RO}^* = \begin{cases} C_{RO} & \textit{if } 0 > c \geq c^* \\ \neg C_{RO} & \textit{otherwise} \end{cases}$$

$$RF^* = \begin{cases} RF & \textit{if } k_3 \leq k_3^* \\ \neg\, RF & \textit{otherwise} \end{cases}$$

$$c^* = -\frac{\ell_3(c^*)q_3(c^*)j}{1-\ell_3(c^*)q_3(c^*)}$$

$$\begin{aligned} k_1^* &= p_1 b + (1-p_1)(\ell_2(c^*)(p_2(c^*)b + (1-p_2(c^*)) \\ &\quad (\ell_3(c^*)(E[q_3(c) \mid c \leq c^*]b - E[k_3 \mid k_3 \leq k_3^*])) - E[k_2 \mid k_2 \leq k_2^*]) \\ k_2^* &= p_2(c^*)b + (1-p_2(c^*))\ell_3(c^*)(E[q_3(c) \mid c \leq c^*]b - E[k_3 \mid k_3 \leq k_3^*]) \\ k_3^* &= E[q_3(c) \mid c \leq c^*]b \end{aligned}$$

$$\begin{aligned} \ell_1(c^*) &= \Pr(k_1 \leq k_1^*) \\ \ell_2(c^*) &= \Pr(k_2 \leq k_2^*) \\ \ell_3(c^*) &= \Pr(k_3 \leq k_3^*) \end{aligned}$$

$$\begin{aligned} p_0 &= \Pr(c \geq 0)(1-q_0) \\ p_1 &= \frac{q_0 \Pr(c \geq 0)}{1-p_0} \\ p_2(c^*) &= \Pr(c \geq c^* \mid c < 0) \end{aligned}$$

Proof: Comparative Statics

Result 1

Proof. To demonstrate $\frac{\partial(1-p_2(c^*))\ell_3(c^*)}{\partial q_0} = 0$, simply note that both $\ell_3(c^*)$ and $p_2(c^*)$ are independent of q_0. We prove the remaining components of the result by first examining how $\ell_3(c^*)$ and then $p_2(c^*)$ change in b and $E[c]$, respectively.

Consider $\frac{\partial \ell_3(c^*)}{\partial b}$. We employ proof by contradiction. Hypothesize $\frac{\partial \ell_3(c^*)}{\partial b} < 0$. Holding c^* constant, because $k_3^* = E[q(c) \mid c \leq c^*]b$, increasing b increases k_3^*, and therefore $\ell_3(c^*) = \Pr(k \leq k_3^*)$ increases for a fixed c^*.

Now consider $c^* = -\frac{\ell_3(c^*)q_3(c^*)j}{1-\ell_3(c^*)q_3(c^*)}$. Increasing $\ell_3(c^*)$, holding $q_3(c^*)$ constant, decreases c^*. Note that $\frac{\partial q_3(c^*)}{\partial c^*} > 0$, and therefore the effect of a change in $\ell_3(c^*)$ on c^* is moderated by $q_3(c^*)$ If this indirect effect yields $\frac{\partial c^*}{\partial \ell_3(c^*)} > 0$ we immediately have a contradiction since that would increase $\ell_3(c^*)$. Otherwise, $\frac{\partial c^*}{\partial \ell_3(c^*)} < 0$.

Recall $\ell_3(c^*) = \Pr(k_3 \leq E[q_3(c) \mid c \leq c^*]b)$. Because $\frac{\partial \ell_3(c^*)}{\partial c^*} > 0$, for $\frac{\partial \ell_3(c^*)}{\partial b} < 0$, as hypothesized, the decrease in $\ell_3(c^*)$ caused by the indirect effect of b on $\ell_3(c^*)$ through c^* must be larger than b's direct increase in $\ell_3(c^*)$. However, if $\frac{\partial \ell_3(c^*)}{\partial b} < 0$, increasing b does not decrease c^* and we have a contradiction. Therefore, $\frac{\partial \ell_3(c^*)}{\partial b} > 0$.

