Supplementary Appendix for "Nation-Building through War" (Sambanis, Skaperdas, and Wohlforth): Additional Cases of Proposition 1 and Interaction of France and Germany under an Indefinite Horizon

Cases III-IV of Proposition 1

Below, we present three of the five cases of the model of inter-state conflict with endogenous social identification. As in the main text, the focus is on whether Peace would be a feasible outcome.

Case III: Group identification in G occurs only after loss in War

This case occurs when $V(s^p)+s^p \geq v > V(s^l)+s^l$. It yields national identification under Peace and after victory in War and ethnic identification after loss in War. The payoff of G then in this case becomes:

$$\begin{aligned} V_P^{G_{III}} &= V(s^p)+s^p+\beta d+t \\ V_W^{G_{III}} &= p(V(s^\nu)+s^\nu+d)+(1-p)v-c \end{aligned}$$

As with the previous cases, Peace is feasible only when $V_P^F + V_P^{G_{III}} \geq V_W^F + V_W^{G_{III}}$ or:

$$\begin{aligned} V(s^p)+s^p+d &\geq p(V(s^\nu)+s^\nu)+(1-p)v+d-2c \\ \text{or } 2c &\geq p(V(s^\nu)+s^\nu)+(1-p)v-V(s^p)-s^p \end{aligned} \tag{1}$$

Given the parameter values for which this case occurs, the right-hand-side of (1) can be positive. As long as that is so, Peace is not feasible for small enough conflict costs (c). However, contrary to case II, War cannot be assured for low enough values of c; the benefits from, and probability of, victory must be high enough.

Case IV: Group identification in G occurs only when there is Peace

This case occurs when $(V(s^\nu)+s^\nu >)V(s^l)+s^l \geq v > V(s^p)+s^p$. It yields national identification after War regardless of whether there is victory or loss; and group identification under Peace. $G's$ payoff becomes:

$$\begin{aligned} V_P^{G_{IV}} &= v+\beta d+t \\ V_W^{G_{IV}} &= p(V(s^\nu)+s^\nu+d)+(1-p)(V(s^l)+s^l)-c \end{aligned}$$

As with the previous cases, Peace is feasible only when $V_P^F + V_P^{G_{IV}} \geq V_W^F + V_W^{G_{IV}}$ or:

$$\begin{aligned} v+d &\geq p(V(s^\nu)+s^\nu)+(1-p)(V(s^l)+s^l)+d-2c \\ \text{or } 2c &\geq p(V(s^\nu)+s^\nu)+(1-p)(V(s^l)+s^l)-v \end{aligned} \tag{2}$$

Given that $V(s^l)+s^l > V(s^p)+s^p$ in this case, the right-hand-side of (2) is always positive and, therefore, War occurs for low enough costs. This case is qualitatively similar to case II.

Case V: National identification always occurs in G

This case occurs when $\min\{V(s^p) + s^p, V(s^l) + s^l\} > v$, and always yields national identification with the following payoffs under Peace and War:

$$\begin{aligned} V_P^{G_V} &= V(s^p) + s^p + \beta d + t \\ V_W^{G_V} &= p(V(s^\nu) + s^\nu + d) + (1-p)(V(s^l) + s^l) - c \end{aligned}$$

Peace is feasible only if:

$$\begin{aligned} V(s^p) + s^p + d &\geq p(V(s^\nu) + s^\nu) + (1-p)v + d - 2c \\ \text{or } 2c &\geq p(V(s^\nu) + s^\nu) + (1-p)(V(s^l) + s^l) - V(s^p) - s^p \end{aligned} \tag{3}$$

The condition for Peace in this case is qualitatively similar to that of case III in (1): Peace is not feasible when the right-hand-side of (3) is positive (with low enough costs of conflict) and that occurs when the convex combination of status under victory and under a loss is higher than status under Peace, with the weights depended on the probability of War.