A nearly identical argument holds for $\frac{\partial \ell(c^*)}{\partial E[c]}$. Hypothesize $\frac{\partial \ell(c^*)}{\partial E[c]} < 0$. Because $E[q_3(c) \mid c \leq c^*]$ is increasing in $E[c]$, holding c^* constant, $\ell_3(c^*)$ is also increasing in $E[c]$, holding c^* constant. The remainder of the proof by contradiction from $\frac{\partial \ell_3(c^*)}{\partial b}$ follows as above. Thus, we have $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$.

Now consider $\frac{\partial p_2(c^*)}{\partial b}$. From the equilibrium proof, $p_2(c^*) = \Pr\left(c \geq \frac{\ell_3 q_3(c^*)j}{1-\ell_3 q_3(c^*)} \mid c < 0\right)$. Let F be the CDF of c. Then, $p_2(c^*) = \frac{1-F(c^*)-(1-F(0))}{F(0)} = \frac{F(0)-F(c^*)}{F(0)}$, or equivalently, $p_2(c^*) = \frac{\Pr(c<0)-\Pr(c<c^*)}{\Pr(c<0)}$. Because $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial c^*}{\partial \ell_3(c^*)} < 0$ (from above), and $\frac{\partial \Pr(c<c^*)}{\partial c^*} > 0$, $\frac{\partial p_2(c^*)}{\partial b} > 0$.

Finally, consider $\frac{\partial p_2(c^*)}{\partial E[c]}$. Because $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$ and $\frac{\partial c^*}{\partial \ell_3(c^*)} < 0$ (from above), $\frac{\partial c^*}{\partial E[c]} < 0$. Since $\frac{\partial c^*}{\partial E[c]} < 0$ and $\frac{\partial \Pr(c<c^*)}{\partial c^*} > 0$, $\frac{\partial \Pr(c<c^*)}{\partial E[c]} < 0$. We also know by definition $\frac{\partial \Pr(c<0)}{\partial E[c]} < 0$. Because $\frac{\partial c^*}{\partial E[c]} < 0$, $\frac{\partial(\Pr(c<0)-\Pr(c<c^*))}{\partial E[c]} > 0$. That, combined with $\frac{\partial \frac{1}{\Pr(c<0)}}{\partial E[c]} > 0$, implies $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$.

Since $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial p_2(c^*)}{\partial b} > 0$, the sign of $\frac{\partial[(1-p_2(c^*))\ell_3(c^*)]}{\partial b}$ is ambiguous. Since $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$ and $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$, the sign of $\frac{\partial[(1-p_2(c^*))\ell_3(c^*)]}{\partial E[c]}$ is ambiguous. □

Result 2

Proof. From the equilibrium proof, $p_1 = \frac{q_0 \Pr(c \geq 0)}{1-p_0} = \frac{q_0(1-\Pr(c<0))}{1-(1-\Pr(c<0))(1-q_0)}$. Then, $\frac{\partial p_1}{\partial q_0} = \frac{\Pr(c<0)-\Pr(c<0)^2}{(\Pr(c<0)+q_0-\Pr(c<0)q_0)^2}$. Both the numerator and denominator are positive; thus, $\frac{\partial p_1}{\partial q_0} > 0$. Note that $\ell_2(c^*)$ does not contain q_0. Thus, $\frac{\partial[(1-p_1)(\ell_2(c^*))]}{\partial q_0} < 0$. □

Result 3

Proof. From Result 2, $\frac{\partial p_1}{\partial q_0} = \frac{\Pr(c<0)-\Pr(c<0)^2}{(\Pr(c<0)+q_0-\Pr(c<0)q_0)^2} > 0$. Taking the cross-partial with respect to $\Pr(c<0)$, we have $\frac{\partial p_1}{\partial q_0 \partial \Pr(c<0)} = \frac{\Pr(c<0)-q_0+\Pr(c<0)q_0}{(\Pr(c<0)q_0-\Pr(c<0)-q_0)^3}$. The sign of $\frac{\partial p_1}{\partial q_0 \partial \Pr(c<0)}$ depends on parameter values; $\frac{\partial p_1}{\partial q_0 \partial \Pr(c<0)} < 0$ when $\Pr(c<0) > \frac{q_0}{1+q_0}$. Thus, the positive effect of q_0 on p_1 is decreasing in $\Pr(c<0)$, which implies that the negative effect of q_0 on $\Pr(RO \mid LFN) = (1-p_1)\ell_2(c^*)$ is decreasing in $\Pr(c<0)$.

Note that q_0 is the probability of unintentional noncompliance conditional on the government choosing to comply. The unconditional probability of unintentional noncompliance is the joint probability that the government chooses to comply and unintentionally commits a violation, $q_0 \Pr(c \geq 0)$. The probability of intentional noncompliance, $\Pr(c<0)$, is greater than the probability of accidental noncompliance when $\Pr(c<0) \geq q_0 \Pr(c \geq 0)$, which is equivalent to the condition under which $\frac{\partial p_1}{\partial q_0 \partial \Pr(c<0)} < 0$. □

Result 4

Proof. From the equilibrium proof, $\ell_2(c^*) = \Pr(k_2 \leq k_2^*)$. Since $\Pr(k_2 \leq k_2^*)$ is increasing in k_2^*, we prove that $\frac{\partial k_2^*}{\partial b} > 0$. Substituting in k_3^*, we have $k_2^* = bp_2(c^*) + \ell_3(c^*)k_3^* - p_2(c^*)\ell_3(c^*)k_3^* - \ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*]$. We then take the derivative with respect to b:

$$\begin{aligned}
\frac{\partial k_2^*}{\partial b} = & \left(\frac{\partial p_2(c^*)}{\partial b}b + p_2(c^*)\right) + \left(\frac{\partial \ell_3(c^*)}{\partial b}k_3^* + \ell_3(c^*)\frac{\partial k_3^*}{\partial b}\right) \\
& - \left(\frac{\partial p_2(c^*)}{\partial b}\ell_3(c*)k_3^* + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial b}k_3^* + p_2(c^*)\ell_3(c^*)\frac{\partial k_3^*}{\partial b}\right) \\
& - \left(\frac{\partial \ell_3(c^*)}{\partial b}E[k_3 \mid k_3 \leq k_3^*] + \ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b}\right) \\
& + \left(\frac{\partial p_2(c^*)}{\partial b}\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial b}E[k_3 \mid k_3 \leq k_3^*] \right. \\
& \left. + p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b}\right).
\end{aligned}$$

Reorganizing terms yields:

$$\frac{\partial k_2^*}{\partial b} = p_2(c^*) + \frac{\partial p_2(c^*)}{\partial b}(b - \ell_3(c^*)k_3^*) + \frac{\partial \ell_3(c^*)}{\partial b}(1 - p_2(c^*))(k_3^* - E[k_3 \mid k_3 \leq k_3^*])$$
$$+ \ell_3(c^*)(1 - p_2(c^*))\left(\frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b}\right) + p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b}.$$

The first term, $p_2(c^*)$, is positive because $p_2(c^*) \in (0, 1)$.

The second term, $\frac{\partial p_2(c^*)}{\partial b}(b - \ell_3(c^*)k_3^*)$, is strictly positive, because b is strictly greater than $\ell_3(c^*)k_3^* = \ell_3(c^*)E[q_3(c) \mid c \leq c^*]b$ and $\frac{\partial p_2(c^*)}{\partial b} > 0$ from Result 1.

The third term, $\frac{\partial \ell_3(c^*)}{\partial b}(1 - p_2(c^*))(k_3^* - E[k_3 \mid k_3 \leq k_3^*])$, is strictly positive, because $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ from Result 1, $(1-p_2(c^*)) > 0$ since $p_2(c^*) \in (0, 1)$, and $(k_3^* - E[k_3 \mid k_3 \leq k_3^*]) > 0$ because k_3 is unbounded.