Interaction of Germany and France under and Indefinite Horizon

Next, we examine an indefinite-horizon version of the model with exogenous probabilities of victory and loss that is adapted to the interaction between France and Germany. We find similar results to those of Proposition 1 but, in addition, we find that when the shadow of the future (the discount) becomes longer, the set of parameter values over which Peace prevails becomes smaller.

The countries are interacting over an indefinite horizon. In each period, each country has the choice of either Peace or War. The sequence of moves in each period are the following:

1. France and Prussia simultaneousy choose either War or Peace. If both choose Peace, then Peace prevails. If at least one country chooses War, then War takes place. Prussia wins with probability $p > 0$ and France wins with probability $1 - p$. The winner keeps the disputed territory for all the subsequent periods.

2. a. After Peace, victory for Prussia after War, or loss for Prussia after War, P and S decide whether to unify or not.

   b. If there is a unified Germany, its elites play a modification of the game in (1); i.e., they decide whether to identify with their region or the nation.

The game we are examining is a Markovian one with three possible states: Peace (induced by both France and Prussia choosing Peace); victory for France and loss for Prussia (induced by either country choosing War and "nature" choosing France as victor); and loss for France and victory for Prussia (induced by either country choosing War and "nature" choosing Prussia as victor). Note that the two latter states are absorbing states (that is, once you reach them you stay there forever). We will first specify and justify the per period payoffs and gradually build on equilibrium behavior.The winner of the war would take possession of the disputed territory not only in the current period but also in all future periods.[1] We suppose that next period's payoff is discounted by both countries by the same discount factor $\delta \in (0, 1)$.

The solution concept we employ in such models is that of Markov Perfect Equilibrium (MPE). Peace is such an equilibrium only if the payoffs under Peace of both countries over the indefinite horizon are higher than those under War. War is always an equilibrium (since it takes only one side to choose War in order to have it and, therefore, trivially War is a best response to War). However, what we are primarily interested in (and show in the end) is whether Peace is *feasible*.

Next, however, we need to specify the payoffs and determine the equilibrium within Germany in stages 2a and 2b (which is the same as stages 3a and 3b in the main text but we produce here for completeness). We first suppose that the per-period material payoffs of P and S, other than what comes from disputed territories are v if Germany were not re-unified or if Germany were to be re-unified and P or S were to choose regional identification, and

[1] This assumption is made for computational simplicity, but it is also consistent with the case. Expectations at the time of the Franco-Prussian war were that disputed territory could be annexed by the victor. None of the qualitative results are affected with alternative assumptions such as that the winner takes possession of the disputed territory for a finite number of periods or has a constant (and high enough) exogenous probability of retaining possesion of the disputed territory.

$V(s^i)+s^i$ $(i=\nu,p,l)$ if Germany were to be re-unified and P and S were to choose national German identification. Thus, the per-period payoffs of P and S in stage 2b are essentially identical to those in (5):

	G	S
G	$V(s^i)+s^i+d^i/2, V(s^i)+s^i+d^i/2$	$v+s^i+d^i/2, v+d^i/2$
P	$v+d^i/2, v+s^i+d^i/2$	$v+d^i/2, v+d^i/2$

where $i=\nu,p,l$ and d^irepresents the payoffs that come from the disputed territories in each state of the world, with $d^\nu=d, d^p=\beta d$, and $d^l=0$. We have assumed that under unification, the per-capita payoffs are distributed equally between Prussian/Northern Germans (P) and Southern Germans (S), providing an incentive for Southern elites to cooperate. Moreover, since the values of $d^i/2$ are the same in each cell, the equilibria we select are the same as in (5): Regional identification when $v>V(s^i)+s^i$ and national identification when $v>V(s^i)+s^i$.

In fact, consistent with the Franco-Prussia case evidence, we suppose in this section that $V(s^\nu)+s^\nu>v\geq\max\{V(s^p)+s^p, V(s^l)+s^l\}$ (corresponding to case II of the previous section) whereby national identification with Germany could occur only after victory in War.