The fourth term, $\ell_3(c^*)(1 - p_2(c^*))\left(\frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3|k_3 \leq k_3^*]}{\partial b}\right)$, is strictly positive on average. First, we know $\ell_3(c^*)(1 - p_2(c^*)) > 0$, because $\ell_3(c^*)$ and $p_2(c^*)$ are probabilities. Second, from above, $k_3^* > E[k_3 \mid k_3 \leq k_3^*]$, which means k_3^* and $E[k_3 \mid k_3 \leq k_3^*]$ can not cross. Because they can not cross, $\frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3|k_3 \leq k_3^*]}{\partial b} > 0$ must hold on average.

The fifth term, $p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3|k_3 \leq k_3^*]}{\partial b}$ is strictly positive because $p_2(c^*)$ and $\ell_3(c^*)$ are probabilities and $\frac{\partial E[k_3|k_3 \leq k_3^*]}{\partial b}$ must be greater than zero since $\frac{\partial k_3^*}{\partial b} > 0$ from Result 1.

Since all terms are strictly positive on average, $\frac{\partial k_2^*}{\partial b} > 0$ on average. More generally, as long as the fourth term is not too negative at some point, the partial derivative is positive everywhere. Note that p_1 does not contain b. Thus, $\frac{\partial[(1-p_1)(\ell_2(c^*))]}{\partial b} > 0$. □

Result 5

Proof. Consider $\frac{\partial \ell_2(c^*)}{\partial E[c]}$. From the equilibrium proof, $\ell_2(c^*) = \Pr(k_2 \leq k_2^*)$. Since $\Pr(k_2 \leq k_2^*)$ is increasing in k_2^*, we prove that $\frac{\partial k_2^*}{\partial E[c]} > 0$. Substituting in k_3^*, we have $k_2^* = bp_2(c^*) + \ell_3(c^*)k_3^* - p_2(c^*)\ell_3(c^*)k_3^* - \ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*]$. We then take the derivative with respect to $E[c]$:

$$\frac{\partial k_2^*}{\partial E[c]} = \left(\frac{\partial p_2(c^*)}{\partial E[c]}b\right) + \left(\frac{\partial \ell_3(c^*)}{\partial E[c]}k_3^* + \ell_3(c^*)\frac{\partial k_3^*}{\partial E[c]}\right)$$
$$- \left(\frac{\partial p_2(c^*)}{\partial E[c]}\ell_3(c*)k_3^* + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial E[c]}k_3^* + p_2(c^*)\ell_3(c^*)\frac{\partial k_3^*}{\partial E[c]}\right)$$
$$- \left(\frac{\partial \ell_3(c^*)}{\partial E[c]}E[k_3 \mid k_3 \leq k_3^*] + \ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial E[c]}\right)$$
$$+ \left(\frac{\partial p_2(c^*)}{\partial E[c]}\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial E[c]}E[k_3 \mid k_3 \leq k_3^*]\right.$$
$$\left. + p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial E[c]}\right).$$

Reorganizing terms yields:

$$\frac{\partial k_2^*}{\partial E[c]} = \frac{\partial p_2(c^*)}{\partial E[c]}(b - \ell_3(c^*)k_3^*) + \frac{\partial \ell_3(c^*)}{\partial E[c]}(1 - p_2(c^*))(k_3^* - E[k_3 \mid k_3 \leq k_3^*])$$
$$+ \ell_3(c^*)(1 - p_2(c^*)) \left(\frac{\partial k_3^*}{\partial E[c]} - \frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial E[c]} \right) + p_2(c^*)\ell_3(c^*) \frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial E[c]}.$$

The first term, $\frac{\partial p_2(c^*)}{\partial E[c]}(b - \ell_3(c^*)k_3^*)$, is strictly positive, because b is strictly greater than $\ell_3(c^*)k_3^* = \ell_3(c^*)E[q_3(c) \mid c \leq c^*]b$ and $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$ from Result 1.

The second term, $\frac{\partial \ell_3(c^*)}{\partial E[c]}(1 - p_2(c^*))(k_3^* - E[k_3 \mid k_3 \leq k_3^*])$, is strictly positive, because $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$ from Result 1, $(1 - p_2(c^*)) > 0$ since $p_2(c^*) \in (0, 1)$, and $(k_3^* - E[k_3 \mid k_3 \leq k_3^*]) > 0$ because k_3 is unbounded.

The third term, $\ell_3(c^*)(1 - p_2(c^*)) \left(\frac{\partial k_3^*}{\partial E[c]} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} \right)$, is strictly positive on average. First, we know $\ell_3(c^*)(1 - p_2(c^*)) > 0$, because $\ell_3(c^*)$ and $p_2(c^*)$ are probabilities. Second, from above, $k_3^* > E[k_3 \mid k_3 \leq k_3^*]$, which means k_3^* and $E[k_3 \mid k_3 \leq k_3^*]$ can not cross. Because they can not cross, $\frac{\partial k_3^*}{\partial E[c]} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} > 0$ must hold on average.

The fourth term, $p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}$ is strictly positive because $p_2(c^*)$ and $\ell_3(c^*)$ are probabilities and $\frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]}$ must be greater than zero since $\frac{\partial k_3^*}{\partial E[c]} > 0$ from Result 1.

Since all terms are strictly positive on average, the derivative is strictly positive on average. More generally, as long as the third term is not too negative at some point, the partial derivative is positive everywhere.

Consider $\frac{\partial p_1}{\partial E[c]}$. From the equilibrium proof, $p_1 = \frac{q_0 \Pr(c \geq 0)}{1 - p_0} = \frac{q_0 \Pr(c \geq 0)}{1 - \Pr(c \geq 0)(1 - q_0)}$. The numerator is increasing in $E[c]$, and the denominator is decreasing in $E[c]$, so $\frac{\partial p_1}{\partial E[c]} > 0$.

Since $\frac{\partial \ell_2(c^*)}{\partial E[c]} > 0$ and $\frac{\partial p_1}{\partial E[c]} > 0$, the sign of $\frac{\partial [(1 - p_1)\ell_2(c^*)]}{\partial E[c]}$ is ambiguous.

We cannot feasibly sign the cross-partial with respect to b. It is

$$\frac{\partial^2 k_2^*}{\partial E[c] \partial b} = \frac{\partial^2 p_2(c^*)}{\partial E[c] \partial b}(b - \ell_3(c^*)X) + \frac{\partial p_2(c^*)}{\partial E[c]}(1 - (\frac{\partial \ell_3(c^*)}{\partial b}X + \ell_3(c^*)\frac{\partial X}{\partial b})$$
$$+ \frac{\partial^2 \ell_3(c^*)}{\partial E[c] \partial b}X + \frac{\partial \ell_3(c^*)}{\partial E[c]}\frac{\partial X}{\partial b} - \frac{\partial p_2(c^*)}{\partial b}\frac{\partial^2 \ell_3(c^*)}{\partial E[c]}X - p_2(c^*)\frac{\partial^2 \ell_3(c^*)}{\partial E[c] \partial b}X$$
$$+ \frac{\partial \ell_3(c^*)}{\partial b}\frac{\partial X}{\partial E[c]} + \ell_3(c^*)\frac{\partial^2 X}{\partial E[c] \partial b} - \frac{\partial p_2(c^*)}{\partial b}\ell_3(c^*)\frac{\partial X}{\partial E[c]}$$
$$- p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial b}\frac{\partial X}{\partial E[c]} - p_2(c^*)\ell_3(c^*)\frac{\partial^2 X}{\partial E[c] \partial b},$$

where $X = E[q_3(c) | c \leq c^*]b - E[k_3 | k_3 \leq k_3^*]$. Thus, we turn to a numeric solution. □