Given that, the decision to unify in stage 2a, would depend on the expectation of the level of identification in stage 2b. In particular, in the case of victory in War, the per-period payoffs of the two sides (P is taken as the row player and S as the column player) woulf be as follows:

	Unify	Not unify
Unify	$V(s^\nu)+s^\nu+d/2, V(s^\nu)+s^\nu+d/2$	$v+s^\nu+d/2, v+d/2$
Not unify	$v+d/2, v+s^\nu+d/2$	$v+d/2, v+d/2$

where we have assumed (but without loss of generality) that in the event of no unification Prussia would receive half of the benefit from the disputed territories. Since $V(s^\nu)+s^\nu>v$, Unification is the Pareto optimal equilibrium and we suppose from now on that Germany would unify after victory in War. Similarly, and without going through all the details, Germany would not unify in the cases of Peace and loss after War. The per-period payoffs of Prussia in the case of Peace would be $v+\beta d$ and in the case of loss after War would be just v.

We are now ready to specify the indefinite horizon payoffs under War and Peace for the two states. If War were to occur in the current period, France's expected payoff over the whole horizon would be the following:

$$\begin{aligned} V_F^w &= (1-p)\sum_{t=0}^{\infty}\delta^t d - c \\ &= (1-p)\frac{d}{1-\delta} - c \end{aligned}$$

Letting τ_t denote a possible transfer from France to Prussia in period t,[2] if Peace were

[2] These transfers would not have to take the form of tribute but could involve the concession of a preferential trade arrangement to the other country or other such indirect mechanisms.

to prevail this period as well as every future period, France's payoff would be:

$$\begin{aligned} V_F^p &= \sum_{t=0}^{\infty} \delta^t((1-\beta)d - \tau_t) \\ &= \frac{(1-\beta)d}{1-\delta} - \sum_{t=0}^{\infty} \delta^t \tau_t \end{aligned}$$

Turning to Prussia, its expected payoff under war would be the following:

$$\begin{aligned} V_P^w &= p\sum_{t=0}^{\infty} \delta^t (V(s^\nu) + s^\nu + d/2) + (1-p)\sum_{t=0}^{\infty} \delta^t v - c \\ &= (1-p)\frac{v}{1-\delta} + p\frac{V(s^\nu) + s^\nu + d/2}{1-\delta} - c \end{aligned}$$

With Peace in the current and all future periods, the payoff of Prussia would be:

$$V_P^p = \sum_{t=0}^{\infty} \delta^t (v + \beta d + \tau_t) = \frac{v + \beta d}{1-\delta} + \sum_{t=0}^{\infty} \delta^t \tau_t$$

Peace can be feasible only the sum of the payoffs under Peace ($V_F^p + V_P^p$) are greater than the sum the payoffs under War ($V_F^w + V_P^w$), or only if:

$$p(V(s^\nu) + s^\nu - v - d/2) \leq 2(1-\delta)c \tag{4}$$

We summarize the implications of this inequality in the following Proposition (noting that $s^\nu \equiv \sigma^\nu - \Delta^\nu$):

Proposition S: *Even if transfers between the countries were possible, there are no transfers that would prevent war if (4) were not to be satisfied. War would be more likely*

(i) the higher is the status of a unified nation after victory in war (σ^ν)

(ii) the lower is the perceived distance between regions after victory in war (Δ^ν)

(iii) the higher are the economic gains from unification ($V(s^\nu)$)

(iv) the higher is the discount factor (δ)

(v) the lower are the costs of war (c)

(vi) the higher is the probability of winning (p).

Note that War may be impossible to avoid even when there are no disputed territories ($d = 0$). In fact, (4) is less likely to be satisfied the smaller d is.[3]

[3] This occurs because Prussia does not completely enjoy the territorial benefits of a victory and shares them, after unification, with S.