Numeric Solution

We estimate the sign of the cross-partial in Result 5 using a numeric solution. In each round of the simulation, we randomly draw values of exogenous parameters from uniform distributions and calculate endogenous parameters numerically, providing functional forms of probability distributions where necessary. We perform one thousand iterations of the simulation, keeping only in-equilibrium combinations of parameter values, and estimate the effect of exogenous parameters on endogenous parameters using OLS models. To simulate changes in $E[c]$, we change the lower bound of its uniform distribution, as changes in the upper bound do not effect endogenous parameters. Replication code is provided below.

```
##################################################
# set up
##################################################

# libraries
library(rootSolve)
library(numDeriv)
library(dplyr)
library(reshape2)

# set parameter values
c.lb <- -7
c.ub <- 7
k3.lb <- -5
k3.ub <- 5
b.par <- 3
j.par <- 1

##################################################
# functions
##################################################

u.pdf <- function(u.lower, u.upper) {
  1 / (u.upper - u.lower)
}

u.cdf <- function(value, u.lower, u.upper) {
  (value - u.lower) / (u.upper - u.lower)
}

q3 <- function(c, z = 1) {
  1 / (1 + exp(-z * c))
}

E.q3 <- function(c.star, c.lb, c.ub) {
```

```
  integrand <- function(c, c.lb, c.ub) {
    q3(c = c) * u.pdf(u.lower = c.lb, u.upper = c.ub)
  }

  integrate(f = Vectorize(integrand), lower = c.lb, upper = c.star, c.lb = c.lb, c
      .ub = c.ub)$value / u.cdf(value = c.star, u.lower = c.lb, u.upper = c.ub)
}

l3 <- function(c.star, b.par, c.lb, c.ub, k3.lb, k3.ub) {

  integrand <- function(k3, k3.lb, k3.ub) {
    u.pdf(u.lower = k3.lb, u.upper = k3.ub)
  }

  integrate(f = Vectorize(integrand), lower = k3.lb, upper = E.q3(c.star = c.star,
       c.lb = c.lb, c.ub = c.ub) * b.par, k3.lb = k3.lb, k3.ub = k3.ub)$value
}

c.star <- function(b.par, j.par, c.lb, c.ub, k3.lb, k3.ub) {

  fun <- function (c.star = c.star, b.par = b.par, j.par = j.par, c.lb = c.lb, c.
      ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) {
    - (l3(c.star = c.star, b.par = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb,
        k3.ub = k3.ub) * q3(c = c.star) * j.par) / (1 - (l3(c.star = c.star, b.par
        = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) * q3(c =
       c.star))) - c.star
  }

  multiroot(fun, start = -1, b.par = b.par, j.par = j.par, c.lb = c.lb, c.ub = c.
      ub, k3.lb = k3.lb, k3.ub = k3.ub)$root
}

p2 <- function(c.star, c.lb, c.ub) {
  (u.cdf(value = 0, u.lower = c.lb, u.upper = c.ub) - u.cdf(value = c.star, u.
      lower = c.lb, u.upper = c.ub)) / u.cdf(value = 0, u.lower = c.lb, u.upper =
      c.ub)
}

E.k3 <- function(c.star, b.par, c.lb, c.ub, k3.lb, k3.ub) {

  integrand <- function(k3, k3.lb, k3.ub) {
    k3 * u.pdf(u.lower = k3.lb, u.upper= k3.ub)
  }

  integrate(f = integrand, lower = k3.lb, upper = E.q3(c.star = c.star, c.lb = c.
      lb, c.ub = c.ub) * b.par, k3.lb = k3.lb, k3.ub = k3.ub)$value
}

k2 <- function(c.star, b.par, j.par, c.lb, c.ub, k3.lb, k3.ub) {
```

```
  p2(c.star = c.star, c.lb = c.lb, c.ub = c.ub) * b.par + (1 - p2(c.star = c.star,
       c.lb = c.lb, c.ub = c.ub)) * l3(c.star = c.star, b.par = b.par, c.lb = c.lb
      , c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) * (E.q3(c.star = c.star, c.lb =
       c.lb, c.ub = c.ub) * b.par - E.k3(c.star = c.star, b.par = b.par, c.lb = c.
      lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub))

}

#################################################
# set up simulation
#################################################

# function to estimate one round of the simulation
sim.round <- function(b.par.range, j.par.range, c.lb.range, c.ub.range, k3.lb.
    range, k3.ub.range) {

  b.par.draw <- runif(1, b.par.range[1], b.par.range[2])
  j.par.draw <- runif(1, j.par.range[1], j.par.range[2])
  c.lb.draw <- runif(1, c.lb.range[1], c.lb.range[2])
  c.ub.draw <- runif(1, c.ub.range[1], c.ub.range[2])
  k3.lb.draw <- runif(1, k3.lb.range[1], k3.lb.range[2])
  k3.ub.draw <- runif(1, k3.ub.range[1], k3.ub.range[2])

  c.star.solution <- c.star(b.par = b.par.draw, j.par = j.par.draw, c.lb = c.lb.
      draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)

  E.q3.sim <- E.q3(c.star = c.star.solution, c.lb = c.lb.draw, c.ub = c.ub.draw)

  l3.sim <- l3(c.star = c.star.solution, b.par = b.par.draw, c.lb = c.lb.draw, c.
      ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)

  p2.sim <- p2(c.star = c.star.solution, c.lb = c.lb.draw, c.ub = c.ub.draw)

  k2.sim <- k2(c.star = c.star.solution, b.par = b.par.draw, j.par = j.par.draw, c
      .lb = c.lb.draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)

  output <- data.frame(b.par = b.par.draw, j.par = j.par.draw, c.lb = c.lb.draw, c
      .ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw,
                     c.star = c.star.solution, E.q3 = E.q3.sim, l3 = l3.sim, p2 =
                         p2.sim, k2 = k2.sim)

  return(output)
}

# function to perform the full simulation
run.sim <- function(iterations, b.par.range, j.par.range, c.lb.range, c.ub.range,
    k3.lb.range, k3.ub.range) {

  output <- list()

  for(i in 1:iterations) {
```

```
    output[[i]] <- sim.round(b.par.range = b.par.range, j.par.range = j.par.range,
         c.lb.range = c.lb.range, c.ub.range = c.ub.range, k3.lb.range = k3.lb.
        range, k3.ub.range = k3.ub.range)

  }

    rbind(output)

    output <- do.call("rbind", output)

    return(output)
}

#################################################
# run simulation
#################################################

# run simulation
output <- run.sim(iterations = 1000, b.par.range = c(0, 10), j.par.range = c(0,
    10), c.lb.range = c(-10, 0), c.ub.range = c(0, 10), k3.lb.range = c(0, 1), k3.
    ub.range = c(1, 10))

# keep in-equilibrium values
output <- filter(output, p2 > 0 & p2 < 1 & l3 > 0 & l3 < 1 & c.star > -10 & c.star
     < 0 & k2 > 0)

# check l3 comparative statics
f.l3 <- (l3 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.l3 <- lm(formula = f.l3, data = output)
summary(mod.l3)

# check p2 comparative statics
f.p2 <- (p2 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.p2 <- lm(formula = f.p2, data = output)
summary(mod.p2)

# estimate first-order derivatives
f.k2 <- (k2 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.k2 <- lm(formula = f.k2, data = output)
summary(mod.k2)

# estimate cross-partial derivative
f.k2 <- (k2 ~ b.par * c.lb + j.par + c.ub + k3.lb + k3.ub)
mod.k2 <- lm(formula = f.k2, data = output)
summary(mod.k2)
```